Prework

Import

```
pip install ucimlrepo
```

```
Collecting ucimlrepo
  Downloading ucimlrepo-0.0.7-py3-none-any.whl.metadata (5.5 kB)
Requirement already satisfied: pandas>=1.0.0 in /usr/local/lib/python3.11/dist-p
Requirement already satisfied: certifi>=2020.12.5 in /usr/local/lib/python3.11/d
Requirement already satisfied: numpy>=1.23.2 in /usr/local/lib/python3.11/dist-p
Requirement already satisfied: python-dateutil>=2.8.2 in /usr/local/lib/python3.
Requirement already satisfied: pytz>=2020.1 in /usr/local/lib/python3.11/dist-pa
Requirement already satisfied: tzdata>=2022.7 in /usr/local/lib/python3.11/dist-
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.11/dist-packag
Downloading ucimlrepo-0.0.7-py3-none-any.whl (8.0 kB)
Installing collected packages: ucimlrepo
Successfully installed ucimlrepo-0.0.7
```

```
import pandas as pd
import numpy as np
import warnings
import seaborn as sns
import matplotlib.pyplot as plt
from ucimlrepo import fetch_ucirepo
```

```
warnings.filterwarnings("ignore")
```

```
  # fetch dataset
communities_and_crime = fetch_ucirepo(id=183)

# data (as pandas dataframes)
X = communities_and_crime.data.features
y = communities_and_crime.data.targets

error_list = ['PolicPerPop', 'LemasSwFTPerPop', 'PctPolicBlack', 'PctPolicHisp', 'Le
communities_and_crime.variables.loc[communities_and_crime.variables.name.isin(error_

# # metadata
# print(communities_and_crime.metadata)

# # variable information
# print(communities_and_crime.variables)
```

Data Understandign

metadata

```
for i, (key, value) in enumerate(communities_and_crime.metadata.items()):
  print(f"Key-{i}: {key}: {value}")
```

```
Key-0: uci_id: 183
Key-1: name: Communities and Crime
Key-2: repository_url: https://archive.ics.uci.edu/dataset/183/communities+and+c
Key-3: data_url: https://archive.ics.uci.edu/static/public/183/data.csv
Key-4: abstract: Communities within the United States. The data combines socio-e
Key-5: area: Social Science
Key-6: tasks: ['Regression']
Key-7: characteristics: ['Multivariate']
Key-8: num_instances: 1994
Key-9: num_features: 127
Key-10: feature_types: ['Real']
Key-11: demographics: ['Race', 'Age', 'Income', 'Occupation']
Key-12: target_col: ['ViolentCrimesPerPop']
Key-13: index_col: None
Key-14: has_missing_values: yes
Key-15: missing_values_symbol: NaN
Key-16: year_of_dataset_creation: 2002
Key-17: last_updated: Mon Mar 04 2024
Key-18: dataset_doi: 10.24432/C53W3X
Key-19: creators: ['Michael Redmond']
Key-20: intro_paper: {'ID': 405, 'type': 'NATIVE', 'title': 'A data-driven softw
Key-21: additional_info: {'summary': "  Many variables are included so that algo
```

```
for i, column in enumerate(communities_and_crime.variables.columns):
  if column == 'name':
    print(f"{i} - {column}: {communities_and_crime.variables[column].shape[0]} count
  else:
    print(f"{i} - {column}: {communities_and_crime.variables[column].value_counts()}
  print()
```

```
0 - name: 128 countries

1 - role: role
Feature    127
Target       1
Name: count, dtype: int64

2 - type: type
Continuous     116
Integer         11
Categorical      1
Name: count, dtype: int64
```

```
3 - demographic: demographic
Race          10
Age            4
Income         2
Occupation     2
Name: count, dtype: int64

4 - description: Series([], Name: count, dtype: int64)

5 - units: Series([], Name: count, dtype: int64)

6 - missing_values: missing_values
no     103
yes     25
Name: count, dtype: int64
```

Target

```
communities_and_crime.variables[communities_and_crime.variables.role=="Target"]
```

	name	role	type	demographic	description	units	missing_v
127	ViolentCrimesPerPop	Target	Continuous	None	None	None	

Type

```
communities_and_crime.variables[communities_and_crime.variables["type"].isin(["Integ
```

	index	name	role	type	demographic	description	units
3	3	communityname	Feature	Categorical	None	None	None
0	0	state	Feature	Integer	None	None	None
1	1	county	Feature	Integer	None	None	None
2	2	community	Feature	Integer	None	None	None
4	4	fold	Feature	Integer	None	None	None
5	15	numbUrban	Feature	Integer	None	None	None
6	16	pctUrban	Feature	Integer	None	None	None
7	54	NumIlleg	Feature	Integer	None	None	None
8	93	MedOwnCostPctIncNoMtg	Feature	Integer	None	None	None
9	94	NumInShelters	Feature	Integer	None	None	None
10	95	NumStreet	Feature	Integer	None	None	None
11	125	LemasPctOfficDrugUn	Feature	Integer	None	None	None

Demographic data

```
communities_and_crime.variables[~communities_and_crime.variables.demographic.isna()]
```

	index	name	role	type	demographic	description	units	miss
4	11	agePct12t21	Feature	Continuous	Age	None	None	
5	12	agePct12t29	Feature	Continuous	Age	None	None	
6	13	agePct16t24	Feature	Continuous	Age	None	None	
7	14	agePct65up	Feature	Continuous	Age	None	None	
8	17	medIncome	Feature	Continuous	Income	None	None	
9	18	pctWWage	Feature	Continuous	Income	None	None	
16	41	PctOccupManu	Feature	Continuous	Occupation	None	None	
17	42	PctOccupMgmtProf	Feature	Continuous	Occupation	None	None	
0	7	racepctblack	Feature	Continuous	Race	None	None	
1	8	racePctWhite	Feature	Continuous	Race	None	None	
2	9	racePctAsian	Feature	Continuous	Race	None	None	
3	10	racePctHisp	Feature	Continuous	Race	None	None	
10	26	whitePerCap	Feature	Continuous	Race	None	None	
11	27	blackPerCap	Feature	Continuous	Race	None	None	
12	28	indianPerCap	Feature	Continuous	Race	None	None	
13	29	AsianPerCap	Feature	Continuous	Race	None	None	
14	30	OtherPerCap	Feature	Continuous	Race	None	None	
15	31	HispPerCap	Feature	Continuous	Race	None	None	

Missing_values

```
communities_and_crime.variables[communities_and_crime.variables.missing_values == "y
```

	index	name	role	type	demographic	description	units
2	30	OtherPerCap	Feature	Continuous	Race	None	None
3	101	LemasSwornFT	Feature	Continuous	None	None	None
4	102	LemasSwFTPerPop	Feature	Continuous	None	None	None
5	103	LemasSwFTFieldOps	Feature	Continuous	None	None	None
6	104	LemasSwFTFieldPerPop	Feature	Continuous	None	None	None
7	105	LemasTotalReq	Feature	Continuous	None	None	None
8	106	LemasTotReqPerPop	Feature	Continuous	None	None	None
9	107	PolicReqPerOffic	Feature	Continuous	None	None	None
10	108	PolicPerPop	Feature	Continuous	None	None	None
11	109	RacialMatchCommPol	Feature	Continuous	None	None	None
12	110	PctPolicWhite	Feature	Continuous	None	None	None
13	111	PctPolicBlack	Feature	Continuous	None	None	None
14	112	PctPolicHisp	Feature	Continuous	None	None	None
15	113	PctPolicAsian	Feature	Continuous	None	None	None
16	114	PctPolicMinor	Feature	Continuous	None	None	None
17	115	OfficAssgnDrugUnits	Feature	Continuous	None	None	None
18	116	NumKindsDrugsSeiz	Feature	Continuous	None	None	None
19	117	PolicAveOTWorked	Feature	Continuous	None	None	None
20	121	PolicCars	Feature	Continuous	None	None	None
21	122	PolicOperBudg	Feature	Continuous	None	None	None
22	123	LemasPctPolicOnPatr	Feature	Continuous	None	None	None
23	124	LemasGangUnitDeploy	Feature	Continuous	None	None	None
24	126	PolicBudgPerPop	Feature	Continuous	None	None	None
0	1	county	Feature	Integer	None	None	None
1	2	community	Feature	Integer	None	None	None

Data Cleaning

```
X.shape
```

```
(1994, 127)
```

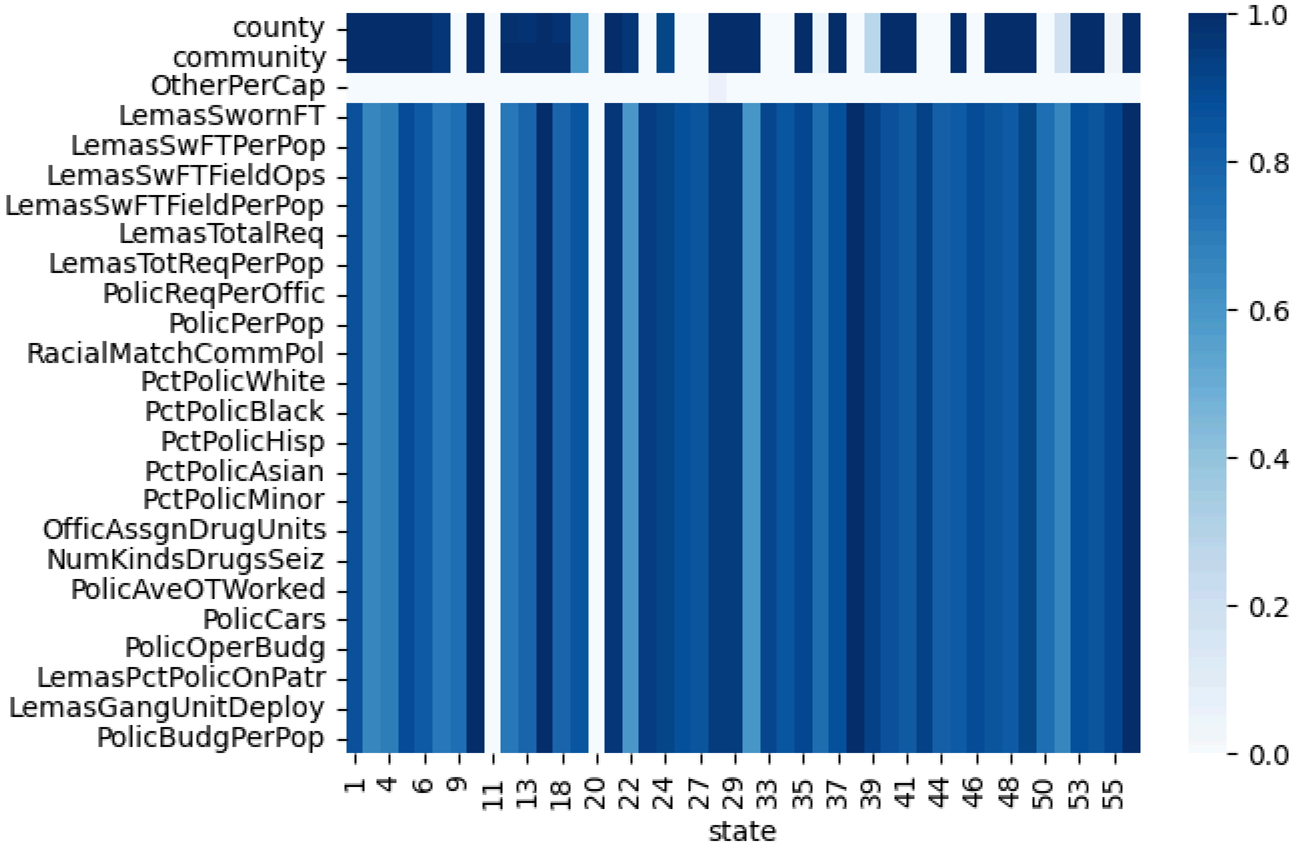
```
missing_features = communities_and_crime.variables[communities_and_crime.variables.m
row_missing_rate = ((X == "?").sum(axis=1) > 0).mean()

for i, column in enumerate(missing_features):
  missing = (X[column] == "?").mean()
  print(f"{i} {column}: {missing:.2f}")
print(f"Total Row missing rate: {row_missing_rate:.2f}")
```

```
0 county: 0.59
1 community: 0.59
2 OtherPerCap: 0.00
3 LemasSwornFT: 0.84
4 LemasSwFTPerPop: 0.84
5 LemasSwFTFieldOps: 0.84
6 LemasSwFTFieldPerPop: 0.84
7 LemasTotalReq: 0.84
8 LemasTotReqPerPop: 0.84
9 PolicReqPerOffic: 0.84
10 PolicPerPop: 0.84
11 RacialMatchCommPol: 0.84
12 PctPolicWhite: 0.84
13 PctPolicBlack: 0.84
14 PctPolicHisp: 0.84
15 PctPolicAsian: 0.84
16 PctPolicMinor: 0.84
17 OfficAssgnDrugUnits: 0.84
18 NumKindsDrugsSeiz: 0.84
19 PolicAveOTWorked: 0.84
20 PolicCars: 0.84
21 PolicOperBudg: 0.84
22 LemasPctPolicOnPatr: 0.84
23 LemasGangUnitDeploy: 0.84
24 PolicBudgPerPop: 0.84
Total Row missing rate: 0.94
```

```
(X.loc[:, missing_features].isna().sum(axis=1) == len(missing_features)).mean()
# There is not a record that miss all 25 features
```

```
0.0
```

```
df_miss_feature_and_location = X.groupby("state").apply(lambda subdf: pd.Series(
    {key : round((subdf[key] == "?").mean(), 3) for key in missing_features}
))
sns.heatmap(df_miss_feature_and_location.T, cmap="Blues")
plt.show()
```




```
drop_X = X.drop(["community", "OtherPerCap", "county",  "communityname",    "fold"],
nan_columns = drop_X.select_dtypes(include="object").columns
drop_X[nan_columns] = drop_X[nan_columns].replace("?", np.nan).astype(float)
drop_X[nan_columns] = drop_X[nan_columns].fillna(drop_X[nan_columns].median())

dummies = pd.get_dummies(drop_X.iloc[:, 0], drop_first=True, prefix="state").astype(
n = dummies.shape[1]
fine_X = pd.concat([dummies, drop_X.iloc[:, 1:]], axis=1)
```

```
from sklearn.preprocessing import OneHotEncoder, StandardScaler
from sklearn.linear_model import ElasticNet
from sklearn.linear_model import LinearRegression
from sklearn.linear_model import Ridge
from sklearn.linear_model import Lasso
import statsmodels as sm
from mlxtend.feature_selection import SequentialFeatureSelector as SFS
from statsmodels.api import OLS
from mlxtend.feature_selection import ExhaustiveFeatureSelector as EFS
```

Problem 1

Question A

- (30 points) What are the most important features? Compare and contrast the top features as determined by:
  - Statistical significance via Least Squares.
  - Best Subsets.
  - Step-wise approaches (and/or Recursive Feature Elimination). • Lasso.
  - Elastic Net.

OLS Statistics Significant

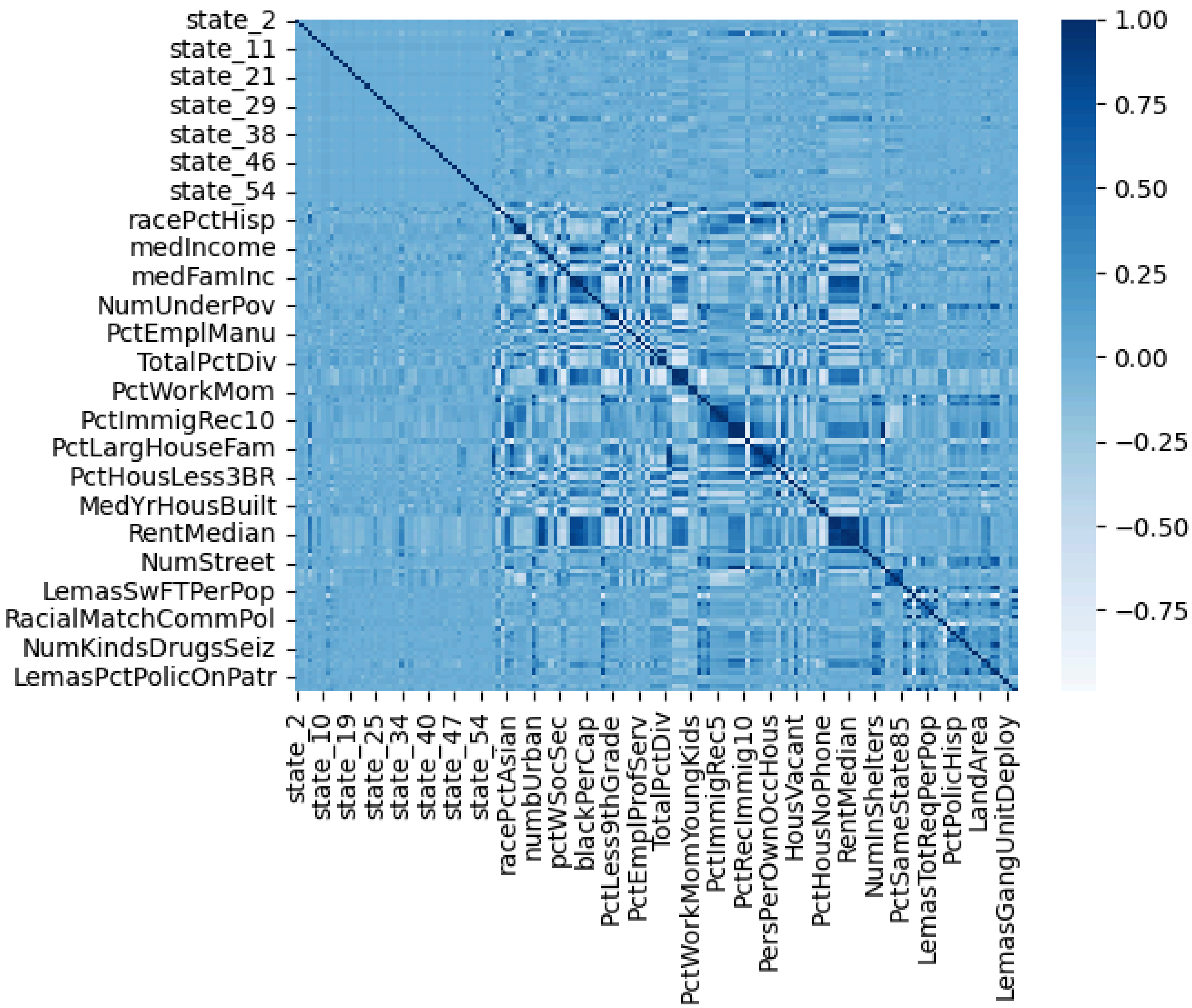
```
column_name = fine_X.columns[n+1:]
centered_Y = pd.DataFrame(StandardScaler(with_mean=True).fit_transform(y), columns=[
stand_X = pd.concat((fine_X.iloc[:, :n], pd.DataFrame(StandardScaler(with_mean=True,
stand_X
```

	state_2	state_4	state_5	state_6	state_8	state_9	state_10	state_11	sta
0	0	0	0	0	1	0	0	0	
1	0	0	0	0	0	0	0	0	
2	0	0	0	0	0	0	0	0	
3	0	0	0	0	0	0	0	0	
4	0	0	0	0	0	0	0	0	
...	...	...	...	...	...	...	...	...	
1989	0	0	0	0	0	0	0	0	
1990	0	0	0	1	0	0	0	0	
1991	0	0	0	0	0	1	0	0	
1992	0	0	0	0	0	0	0	0	
1993	0	0	0	1	0	0	0	0	

1994 rows × 165 columns

```
import seaborn as sns

corr = stand_X.corr()
```

```
sns.heatmap(corr, cmap="Blues")
plt.show()
```




```
np.fill_diagonal(corr.values, np.nan)
lower_tri_indices = np.tril_indices_from(corr.values, k=-1)
lower_corr_values = corr.values[lower_tri_indices]
selected_pairs = pd.DataFrame({
    "Feature 1": corr.index[lower_tri_indices[0]],
    "Feature 2": corr.columns[lower_tri_indices[1]],
    "Correlation": lower_corr_values
})

# Filter correlations > 0.75
selected_pairs = selected_pairs[selected_pairs["Correlation"].abs() > 0.9].res
selected_pairs.shape
```

```
(56, 3)
```

```
correlations = np.zeros((stand_X.shape[1], 1))
for i, column in enumerate(stand_X.columns):
  correlations[i, 0] = stand_X[column].corr(centered_Y.iloc[:, 0])
cor_df = pd.DataFrame(correlations, index=stand_X.columns, columns=["correlation"])
top_cor = abs(cor_df).sort_values(by="correlation", ascending=False).head(10)
last_cor = abs(cor_df).sort_values(by="correlation", ascending=False).tail(10)



intercept = pd.DataFrame(np.ones((stand_X.shape[0], 1)), columns=["intercept"])
# not top_cor_features
lsmodel_not = OLS(centered_Y, stand_X).fit()
result_ls_not = np.column_stack((lsmodel_not.params.index, lsmodel_not.params.values
result_ls_not = result_ls_not[result_ls_not[:, 2].argsort()][:10, :2]

# top_10_cor_features
lsmodel_yes = OLS(centered_Y, stand_X.loc[:, top_cor.index]).fit()
result_ls_yes = np.column_stack((lsmodel_yes.params.index, lsmodel_yes.params.values
result_ls_yes = result_ls_yes[result_ls_yes[:, 2].argsort()][:, :2]

results_ls = np.column_stack((result_ls_not, result_ls_yes))
params_ls = pd.DataFrame(results_ls, columns=["Name_Not", "Params_Not", "Name_Yes",
print("Top Cor Params:\n", list(top_cor.index))
params_ls
```

```
Top Cor Params:
 ['PctKids2Par', 'PctIlleg', 'PctFam2Par', 'racePctWhite', 'PctYoungKids2Par', '
```

	Name_Not	Params_Not	Name_Yes	Params_Yes
0	state_28	-0.978944	PctIlleg	0.297512
1	state_12	0.582829	racePctWhite	-0.215177
2	state_13	-0.545023	FemalePctDiv	0.162579
3	state_51	-0.540313	PctKids2Par	-0.660482
4	state_42	-0.3721	PctFam2Par	0.379547
5	state_6	0.376383	pctWInvInc	0.035502
6	pctUrban	0.071043	racepctblack	-0.033854
7	MedRent	0.423081	PctYoungKids2Par	0.022763
8	PctWorkMom	-0.134557	pctWPubAsst	0.011648
9	state_39	-0.227767	PctTeen2Par	0.014326

后续步骤： 使用 params_ls 生成代码 查看推荐的图表 New interactive sheet

Best Subsets

```
model_linear = LinearRegression(fit_intercept=False)
efs = EFS(
    model_linear,
    min_features=1,
    max_features=10,
    scoring="neg_mean_squared_error", #we use negative mse since the sfs method maxi
    cv=0
)

efs.fit(stand_X.loc[:, top_cor.index], centered_Y)
efs_summary = pd.DataFrame.from_dict(efs.get_metric_dict()).T[["feature_idx", "avg_s
```

```
Features: 1023/1023
```

```
efs_size = np.zeros(len(efs_summary))
for i in range(len(efs_summary)):
    efs_size[i] = len(efs_summary["feature_idx"][i])
efs_summary['model size'] = efs_size
efs_summary.rename(columns={'avg_score': 'neg_mse'}, inplace=True)
best_features = efs_summary.sort_values(by="neg_mse", ascending=False).iloc[0]
result_efs = top_cor.iloc[list(best_features["feature_idx"]), :]
params_best = result_efs.reset_index().rename(columns={"index": "Name_Yes", "correla
params_best
```

	Name_Yes	value_Yes(Correlation)
0	PctKids2Par	0.738424
1	PctIlleg	0.737957
2	PctFam2Par	0.706667
3	racePctWhite	0.684770
4	PctYoungKids2Par	0.666059
5	PctTeen2Par	0.661582
6	racepctblack	0.631264
7	pctWInvInc	0.576324
8	pctWPubAsst	0.574665
9	FemalePctDiv	0.556032

后续步骤： 使用 params_best 生成代码 查看推荐的图表 New interactive sheet

Step-Wise Approaches

```
#defining aic evaluation functions compatible with mlxtend.feature_selection.Sequent
def calculate_aic(estimator, X, y):
    """
    Custom AIC scorer for SequentialFeatureSelector.
    Args:
        estimator: A fitted sklearn-compatible estimator.
        X: Features (numpy array).
        y: Target variable (numpy array).
    Returns:
        Negative AIC value for compatibility with SFS (higher is better).
    """
    n, k = X.shape  # n: number of samples, k: number of predictors
    y_pred = estimator.predict(X)
    residual_sum_of_squares = np.sum((y - y_pred) ** 2)
    aic = n * np.log(residual_sum_of_squares / n) + 2 * k
    return -aic  # SFS maximizes the score

def aic_scorer_wrapper(estimator, X, y):
    estimator.fit(X, y)
    return calculate_aic(estimator, X, y)

sfs = SFS(
    estimator=LinearRegression(fit_intercept=False),
    k_features=(1, 10),
    forward=True,
    floating=False,
    scoring=aic_scorer_wrapper,
    cv=0
)

sfs.fit(stand_X.loc[:, :], centered_Y)
results_Not = max([value for _, value in sfs.get_metric_dict().items()], key=lambda
results_Not = np.array(results_Not)

sfs.fit(stand_X.loc[:, top_cor.index], centered_Y)
results_Yes = max([value for _, value in sfs.get_metric_dict().items()], key=lambda
results_Yes = {
    "Name_Yes": pd.Series(results_Yes)
}

params_step = pd.concat((pd.DataFrame(results_Yes), pd.DataFrame(results_Not, column
params_step
```

	Name_Yes	Name_Not
0	PctKids2Par	state_12
1	PctIlleg	state_13
2	PctFam2Par	state_25
3	racePctWhite	state_28
4	FemalePctDiv	racePctWhite
5	NaN	MalePctDivorce
6	NaN	PctKids2Par
7	NaN	PctWorkMom
8	NaN	PctIlleg
9	NaN	HousVacant

后续步骤： 使用 params_step 生成代码 查看推荐的图表 New interactive sheet

LASSO

```
lasso = Lasso(fit_intercept=False, alpha=0.07)
lasso.fit(stand_X.loc[:, :], centered_Y.iloc[:, 0])
# print(pd.DataFrame(np.stack((fine_X.columns.values, fit0.coef_, lasso.coef_), axis
index_lasso_not = np.where(lasso.coef_ != 0)
result_lasso_not = np.column_stack((stand_X.columns[index_lasso_not], lasso.coef_[in
temp1 = pd.DataFrame(result_lasso_not, columns=["Name_Not", "Value_Not"], index=rang

lasso.fit(stand_X.loc[:, top_cor.index], centered_Y.iloc[:, 0])
index_lasso_yes = np.where(lasso.coef_ != 0)
result_lasso_yes = {
    "Name_Yes": pd.Series(top_cor.index[index_lasso_yes]),
    "Value_Yes": pd.Series(lasso.coef_[index_lasso_yes])
}

params_lasso = pd.concat((pd.DataFrame(result_lasso_yes), temp1), axis=1)
params_lasso
```

	Name_Yes	Value_Yes	Name_Not	Value_Not
0	PctKids2Par	-0.266173	racePctWhite	-0.185912
1	PctIlleg	0.224178	pctUrban	0.006777
2	racePctWhite	-0.212139	MalePctDivorce	0.064956
3	FemalePctDiv	0.079820	PctKids2Par	-0.266554
4	NaN	NaN	PctWorkMom	-0.004336
5	NaN	NaN	PctIlleg	0.192499
6	NaN	NaN	PctPersDenseHous	0.04448
7	NaN	NaN	HousVacant	0.08634
8	NaN	NaN	PctVacantBoarded	0.005694
9	NaN	NaN	NumStreet	0.035718

后续步骤： 使用 params_lasso 生成代码 查看推荐的图表 New interactive sheet

Elastic Net

```
elasticnet = ElasticNet(alpha=1, l1_ratio = 0.29, fit_intercept=False)
elasticnet.fit(stand_X.loc[:, :], centered_Y.iloc[:, 0])
index_en_not = np.where(~np.isclose(elasticnet.coef_, 0))
result_en_not = np.column_stack((stand_X.columns.values[index_en_not], elasticnet.co

elasticnet.fit(stand_X.loc[:, top_cor.index], centered_Y.iloc[:, 0])
index_en_yes = np.where(~np.isclose(elasticnet.coef_, 0))
result_en_yes = {
    "Name_Yes": pd.Series(top_cor.index[index_en_yes]),
    "Value_Yes": pd.Series(elasticnet.coef_[index_en_yes])
}
params_en = pd.concat((pd.DataFrame(result_en_yes), pd.DataFrame(result_en_not, colu
params_en
```

	Name_Yes	Value_Yes	Name_Not	Value_Not
0	PctKids2Par	-0.095074	racepctblack	0.018048
1	PctIlleg	0.102120	racePctWhite	-0.089296
2	PctFam2Par	-0.063209	pctWInvInc	-0.001308
3	racePctWhite	-0.089257	FemalePctDiv	0.003679
4	PctYoungKids2Par	-0.030703	TotalPctDiv	0.001234
5	PctTeen2Par	-0.028707	PctFam2Par	-0.06309
6	racepctblack	0.018050	PctKids2Par	-0.094948
7	pctWInvInc	-0.001408	PctYoungKids2Par	-0.030612
8	FemalePctDiv	0.004179	PctTeen2Par	-0.0286
9	NaN	NaN	PctIlleg	0.102142

后续步骤： 使用 params_en 生成代码 查看推荐的图表 New interactive sheet

Results:

```
params_dfs = [params_ls, params_best, params_step, params_lasso, params_en]

params_ls.columns = pd.MultiIndex.from_product([["Least_Square"], params_ls.columns]
params_best.columns = pd.MultiIndex.from_product([["Best_Subsets"], params_best.colu
params_step.columns = pd.MultiIndex.from_product([["Step_Wise"], params_step.columns
params_lasso.columns = pd.MultiIndex.from_product([["Lasso"], params_lasso.columns])
params_en.columns = pd.MultiIndex.from_product([["Elastic_Net"], params_en.columns])
print("'Not' means 'selected features from all', 'Yes' means 'selected features from
result = pd.concat([params_ls, params_best, params_step, params_lasso, params_en], a
result
```

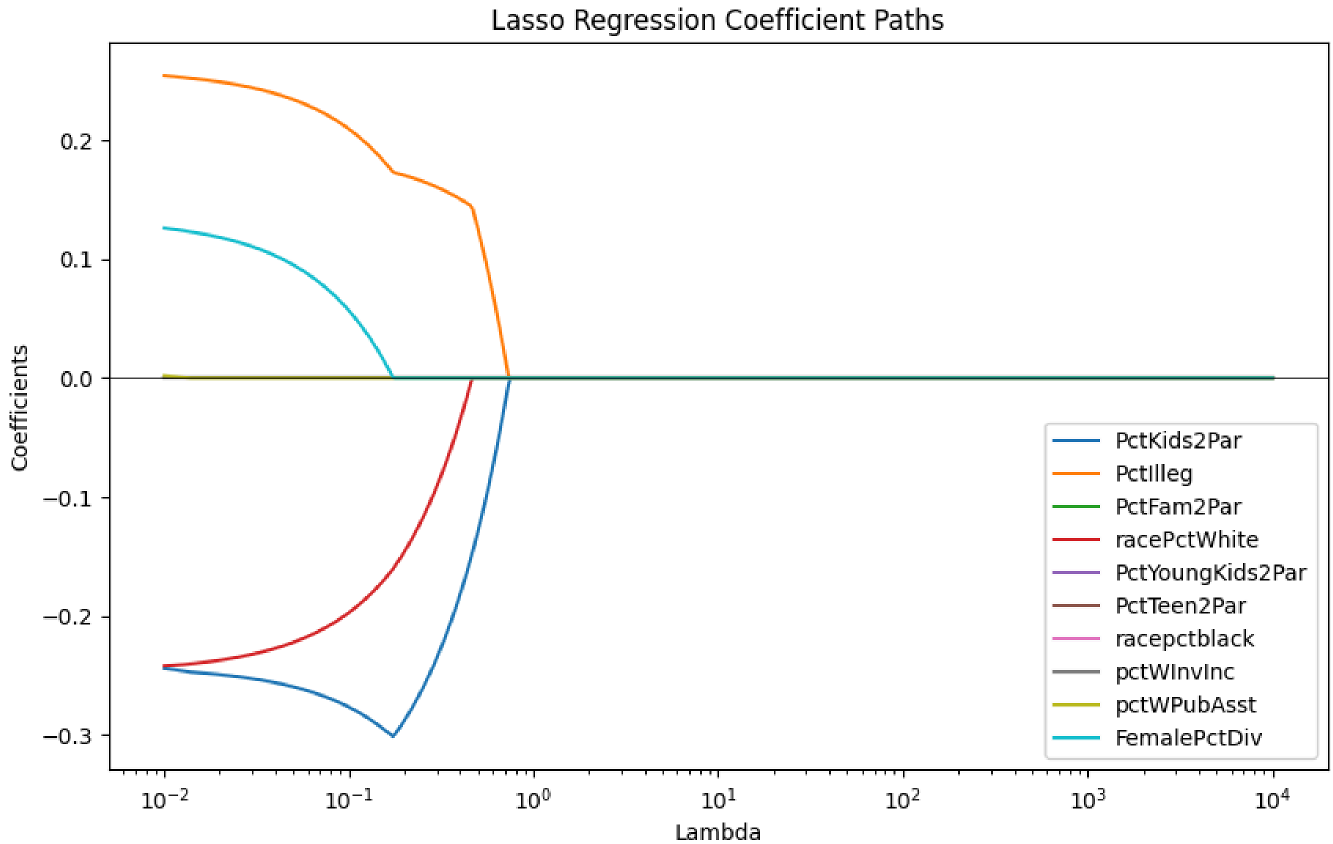
后续步骤： 使用 result 生成代码 查看推荐的图表 New interactive sheet

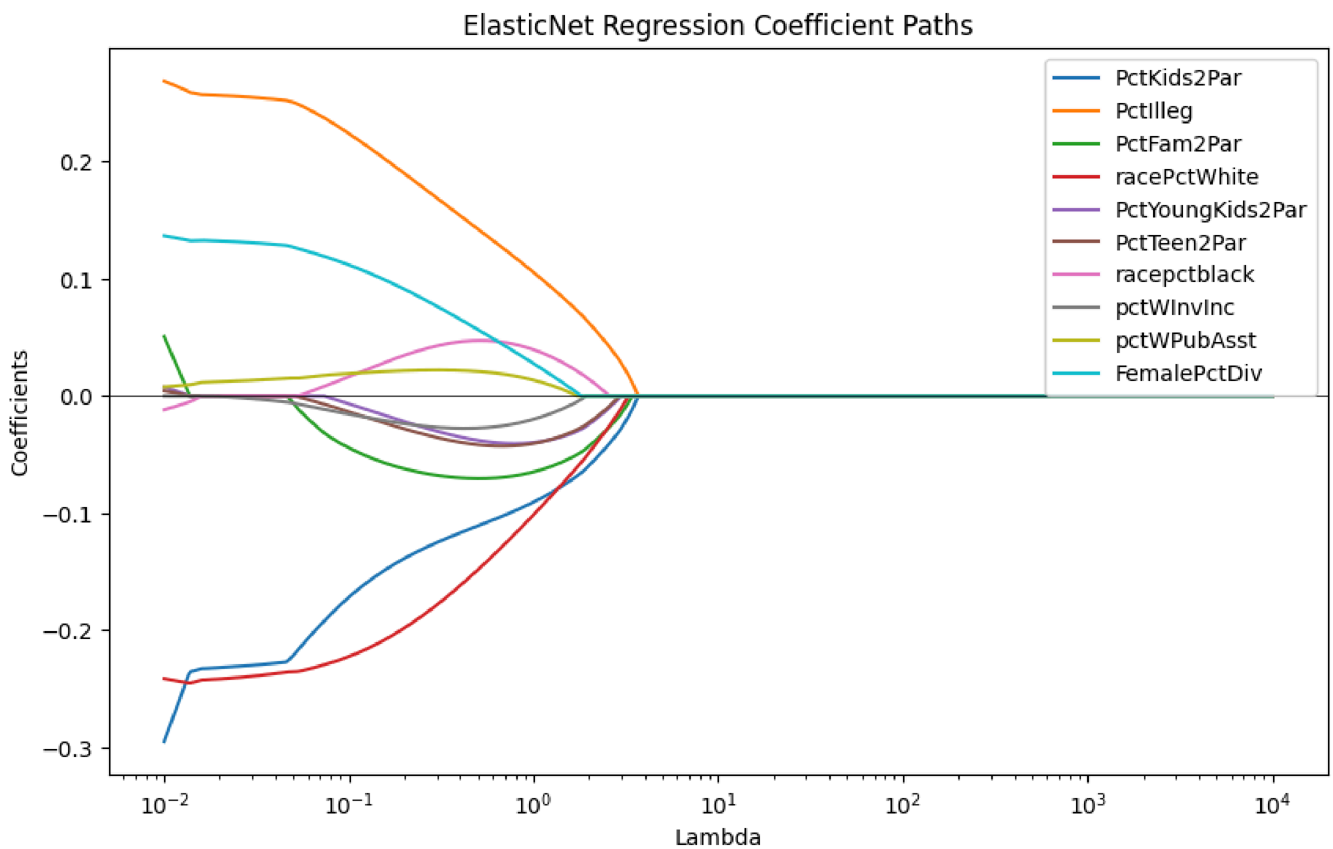
(ii) Fit and visualize regularization paths for the following methods:

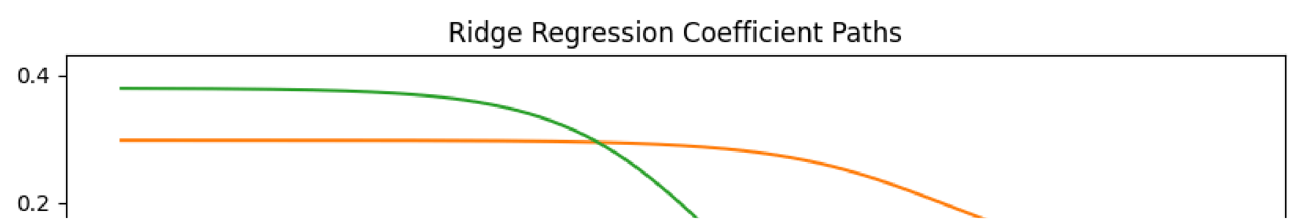
- Lasso
- Elastic Net (at two separate α's).
- Ridge.

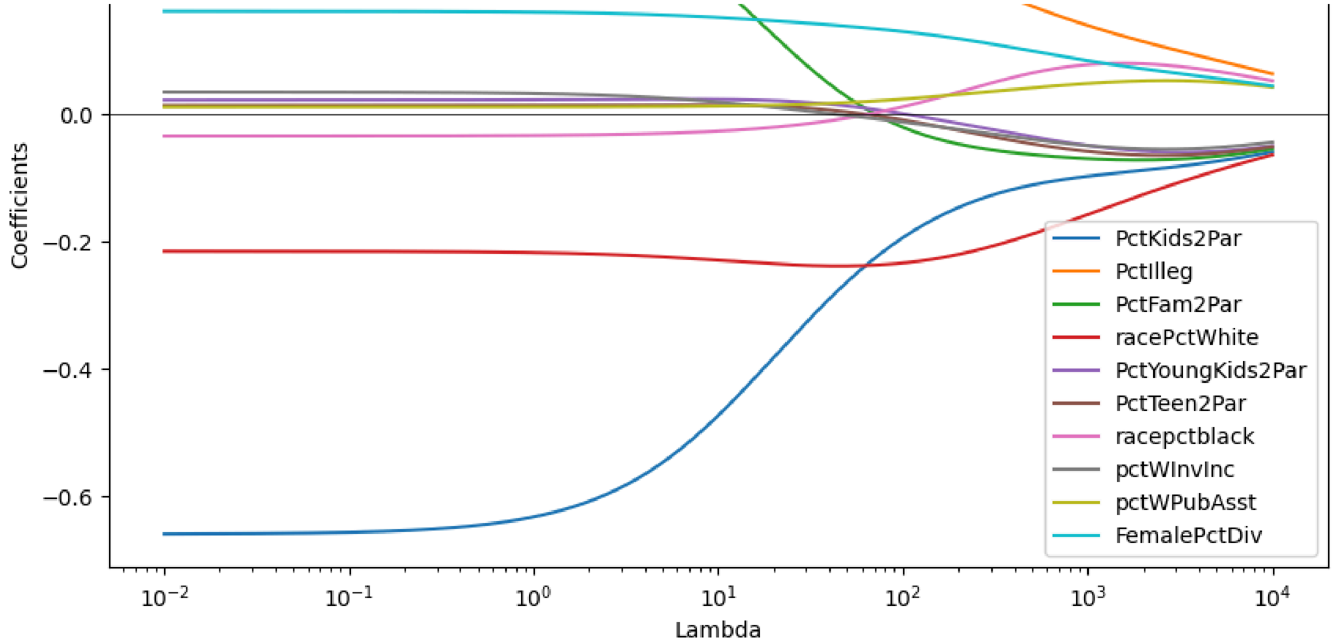
```
def model_features_returner(column):
  i, j = column
  model, feature = None, None
  if i == "Least_Square":
    return LinearRegression, result[column].dropna().values
  elif i == "Best_Subsets":
    return LinearRegression, result[column].dropna().values
  elif i == "Step_Wise":
    return LinearRegression, result[column].dropna().values
  elif i == "Lasso":
    return Lasso, result[column].dropna().values
  elif i == "Elastic_Net":
    return ElasticNet, result[column].dropna().values
  else:
    return Ridge, top_cor.index

def regularization_path(X, Y, model, features, l1_ratio=0.1):
```

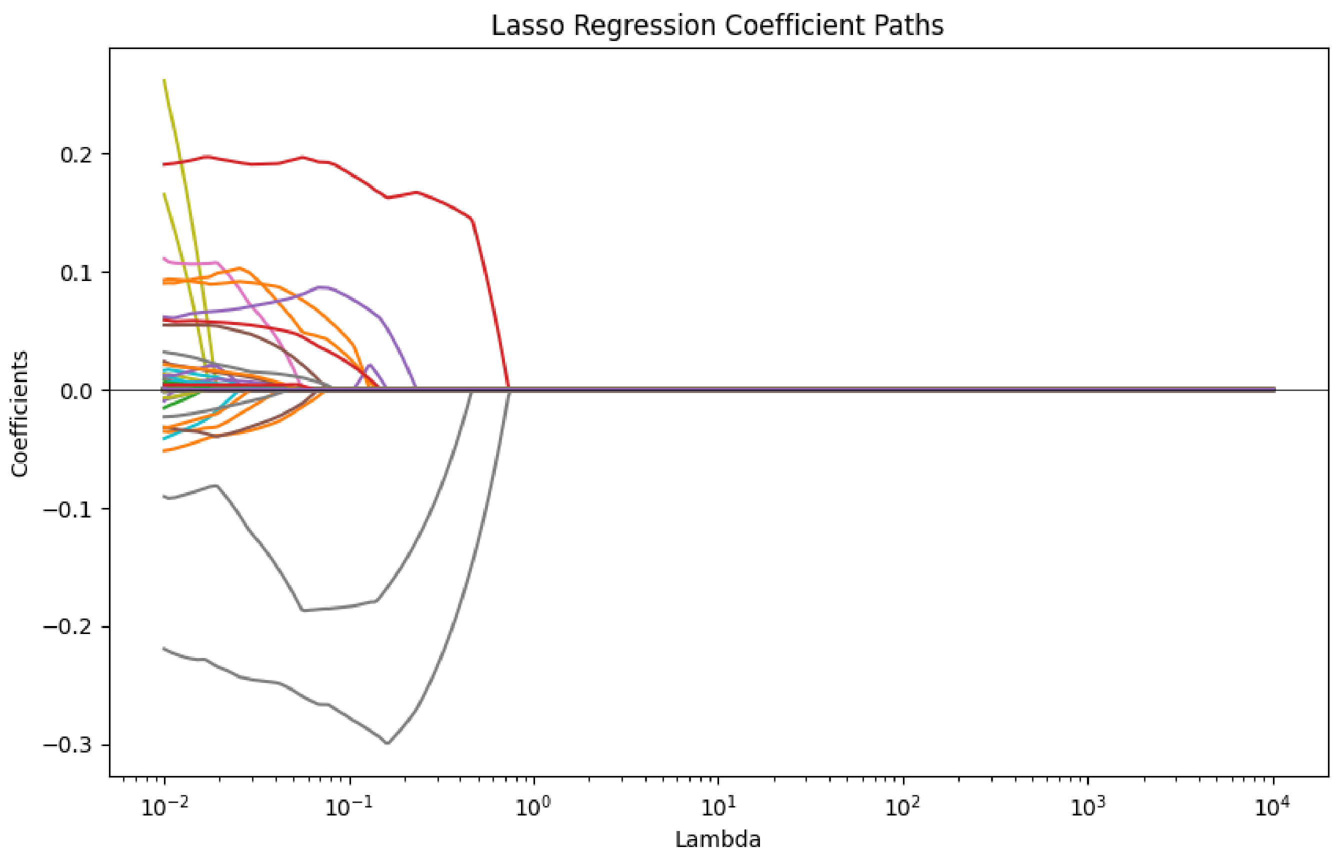
```
    lambdas = np.exp(np.linspace(np.log(0.01), np.log(10000), 500))
    betasl = np.zeros((len(lambdas), X.loc[:, features].shape[1]))
    for i, lamb in enumerate(lambdas):
      if model == ElasticNet:
        m = model(alpha = lamb, l1_ratio = l1_ratio, fit_intercept=False)
      else:
        m = model(alpha = lamb)
      m.fit(X.loc[:, features], Y)
      betasl[i, :] = m.coef_

    # Plot Lasso paths (log-scale)
    plt.figure(figsize=(10, 6))
    for j in range(X.loc[:, features].shape[1]): #for each variable
        plt.plot(lambdas, betasl[:, j], label=f"Variable {j+1}")
    plt.xscale("log")
    plt.xlabel("Lambda")
    plt.ylabel("Coefficients")
    plt.title(f"{type(m).__name__} Regression Coefficient Paths")
    if len(features) <= 10:
      plt.legend(features)
    plt.axhline(y=0, color="black", linewidth=0.5)
    plt.show()

  # columns = list(filter(lambda x: (x[0] in ["Lasso", "Elastic_Net"]) and (x[1] in ['
  # for column in columns:
  #   model, features = model_features_returner(column)
  #   regularization_path(stand_X, centered_Y, model, features)

  regularization_path(stand_X, centered_Y, Lasso, top_cor.index)
  regularization_path(stand_X, centered_Y, ElasticNet, top_cor.index, l1_ratio=0.2)
  regularization_path(stand_X, centered_Y, Ridge, top_cor.index)

  regularization_path(stand_X, centered_Y, Lasso, stand_X.columns)
  regularization_path(stand_X, centered_Y, ElasticNet, stand_X.columns, l1_ratio=0.2)
  regularization_path(stand_X, centered_Y, Ridge, stand_X.columns)
```

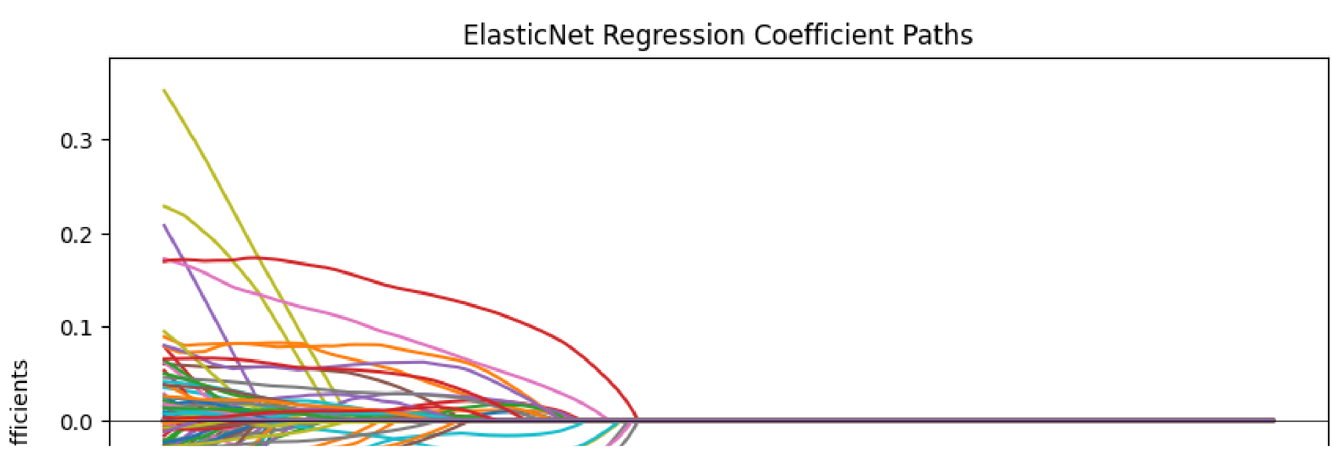

Lasso Regression Coefficient Paths
Coefficients
Lambda
PctKids2Par
PctIlleg
PctFam2Par
racePctWhite
PctYoungKids2Par
PctTeen2Par
racepctblack
pctWInvInc
pctWPubAsst
FemalePctDiv

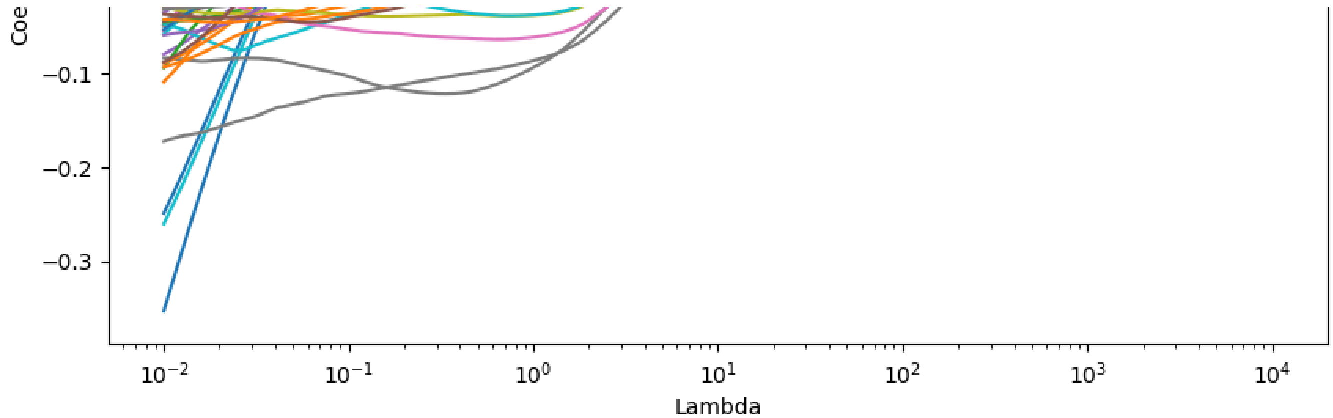

ElasticNet Regression Coefficient Paths
Coefficients
Lambda
PctKids2Par
PctIlleg
PctFam2Par
racePctWhite
PctYoungKids2Par
PctTeen2Par
racepctblack
pctWInvInc
pctWPubAsst
FemalePctDiv

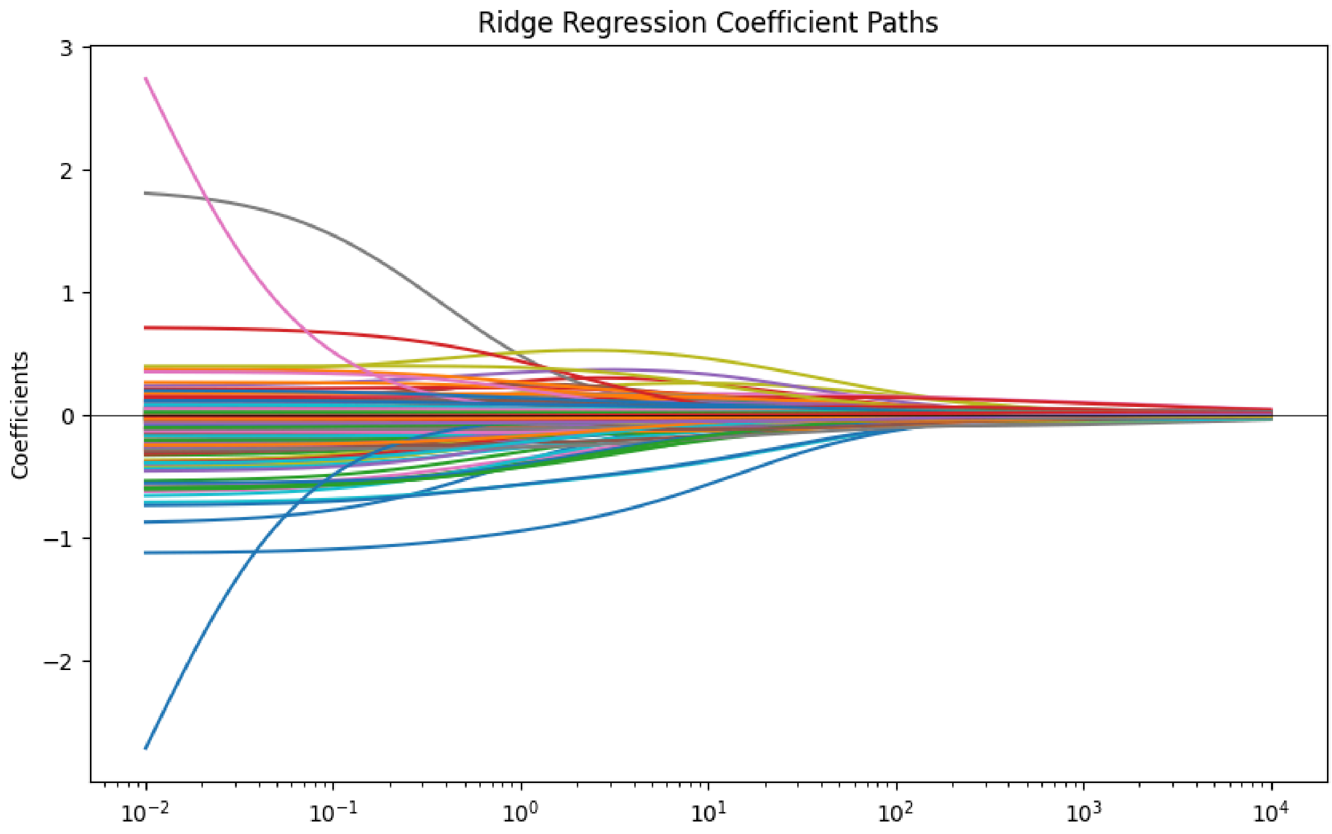

Ridge Regression Coefficient Paths


Coefficients
0.0
−0.2
−0.4
−0.6
PctKids2Par
PctIlleg
PctFam2Par
racePctWhite
PctYoungKids2Par
PctTeen2Par
racepctblack
pctWInvInc
pctWPubAsst
FemalePctDiv
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Lambda


Lasso Regression Coefficient Paths
Coefficients
0.2
0.1
0.0
−0.1
−0.2
−0.3
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Lambda


ElasticNet Regression Coefficient Paths
fficients
0.3
0.2
0.1
0.0


Coe
−0.1
−0.2
−0.3
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Lambda


Ridge Regression Coefficient Paths
Coefficients
3
2
1
0
−1
−2
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}

```
stand_X["FemalePctDiv"].corr(stand_X["PctKids2Par"])
```

-0.7299217835356265

(iii) Reflect on these results.

- Are the top features different for each method? Why or why not? If there are tuning parameters, how did you determine these? Do different tuning parameters yield different important features? Are there any features consistently selected by all methods? What are the most important features and how did you determine this? Explain and expand on your responses.

```
result
```

	Least_Square				Best_Subsets	
	Name_Not	Params_Not	Name_Yes	Params_Yes	Name_Yes	value_Yes(C
0	state_28	-0.978944	PctIlleg	0.297512	PctKids2Par	
1	state_12	0.582829	racePctWhite	-0.215177	PctIlleg	
2	state_13	-0.545023	FemalePctDiv	0.162579	PctFam2Par	
3	state_51	-0.540313	PctKids2Par	-0.660482	racePctWhite	
4	state_42	-0.3721	PctFam2Par	0.379547	PctYoungKids2Par	
5	state_6	0.376383	pctWInvInc	0.035502	PctTeen2Par	
6	pctUrban	0.071043	racepctblack	-0.033854	racepctblack	
7	MedRent	0.423081	PctYoungKids2Par	0.022763	pctWInvInc	
8	PctWorkMom	-0.134557	pctWPubAsst	0.011648	pctWPubAsst	
9	state_39	-0.227767	PctTeen2Par	0.014326	FemalePctDiv	

后续步骤： 使用 result 生成代码 查看推荐的图表 New interactive sheet

```
count = dict()
for i in result:
  if i[1] != "Name_Not":
    continue
  for j in range(10):
    if result.loc[j, i] not in count.keys():
      count[result.loc[j, i]] = 1
    else:
```

```
        count[result.loc[j, i]] += 1
  # for i in top_cor.index:
  #   if i not in count.keys():
  #     count[i] = 1
  #   else:
  #     count[i] += 1

  r = sorted(count.items(), key=lambda x: x[1], reverse=True)
  final = pd.DataFrame(r, columns=["Feature", "Count"])
```

Reflecting on the feature selection results, we observe that different methods prioritize different features due to their unique selection criteria.

- **OLS identifies features based on statistical significance**, selecting those with low p-values. In my selected params, most are state dummies. However, other models do not quit find state dummies significant, except for step-wise model. And ls model does not choose features ('PctKids2Par', 'PctIlleg', 'racePctWhite') which are chosen by all other models.
- **LASSO selects a sparse set of features**, dropping those with high collinearity. Based on the regularizatoin paths with top correlated features, it shows that "PctKids2Par" increases as "FemalePctDiv" decreases to zero, indicating their high correlation. Finally, all betas shrink to zero. Different lambda to be set will lead to different number and value of betas.
- **Ridge retains all features but shrinks them differently depending on multicollinearity.** Unlike Lasso and Elastic_Net, Ridge won't make betas ultimately to zeros. Instead, it force betas to converge into several groups.
- **Best Subsets chooses a feature set optimizing predictive accuracy.** It compares all combination of betas, which takes much much longer time.
- **Stepwise approaches iteratively adding features to improve model fit.** It return the results much faster than the Best Subset model, but it wont check all possible combinations.
- **Elastic Net balances LASSO and Ridge behaviors, selecting features based on both sparsity and correlation handling.** Because it contains norm 1 penalty, it did shrink betas to zero finally, but due to the l1_ratio it shrink betas with slow pace.

Different tuning parameters led to different selected features, especially in **LASSO and Elastic Net**, where higher regularization forces more coefficients to zero. However, some features were **consistently selected across all methods**, indicating their strong predictive power.

Most Important Features: After comparing all methods and getting 27 significant features. The most consistently selected and impactful features are:

- **'PctKids2Par', 'PctIlleg', 'racePctWhite':** Selected in four models.
- **'PctWorkMom':** Selected in three models.

- **'state_28', 'state_12', 'state_13', 'pctUrban', 'MalePctDivorce','HousVacant', 'racepctblack', 'pctWInvInc', 'FemalePctDiv','PctFam2Par', 'PctYoungKids2Par', 'PctTeen2Par':** Selected in two models.
- **'state_51', 'state_42', 'state_6', 'MedRent', 'state_39','state_25', 'PctPersDenseHous', 'PctVacantBoarded', 'NumStreet', 'TotalPctDiv', 'pctWPubAsst':** Selected in one model.

Thus, "PctKids2Par", "PctIlleg", "racePctWhite can be considered the most critical features, as they remain important regardless of the selection method used.

Problem b

```
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression, RidgeCV, LassoCV, ElasticNetCV
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import StandardScaler
import numpy as np

# Split data into 60% train, 20% validation, 20% test
def split_data(X, y):
    X_train, X_temp, y_train, y_temp = train_test_split(X, y, test_size=0.4, random_
    X_val, X_test, y_val, y_test = train_test_split(X_temp, y_temp, test_size=0.5, r
    return X_train, X_val, X_test, y_train, y_val, y_test

# Number of repetitions
n_repeats = 10
test_errors = {
    "OLS": {"validation": [], "test": []},
    "Ridge": {"validation": [], "test": []},
    "Best": {"validation": [], "test": []},
    "Step":{"validation": [], "test": []},
    "LASSO": {"validation": [], "test": []},
    "ElasticNet": {"validation": [], "test": []},
    "Mean": {"validation": [], "test": []}
               }

for _ in range(n_repeats):
    # Split the data
    X_train, X_val, X_test, y_train, y_val, y_test = split_data(stand_X, centered_Y)

    # Mean
    test_errors["Mean"]["validation"].append(mean_squared_error(y_val, np.mean(y_tra
    test_errors["Mean"]["test"].append(mean_squared_error(y_test, np.mean(y_train) *

    # OLS
    ols = LinearRegression().fit(X_train, y_train)
    test_errors["OLS"]["validation"].append(mean_squared_error(y_val, ols.predict(X_
    test_errors["OLS"]["test"].append(mean_squared_error(y_test, ols.predict(X_test)
```

```
    # Ridge (chooses best alpha using validation set)
    ridge = RidgeCV(alphas=np.logspace(-3, 3, 10), store_cv_values=True).fit(X_train
    test_errors["Ridge"]["validation"].append(mean_squared_error(y_val, ridge.predic
    test_errors["Ridge"]["test"].append(mean_squared_error(y_test, ridge.predict(X_t

    # # LASSO (chooses best alpha using validation set)
    lasso = LassoCV(alphas=np.logspace(-3, 3, 10), cv=5).fit(X_train, y_train)
    test_errors["LASSO"]["validation"].append(mean_squared_error(y_val, lasso.predic
    test_errors["LASSO"]["test"].append(mean_squared_error(y_test, lasso.predict(X_t

    # # Elastic Net (chooses best alpha and L1 ratio using validation set)
    elastic_net = ElasticNetCV(l1_ratio=[0.1, 0.5, 0.9], alphas=np.logspace(-3, 3, 1
    test_errors["ElasticNet"]["validation"].append(mean_squared_error(y_val, elastic
    test_errors["ElasticNet"]["test"].append(mean_squared_error(y_test, elastic_net.

    # # Step Wise
    sfs.fit(X_train, y_train)
    best_features = list(sfs.k_feature_idx_)
    step = LinearRegression().fit(X_train.iloc[:, best_features], y_train)
    test_errors["Step"]["validation"].append(mean_squared_error(y_val, step.predict(
    test_errors["Step"]["test"].append(mean_squared_error(y_test, step.predict(X_tes

    # # Best Subsets
    efs.fit(X_train.loc[:, top_cor.index], y_train)
    best_features = list(efs.best_idx_)
    best = LinearRegression().fit(X_train.iloc[:, best_features], y_train)
    test_errors["Best"]["validation"].append(mean_squared_error(y_val, best.predict(
    test_errors["Best"]["test"].append(mean_squared_error(y_test, best.predict(X_tes
    print(f"\nRound-{_+1} Done.")

# Compute average test error for each method
avg_test_errors = {
    model: {types: np.mean(values) for types, values in items.items()}
    for model, items in test_errors.items()
}
avg_test_errors = pd.DataFrame.from_dict(avg_test_errors).T
```

```
Features: 1023/1023
Round-1 Done.
Features: 1023/1023
Round-2 Done.
Features: 1023/1023
Round-3 Done.
Features: 1023/1023
Round-4 Done.
Features: 1023/1023
Round-5 Done.
Features: 1023/1023
Round-6 Done.
Features: 1023/1023
Round-7 Done.
Features: 1023/1023
Round-8 Done.
Features: 1023/1023
Round-9 Done.
Features: 1023/1023
Round-10 Done.
```

	validation	test
OLS	0.335173	0.367681
Ridge	0.315781	0.352137
Best	0.909232	0.962026
Step	0.321264	0.358342
LASSO	0.314434	0.347623
ElasticNet	0.312482	0.349480
Mean	0.990844	1.056437

后续步骤： 使用 avg_test_errors 生成代码 查看推荐的图表 New interactive sheet

```
avg_test_errors.sort_values(by="test", ascending=False)
```

	validation	test
Mean	0.990844	1.056437
Best	0.909232	0.962026
OLS	0.335173	0.367681
Step	0.321264	0.358342
Ridge	0.315781	0.352137
ElasticNet	0.312482	0.349480
LASSO	0.314434	0.347623

```
avg_test_errors.sort_values(by="validation", ascending=False)
```

	validation	test
Mean	0.990844	1.056437
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OLS	0.335173	0.367681
Step	0.321264	0.358342
Ridge	0.315781	0.352137
LASSO	0.314434	0.347623
ElasticNet	0.312482	0.349480

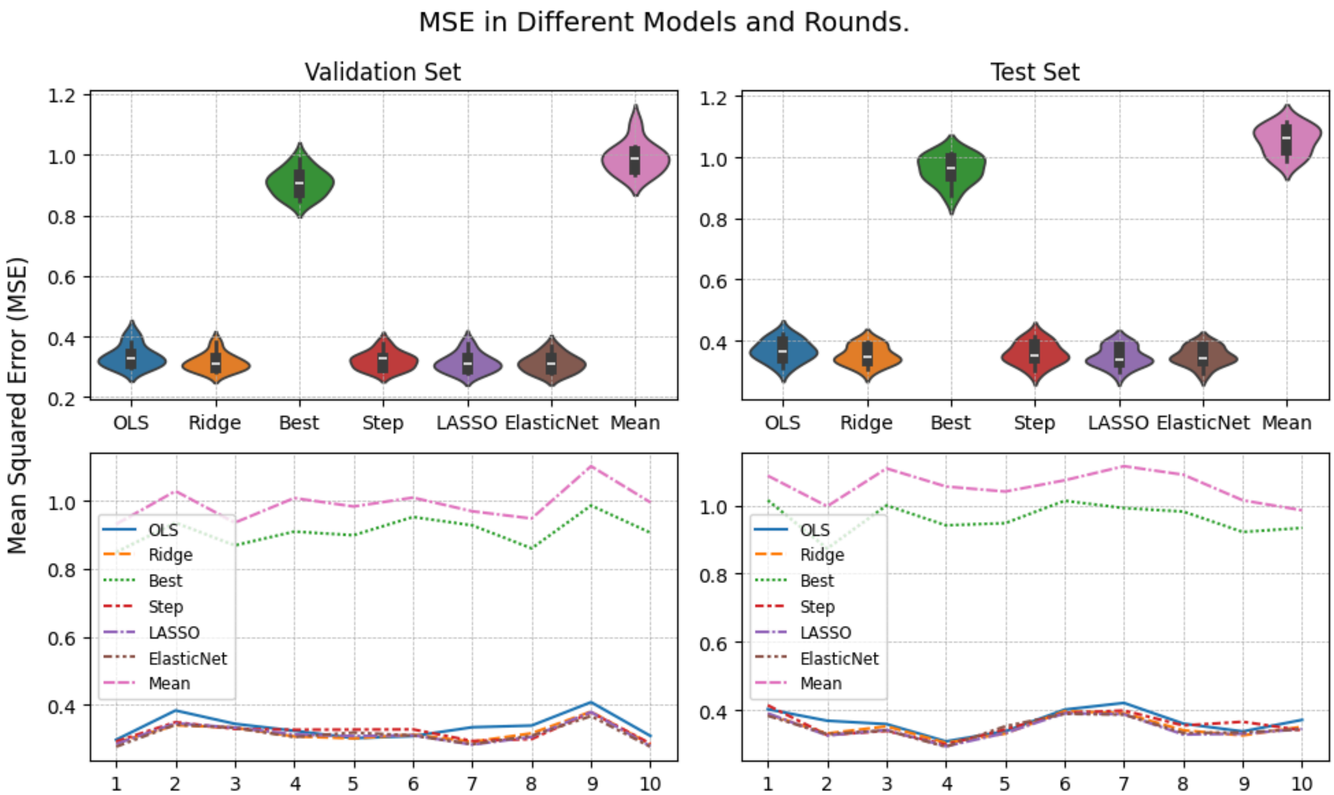
```
import matplotlib.pyplot as plt
import seaborn as sns

validation_sets = {model: values["validation"] for model, values in test_errors.item
test_sets = {model: values["test"] for model, values in test_errors.items()}
df_va = pd.DataFrame.from_dict(validation_sets)
df_te = pd.DataFrame.from_dict(test_sets)

fig, ax = plt.subplots(2, 2, figsize=(10, 6))
fig.suptitle("MSE in Different Models and Rounds.", fontsize=14)
fig.supylabel("Mean Squared Error (MSE)")

sns.violinplot(data=df_va, ax=ax[0, 0])
sns.violinplot(data=df_te, ax=ax[0, 1])
sns.lineplot(data=df_va, ax=ax[1, 0])
sns.lineplot(data=df_te, ax=ax[1, 1])

ax[0, 0].set_title("Validation Set")
ax[0, 1].set_title("Test Set")
ax[1, 1].legend(loc="center left", fontsize="small")
ax[1, 0].legend(loc="center left", fontsize="small")
ax[1, 0].set_xticks(range(0, 10))  # Keep the original tick positions
ax[1, 0].set_xticklabels(range(1, 11))  # Change labels to start from 1
ax[1, 1].set_xticks(range(0, 10))  # Keep the original tick positions
ax[1, 1].set_xticklabels(range(1, 11))  # Change labels to start from 1
for i in range(2):
  for j in range(2):
    ax[i, j].grid(linestyle="--", linewidth=0.5)
plt.tight_layout()
plt.show()
```




```
print(f"""\
The MSE of these models based on validation starting at least to highest is:
ElasticNet < Lasso < Ridge < Step < OLS < Best < Mean
The MSE of these models based on test starting at least to highest is:
Lasso < ElasticNet < Ridge < Step < OLS < Best < Mean

So based on these results, we can conclude that:
Elastic Net and LASSO achieve the best prediction error because they
effectively combine the advantages of LASSO (feature selection) and Ridge
(handling multicollinearity). Additionally, there are {selected_pairs.shape[0]}
highly correlated features in the dataset, where the absolute correlation
exceeds 0.9. This makes regularization methods particularly useful in
reducing overfitting.

From the graphs, it is difficult to clearly identify overfitting. However, OLS
tends to overfit when the number of parameters exceeds the number of records
( p > n ). In this case, where  X  has shape {stand_X.shape}, this is not an
issue.

Since LASSO, Ridge, and Elastic Net include regularization, they cannot perform
worse than OLS in terms of generalization. However, Stepwise Selection and Best
Subset Selection are more prone to overfitting because they may select too many
parameters. Additionally, these methods are computationally expensive, so I
limited their result size to ensure efficiency.

Conclusion:
Lasso is the overall best method for prediction on this dataset.
""")
```

```
The MSE of these models based on validation starting at least to highest is:
ElasticNet < Lasso < Ridge < Step < OLS < Best < Mean
The MSE of these models based on test starting at least to highest is:
Lasso < ElasticNet < Ridge < Step < OLS < Best < Mean

So based on these results, we can conclude that:
Elastic Net and LASSO achieve the best prediction error because they
effectively combine the advantages of LASSO (feature selection) and Ridge
(handling multicollinearity). Additionally, there are 56
highly correlated features in the dataset, where the absolute correlation
exceeds 0.9. This makes regularization methods particularly useful in
reducing overfitting.

From the graphs, it is difficult to clearly identify overfitting. However, OLS
tends to overfit when the number of parameters exceeds the number of records
( p > n ). In this case, where  X  has shape (1994, 165), this is not an
issue.

Since LASSO, Ridge, and Elastic Net include regularization, they cannot perform
worse than OLS in terms of generalization. However, Stepwise Selection and Best
Subset Selection are more prone to overfitting because they may select too many
parameters. Additionally, these methods are computationally expensive, so I
limited their result size to ensure efficiency.

Conclusion:
Lasso is the overall best method for prediction on this dataset.
```

Question 2

(a) (10 points) Empirically demonstrate or mathematically show that that fitting linear regression with an intercept term is equivalent to

- (i) fitting linear regression when centering Y and centering the columns of X, and
- (ii) fitting linear regression when adding a column of ones to X.

```
import numpy as np
import pandas as pandas
import statsmodels.api as sm
import sklearn.metrics as metric

np.random.seed(89)
beta_true = np.array([2, 3, 5, 4]).reshape(-1, 1)
x_emperical = np.column_stack((np.random.uniform(0, 100, 1000), np.random.norm
x_emperical = sm.add_constant(x_emperical)
y_emperical = x_emperical @ beta_true + np.random.normal(0, 500, size=(1000, 1
y_emperical_center = y_emperical - y_emperical.mean()
```

```
x_emperical_center = (x_emperical - x_emperical.mean(axis=0))

model1 = sm.OLS(y_emperical, x_emperical)
result1 = model1.fit()

model2 = sm.OLS(y_emperical_center, x_emperical_center)
result2 = model2.fit()

result2.params, result1.params

y_emperical_pred_1 = x_emperical @ result1.params.reshape(-1, 1)
y_emperical_pred_2 = x_emperical_center @ result2.params.reshape(-1, 1)

if np.isclose((result1.params[1:] - result2.params[1:]), 0).mean() == 1:
  print("these models predict the slope in same values.\n")
else:
  print("these models dont predict the slope in same values.\n")
print(f"true_beta: {beta_true.flatten()}")
print(f"(i)\tmodel1: {result2.params}")
print(f"(ii)\tmodel2: {result1.params}")

print(f"MSE of model1: {metric.mean_squared_error(y_emperical, y_emperical_pre
print(f"MSE of model2: {metric.mean_squared_error(y_emperical_center, y_emperi
```

```
these models predict the slope in same values.

true_beta: [2 3 5 4]
(i)     model1: [0.          4.09753977 5.17005687 9.37841763]
(ii)    model2: [-86.18920724   4.09753977   5.17005687   9.37841763]
MSE of model1: 243404.143
MSE of model2: 243404.143
```

(b) (10 points) Empirically demonstrate or mathematically show that the least squares solution has zero training error when p > n.

```
import random

np.random.seed(89)
x_emperical = np.random.normal(random.random()*100, random.random()*1000, size
for i in range(14):
  x_emperical = np.column_stack((x_emperical, np.random.normal(random.random()

beta_true2 = np.random.randint(5, 10, size=(16, 1))
x_emperical = sm.add_constant(x_emperical)
y_emperical = x_emperical @ beta_true2 + np.random.normal(0, 50, size=(10, 1))
y_emperical_center = y_emperical - y_emperical.mean()
x_emperical_center = x_emperical - x_emperical.mean(axis=0)
```