1. (a) $\alpha^k = \underset{\alpha}{\arg\min}\, f(x^k + \alpha d^k)$ with $d^k = -\nabla f(x^k)$

Since $\alpha^k$ minimizes $h(\alpha) = f(x^k + \alpha d^k)$, $h'(\alpha)$ must be $0$ at $\alpha = \alpha^k$

Then, we have $h'(\alpha) = \dfrac{d f(x^k + \alpha d^k)}{d\alpha} = \langle d^k, \nabla f(x^k + \alpha d^k) \rangle$

Thus, $h'(\alpha^k) = \langle d^k, \nabla f(x^k + \alpha^k d^k) \rangle = 0$

(b) Since $d^k = -\nabla f(x^k)$,

$$\langle d^k, \nabla f(x^k + \alpha^k d^k) \rangle = \langle -\nabla f(x^k), \nabla f(x^k + \alpha^k d^k) \rangle = 0$$

$x_k$ and $x_{k+1}$ are two consecutive points generated by the steepest descent algorithm, then we have

$$x^{k+1} = x^k + \alpha^k d^k$$

Thus, $\langle -\nabla f(x^k), \nabla f(x^{k+1}) \rangle = 0$

$$\langle \nabla f(x^k), \nabla f(x^{k+1}) \rangle = 0$$