Washington University in St. Louis

$$\min_x \quad f(x) \qquad \text{s.t.} \quad x \in X \subset \mathbb{R}^n$$

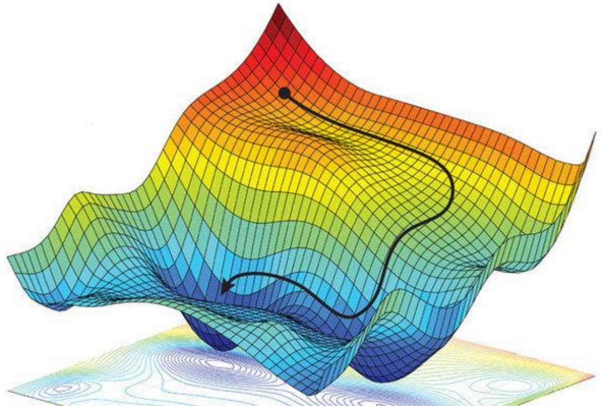
$$\nabla f(x) = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} \\ \vdots \\ \frac{\partial f(x)}{\partial x_n} \end{bmatrix}$$

ESE 415

Optimization

Instructor: Shen Zeng

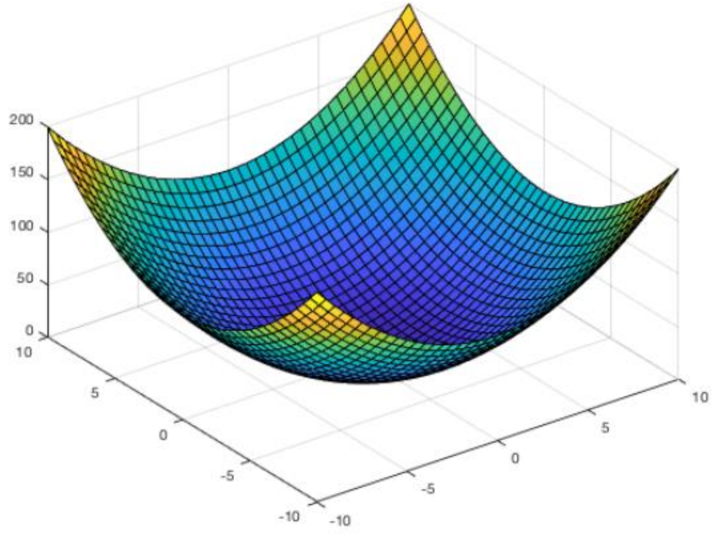
ESE Department, Washington University in St. Louis



$$f(x + \alpha d) \approx f(x) + \alpha(\nabla f(x))^T d.$$

Mathematical Optimization in a Nutshell

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) \\ \text{subject to} & \mathbf{x} \in \mathcal{X} \end{array}$$



- Optimization variable: $\mathbf{x} = (x_1, \ldots, x_n)$
- Objective function: $f : \mathbb{R}^n \to \mathbb{R}$
- Contraint set: $\mathcal{X} = \{\mathbf{x} \mid \mathbf{h}(\mathbf{x}) = 0\} \cap \{\mathbf{x} \mid \mathbf{g}(\mathbf{x}) \leq 0\}$
- Optimal solution $\mathbf{x}^*$ has smallest value of f among all vectors that satisfy the constraints

The goal in mathematical optimization is to find the smallest value of a function under certain constraints.

Optimization is finding extensive use everywhere

- Optimization ideas and principles can be found in almost all engineering domains nowadays. They are also deeply rooted in several mathematical and physical disciplines.
- Optimization is still an active research area with many open questions

The goal of ESE 415 is to help you understand and apply the basics of optimization theory!

By the end of the semester, you should be able to:

- recognize and formulate problems as optimization problems
- derive and characterize optimal solutions
- develop code for optimization algorithms

Rough List of Specific Topics:

- optimality conditions
- convex sets and functions
- constrained and unconstrained optimization
- optimization algorithms
- analysis of optimization algorithms
- examples and applications

Organizational Matters

Anything relevant to ESE 415 can be found at the course website:

https://wustl.instructure.com/courses/132084

No fixed textbook requirement, but a list of recommended textbooks can be found at the above course website.

Grading policy:

- Homework: 35%. Biweekly; theoretical and computational exercises.
  Late submission policy: deduction of 20 points per late day.
- Midterm Exam: 30%. Monday, **September 30, 2024**, taken during class hours.
- Final Exam: 35%. Tuesday, **December 17, 2024**, 10:30am-12:30pm, Lopata 101.

List of Assistants to the Instructor & Graders

<u>Head Assistant to the Instructor:</u>

- Nia Hodges: h.nia@wustl.edu (2nd year Systems Science & Mathematics Ph.D. student, ESE)

<u>Assistants to the Instructor & Graders:</u>

- Xiaozhu Cao: c.xiaozhu@wustl.edu
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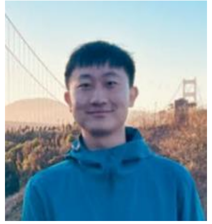
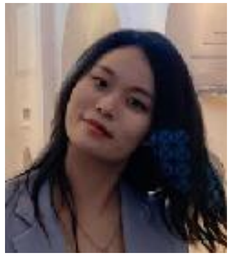
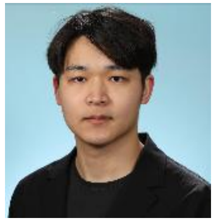

<u>ESE 415 Office Hours</u>:

Every **Tuesday**, **Thursday**, and **Friday** from **2:30-3:30pm** via Zoom (access through Canvas).

Brief history of optimization

Antiquity: Optimization questions pertaining to geometry

- Euclid (300bc): minimal distance between a point and a line
- Heron (100bc): light travels between two points through the path with shortest length when reflecting from a mirror

17-18th century: Analytical treatments via calculus

- Kepler (1615): Optimal dimensions of wine barrel
- Fermat (1636): derivative of function vanishes at extreme points
- Newton (1660s) and Leibniz (1670s) create mathematical analysis that forms the basis of <u>calculus of variations</u>*

*huge significance in mathematical and physical research areas

Brief history of optimization

19th century: the first optimization algorithms are presented

- Gauss & Legendre (~1800): least squares method
- Cauchy (1847): gradient method
- Gibbs shows that chemical equilibrium is an energy minimum

20th century: algorithmic research expands as computers become ubiquitous

- von Neuman and Morgenstern (1944): dynamic programming for solving sequential decision problems
- Dantzig (1947): simplex method for solving Linear Programs
- Kuhn & Tucker (1951): optimality conditions for nonlinear problems. Karush (1939): Similar conditions
- Nesterov (1983): accelerated gradient method
- Karmarkar (1984): polynomial time algorithm for Linear Programs;
  creates a boom of interior point methods.
- **Modern era**: non-smooth analysis, stochastic optimization, optimal control of dynamical systems, ...