ESE 415: Optimization (FL2024) Instructor: Shen Zeng
https://wustl.instructure.com/courses/132084 **Due:** September 25, 2024

Homework 3

1. Consider the steepest descent algorithm with an exact line search via

$$\alpha^k = \arg\min f(x^k + \alpha d^k)$$

with $d^k = -\nabla f(x^k)$.

(a) Let α^k denote the minimizer of the function $h(\alpha) := f(x^k + \alpha d^k)$. Show that

$$\langle d^k, \nabla f(x^k + \alpha^k d^k)\rangle = 0.$$

(b) Now suppose that x^k and x^{k+1} are two consecutive points generated by the steepest descent algorithm with an exact line search as introduced in the beginning of this problem. Show that

$$\langle \nabla f(x^k), \nabla f(x^{k+1})\rangle = 0.$$

2. **(Programming Problem)** Consider the function

$$f(x_1, x_2) = x_1^2 + 5x_2^2 + 4x_1x_2 - 6x_1 - 14x_2 + 20.$$

(a) Implement the Steepest Descent algorithm with an exact line search as introduced in the previous problem to minimize the function. Show the contour plot of the function and overlay the path traced out by the iterations $x^0, x^1, x^2, \ldots$. Furthermore provide a table with columns for

$$x_1^k, x_2^k, d_1^k, d_2^k, \|d^k\|, \alpha^k, f(x^k)$$

for all iterations $k = 0, 1, 2, \ldots$ until the tolerance $\|d^k\| < \epsilon := 10^{-6}$ is achieved.

(b) Solve the minimization problem with the MATLAB optimization solver "fminunc" using the same initialization x^0 and the same tolerance for $\|d^k\|$ as in the foregoing subproblem. Print out the iterations. Hint: Set the options to

```
opt = optimoptions(@fminunc,'Display','iter-detailed','Algorithm',...
    'quasi-newton','HessUpdate','steepdesc','MaxFunctionEvaluations',2000);
```

and run the solver via

```
[x,fval,exitflag,output] = fminunc(fun,x0,opt);
```

after having defined the function `fun` appropriately.

3. Consider the cost functional $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x) = \frac{x_1^4}{4} - x_1^2 + 2x_1 + (x_2 - 1)^2.$$

(a) Examine the first step of Newton's method in the minimization of the above cost. Are there initializations of x^0 for which the method breaks down?

(b) **(Programming Problem)** Implement Newton's method for the minimization of f with an initialization that works. Show the contour plot and the path traced out by the iterations $x^0, x^1, x^2, \ldots$