ESE 415

Homework 1 Solution:

1. — (0pts)

2. Gross profit from sales: $100x_1 + 150x_2 + 200x_3 + 400x_4$

(20/15) Constraints: wood: $10x_1 + 12x_2 + 25x_3 + 20x_4 \leq 5000$

labor: $2x_1 + 4x_2 + 8x_3 + 12x_4 \leq 1500$

Optimization problem:

(5pts) $\underset{(x_1, x_2, x_3, x_4)}{\text{minimize}} \quad -100x_1 - 150x_2 - 200x_3 - 400x_4$

subject to (5pts) $10x_1 + 12x_2 + 25x_3 + 20x_4 \leq 5000$

(5pts) $2x_1 + 4x_2 + 8x_3 + 12x_4 \leq 1500$

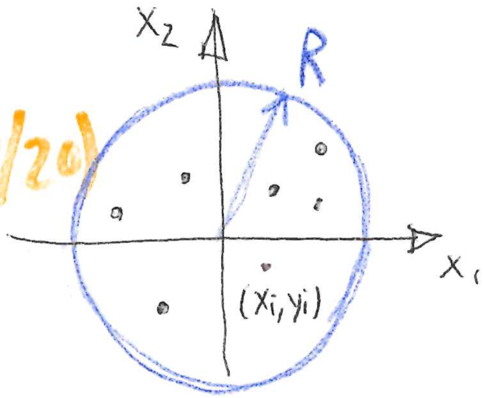
optional $\begin{cases} x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0 \\ x_1, x_2, x_3, x_4 \in \mathbb{N}_0 \end{cases}$

Linear program! (+5 BONUS PT's)

Optional: Integer Linear Program.

3.

(20/20)



Implicitly assumed that circle centered at $(0,0)$.

Optimization problem:

$$\underset{R}{\text{minimize}} \quad R$$

10 pts

$$\text{subject to} \quad R^2 \geq x_i^2 + y_i^2 \,, \quad i = 1, 2, \ldots, N$$
$$(R^2 \geq 0)$$

Solution can be easily obtained by just calculating all values of $x_i^2 + y_i^2$ and selecting the largest occurring value and that will be R^* (common sense analysis).

10 pts

4.

(a) (30/30) $\underline{n=1}$: $f(x) = f(x^*) + \cancel{\underbrace{f'(x^*)}_{=0 \text{ by assumption}}(x-x^*)} + \underbrace{\frac{f''(z)}{2}}_{>0 \text{ by assumption}} \underbrace{(x-x^*)^2}_{>0 \text{ for } x \neq x^*}$ (5 pts)

5 pts for formula

with z between x and x^*. (5 pts)

$\Rightarrow$ $f(x) = f(x^*) +$ "something > 0"

for all $x \in I$ with $x \neq x^*$.

(5 pts) $\Rightarrow$ $f(x^*) < f(x)$ for all $x \in I$, $x \neq x^*$.

(b) $f(x) = e^{x^2}$

(chain rule) $f'(x) = e^{x^2} \frac{d}{dx} x^2 = 2x e^{x^2}$

stationary points: $f'(x) = 0 \rightarrow x = 0$ only

(5 pts) stationary point

product rule & chain rule $f''(x) = 2e^{x^2} + 2x(2xe^{x^2})$

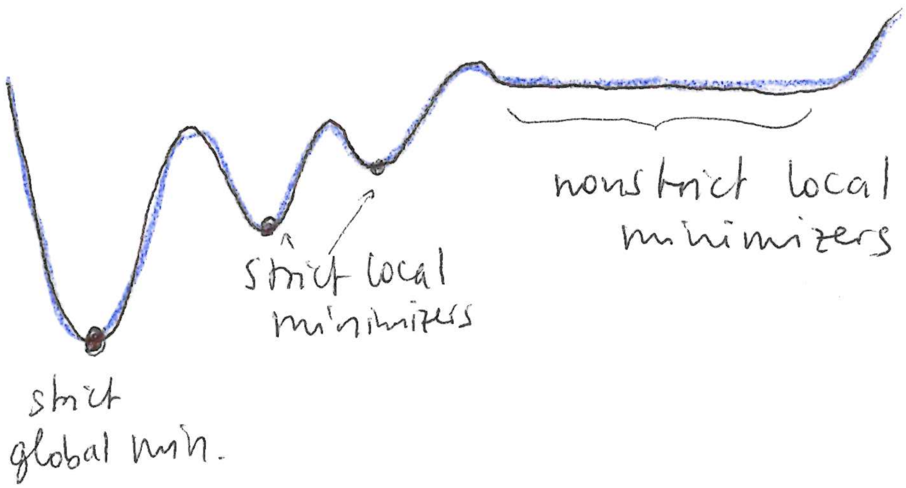
$= 2e^{x^2} + 4x^2 e^{x^2}$

$= \underbrace{(2+4x^2)}_{>2}\underbrace{e^{x^2}}_{>0} > 0$ for all $x \in \mathbb{R}$

(5 pts) $\Rightarrow$ $x^* = 0$ is strict global minimizer on $\mathbb{R}$.

5.

(15/15)



15 pts

6. $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

(20/20)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longmapsto \begin{pmatrix} xz + y^2 \\ \sin(xyz) + z \end{pmatrix}$$

(a) $$J_f(x) = \begin{pmatrix} z, & 2y, & x \\ yz\cos(xyz), & xz\cos(xyz), & xy\cos(xyz)+1 \end{pmatrix}$$ (10 pts)

<u>at $x = (0,-1,1)$:</u> (5 pts)

$$J_f\left(\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}\right) : \begin{pmatrix} 1, & -2, & 0 \\ -1, & 0, & 1 \end{pmatrix}$$

(b) Directions $v \in \mathbb{R}^3$ for which <u>directional derivative</u> of f is zero are equal to solutions to

(5 pts) $J_f\left(\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}\right) v = 0$. $\rightsquigarrow$ null space of $\begin{pmatrix} 1, & -2, & 0 \\ -1, & 0, & 1 \end{pmatrix}$

YES! $\Leftarrow$ obviously non-trivi