A **batch** training algorithm (batch size = 4) for Perceptron model is run on the following data set:

x_1	x_2	y
0	0	-1
0	1	-1
1	0	-1
1	1	1

Where, x_1 and x_2 are the features (inputs) and y is the target value (output)

The weight vector is $\boldsymbol{w} = [w_0 \quad w_1 \quad w_2]^T$, the expended feature vector is $\boldsymbol{x} = [1 \quad x_1 \quad x_2]^T$. Let's set the initial value of $\boldsymbol{w}$ as $\boldsymbol{w^0} = [0.1 \quad 0.1 \quad 0.1]^T$.

(1) Calculate the loss function $E_p(\boldsymbol{w}) = -\sum_{i \in M} \boldsymbol{w}^T \boldsymbol{x}_i y_i$ at the end of the first epoch of the batch training algorithm.

(2) Update the weight vector with learning rate $\eta = 0.01$ at the end of the first epoch.

(1) first epoch: $w^0 = (0.1 \quad 0.1 \quad 0.1)^T$, $\vec{x_1} = (1, 0, 0)^T$, $\vec{x_2} = (1, 0, 1)^T$.

$\vec{x_3} = (1, 1, 0)^T$. $\vec{x_4} = (1, 1, 1)^T$ $y_1 = -1, y_2 = -1, y_3 = -1, y_4 = 1$

$w^{0^T} \vec{x_1} = (0.1 \ 0.1 \ 0.1)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0.1$

$\hat{y_1} = sign(w^{0^T}\vec{x_1}) = sign(0.1) = 1$ \qquad misclassified

$w^{0^T} \vec{x_2} = (0.1, 0.1, 0.1)\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 0.2$

$\hat{y_2} = sign(w^{0^T}\vec{x_2}) = sign(0.2) = 1$ \qquad misclassified

$w^{0^T}\vec{x_3} = (0.1, 0.1, 0.1)\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0.2$, $\hat{y_3} = 1$ misclassified

$w^{0^T} \vec{x_4} = (0.1, 0.1, 0.1)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0.3$ $\hat{y_4} = 1$ \qquad correct

$E_p(w) = -\sum_{i \in m} w^T x_i y_i = -\left(w^T x_1 y_1 + w^{0^T} x_2 y_2 + w^{0^T} x_3 y_3 \right)$

$= -\left[(0.1, 0.1, 0.1)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot (-1) + (0.1, 0.1, 0.1)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot (-1) \right.$

$$+ (0.1, 0.1, 0.1) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot (-1) \Big]$$

$$= \quad 0.1 + 0.2 + 0.2 = 0.5$$

(2) $\quad w' = w^0 + \eta \sum_{i \in M_k} x_i y_i = (0.1, 0.1, 0.1)^T + \eta (x_1 y_1 + x_2 y_2 + x_3 y_3)$

$$= (0.1, 0.1, 0.1)^T + 0.01 \left(\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \right)$$

$$= \begin{pmatrix} 0.1 \\ 0.1 \\ 0.1 \end{pmatrix} + 0.01 \begin{pmatrix} -3 \\ -1 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.07 \\ 0.09 \\ 0.09 \end{pmatrix}$$