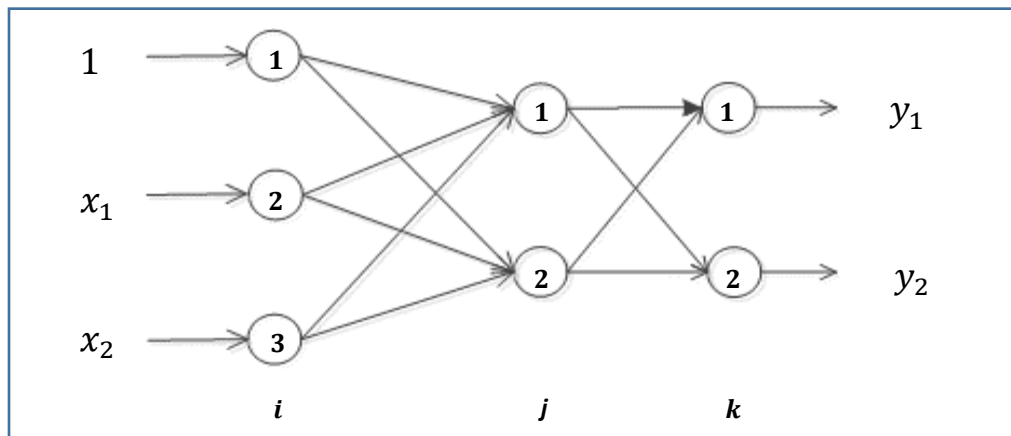


Consider the following artificial neural network model:



This model has input layer, hidden layer, and output layer with 3, 2 and 2 nodes respectively.

This model is trained to solve the XOR problem with the following truth table:

$x_1$	$x_2$	$\mathbf{t} = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$
-1	1	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
1	-1	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
1	1	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
-1	-1	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Where,  $x_1$  and  $x_2$  are the features and  $\mathbf{t}$  is the target value in one-hot code.

**Sigmoid function** is used in the hidden nodes and output nodes as the activation functions.

Assuming the initial values of the weights are all set as 0.1. The first instance in the table is fed into the model:

- (1) Find the net activations and outputs of the hidden nodes
- (2) Find the net activations and outputs of the output nodes
- (3) Evaluate the **training error** of this instance

$$(1) \begin{pmatrix} net_1 \\ net_2 \end{pmatrix} = \begin{pmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{pmatrix} \begin{pmatrix} 1 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0.1 + 0.1x_1 + 0.1x_2 \\ 0.1 + 0.1x_1 + 0.1x_2 \end{pmatrix}$$

$$z_j = \text{sigmoid}(net_j) = \frac{1}{1 + e^{-net_j}}$$

$$\begin{pmatrix} net_1 \\ net_2 \end{pmatrix} = \begin{pmatrix} 0.1 - 0.1 + 0.1 \\ 0.1 - 0.1 + 0.1 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.1 \end{pmatrix}$$

$$z_1 = \frac{1}{1 + e^{-0.1}}$$

$$z_2 = \frac{1}{1 + e^{-0.1}} \approx 0.525$$

$$(2) \begin{pmatrix} net_1 \\ net_2 \end{pmatrix} = \begin{pmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 0.1z_1 + 0.1z_2 \\ 0.1z_1 + 0.1z_2 \end{pmatrix}$$

$$y_k = \text{sigmoid}(net_k) = \frac{1}{1 + e^{-net_k}}$$

$$\begin{pmatrix} net_1 \\ net_2 \end{pmatrix} = \begin{pmatrix} \frac{0.2}{1 + e^{-0.1}} \\ \frac{0.2}{1 + e^{-0.1}} \end{pmatrix} \approx \begin{pmatrix} 0.105 \\ 0.105 \end{pmatrix}$$

$$y_1 = \frac{1}{1 + e^{-net_1}} = \frac{1}{1 + e^{-0.105}} \approx 0.526$$

$$y_2 = \frac{1}{1 + e^{-net_2}} = \frac{1}{1 + e^{-0.105}} \approx 0.526$$

$$(3) E = \frac{1}{2} \left( (0.526 - 1)^2 + (0.526 - 0)^2 \right) \approx 0.251$$