

Problem 2 (1)

$$f_1(x) = \frac{1}{1+e^{-x}}, \quad f_2(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\text{Let } y = 1 + e^{-x}, \quad f_1(x) = \frac{1}{y}$$

$$f_1'(x) = \frac{d}{dx} f_1(x) = \frac{df_1}{dy} \frac{dy}{dx} = -\frac{1}{y^2} \cdot (-e^{-x}) = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$f_1(x)(1-f_1(x)) = \frac{1}{1+e^{-x}} \left(1 - \frac{1}{1+e^{-x}}\right) = \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$\text{Thus, } f_1'(x) = f_1(x)(1-f_1(x))$$

$$1 - f_2^2(x) = 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2 = 1 - \frac{(e^x - e^{-x})^2}{(e^x + e^{-x})^2} = 1 - \frac{e^{2x} - 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}} = \frac{4}{e^{2x} + 2 + e^{-2x}}$$

$$\begin{aligned} f_2'(x) &= \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2} = \frac{(e^{2x} + 2 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})}{e^{2x} + 2 + e^{-2x}} \\ &= \frac{4}{e^{2x} + 2 + e^{-2x}} \end{aligned}$$

$$\text{Thus, } f_2'(x) = 1 - f_2^2(x)$$