ESE 520 Probability and Stochastic Processes

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EXAM 1 - Practice

Instructions:

1. You must show all work to completely justify your answers in order to receive any credit.
2. You can use One one-sided sheet of paper with your own formulas.

1. a) Let A_1, A_2, and A_3 be three arbitrary events. What is the formula for $P(A_1 \cup A_2 \cup A_3) =$

b) Suppose we roll three (fair) dice. What is the probability that we get at least one 6?

2. a) Let X has an exponential distribution with parameter $\lambda = 1$. That is, X has the density $f(x) = e^{-x}, x > 0$ (and zero otherwise). Define $Y = F(X)$ where $F(x)$ is the cdf of X. Show that the new random variable Y has the uniform distribution on the interval $[0, 1]$.

b) Now, assume that U a uniform distribution on the interval $[0, 1]$ and F^{-1} is the inverse of the cdf F of the exponential distribution from a), that is $F^{-1}(F(x)) = x$. Let $Y = F^{-1}(U)$. Show that Y has the exponential distribution.

3. Let $\Omega = \{1, 2, 3, 4, 5, 6\}$ and consider a random variable $X = 2\mathbf{1}_{\{1,2\}} + 3\mathbf{1}_{\{1,3,5\}} - 2\mathbf{1}_{\{3,5\}}$ where

$$\mathbf{1}_A(w) = \begin{cases} 1, & w \in A \\ 0, & w \notin A. \end{cases}$$

is an indicator function of a set A.

a) Identify the partition of Ω genetated by X.

b) Is X a measurable map with respect to the σ-algebra generated by the events from $\mathcal{A} = \{1\}, \{2\}, \{3, 5\}, \{4\}$, and $\{6\}$?

4. Suppose $X_1, ..., X_n$ are independent random variables each having an exponential distribution with parameter $\lambda > 0$. That is, their pdf has the form $f(x) = \lambda e^{-\lambda x}, x > 0$. Show that $\min\{X_1, ..., X_n\}$ has an exponential distribution with parameter $n\lambda$.

5. Let (X, Y) has the jointly continuous density

$$f(x,y) = \begin{cases} e^{-y}, & 0 < x < y < \infty \\ 0, & \text{otherwise.} \end{cases}$$

Find the conditional densities $f_{X|Y=y}(x,y)$ and $f_{Y|X=x}(x,y)$.

6. Suppose U and V are independent random variables having both uniform distribution on the interval $[0, 1]$. Find $E(U - V)^2$.

7. Let X has a Poisson distribution with parameter $\lambda > 0$, that is

$$P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}, x = 0, 1, 2, ...$$

Find the variance of X.