ESE 520 Probability and Stochastic Processes

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EXAM 2-Practice

Name (Please print):

Total: 70 points (each problem is 10 points worth)

Instructions:

1. You must show all work to completely justify your answers in order to receive any credit.

2. You can use One one-sided sheet of paper with your own formulas.

1. Let (X, Y) be a randomly selected point on $\mathbb{R}^2$ where both coordinates X and Y have $\mathcal{N}(0, 1)$ distribution and are independent. Also, the polar coordinates for (X, Y) are (R, θ) and are defined through $X = R\cos\theta, Y = R\sin\theta$. Calculate $E(X|\theta)$.

2. Let X_1 X_2 be independent random variables each of which has a normal distribution $\mathcal{N}(\mu_1, \sigma_1^2)$ and $\mathcal{N}(\mu_2, \sigma_2^2)$, respectively. Define the random variable $Y := X_1 + X_2$. Using characteristic functions, show that Y also has a normal distribution $\mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$. Hint: The characteristic function of a normal rv with mean μ and variance σ^2 has the form $\phi(t) = e^{i\mu t - \frac{1}{2}\sigma^2 t^2}$.

3. Two measurements $y_1 = 2$ and $y_2 = 5$ are taken to estimate the value x of a random variable X. Assume that the joint distribution of the full random vector (X, Y_1, Y_2) is Gaussian and that X has expectation $\mu_X = 3$, variance $\sigma_X^2 = 4$, and the following covariances with the random variables Y_1 and Y_2 which describe the first and second measurement, $cov(X, Y_1) = -1$ and $cov(X, Y_2) = +1$. Also suppose the measurements Y_1 and Y_2 have means $\mu_{Y_1} = \mu_{Y_2} = 3$, variances $\sigma_{Y_1}^2 = 3$ and $\sigma_{Y_2}^2 = 5$ and covariance $cov(Y_1, Y_2) = -2$. Find the best mean square estimate for X in terms of the measurements (y_1, y_2).

4. Let $X_1, X_2, ..., X_n$ be independent and exponentially distributed (with parameter $\lambda > 0$) random variables. Define $Y := \max\{X_1, ..., X_n\}$. Find the density of Y.

5. Assume that a jointly continuous vector (X, Y) has the joint pdf $f(x, y) = \frac{1}{x}, 0 < y < x < 1$ (and equal to zero otherwise). Find $cov(X, Y)$. Hint: Use integration by parts when calculating $E(Y)$.

6. Let X be a continuous rv X with the density $f(x) = \frac{1}{2}e^{-\frac{1}{2}|x|}, x \in \mathbb{R}$. Find $\psi(t) = E(e^{tX})$.

7. With (X, Y) and (R, θ) defined as in Problem 1, show that random variables X and R are *uncorrelated*.