

ESE 520 Probability and Stochastic Processes

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EXAM 2-Sample

Name (Please print):

Instructions:

1. You must show all work to completely justify your answers in order to receive any credit.
2. You can use One one-sided sheet of paper with your own formulas.

1. a) Assume X has a normal distribution with mean μ and variance σ^2 . Find the moment-generating function of X .

b) Use part a) to find $E(X)$;

2. Let X and Y be two random variables having jointly continuous distribution with the joint pdf given as

$$f(x, y) = \begin{cases} 8xy, & 0 < y < x < 1 \\ 0, & \text{otherwise} . \end{cases}$$

Find the best linear mean-square estimate of Y by X .

3. Suppose X and Y have joint density e^{-x} for $0 < y < x$. Find $E(X|Y)$ and $Var(X|Y)$.

4. Suppose you play roulette and bet \$1 on black each time. Your net winnings on the i th play, X_i , are 1 with probability $1/3$ and -1 with probability $2/3$. What is the probability your net winnings are ≥ 0 after 100 plays?

Use the Central Limit Theorem and the enclosed normal table to find the approximate probability.

5. Suppose U, V, W are independent and have mean μ and variance σ^2 . Find $\rho(U + V, V + W)$.

6. Suppose X and Y are independent exponential random variables with the same parameter $\lambda > 0$. Find the density of $\min\{X, Y\}$.

7. Let X_1, X_2 be independent random variables each of which has a normal distribution $\mathcal{N}(\mu_1, \sigma_1^2)$ and $\mathcal{N}(\mu_2, \sigma_2^2)$, respectively. Define the random variable $Y := X_1 + X_2$. Using characteristic functions, show that Y also has a normal distribution $\mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$. Hint: The characteristic function of a normal rv with mean μ and variance σ^2 has the form $\phi(t) = e^{i\mu t - \frac{1}{2}\sigma^2 t^2}$.