ESE 520 Probability and Stochastic Processes

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EXAM 3- Practice

Name (Please print):

Instructions:

1. You must show all work to completely justify your answers in order to receive any credit.

2. You can use One one-sided sheet of paper with your own formulas.

1. Let (N_t) be a Poisson process with parameter $\lambda > 0$. For times $0 < t_1 < t_2 < t_3$ and non-negative integers n_1, n_2 and n_3 ($n_1 \leq n_2 \leq n_3$) calculate

$$P(N_{t_1} = n_1, N_{t_2} = n_2, N_{t_3} = n_3)$$

2. Let a and b be independent $\mathcal{N}(0,1)$ distributed random variables and define

$$X_t = r\cos(2\pi t + \theta)$$

where

$$r = \sqrt{a^2 + b^2} \text{ and } \theta = \arctan\frac{b}{a}.$$

Is (X_t) a WSS process or not?

3. Internet packets arrive at a router according to a Poisson process with intensity $\lambda(\lambda > 0)$. Find the variance of the time it takes for the first $n(n \geq 1)$ packets to arrive.

4. Let $(W_t), t \geq 0$ be a Wiener process. Find the density of $W_t^2, t > 0$.

5. Let $(X_t), t \in \mathbb{R}$ be a white noise process (mean zero Gaussian process with $R_X(\tau) = \delta_0(\tau)$). Define $Y_t := \int_0^t X_\tau d\tau$. For $s < t$, show that $E(Y_s Y_t) = s$.

6. Let $W_t), t \geq 0$ be a Wiener process. For a constant $c > 0$, define $V_t := \frac{1}{c} W_{c^2 t}$. Show that $(V_t), t \geq 0$ is again a Wiener process.

7. In a communication system, the carrier signal at the receiver is modeled by $X_t = \cos(2\pi vt + \theta), t \in \mathbb{R}$, where θ has uniform distribution $U[-\pi, \pi]$ and v is a parameter. Find the the covariance function $R_X(s,t)$.