ESE 520 Probability and Stochastic Processes

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EXAM 1 - Practice

Name (Please print):

Instructions:

1. You must show all work to completely justify your answers in order to receive any credit.
2. You can use One one-sided sheet of paper with your own formulas.

1. Let X and Y be two independent random variables, both uniformly distributed on the interval $[0, 1]$. Calculate $E|X - Y|$.

2. Let X and Y be two continuous random variables with joint pdf $f_{XY}(x, y)$. Let $U = XY$. Prove that f_U, the pdf of the random variable U, is given by

$$f_U(u) = \int_{-\infty}^{\infty} f_{XY}(\frac{u}{v}, v)|\frac{1}{v}|dv.$$

Hint: Use the transformation $(x, y) \rightarrow (u, v)$ defined as $u = xy, v = y$ to find first the joint pdf $f_{UV}(u, v)$ of the random vector (U, V).

3. Prove or disprove that a union of two σ - algebras is again a σ - algebra.

4. Suppose that X_1 and X_2 are two independent random variables each having an exponential distribution with parameter $\lambda = 2$. That is, their pdfs have the form $f(x_i) = 2e^{-2x_i}, x_i > 0, i = 1, 2$.

a) Find the pdf of Y defined as $Y = \max\{X_1, X_2\}$.

b) Find $P(Y > 1)$.

5. Let (X, Y) has the jointly continuous density

$$f(x,y) = \begin{cases} 8xy, & 0 < y < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

a) Verify that $f(x, y)$ is a correct density.

b) Find the conditional densities $f_{X|Y=y}(x, y)$ and $f_{Y|X=x}(x, y)$.

6. Show that if $A_1, A_2, ...$ is an *expanding* sequence of events, that is,

$$A_1 \subset A_2 \subset .....,$$

then

$$P(A_1 \cup A_2 \cup ....) = \lim_{n \to \infty} P(A_n).$$

7. Let X be a random variable with the standard normal distribution, That is, its pdf is given by $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, x \in \mathbb{R}$.

a) Show that $\int_{\mathbb{R}} f(x)dx = 1$.

b) Calculate $E(X^3)$.