ESE 520 Probability and Stochastic Processes

Exam3-Outline

1. Definition of a random process. Finite dimensional ditributions of a random process. Consistency properties. Equivalence of random processes. The Theorem of Kolmogorov about the existence of a stochastic process. σ-algebra of cyllindrical sets. The Theorem of Kolmogorov about the existence of continuous verison of a stochastic process.

2. The Poisson process $(N_t), t \geq 0$: its definition and properties. The probability distribution of the random variable N_t.

3. The Poisson process as the counting process. The relation between (N_t) and the corresponding sequence $(T_i), i = 0, 1, 2...$ of exponentially distributed random variables.

4. Gaussian processes: definition and properties. The role of mean and covariance functions. Properties of covariance function.

5. Wiener process (Brownian motion) as an example of an important Gaussian process. Its covariance function.

6. Equivalent characterization of the Wiener process as a process with independent increments. Know how to verify if a given process is a Wiener process as a process with stationary, independent, and normally distributed increments.

7. Path properties of the Wiener process.

8. WSS processes. Stationary WSS processes. Know how to verify if a given process is a WSS process.

9. Construction of "white noise" as an important example of a WSS process. Dirac delta function: its definiton and properties.

10. Spectral density $S(v)$ and the covariance function $R(\tau)$ as two important characteristics of a WSS process. The relation between them. Know how to find one characteristic given another.

11. The example of a linear representation model using "white noise" as the input process. The impulse-response and transfer functions.

~~12~~. Markov processes: definition, probability transition function, Chapman-Kolmogorov equations.

~~13~~. Homogeneous Markov processes.

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- Poisson process (N_t) $t \in [0, +\infty)$ $\longrightarrow$ 2 ×
- Wiener process (W_t), $t \in [0, +\infty)$ $\longrightarrow$ 2 ×
- Gaussian process $\longrightarrow$
- white noise $\longrightarrow$
- WSS-processes $\Longrightarrow$

2-3 ×

- LDS-applications (LN21) $\longrightarrow$ 1-2 ×

7 problems

$$\int_{-\infty}^{+\infty} R(\tau) e^{-2\pi i \nu \tau} d\tau$$

$$R(\tau) = \begin{cases} |\tau| & , |\tau|<1 \\ 0 & , |\tau| \geq 1 \end{cases}$$

$$\Rightarrow \int_{-1}^{1} |\tau| e^{-2\pi\nu\pi i} d\tau = 2\int_0^1 \tau \cdot \cos(2\pi\nu\tau)\, d\tau$$

$$= 2\left(\frac{i\sin(2\pi\nu\tau)}{2\pi\nu}\Big|_0^1 - \frac{1}{2\pi\nu}\int_0^1 \sin(2\pi\nu\tau)\, d\tau\right)$$

$$\downarrow$$

$$e^{-2\pi\nu\pi i} = \underbrace{\cos(2\pi\nu\tau)}_{\text{even}} - \underbrace{i\sin(2\pi\nu\tau)}_{\text{odd}}$$

$$= \frac{\sin(2\pi\nu)}{\pi\nu} + \frac{2}{(2\pi\nu)^2} \cos(2\pi\nu\tau)\Big|_0^1$$