ESE 520 Probability and Stochastic Processes

Instructor: Vladimir Kurenok

EXAM 3 - Practice

Name (Please print):

1. You must show all work to completely justify your answers in order to receive any credit.

2. You can use One one-sided sheet of paper with your own formulas.

1. Let $(W_t), t \geq 0$, be a one-dimensional Wiener process. For $t \geq 0$, define new process $Z_t := e^{W_t - \frac{1}{2}t}$. One can show that $E(Z_t) = 1$. Use this result to compute the covariance function of Z_t. Is this process WSS process?

2. Let $(W_t), t \geq 0$, be a one-dimensional Wiener process. For a positive constant α define a new stochastic process

$$X_t := \alpha W_{\frac{t}{\alpha^2}}.$$

Prove that X_t is also a Wiener process.

3. Space shuttles are lunched according to a Poisson process. The average time between lunches is 2 months.

a) Find the probability that there are no lunches during a 4 month period.

b) Find the probability that during at least 1 month out of three consecutive months, there are at least two lunches.

4. In a linear dynamical system with input process (X_t) being white noise, the transfer function is given by

$$H(\nu) = 1 - |\nu|^2, |\nu| \leq 1,$$

(and equal to zero for $|\nu| > 1$).

Find $E(Y^2)$ (output power of the system), where (Y_t) is the output process.

5. In a communication system, the carrier signal at the receiver is modeled by $X_t = \cos(2\pi v t + \theta), t \in \mathbb{R}$, where θ has uniform distribution $U[-\pi, \pi]$ and v is a parameter. Find the the covariance function $R_X(s,t)$.

6. Let (W_t) be a Wiener process. For $\lambda > 0$, define

$$Y_t := \frac{e^{-\lambda t}}{\sqrt{\lambda}} W_{e^{2\lambda t}}.$$

Is Y_t a WSS process?

7. Let $(W_t), t \geq 0$ be a Wiener process. For $0 < s < t$, calculate $E\Big((W_t - W_s)^4 W_t^2\Big)$.