ESE 520 Probability and Stochastic Processes

HW 6: due by 11pm CST on Tuesday, December 3, 2024

Total: 40 points

Pr. 1. [5pt] Let (X_t) be a zero-mean, WSS process with correlation function $R(\tau)$. Let $Y_t := X_t \cos(2\pi f t + \Theta)$, where Θ has $U(-\pi, \pi)$ distribution and Θ is independent of the process (X_t).

a) Find the correlation function of (Y_t);

b) Find the cross-correlation function of (X_t) and (Y_t);

c) Is (Y_t) WSS?

Pr. 2 [5p.] If a WSS process (X_t) has correlation function $R_X(\tau) = \frac{1}{1+\tau^2}$, find $S_X(f)$.

Pr. 3 [5p.] Find the correlation function corresponding to each of the following spectral densities (τ_0 is a fixed value):

a) $\delta(\tau)$;

b) $\delta(\tau - \tau_0) + \delta(\tau + \tau_0)$;

c) $e^{-\tau^2/2}$;

d) $e^{-|\tau|}$.

Pr. 4 [5p.] Let (W_t) be a Wiener process, and let

$$Y_t := \frac{e^{-\lambda t}}{\sqrt{2\lambda}} W_{e^{2\lambda t}}.$$

Show that

$$R_Y(t_1, t_2) = \frac{1}{2\lambda} e^{-\lambda |t_1 - t_2|}.$$

Pr. 5 [5p.] Let (W_t) be a Wiener process with $E[W_t^2] = t$. Let $Y_t := e^{W_t}$. Find the correlation function $R_Y(t_1, t_2) := E[Y_{t_1} Y_{t_2}]$ for $t_2 > t_1$.

Pr. 6 [5p.] Let (W_t) be a Wiener process, i.e. $W_0 = 0$, it has independent increments and $W_t - W_s$ has $\mathcal{N}(0, t-s)$ distribution, where $t > s$. Show that (W_t) is a Gaussian process.

Pr. 7 [5p.] White noise (with spectral density equal to 1) is passed through a linear dynamical system with impulse response function $h(t) = \frac{1}{1+t^2}$. If (Y_t) denotes the output of the system, find $E[Y_{t+1/2} Y_t]$.

Pr. 8 [5p.] Let $P = (p_{ji}), i, j \in S$ be a stochastic matrix, where S is a finite or a countable set. Show that any natural power P^n of P is a stochastic matrix as well. Recall: P is called a stochastic matrix if and only if $p_{ji} \geq 0$ for all $i, j \in S$ and the sum of entries in each column is 1.