

# ESE 520 Probability and Stochastic Processes

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## **EXAM 2-Sample**

Name (Please print):

Instructions:

1. You must show all work to completely justify your answers in order to receive any credit.
2. You can use One one-sided sheet of paper with your own formulas.

1. a) Assume  $X$  has a normal distribution with mean  $\mu$  and variance  $\sigma^2$ . Find the moment-generating function of  $X$ .

b) Use part a) to find  $E(X)$ ;

2. Let  $X$  and  $Y$  be two random variables having jointly continuous distribution with the joint pdf given as

$$f(x, y) = \begin{cases} 8xy, & 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the best linear mean-square estimate of  $Y$  by  $X$ .

3. Suppose  $X$  and  $Y$  have joint density  $e^{-x}$  for  $0 < y < x$ . Find  $E(X|Y)$  and  $Var(X|Y)$ .

4. Suppose you play roulette and bet \$1 on black each time. Your net winnings on the  $i$ th play,  $X_i$ , are 1 with probability  $1/3$  and -1 with probability  $2/3$ . What is the probability your net winnings are  $\geq 0$  after 100 plays?

Use the Central Limit Theorem and the enclosed normal table to find the approximate probability.

5. Suppose  $U, V, W$  are independent and have mean  $\mu$  and variance  $\sigma^2$ . Find  $\rho(U + V, V + W)$ .

6. Suppose  $X$  and  $Y$  are independent exponential random variables with the same parameter  $\lambda > 0$ . Find the density of  $\min\{X, Y\}$ .

7. Let  $X_1, X_2$  be independent random variables each of which has a normal distribution  $\mathcal{N}(\mu_1, \sigma_1^2)$  and  $\mathcal{N}(\mu_2, \sigma_2^2)$ , respectively. Define the random variable  $Y := X_1 + X_2$ . Using characteristic functions, show that  $Y$  also has a normal distribution  $\mathcal{N}(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ . Hint: The characteristic function of a normal rv with mean  $\mu$  and variance  $\sigma^2$  has the form  $\phi(t) = e^{i\mu t - \frac{1}{2}\sigma^2 t^2}$ .