ESE 520 Probability and Stochastic Processes

Instructor: Vladimir Kurenok

# EXAM 3 - Practice

Name (Please print):

**Total: 70 points** (each problem is 10 points worth)

Instructions:

1. You must show all work to completely justify your answers in order to receive any credit.
2. You can use One one-sided sheet of paper with your own formulas.

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1. Let $(X_t), t \in \mathbb{R}$ be a WSS process with covariance function $R(\tau) = e^{-\tau^2}$. Find the corresponding spectral density $S_X(v)$.

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2. Let $a$ and $b$ be independent $\mathcal{N}(0,1)$ distributed random variables and define

$$X_t = r\cos(2\pi t + \theta)$$

where

$$r = \sqrt{a^2+b^2} \text{ and } \theta = \arctan\frac{b}{a}.$$

Is $(X_t)$ a WSS process or not?

Hint: You might want to use the identity $\cos(\alpha+\beta) = \cos\alpha\sin\beta - \sin\alpha\cos\beta$.

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3. Let $(W_t, V_t), t \geq 0$ be a 2-dimensional Wiener process. That is, both $W$ and $V$ are 1-dimensional independent Wiener processes. Calculate $P(W_t^2 + V_t^2 < 1)$.

Hint: You might want to use polar coordinates.

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4. Let $(N_t)$ be a Poisson process with parameter $\lambda > 0$. For non-negative integers $a, b$ and $c$ ($a \leq b \leq c$), calculate

$$P(N_1 = a, N_2 = b, N_3 = c)$$

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5. Let $(W_t), t \geq 0$ be a Wiener process. Consider a random vector $(W_1, W_2)$. Write down the joint distribution function of this vector. In other words, find the joint density $f_{W_1,W_2}(x, y)$.

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6. Let $(W_t), t \geq 0$ be a Wiener process. For $0 < s < t$, calculate $E\Big((W_t - W_s)^4 W_t^2\Big)$.

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7. Let $(X_t)$ be a white noise as the input process in a linear dynamical system (we considered in lecture) and the output process $Y$ is defined as $Y_t = \int_0^t X_s ds$ (integrated white noise). *Formally*, calculate the mean and covariance functions of $Y$.

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