ESE 520 Probability and Stochastic Processes

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EXAM 3 - Sample

Name (Please print):

Total: 70 points

Instructions:

1. You must show all work to completely justify your answers in order to receive any credit.
2. You can use One one-sided sheet of paper with your own formulas.

1. Let $(N_t), t \geq 0$ be a Poisson process with parameter $\lambda > 0$. For any $s, t \geq 0$, compute the covariance function

$$cov(N_s, N_t) = E[(N_s - E(N_s))(N_t - E(N_t))].$$

Hint: You can use the fact that $E(N_t) = t\lambda$ and $Var(N_t) = \lambda t$ for any $t \geq 0$.

2. Let $(N_t), t \geq 0$ be a Poisson process with parameter $\lambda > 0$ and $(W_t), t \geq 0$ be a Wiener process. Assume that they are *independent* from each other. Calculate

$$E(e^{N_1+W_1}).$$

3. Let W_t be a 2-dimensional Wiener process and define $D := \{x \in \mathbb{R}^2 : |x| < 1\}$. Compute

$$P(W_t \in D).$$

4. A WSS process (X_t) is the input process in the linear system (discussed in lecture) with the transfer function $H(\nu)$ defined as

$$H(\nu) = \begin{cases} \sqrt{|\nu|}, & -1 \leq \nu \leq 1 \\ 0, & \nu \notin [-1, 1]. \end{cases}$$

Let Y be the output process in the system where $S_X(\nu) = \frac{e^{-\nu^2/2}}{\sqrt{2\pi}}$. Find $E[Y_t^2]$, the expected output power of the system.

Hint: $E(Y_t^2) = R_Y(0)$.

5. Let (W_t) be a Wiener process. For $\lambda > 0$, define

$$Y_t := \frac{e^{-\lambda t}}{\sqrt{\lambda}} W_{e^{2\lambda t}}.$$

Is Y_t a WSS process?

6. Consider the process $X_t := \cos(t + \theta)$ where θ is a random variable uniformly distributed on the interval $[-\pi, \pi]$. Show that for any function $g(x)$, the vaule $E[g(X_t)]$ does not depend on t.

7. A sports clothing store sells football yerseys with a certain very popular number on them according to a Poisson process of rate three crates per day. Find the probability that on 3 days in a row, the store sells at least two crates each day.