ESE 520 Probability and Stochastic Processes

Exam1-Outline

1. Probability model $(\Omega, \mathcal{F}, \mathbf{P})$ of a random experiment with countably many outcomes. The construction of probability $\mathbf{P}$ in this case and the choice of $\mathcal{F}$.

2. Probability model $(\Omega, \mathcal{F}, \mathbf{P})$ of a random experiment with uncountably many outcomes. The choice of $\mathcal{F}$ in this case. σ-algebras of events. The general definition of probability $\mathbf{P}$ (due to A. N. Kolmogorov) on a general Ω.

3. Elementary properties of probability function $\mathbf{P}$ including the complement rule, addition formula etc. Continuity of $\mathbf{P}$ from "below" and "above".

4. Conditional probability. Bayes' formula and formula of total probability. Independent events. Independent σ-algebras.

5. Borel σ-algebras on $\mathbb{R}$. Some examples of probability measures on $\Omega = \mathbb{R}$. Cumulative distribution functions and probability density functions as realvalued functions defined on $\mathbb{R}$.

6. A random variable X as a measurable function $X : (\Omega, \mathcal{F}, \mathbf{P}) \to (\mathbb{R}, \mathcal{B}(\mathbb{R}))$. Discrete and continuous random variables and their probability distributions. Examples of classical discrete and continuous one-dimensional random variables (binomial, Poisson, geometric, uniform, exponential, normal, gamma, Rayleigh).

7. Random vectors and their probability distributions. Joint cdf's and joint pdf's of a random vector and their properties. Independent random variables and their characterization in terms of their joint cdf and/or pdf. Jointly Gaussian random vector and its pdf.

8. Transformations of random vectors and the formula for the pdf of a transformed continuous random vector. Conditional densities. Marginal densities.

9. Expectation of a random variable and its elementary properties as a linear operator including the expectation of the product of independent random variables.