

# DSI First Day (Baseline) Assessment: Math & Statistics

## 1. Multivariate calculus

What is the partial derivative of  $y$  with respect to  $x$  ?  
$$y = x \sin(z) e^{-x}$$

## 2. Vectors and matrices

Consider the matrix  $\mathbf{X}$  and the vector  $\mathbf{y}$  below

$$\mathbf{X} = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

What is the product  $\mathbf{Xy}$  ?

Is  $\mathbf{X}$  invertible? If so, give the inverse, if not explain why not.

What is the rank of  $\mathbf{X}$  ?

## 3. Probability and statistics

Consider a sample of data  $S$  obtained by flipping a coin  $x$ , where 0 denotes the coin turned up heads, and 1 denotes that it turned up tails.

$$S = \{1, 1, 0, 1, 0\}$$

What is the sample mean for this data ?

What is the sample variance ?

What is the probability of observing this data assuming that a coin with an equal probability of heads and tails was used (i.e., by the probability distribution  $p(x=1)=0.5$ ,  $p(x=0)=0.5$ ).

Note the probability of this data sample would be greater if the value of  $p(x=1)$  was not 0.5, but some other value. What is the value that maximizes the probability of sample  $S$ ? [optional: can you prove your answer is correct?]

Given the following joint distribution between  $x$  and  $y$ , what is  $P(x=T | y=b)$ ?

$P(x,y)$	$y$	a	b	c
$x$				
T		0.2	0.1	0.2
F		0.05	0.15	0.3