Complexity Theory

TM-Halt — in RE

SAT-FO-String — in R

Input: A first-order formula φ for strings
Decision: Is φ satisfiable?

2-PlayerCorridorTiling — EXP-C

Input: Tilings T and integer k (unary)
Decision: Does player one has a winning strategy in the tiling game?

QBF — PSPACE-C

Input: Quantified Boolean formula φ (in prenex normal form and CNF)
Decision: Is φ satisfiable?

CorridorTiling — PSPACE-C

Input: Tilings T and integer k (unary)
Decision: Is there a $k \times l$ tiling for T?

SAT-UNSAT — DP-C

Input: Boolean formulas φ_1, φ_2 in CNF
Decision: Is φ_1 satisfiable and φ_2 unsatisfiable?

SAT — NP-C

Input: Boolean formula φ in CNF
Decision: Is φ satisfiable?

Tiling — NP-C

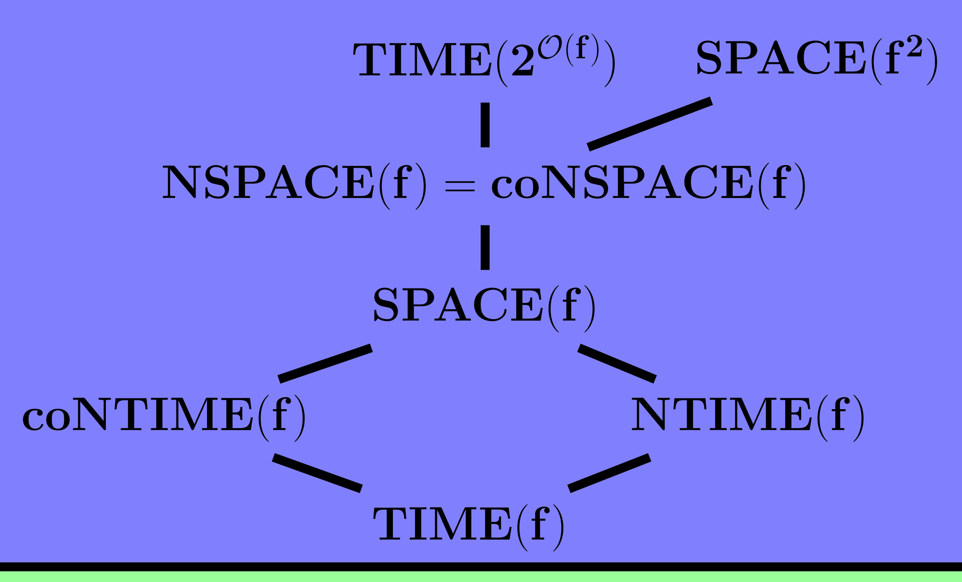
Input: Tilings T und integers c, r (unary)
Decision: Is there a $c \times r$ tiling for T?

TSP — NP-C

Input: Weighted Graph G and $k \in \mathbb{N}$
Decision: Is there a circle in G with a weight smaller or equal to k that visits all nodes?

Small Hierarchy

For $f(n) \geq \log(n)$



HornSAT — P-C

Input: Finite set of Horn-clauses K
Decision: Is K satisfiable?

Determinant — in $\mathbf{NC^2}$

AcyclicCS — LOGCFL-C

Input: Variables V, domain U, constraints C, such that (V, C) is acyclic
Decision: Is there a solution $\sigma \subseteq V \times U$ such that $\sigma(\mathcal{C})$ is true?

REACH — NL-C

Input: Graph $G = (V, E)$ and $s, t \in V$
Decision: Is there a path from s to t in G?

UREACH — L-C

Input: Undirected graph $G=(V,E)$ and $s, t \in V$
Decision: Is there a path from s to t in G?

FormulaValueProblem — $\mathbf{NC^1}$-C

Input: Variable free Boolean formula B
Decision: Does B evaluate to true?

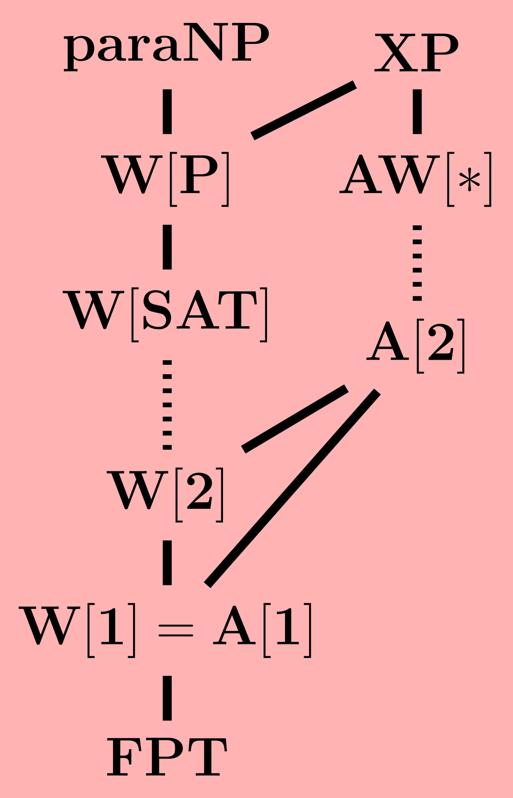
Multiplication — $\mathbf{TC^0}$-C

Addition — in $\mathbf{AC^0}$

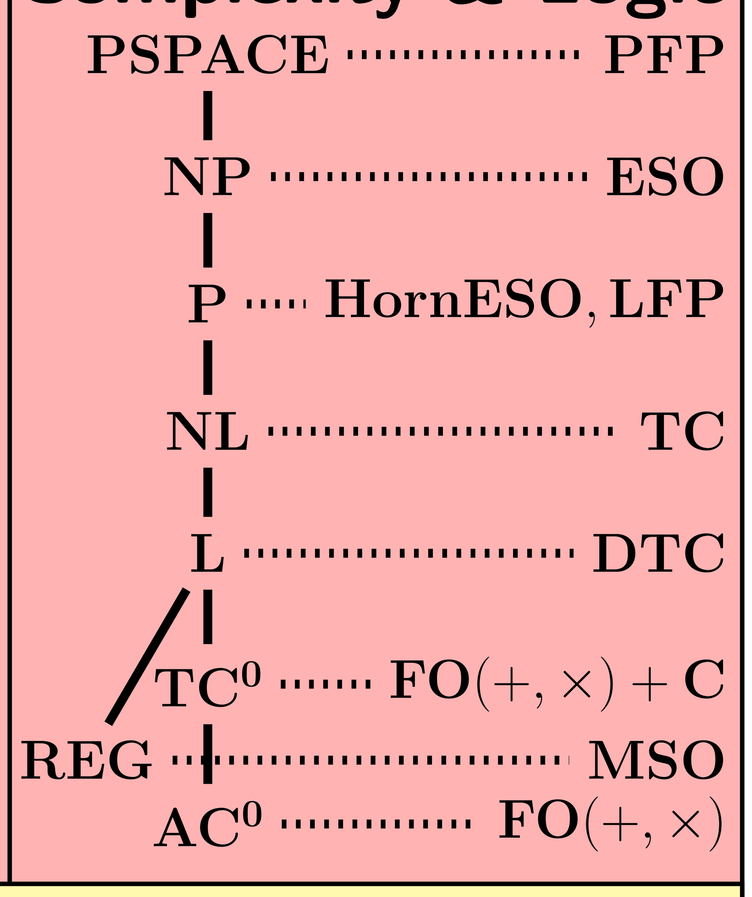
Parameterized Problems

A tuple (Q, κ) is a parameterized problem, if $Q \subseteq \Sigma^*$ and $\kappa : \Sigma^* \to \mathbb{N}$ is computable in polynomial time.

Descriptive Complexity & Parametrization



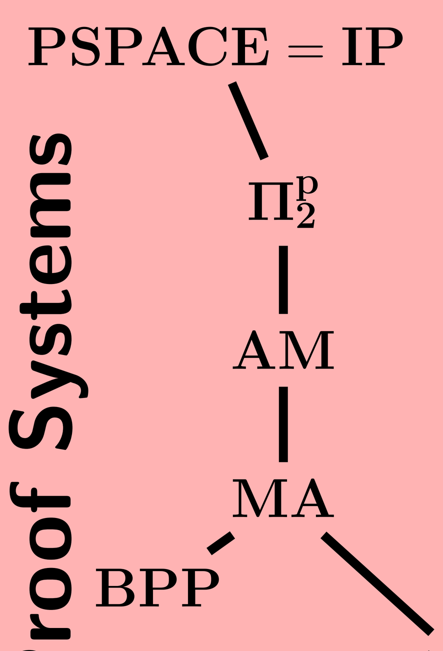
Complexity & Logic



Complexity Classes and Logic

A logic $\mathcal{L}$ has complexity $\mathcal{C}$, if for each sentence $\varphi \in \mathcal{L}$ it holds $\{ \text{enc}(\mathfrak{A}) \mid \mathfrak{A} \models \varphi \} \in \mathcal{C}$.

Proof Systems



$\mathbf{NP} = \mathbf{PCP}(\log, \text{poly}) = \mathbf{PCP}(0, \text{poly}) = \mathbf{PCP}(\log, 1)$

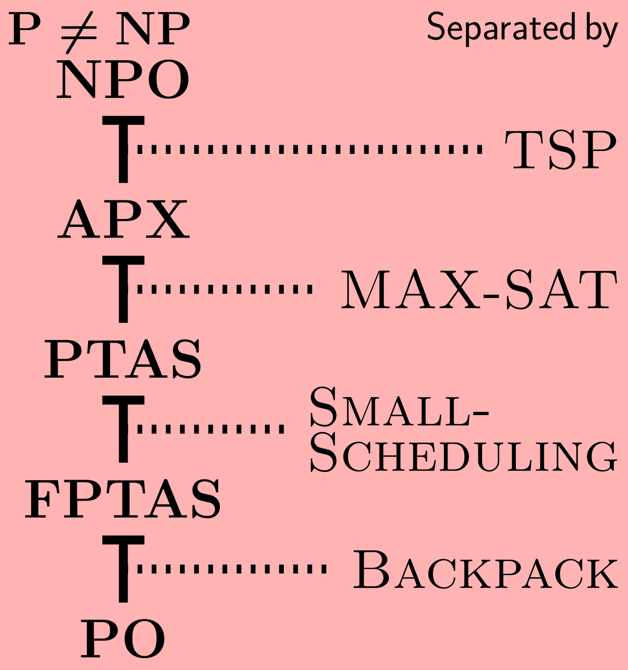
PCP *(probabilistically checkable proof)*

$PCP(\mathcal{F}, \mathcal{G})$ is the class of languages L such there is a verifier V that

- uses $\leq r(n)$ random bits, $(r \in \mathcal{F})$
- reads $\leq q(n)$ bits of the proof, $(q \in \mathcal{G})$
- accepts for a proof π everything from L and does not accept any $x \notin L$ with probability $> \frac{1}{2}$.

Optimization

For P ≠ NP — Separated by



Class operators

For every $\mathcal{C}$ which is closed under Turing-reduction holds:

$\mathbf{BP} \cdot \mathbf{BP} \cdot \mathcal{C} \subseteq \mathbf{BP} \cdot \mathcal{C}$
$\oplus \cdot \oplus \cdot \mathcal{C} \subseteq \oplus\mathcal{C}$
$\oplus \cdot \mathbf{BP} \cdot \mathcal{C} \subseteq \mathbf{BP} \cdot \oplus \cdot \mathcal{C}$
$\exists \cdot \mathcal{C} \subseteq \mathbf{BP} \cdot \oplus \cdot \mathcal{C}$
$\exists \cdot \mathbf{BP} \cdot \mathcal{C} \subseteq \mathbf{BP} \cdot \exists \cdot \mathcal{C}$
$\exists \cdot \mathbf{BP} \cdot \mathbf{P} = \mathbf{MA}$
$\mathbf{BP} \cdot \exists \cdot \mathbf{P} = \mathbf{AM}$

Theorem — Chandra, Kozen, Stockmeyer

(a) For $S(n) = \Omega(\log n)$ holds:
$\mathbf{ASPACE(S)} \subseteq \mathbf{TIME}(2^{\mathcal{O}(S)})$

(b) For $T(n) = \Omega(n)$ holds:
$\mathbf{TIME(T)} \subseteq \mathbf{ASPACE}(\log \mathbf{T})$

Immerman–Szelepcsényi theorem

For space constructible S with $S(n) \geq \log n$ holds:

$$\mathbf{NSPACE(S)} = \mathbf{coNSPACE(S)}$$

Savitch's theorem — Savitch

For space constructible S with $S(n) \geq \log n$ holds:

$$\mathbf{NSPACE(S)} \subseteq \mathbf{SPACE}(\mathbf{S}^2)$$

Space hierarchy theorem — Stearns, Szepietowski

For time constructible f, g with $f(n) \in \Omega(\log n)$ and $g(n) \in \omega(f(n))$ holds:

$$\mathbf{SPACE(f)} \subsetneq \mathbf{SPACE(g)}$$
$$\mathbf{NSPACE(f)} \subsetneq \mathbf{NSPACE(g)}$$

Time hierarchy theorem — Hartmanis, Stearns

For time constructible f, g with $f(n) \in \Omega(n)$ and $g(n) \in \omega(f(n) \cdot \log(f(n)))$ holds:

$$\mathbf{TIME(f)} \subsetneq \mathbf{TIME(g)}$$

FernUniversität in Hagen
AIG
Poster by Dr. Kai Sauerwald (Artificial Intelligence Group, FernUniversität in Hagen) – https://kai-sauerwald.de
based on a lecture by Prof. Dr. Schwentick (TU Dortmund University)