Complexity Theory

Poster by Dr. Kai Sauerwald (Artificial Intelligence Group, FernUniversität in Hagen); based on a lecture by Prof. Dr. Schwentick (LogiDAC, TU Dortmund University).

TM-Halt — in $\mathbf{RE}$

SAT-FO-String — in $\mathbf{R}$

Input: A first-order formula φ for strings
Decision: Is φ satisfiable?

2-PlayerCorridorTiling — $\mathbf{EXP}$-C

Input: Tilings T and integer k (unary)
Decision: Does player one has a winning strategy in the tiling game?

QBF — $\mathbf{PSPACE}$-C

Input: Quantified Boolean formula φ (in prenex normal form and CNF)
Decision: Is φ satisfiable?

CorridorTiling — $\mathbf{PSPACE}$-C

Input: Tilings T and integer k (unary)
Decision: Is there a $k \times l$ tiling for T?

SAT-UNSAT — $\mathbf{DP}$-C

Input: Boolean formulas φ_1, φ_2 in CNF
Decision: Is φ_1 satisfiable and φ_2 unsatisfiable?

SAT — $\mathbf{NP}$-C

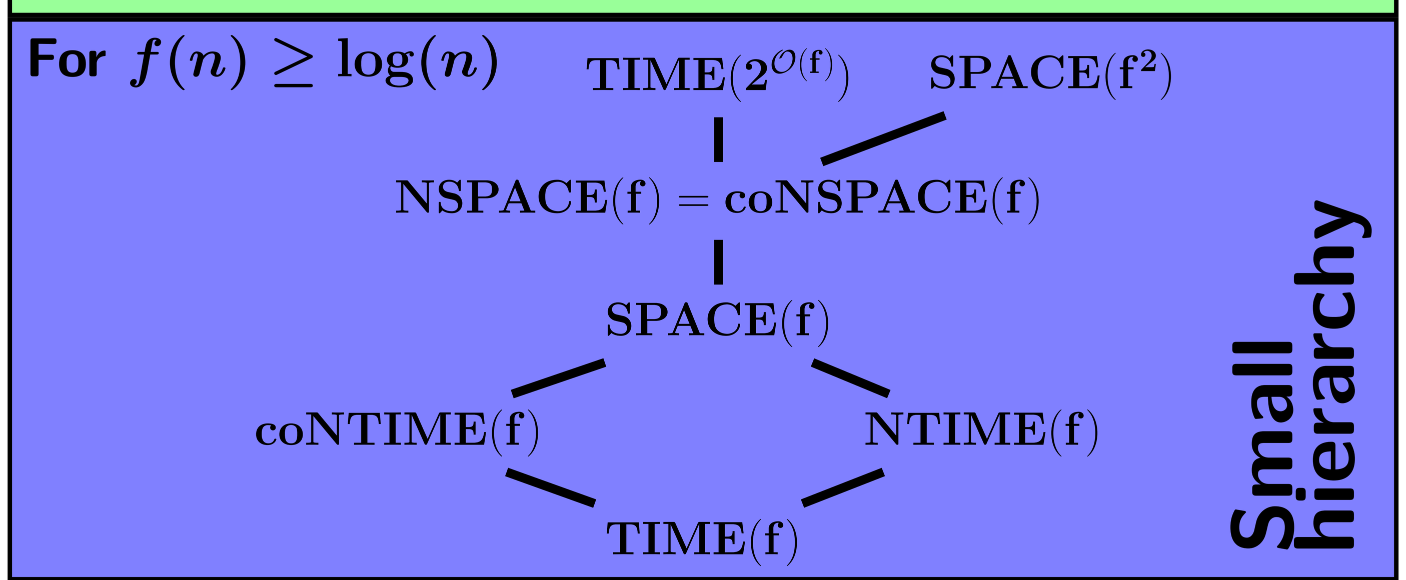
Input: Boolean formula φ in CNF
Decision: Is φ satisfiable?

Tiling — $\mathbf{NP}$-C

Input: Tilings T und integers c, r (unary)
Decision: Is there a $c \times r$ tiling for T?

TSP — $\mathbf{NP}$-C

Input: Weighted Graph G and $k \in \mathbb{N}$
Decision: Is there a circle in G with a weight smaller or equal to k that visits all nodes?



HornSAT — $\mathbf{P}$-C

Input: Finite set of Horn-clauses K
Decision: Is K satisfiable?

Determinant — in $\mathbf{NC}^2$

AcyclicCS — $\mathbf{LOGCFL}$-C

Input: Variables V, domain U, constraints C, such that (V, C) is acyclic
Decision: Is there a solution $\sigma \subseteq V \times U$ such that $\sigma(\mathcal{C})$ is true?

REACH — $\mathbf{NL}$-C

Input: Graph $G = (V, E)$ and $s, t \in V$
Decision: Is there a path from s to t in G?

UREACH — $\mathbf{L}$-C

Input: Undirected graph $G{=}(V, E)$ and $s, t \in V$
Decision: Is there a path from s to t in G?

FormulaValueProblem — $\mathbf{NC}^1$-C

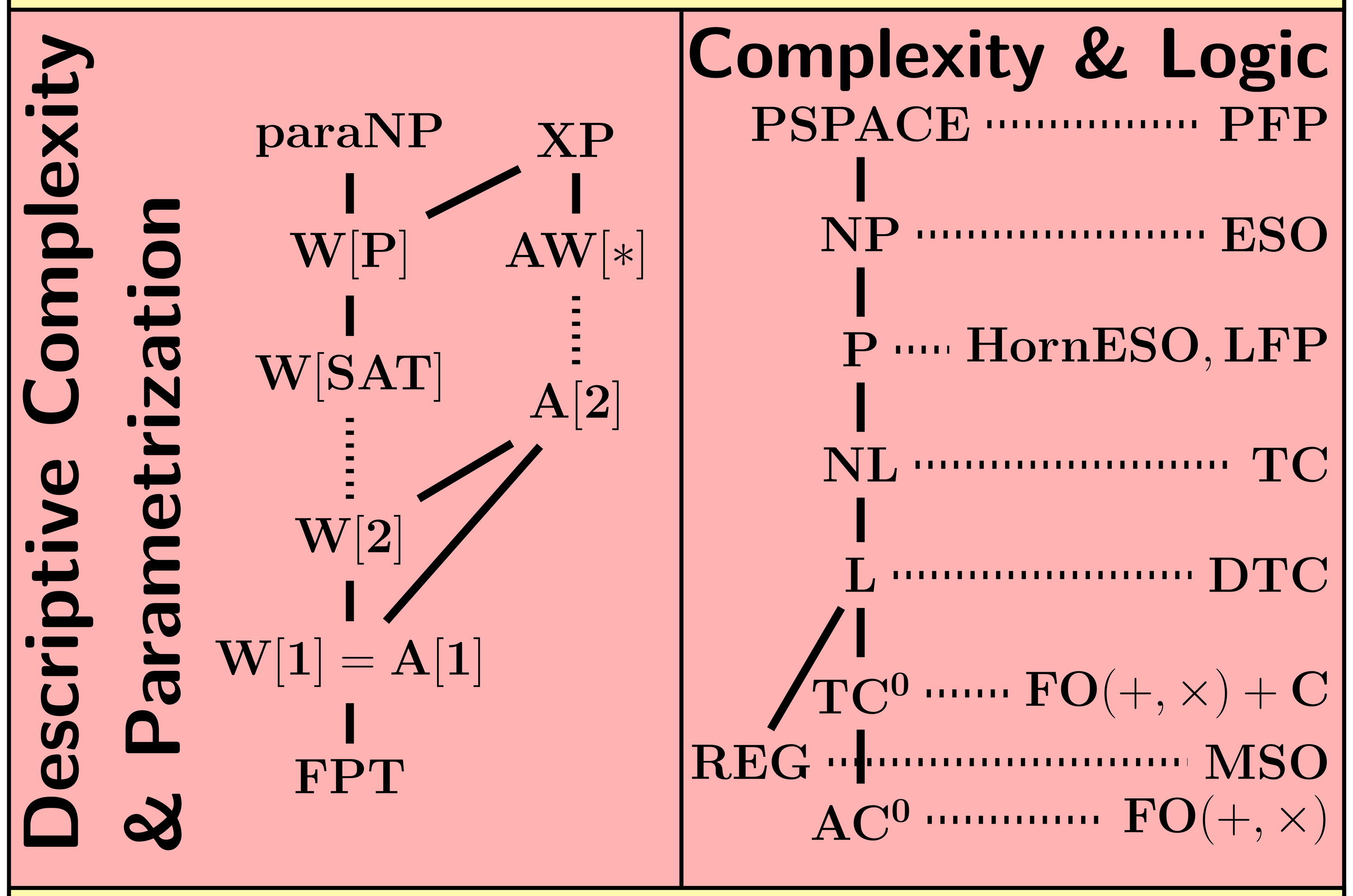
Input: Variable free Boolean formula B
Decision: Does B evaluate to true?

Multiplikation — $\mathbf{TC}^0$-C

Addition — in $\mathbf{AC}^0$

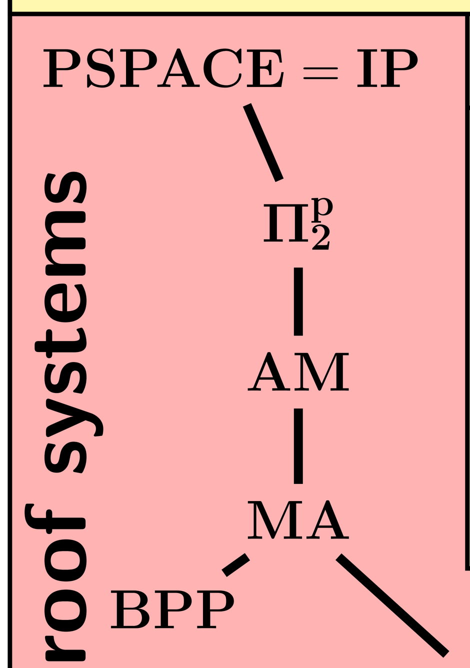
Parameterized Problems

A tuple (Q, κ) is a parameterized problem, if $Q \subseteq \Sigma^*$ and $\kappa : \Sigma^* \to \mathbb{N}$ is computable in polynomial time.



Complexity Classes and Logic

A logic $\mathcal{L}$ has complexity $\mathcal{C}$, if for each sentence $\varphi \in \mathcal{L}$ it holds $\{ \mathrm{enc}(\mathfrak{A}) \mid \mathfrak{A} \models \varphi \} \in \mathcal{C}$.

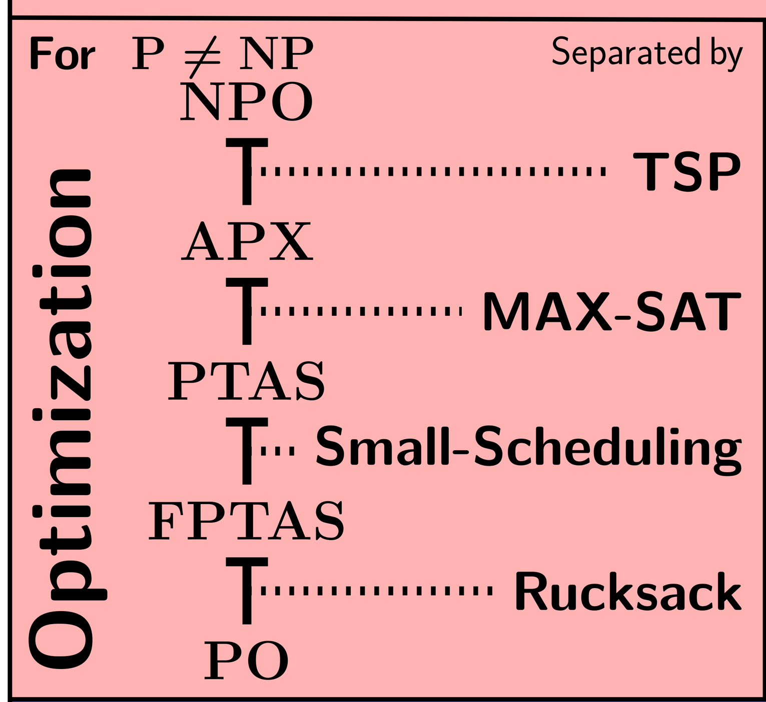


PCP *(probabilistically checkable proof)*

$PCP(\mathcal{F}, \mathcal{G})$ is the class of languages L such there is a verifier V that

- uses $\leq r(n)$ random bits, $(r \in \mathcal{F})$
- reads $\leq q(n)$ bits of the proof, $(q \in \mathcal{G})$
- accepts for a proof π everything from L and does not accept any $x \notin L$ with probability $> \frac{1}{2}$.

$\mathbf{NP} = \mathbf{PCP}(\log, \mathrm{poly}) = \mathbf{PCP}(0, \mathrm{poly}) = \mathbf{PCP}(\log, 1)$



Satz: Klassenoperatoren

Sei $\mathcal{C}$ unter Turing-Reduktion abgeschlossen. Dann gilt:

$\mathbf{BP} \cdot \mathbf{BP} \cdot \mathcal{C} \subseteq \mathbf{BP} \cdot \mathcal{C}$
$\oplus \cdot \oplus \cdot \mathcal{C} \subseteq \oplus\mathcal{C}$
$\oplus \cdot \mathbf{BP} \cdot \mathcal{C} \subseteq \mathbf{BP} \cdot \oplus \cdot \mathcal{C}$
$\exists \cdot \mathcal{C} \subseteq \mathbf{BP} \cdot \oplus \cdot \mathcal{C}$
$\exists \cdot \mathbf{BP} \cdot \mathcal{C} \subseteq \mathbf{BP} \cdot \exists \cdot \mathcal{C}$
$\exists \cdot \mathbf{BP} \cdot \mathbf{P} = \mathbf{MA}$
$\mathbf{BP} \cdot \exists \cdot \mathbf{P} = \mathbf{AM}$

Theorem — Chandra, Kozen, Stockmeyer

(a) Ist $S(n) = \Omega(\log n)$, so gilt
$$\mathbf{ASPACE}(\mathbf{S}) \subseteq \mathbf{TIME}(2^{\mathcal{O}(\mathbf{S})})$$
(b) Ist $T(n) = \Omega(n)$, so gilt
$$\mathbf{TIME}(\mathbf{T}) \subseteq \mathbf{ASPACE}(\log \mathbf{T})$$

Immerman–Szelepcsényi theorem

For space constructible S with $S(n) \geq \log n$ holds:
$$\mathbf{NSPACE}(\mathbf{S}) = \mathbf{coNSPACE}(\mathbf{S})$$

Savitch's theorem — Savitch

For space constructible S with $S(n) \geq \log n$ holds:
$$\mathbf{NSPACE}(\mathbf{S}) \subseteq \mathbf{SPACE}(\mathbf{S}^2)$$

Space hierarchy theorem — Stearns, Szepietowski

For time constructible f, g with $f(n) \in \Omega(\log n)$ and $g(n) \in \omega(f(n))$ holds:
$$\mathbf{SPACE}(\mathbf{f}) \subsetneq \mathbf{SPACE}(\mathbf{g})$$
$$\mathbf{NSPACE}(\mathbf{f}) \subsetneq \mathbf{NSPACE}(\mathbf{g})$$

Time hierarchy theorem — Hartmanis, Stearns

For time constructible f, g with $f(n) \in \Omega(n)$ and $g(n) \in \omega(f(n) \cdot \log(f(n)))$ holds:
$$\mathbf{TIME}(\mathbf{f}) \subsetneq \mathbf{TIME}(\mathbf{g})$$