Nambu-Goldstone mode for supersymmetry breaking in QCD and Bose-Fermi cold atom system at BEC phase

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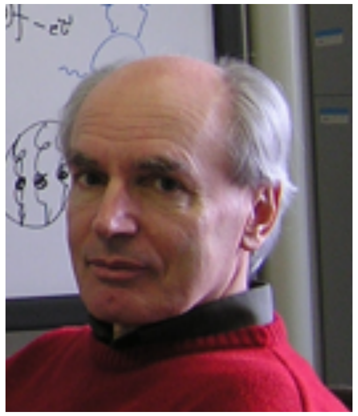
QCD: Y. Hidaka, DS, and T. Kunihiro, Nucl. Phys. A **876**, 93 (2012).
DS, Phys. Rev. D **87**, 096011 (2013).
Cold atom: J-P. Blaizot, Y. Hidaka, DS, Phys.Rev. A 92, 063629 (2015).
J-P. Blaizot, Y. Hidaka, DS, arXiv:1707.05634 [cond-mat.quant-gas].

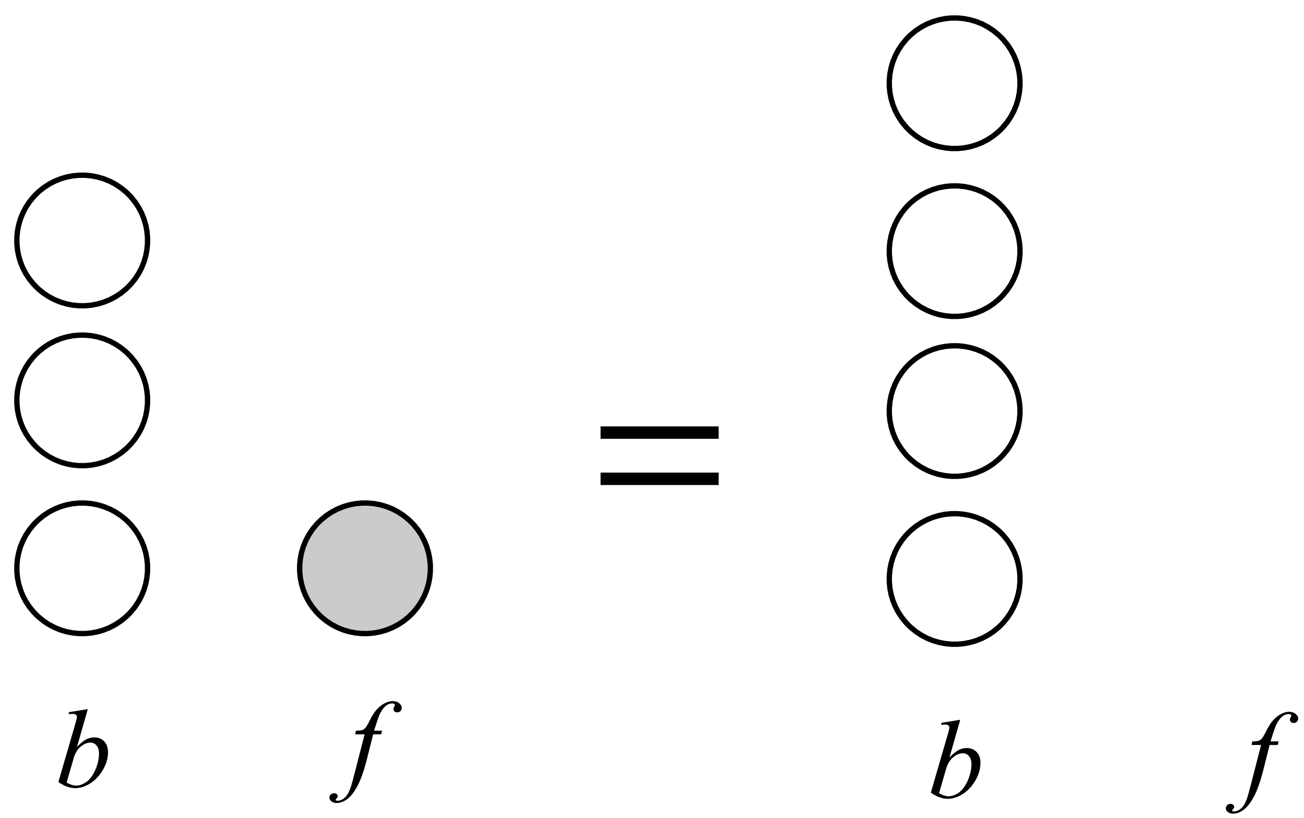


Outline

- Introduction (Supersymmetry, Nambu-Goldstone mode for SUSY-breaking)
- Relativistic system (Wess-Zumino model, QCD)
- Cold atom system (No BEC)
- Cold atom system (BEC) ←**Main topic**
- Summary

Supersymmetry (SUSY)

Symmetry related to interchanging a boson and a fermion

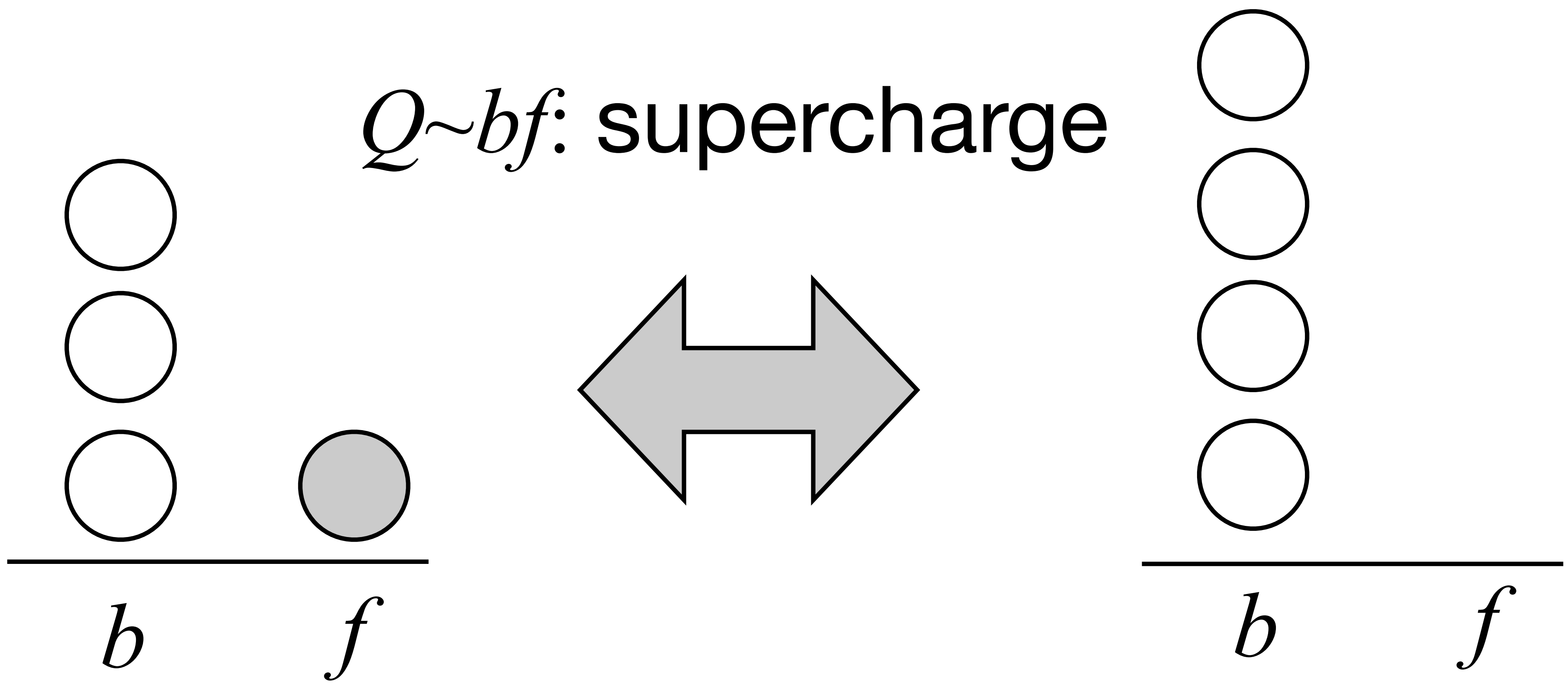


Same energy

SUSY and supercharge

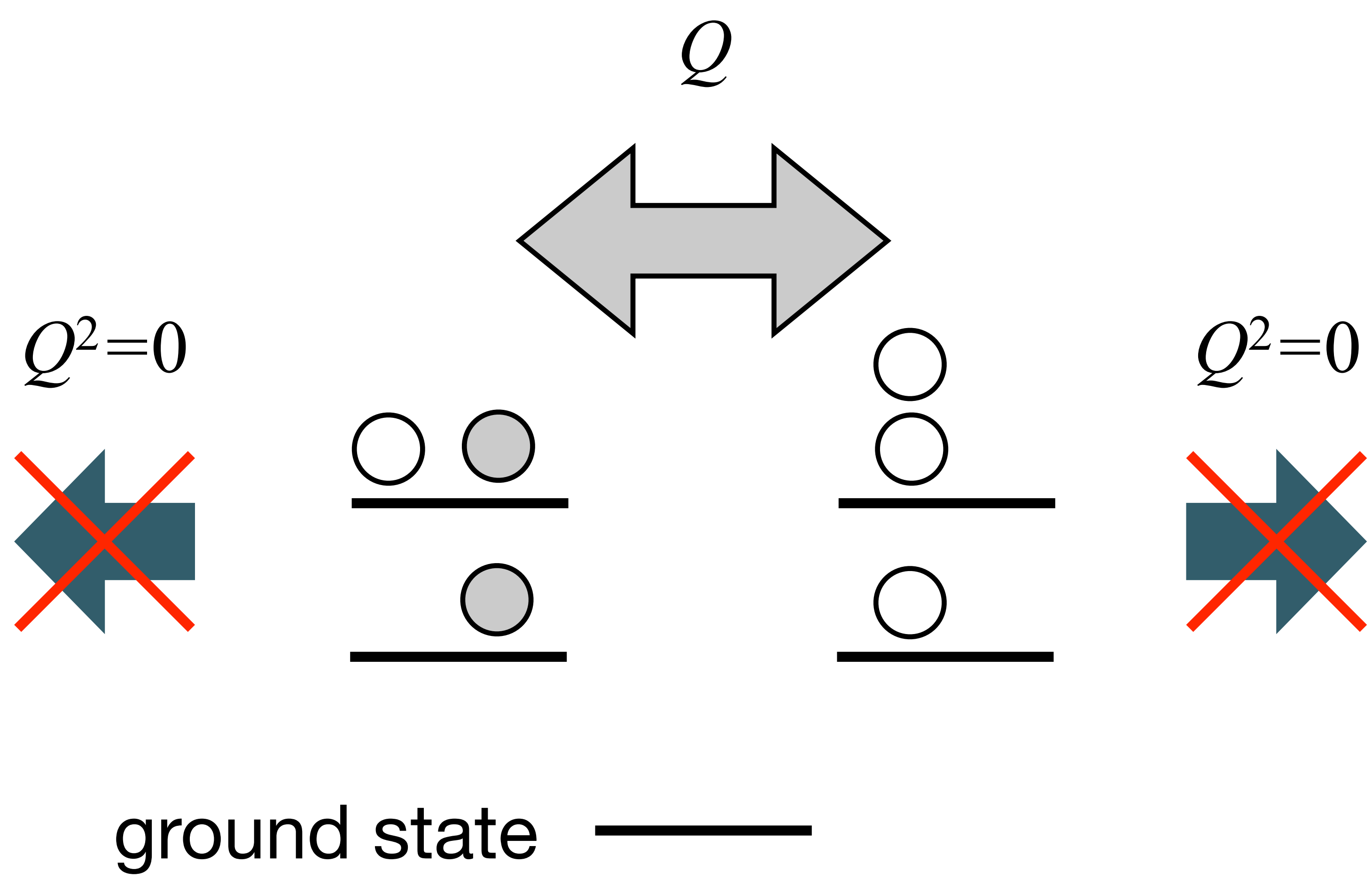
Mathematically, $[Q, H]=0$

$Q \sim bf$: supercharge



Supercharge operator: annihilate one fermion and create one boson (and its inverse process)

SUSY breaking

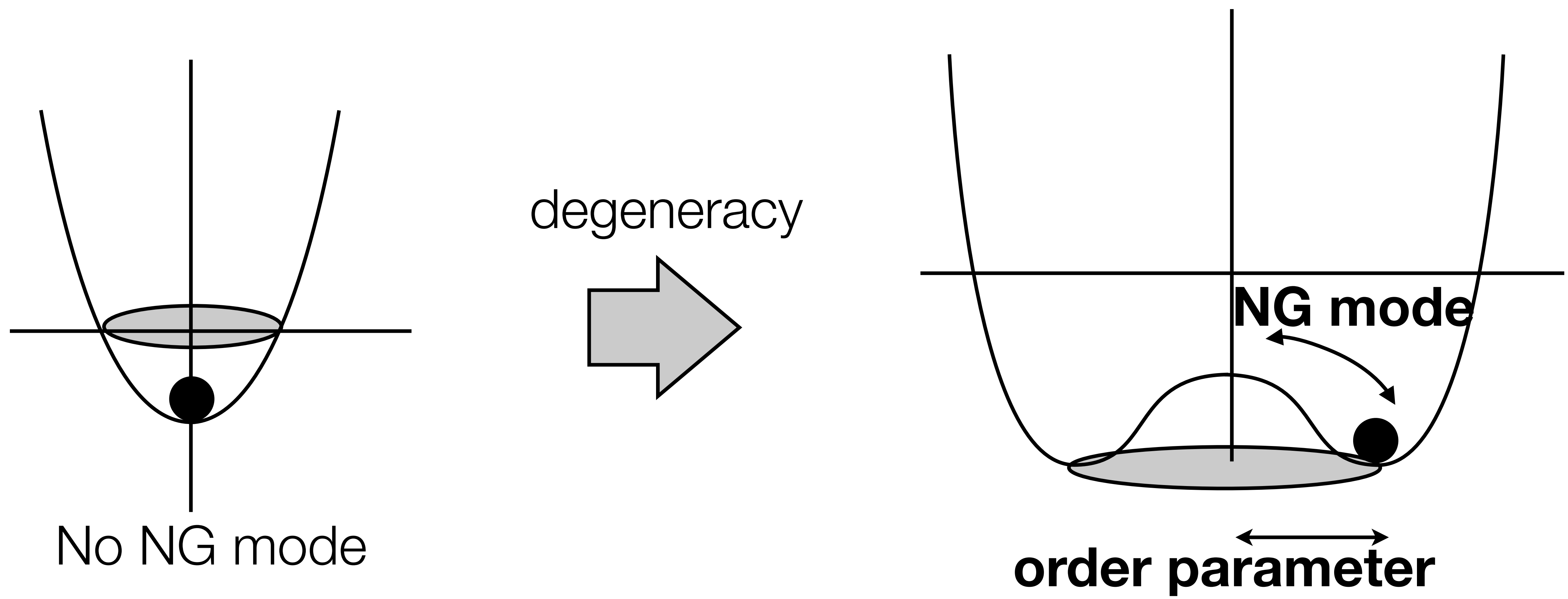


As long as the system is not at vacuum, the states always contain double degeneracy.

examples: High Temperature (QCD), Fermi sea+BEC (cold atom)

NG “*fermion*” which is related to SUSY breaking

Generally, degeneracy generates zero energy excitation (**Nambu-Goldstone (NG) mode**).



We expect that NG mode appears in SUSY case.

V. V. Lebedev and A. V. Smilga, Nucl. Phys. B **318,** 669 (1989)

NG "*fermion*" which is related to SUSY breaking

Nambu-Goldstone theorem:

(fermion ver.)

J^μ: supercurrent

Q=J^0: supercharge

Broken symmetry

$$-ip_\mu \int d^4x e^{ip\cdot(x-y)} \langle T J^\mu(x) O(y) \rangle = \langle \{Q, O\} \rangle$$

NG mode

Order parameter

"When the order parameter is finite, the propagator in the left-hand side has a pole at $p \to 0$."

V. V. Lebedev and A. V. Smilga, Nucl. Phys. B **318,** 669 (1989)

NG "*fermion*" which is related to SUSY breaking

Broken symmetry

$$-ip_\mu \int d^4x e^{ip\cdot(x-y)} \langle T J^\mu(x) O(y) \rangle = \langle \{Q, O\} \rangle$$

NG mode

Order parameter

If we set O=Q, order parameter is energy-momentum tensor ($T^{\mu\nu}$) in the relativistic system.

In the presence of medium, **SUSY is always broken**.

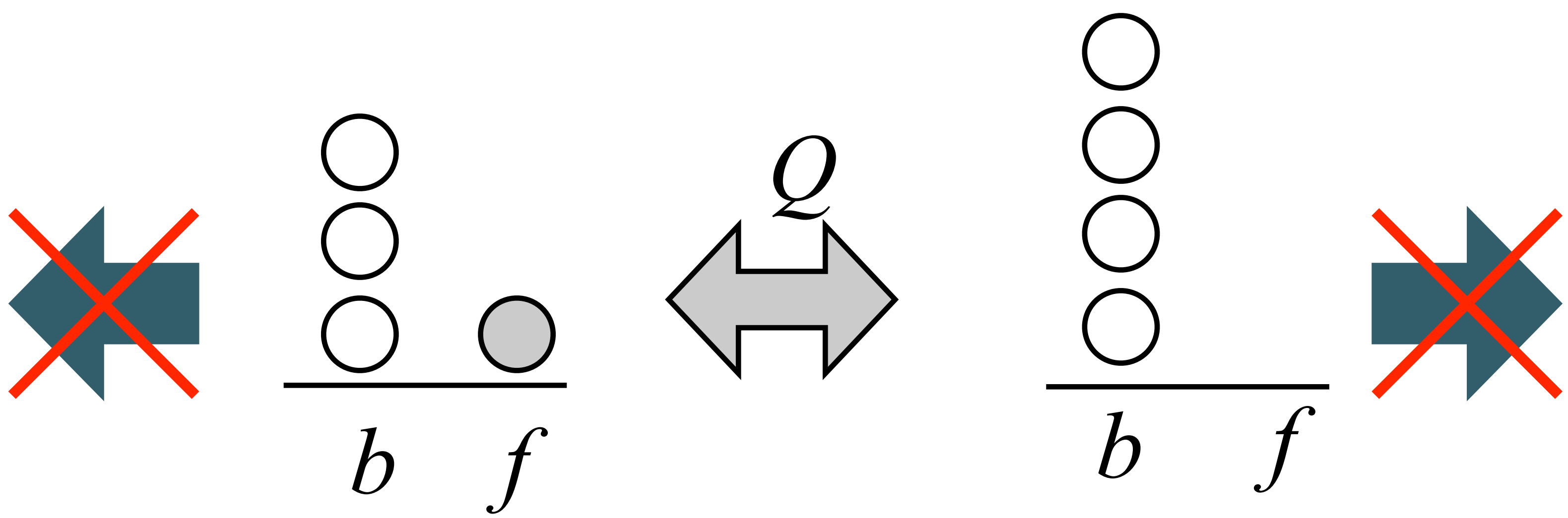
$$\langle T^{\mu\nu} \rangle = \mathrm{diag}(\rho, p, p, p)$$

SUSY breaking

$$-ip_\mu \int d^4x e^{ip\cdot(x-y)} \langle T J^\mu(x) O(y) \rangle = \langle \{ Q, O \} \rangle$$

NG mode

NG mode appears in <*QQ*>.



SUSY is fermionic symmetry,
so the NG mode is **fermion** (**Goldstino**).

Rare fermionic zero mode

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V. V. Lebedev and A. V. Smilga, Nucl. Phys. B **318,** 669 (1989).

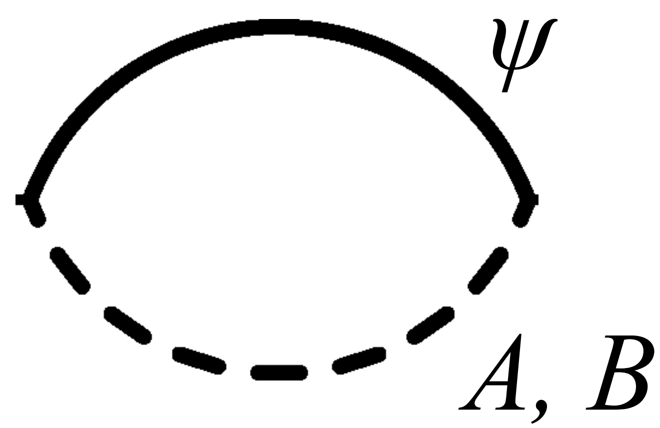
Goldstino in Wess-Zumino model

In Wess-Zumino model, the goldstino exists.

(Weyl fermion+Scalar and Pseudo-scalar bosons)

$$Q \sim (A,B) \times \Psi$$

$$<QQ> \sim$$

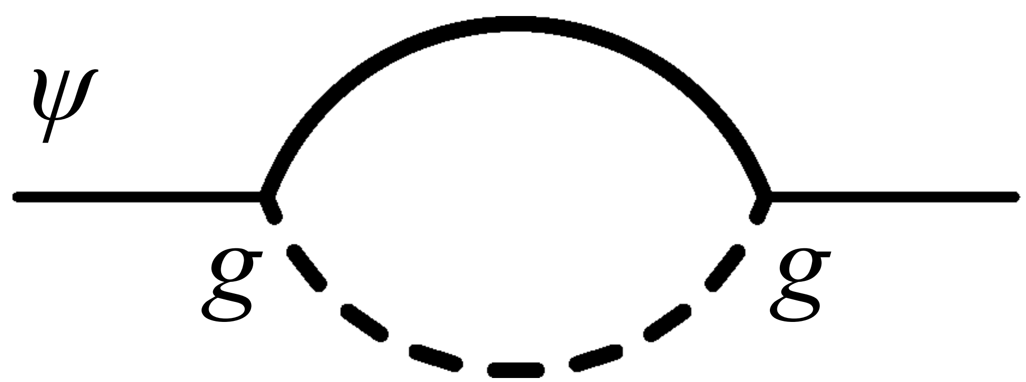


Pole appears in the Green function.

V. V. Lebedev and A. V. Smilga, Nucl. Phys. B **318,** 669 (1989).

Goldstino in Wess-Zumino model

Through the fermion-boson coupling, it appears also in fermion propagator.



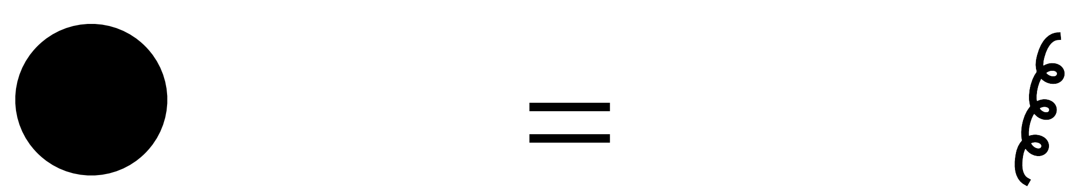
dispersion relation	$\mathrm{Re}\,\omega = p/3$
Residue	g^2/π^2

V. V. Lebedev and A. V. Smilga, Annals Phys. **202**, 229 (1990)

(Quasi) SUSY in QCD

Both of the quark and the gluon are regarded as massless at high T

There is SUSY approximately if we neglect the interaction.



q g

Quasi-goldstino in QCD

In weak coupling regime, we established the existence of the (quasi) goldstino in QCD.

Dispersion relation	$\mathrm{Re}\,\omega = p/3$
Damping rate	$\mathrm{Im}\,\omega = \zeta_q + \zeta_g = O(g^2 T)$
Residue	$\dfrac{g^2 N}{8\pi^2 (N^2-1)} \left(\dfrac{5}{6} N + \dfrac{1}{2N} + \dfrac{2}{3} N_f \right)^2$

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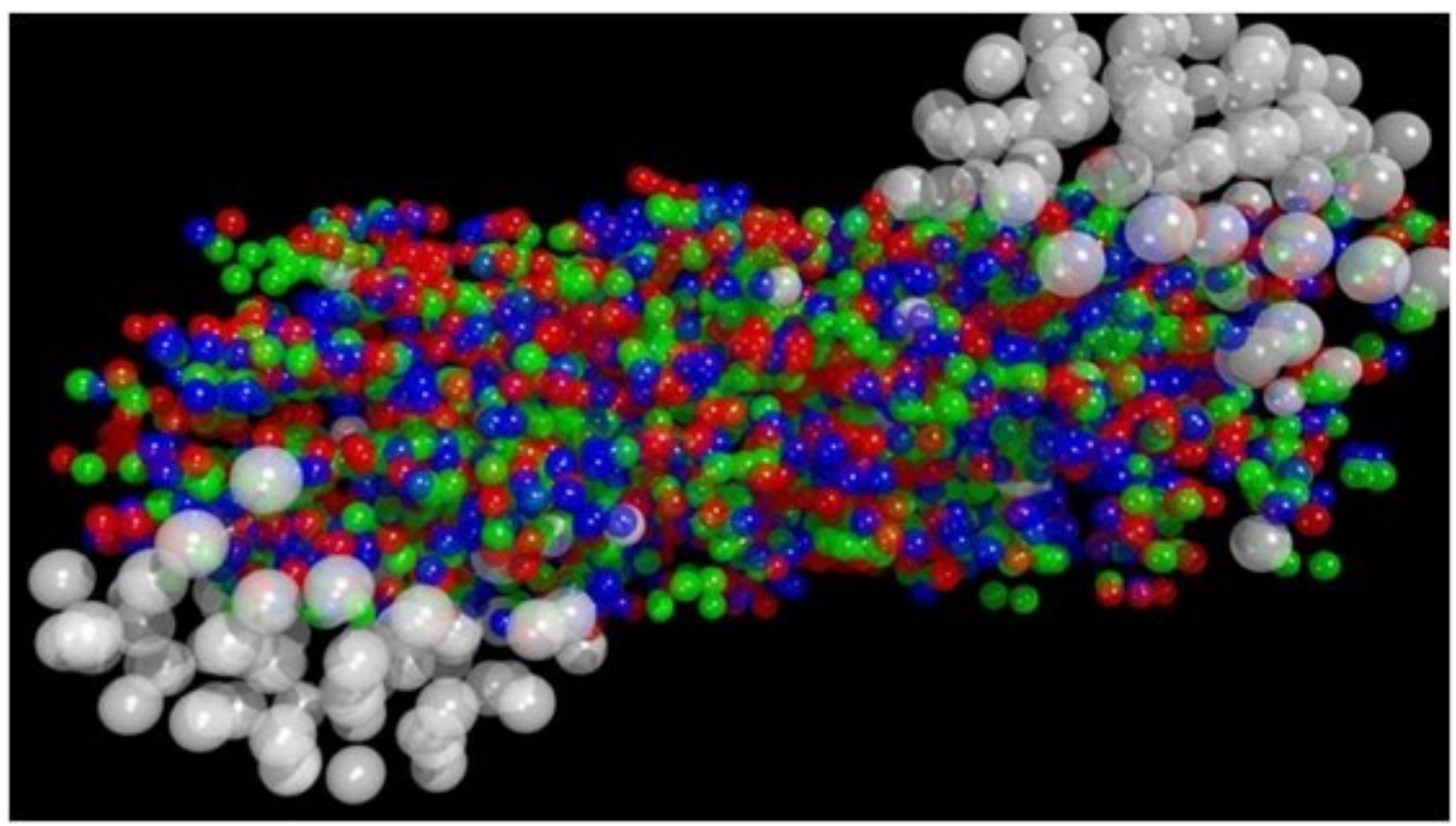
Experimental detection

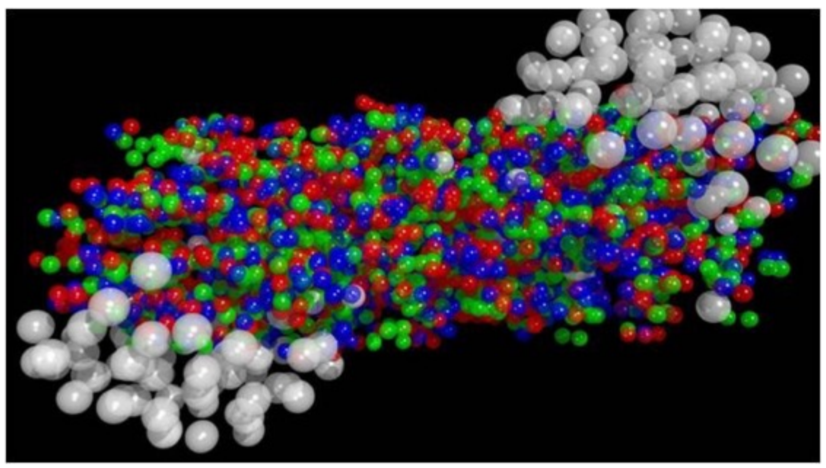
How can we detect goldstino in experiment?

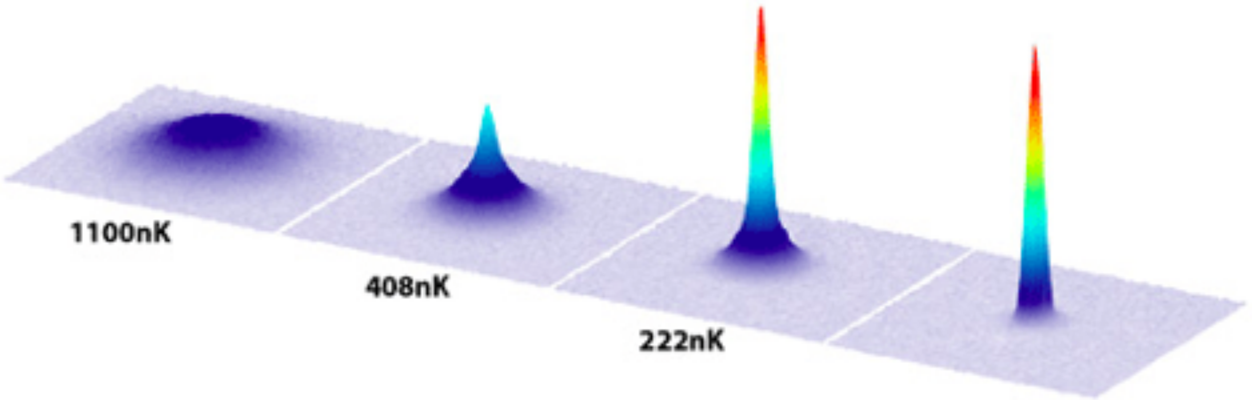
So far, detecting quark spectrum in heavy ion collision has not been done…



Picture: UrQMD group, Frankfurt.

SUSY in Cold Atom System





Picture: Ferlaino group, Innsbruck.

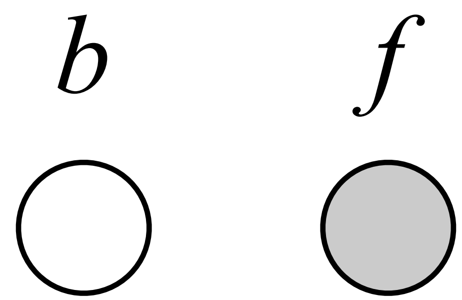
Cold atom system is
easier to realize, and its experiment is cleaner.
(**Test site of many-body physics**)

cf: T. Ozawa, Nature Physics **11**, 801 (2015),
Wess-Zumino model: Y. Yu, and K. Yang, PRL **105**, 150605 (2010),
Dense QCD: K. Maeda, G. Baym and T. Hatsuda, PRL **103**, 085301 (2009),
Relativistic QED: Kapit and Mueller, PRA **83**, 033625 (2011).

SUSY in Cold Atom System

Y. Yu and K. Yang, PRL **100**, 090404.

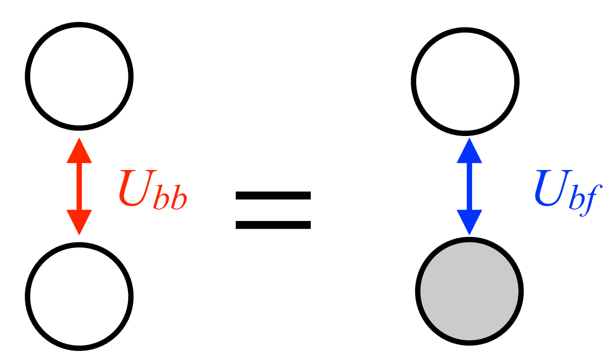
Prepare **Bose-Fermi mixture.**



1. Same mass: $m = m_b = m_f$

$$\frac{U_{bb}}{2}\sum_i n_i^b\left(n_i^b - 1\right) + U_{bf}\sum_i n_i^b n_i^f$$

2. Same interaction: $U = U_{bb} = U_{bf}$



SUSY in Cold Atom System

Y. Yu and K. Yang, PRL **100**, 090404.

$m = m_b = m_f$
$U_{bb} = U_{bf}$

$\Rightarrow$

$$[Q, H] = 0$$

$$(Q = \int dx\, b^\dagger_x f_x,\ Q^\dagger = \int dx\, b_x f^\dagger_x)$$

S. Endo, private communication.

Possible candidates: $^{6}\mathrm{Li}$-$^{7}\mathrm{Li}$, $^{173}\mathrm{Yb}$-$^{174}\mathrm{Yb}$

($m_b/m_f = 1.17$, U is easy to tune) ($U_{bb}/U_{bf} = 1.32$, m is almost same)

Goldstino in cold atom systems

Broken symmetry

$$-ip_\mu \int d^4x e^{ip\cdot(x-y)} \langle T J^\mu(x) O(y) \rangle = \langle \{Q, O\} \rangle$$

NG mode

Order parameter

$Q = bf^\dagger$

$Q^\dagger = b^\dagger f$

If we set $O=Q^\dagger$, **NG mode appears in <$QQ^\dagger$>**.

Order parameter is density (<{Q, $Q^\dagger$}>=ρ) in this case.

SUSY is always broken when ρ is finite.

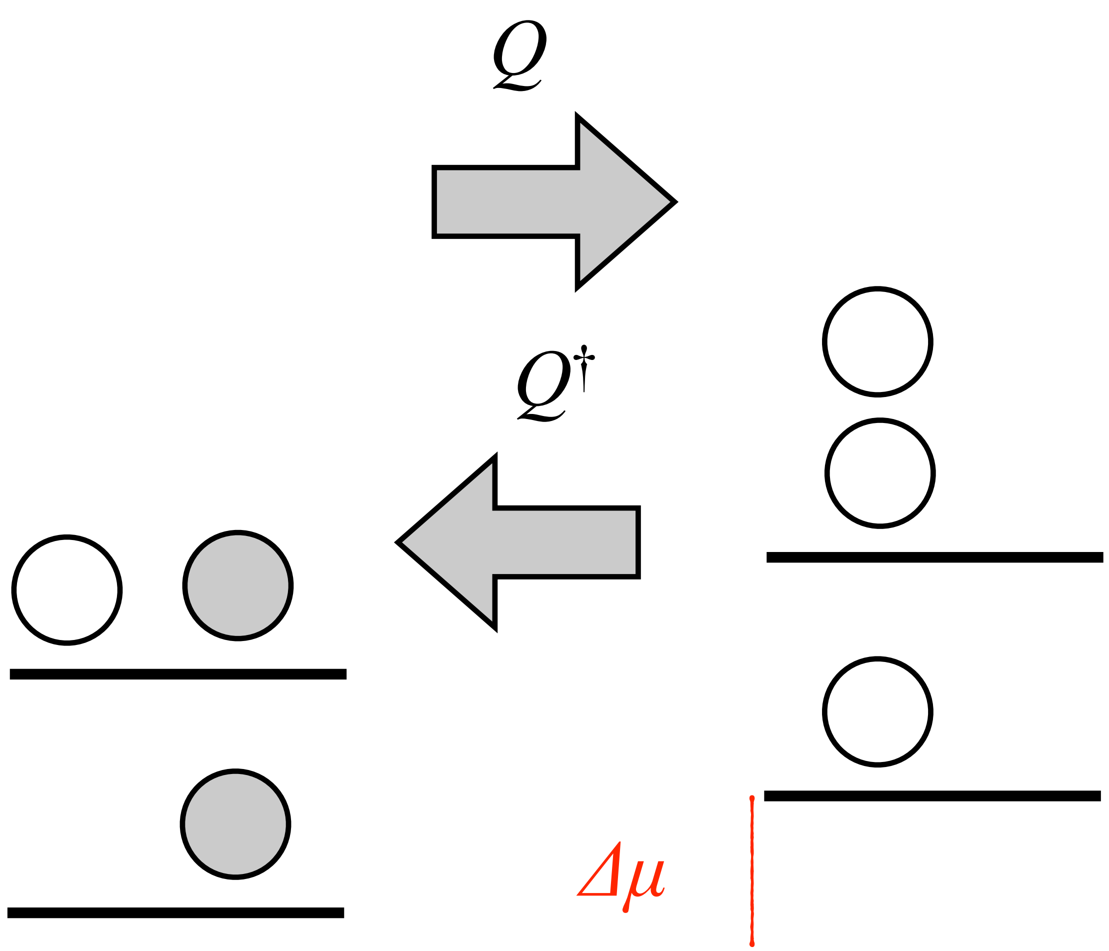
Explicit breaking of SUSY

Y. Yu and K. Yang, PRL **100**, 090404.

Finite density causes **explicit SUSY breaking**.

$$[Q, H-\mu_f N_f-\mu_b N_b]=-\Delta\mu Q$$

Grand Canonical Hamiltonian $\mu_f - \mu_b$

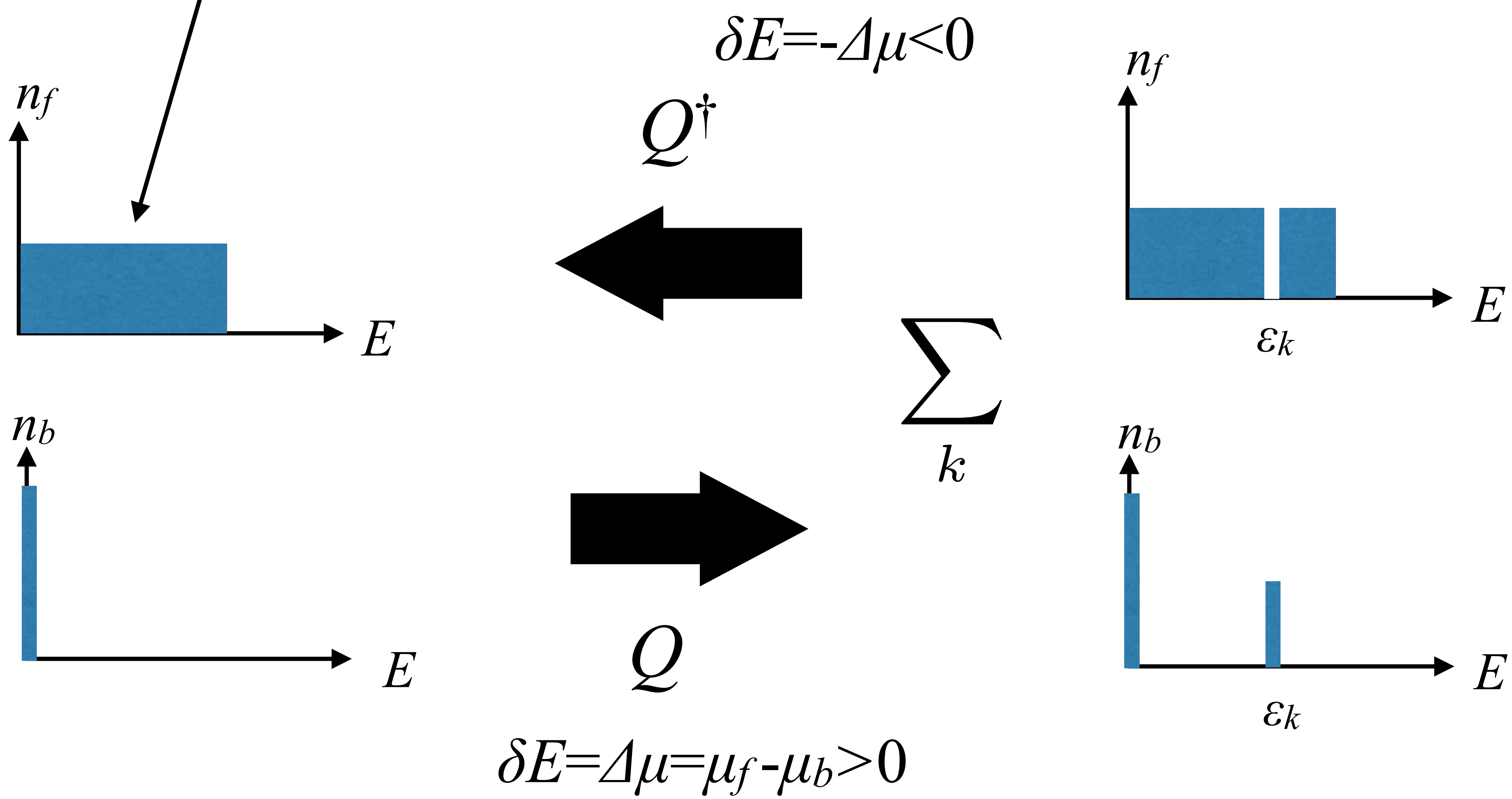


Gapped Goldstino
(ω=-$\Delta\mu$)

Explicit breaking of SUSY

(T=0, weak coupling) $\mu_f=\varepsilon_f$, $\mu_b=0$

Only this state (Fermi sea+BEC) is realized as ground state.



Goldstino spectrum

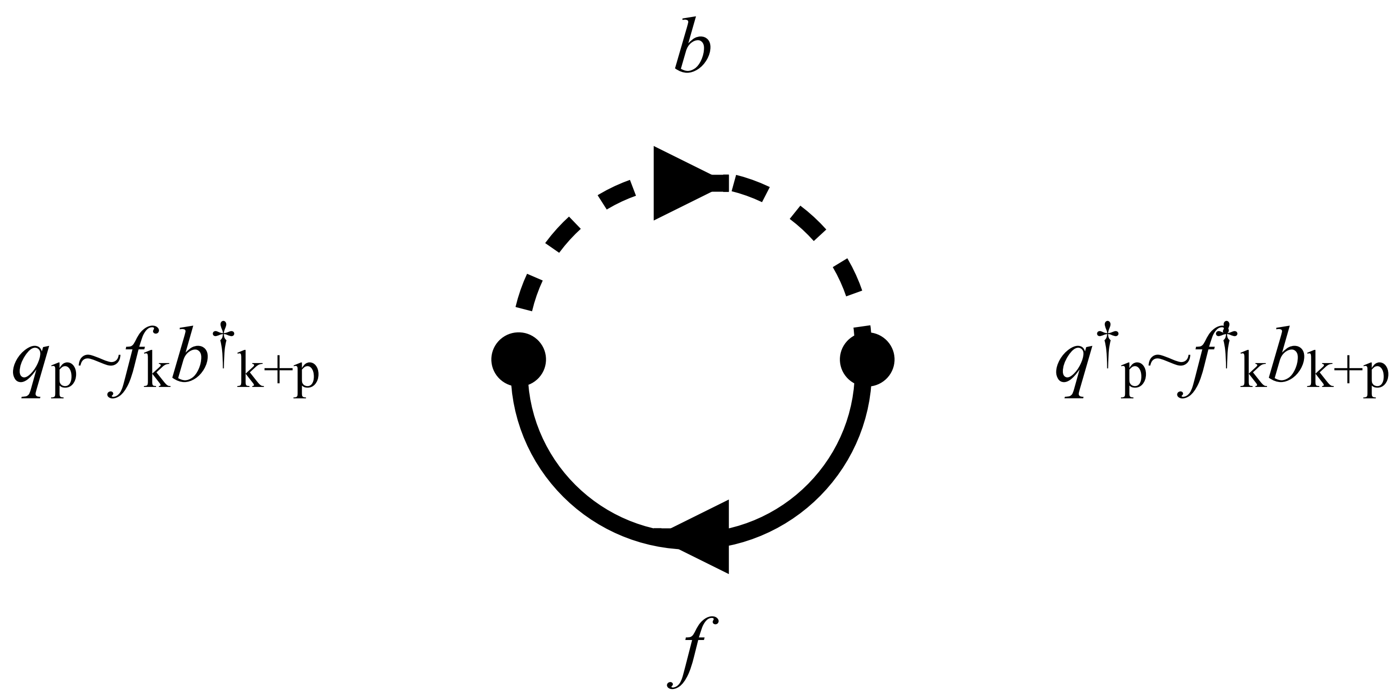
Calculate the spectrum of the Goldstino.

$$G^R(p) = i \int dt \int d^3\mathbf{x}\, e^{i\omega t - i\mathbf{p}\cdot\mathbf{x}} \theta(t) \langle \{ q(t,\mathbf{x}), q^\dagger(0) \} \rangle$$

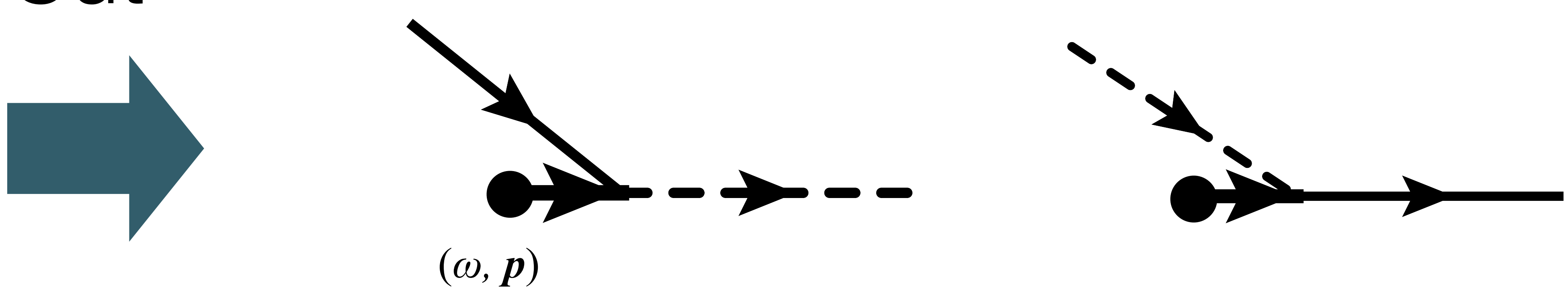
$$\sigma(\omega, \mathbf{p}) = 2\mathrm{Im}\, G(\omega, \mathbf{p})$$

Goldstino spectrum (free case)

Free case (U=0)

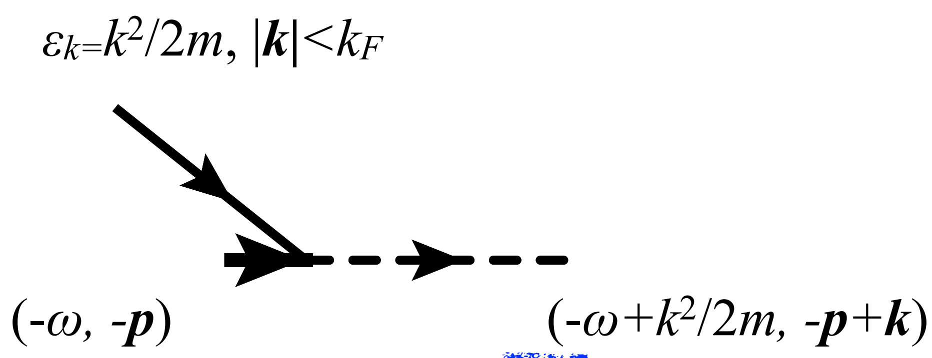


Cut



Goldstino spectrum (free case)

Landau damping

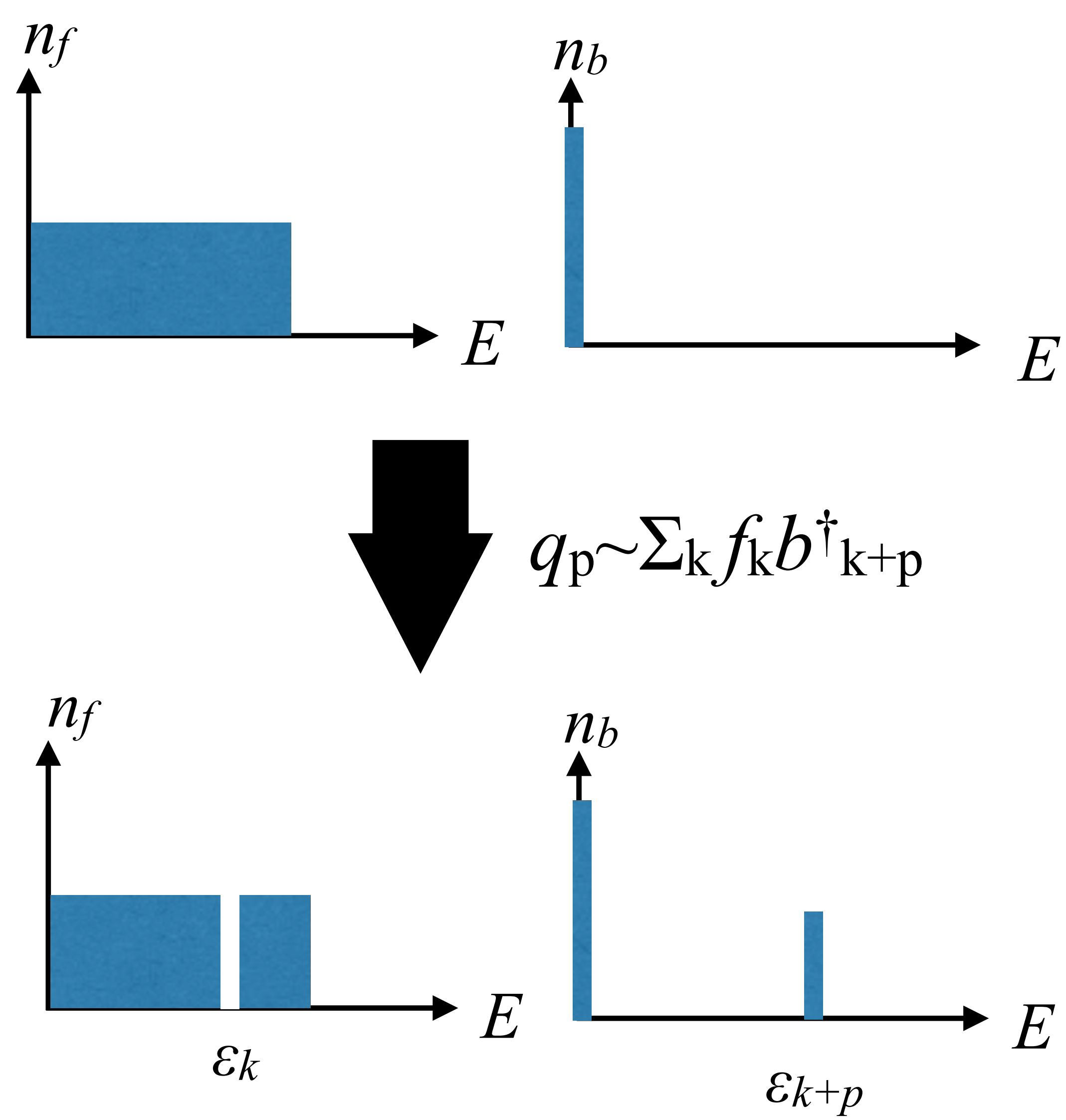


should be on-shell: $\overline{\omega}=-p^2/2m-p\,k\cos\theta/m$

$0\sim k_F$ $-1\sim 1$

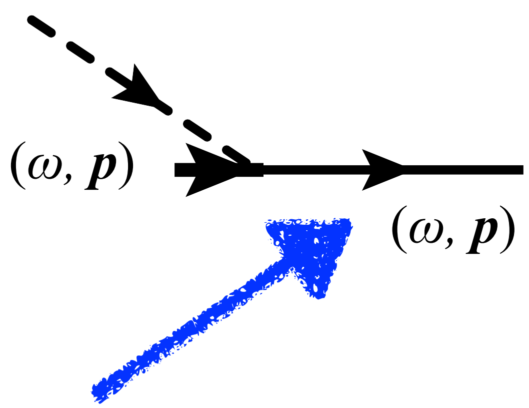
Width is $\Delta\omega=pk_F/m$

Continuum.



Goldstino spectrum (free case)

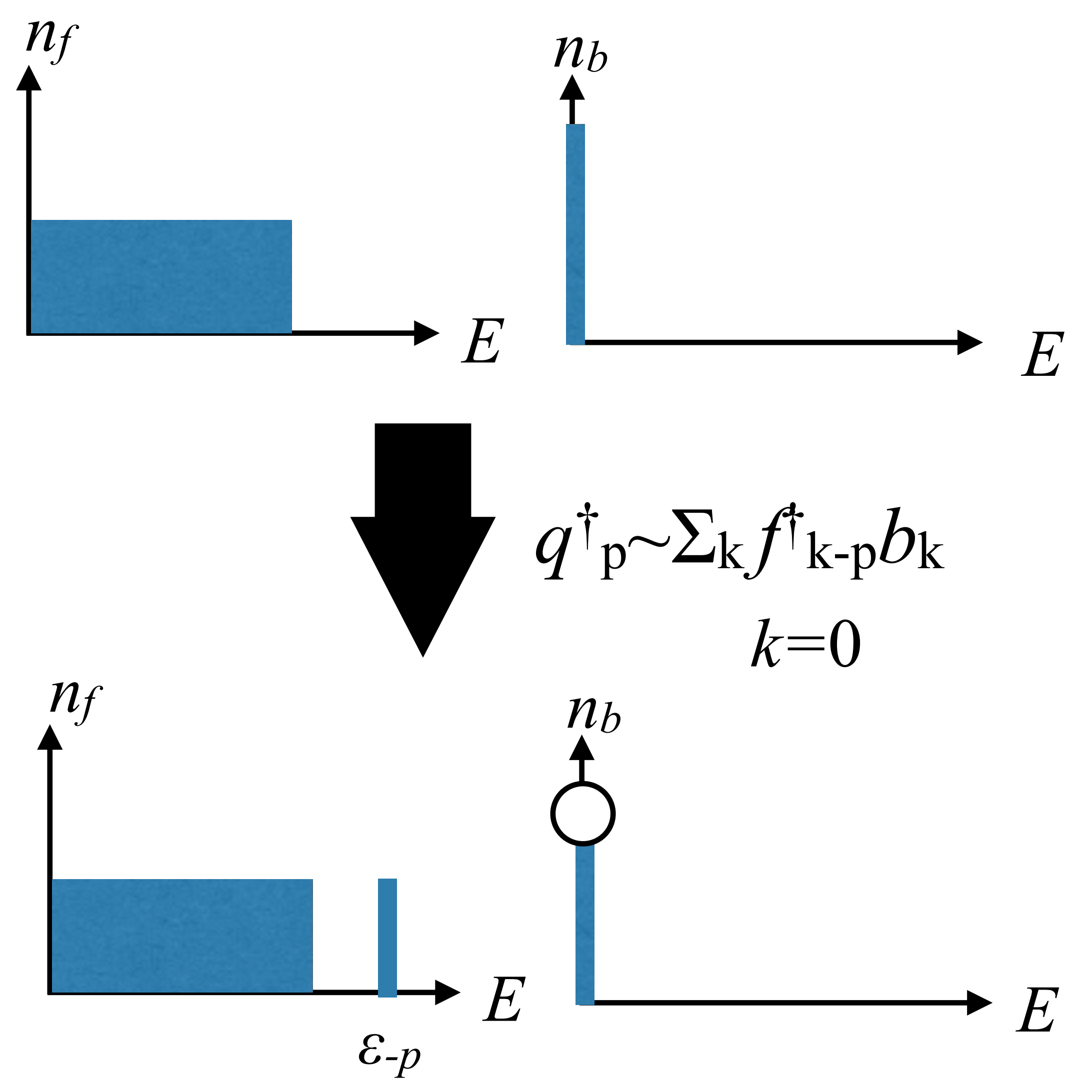
$\varepsilon_k=0,\ k=0$



should be on-shell: $\overline{\omega}=p^2/2m$

Other value of ω is not allowed!

Pole, No width.

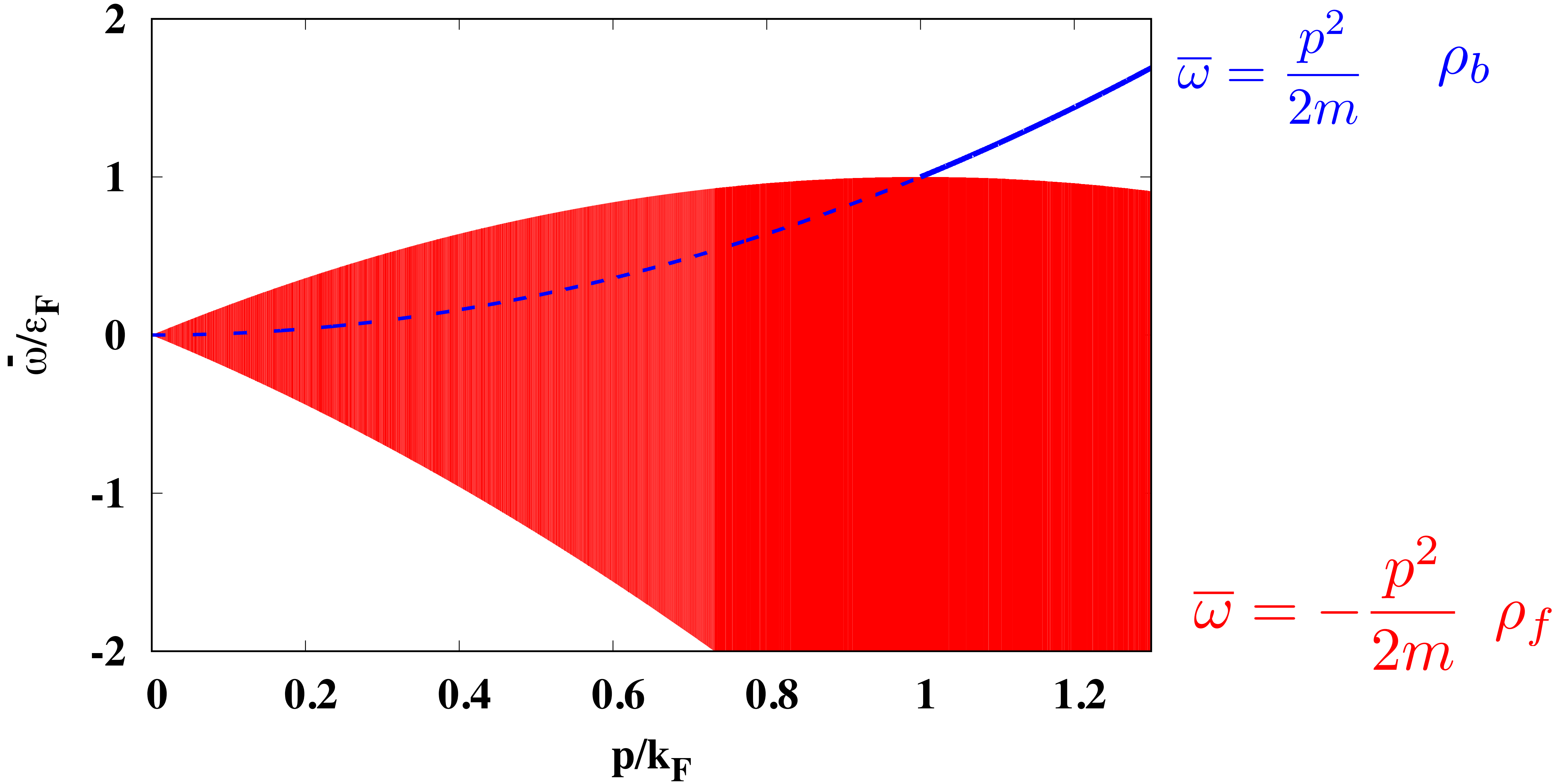


Continuum Pole

$n_{\mathrm{f}}(1+\cancel{n_{\mathrm{b}}})+(1-\cancel{n_{\mathrm{f}}})n_{\mathrm{b}}$

Dispersion Relation	$\overline{\omega}=p^2/2m$
Strength	ρ_b

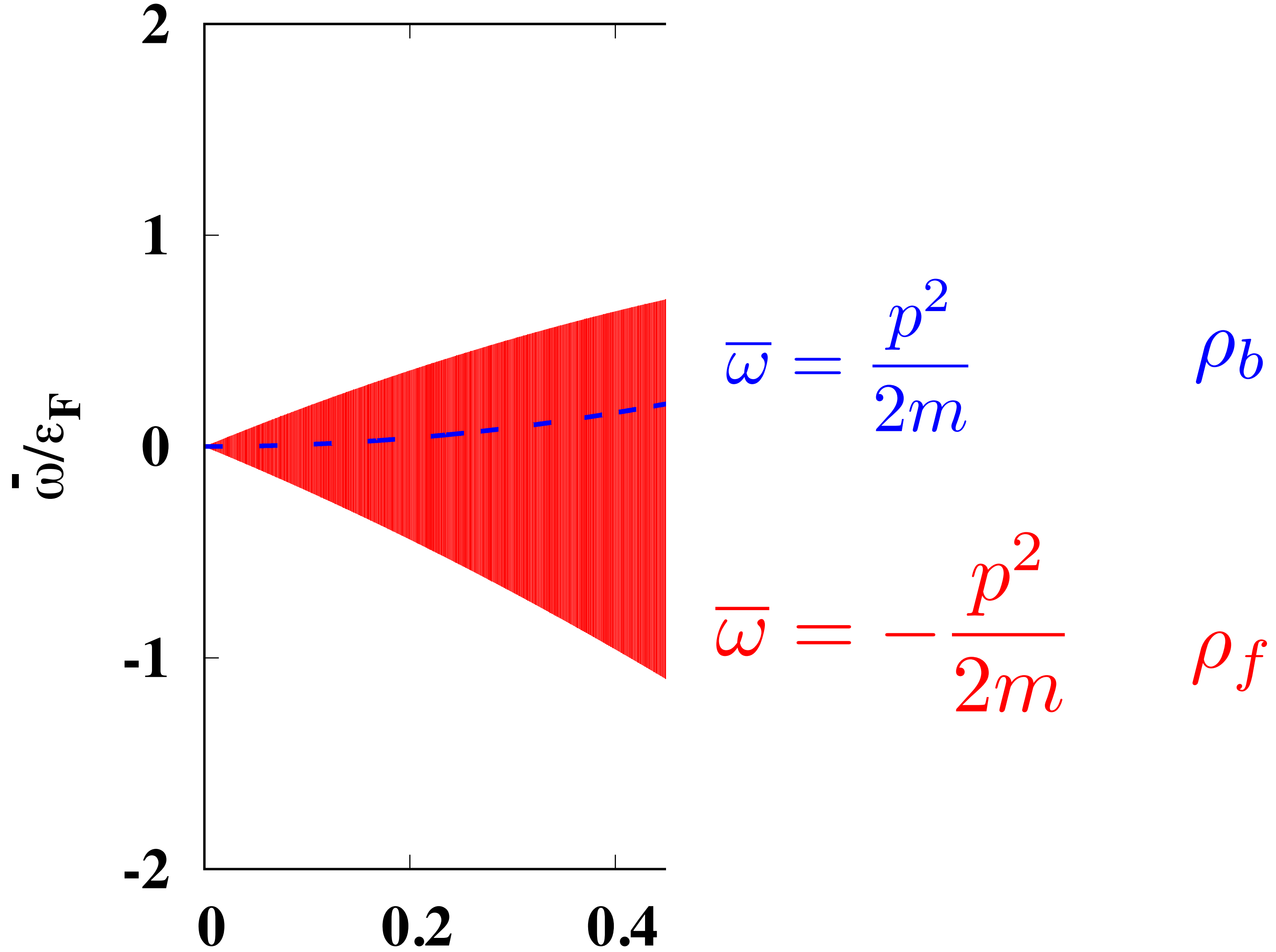
Goldstino spectrum (free case)



Continuum+Pole.

Goldstino spectrum (free case)

These pole and continuum satisfies the NG theorem.



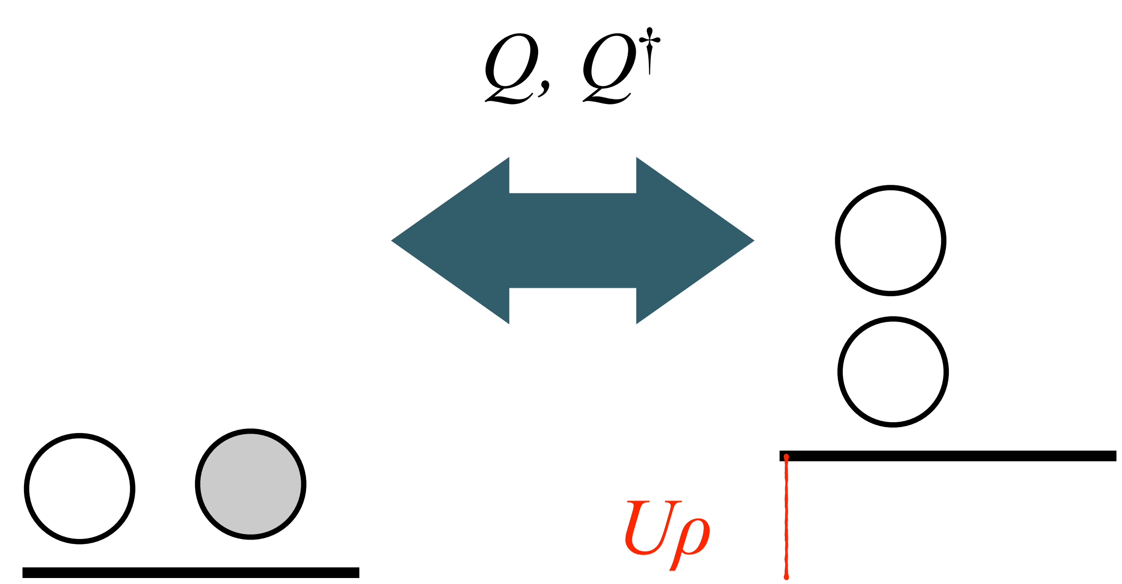
Goldstino spectrum (interacting case)

Switch on the interaction.

For simplicity, we start with two-dimension case, in which there are no BEC.

Mean field approximation

Different MF correction



fermion: $U\rho_b$

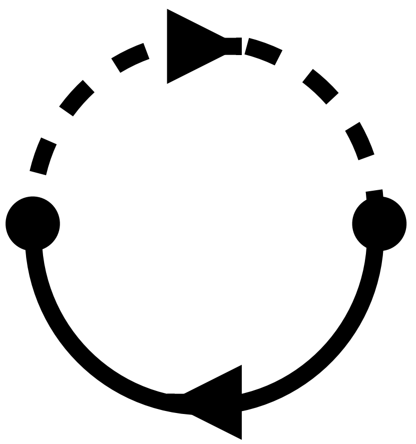
boson: $U\rho_f$+$2U\rho_b$

Gap in goldstino spectrum?

Goldstino spectrum (interacting case)

Actually, it is the case in Green function

$$G^0(p) = -\int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{n_F(\epsilon^f_k) + n_B(\epsilon^b_{k+p})}{\overline{\omega} + [2\mathbf{k}\cdot\mathbf{p} + \mathbf{p}^2]/2m + U\rho}$$



At p=0

$$\Rightarrow -\frac{\rho}{\overline{\omega} + U\rho} \quad \sim U^{-1}$$

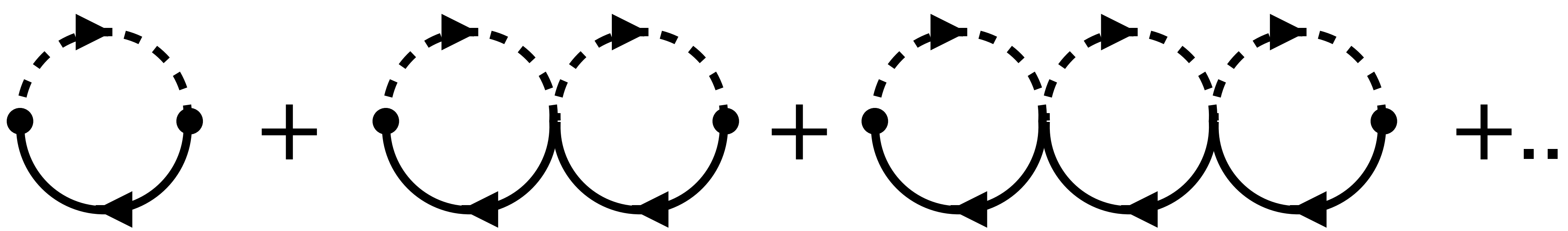
It contradicts with the exact result (Gapless NG mode).
We should have missed something…

T. Shi, Y. Yu, and C. P. Sun, PRA **81**, 011604(R) (2010)

Goldstino spectrum (interacting case)

All ring diagrams contributes at the same order.

U^{-1} $\qquad$ $U^{-1} \times U \times U^{-1} = U^{-1}$



We need to sum up infinite ring diagrams.

Random Phase Approximation

Goldstino spectrum (interacting case)

Result 1. Goldstino Pole

$$G_{\mathrm{RPA}}(p) = \frac{1}{[G^0(p)]^{-1} + U}$$

At $p=0$

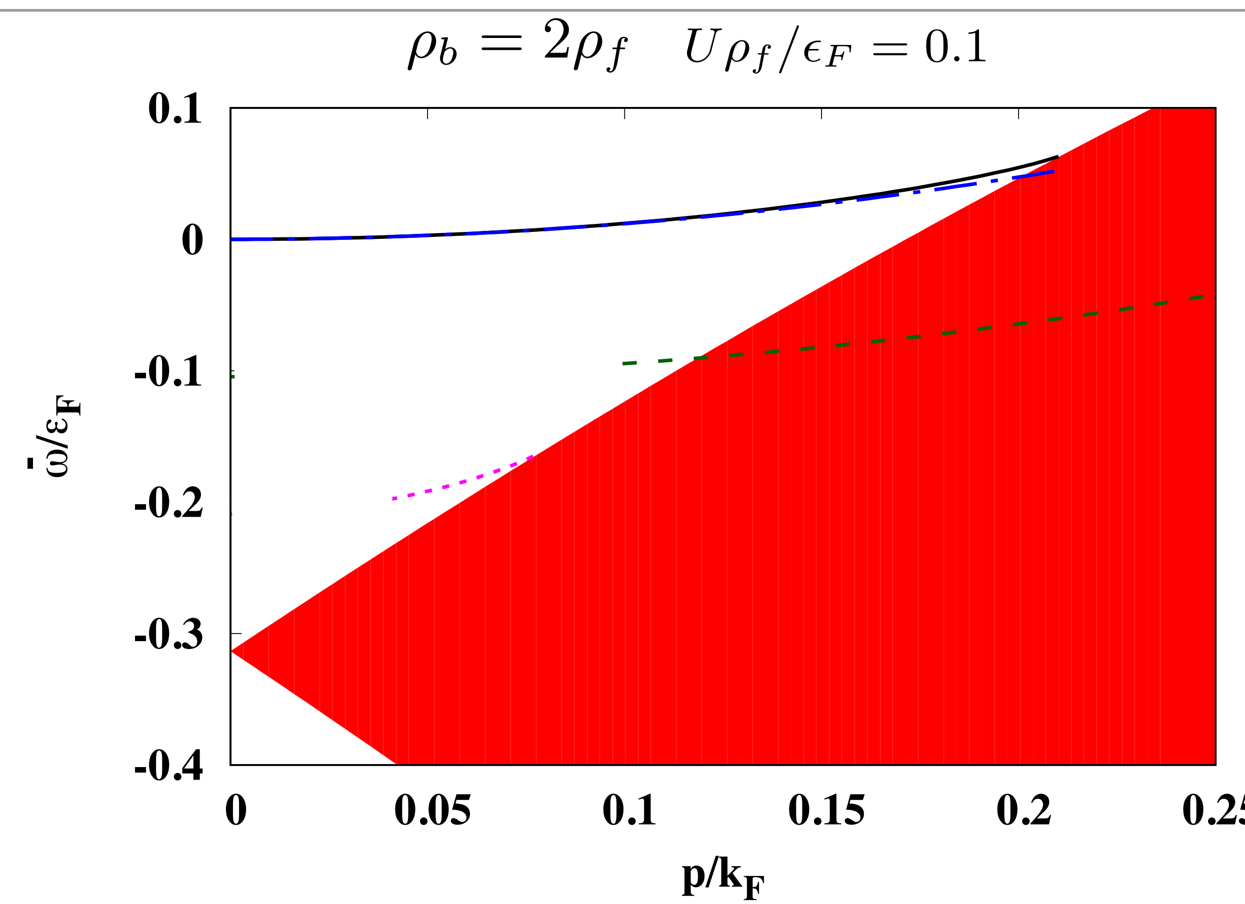
$$G^0 = -\frac{\rho}{\overline{\omega} + U\rho}$$

$$\Rightarrow \quad G_{\mathrm{RPA}}(\omega, 0) = -\frac{\rho}{\overline{\omega}}$$

Gap disappears!

Goldstino spectrum (interacting case)

Expression at finite p



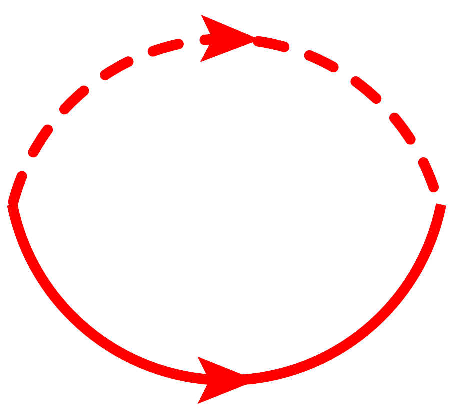
Dispersion Relation	$\omega = -\Delta\mu + \alpha p^2/2m$
Strength	$Z = \rho - \mathbf{p}^2 \frac{1}{4\pi} \left(\frac{\epsilon_F}{U\rho}\right)^2$

$$\alpha = \frac{\rho_b - \rho_f}{\rho} + \frac{\epsilon_F}{U\rho}\frac{\rho_f}{\rho}$$

(p=0: maximum value allowed by sum rule.
The sum rule is saturated by the pole)

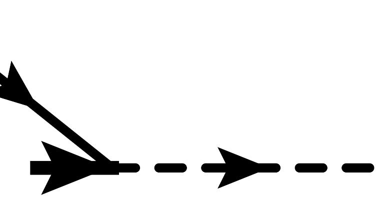
Goldstino spectrum (interacting case)

Result 2. Continuum is shifted.



Cut

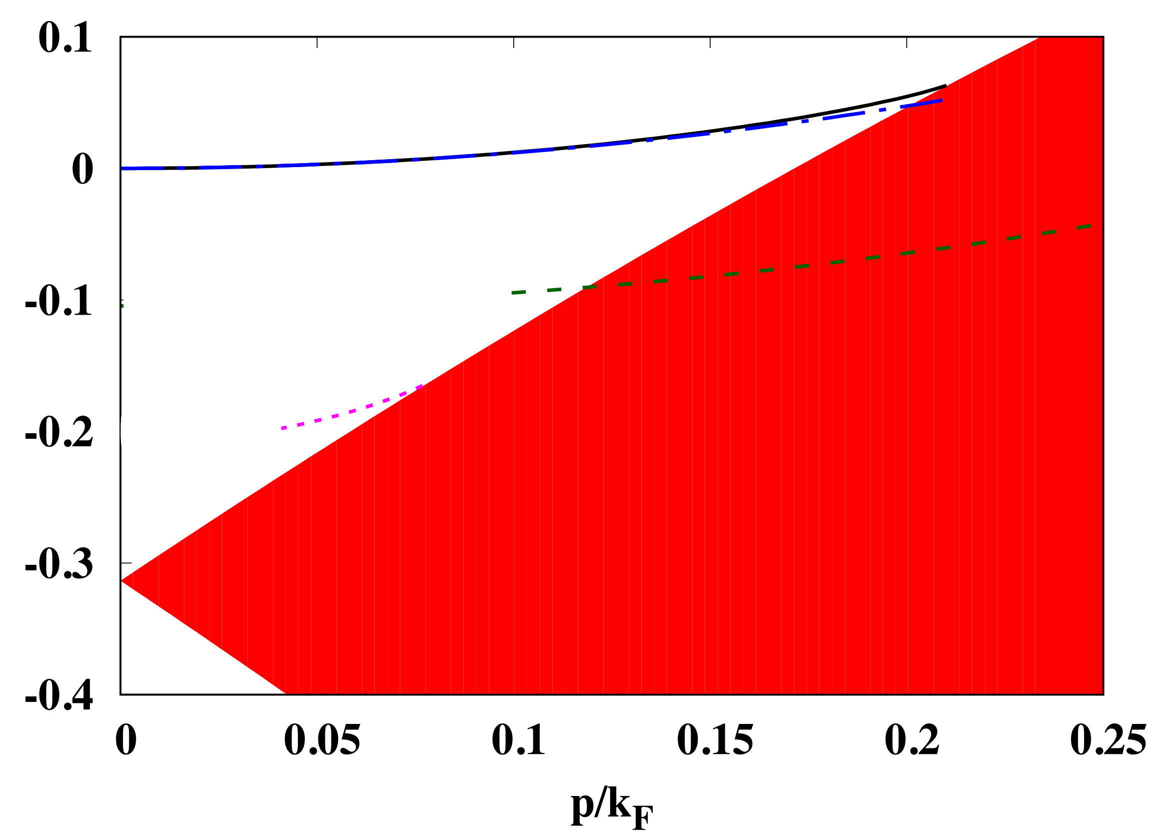
($\varepsilon_k = k^2/2m + U\rho_b$, $\boldsymbol{k}$)



($-\boldsymbol{\omega}$, $-\boldsymbol{p}$) ($-\omega + k^2/2m + U\rho_b$, $-\boldsymbol{p}+\boldsymbol{k}$)

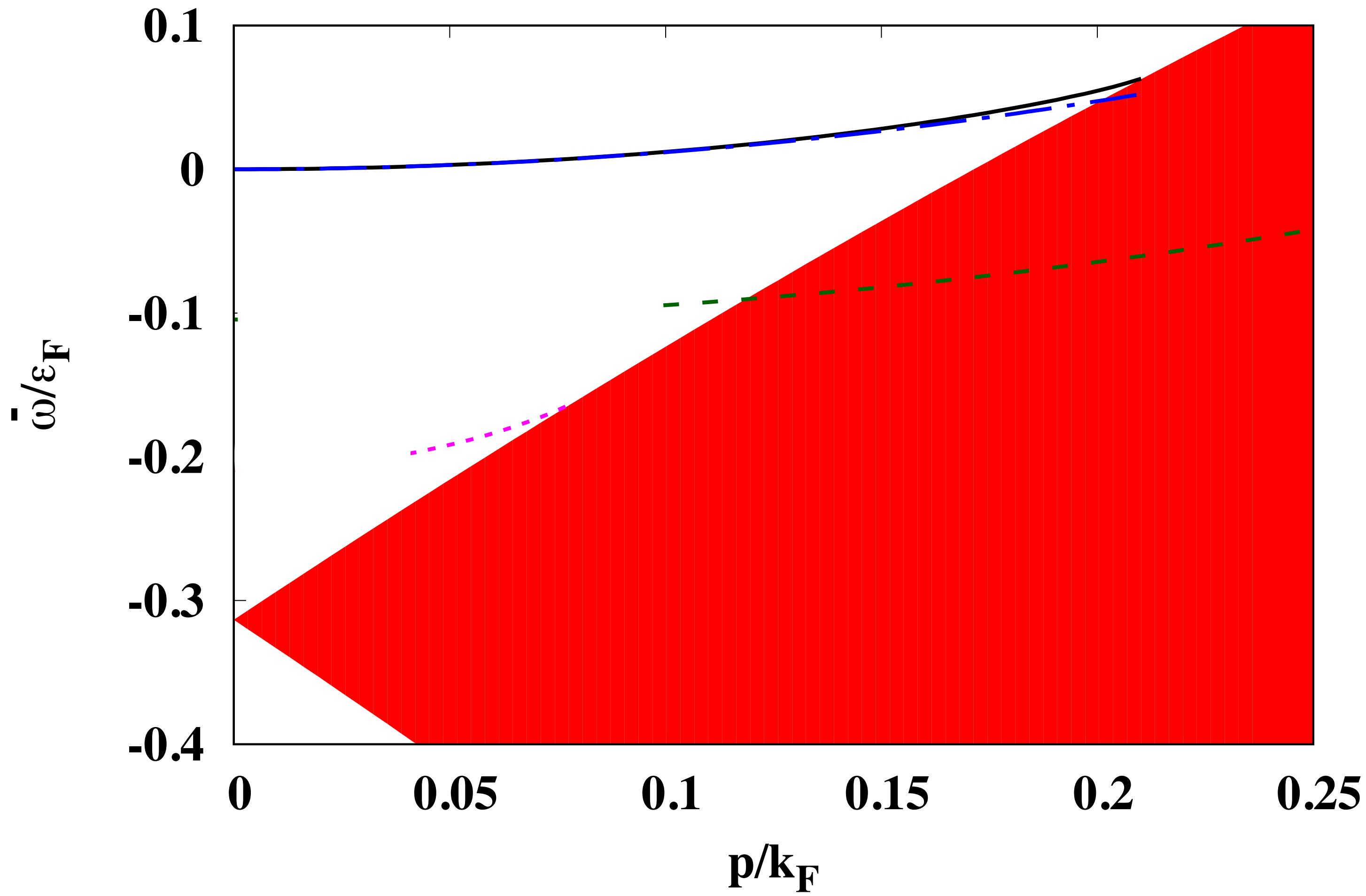
should be on-shell: $\overline{\omega} = -p^2/2m - p\ k\ \cos\theta/m + U\rho$

shift: $U\rho$



Goldstino spectrum (interacting case)

Summary



- **Continuum+Pole** (as U=0 case), but the continuum is shifted so that the pole is out of the continuum at small p.

- At p=0, all the spectral weights are given to the pole.

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Goldstino spectrum (interacting case)

What happens in BEC phase?

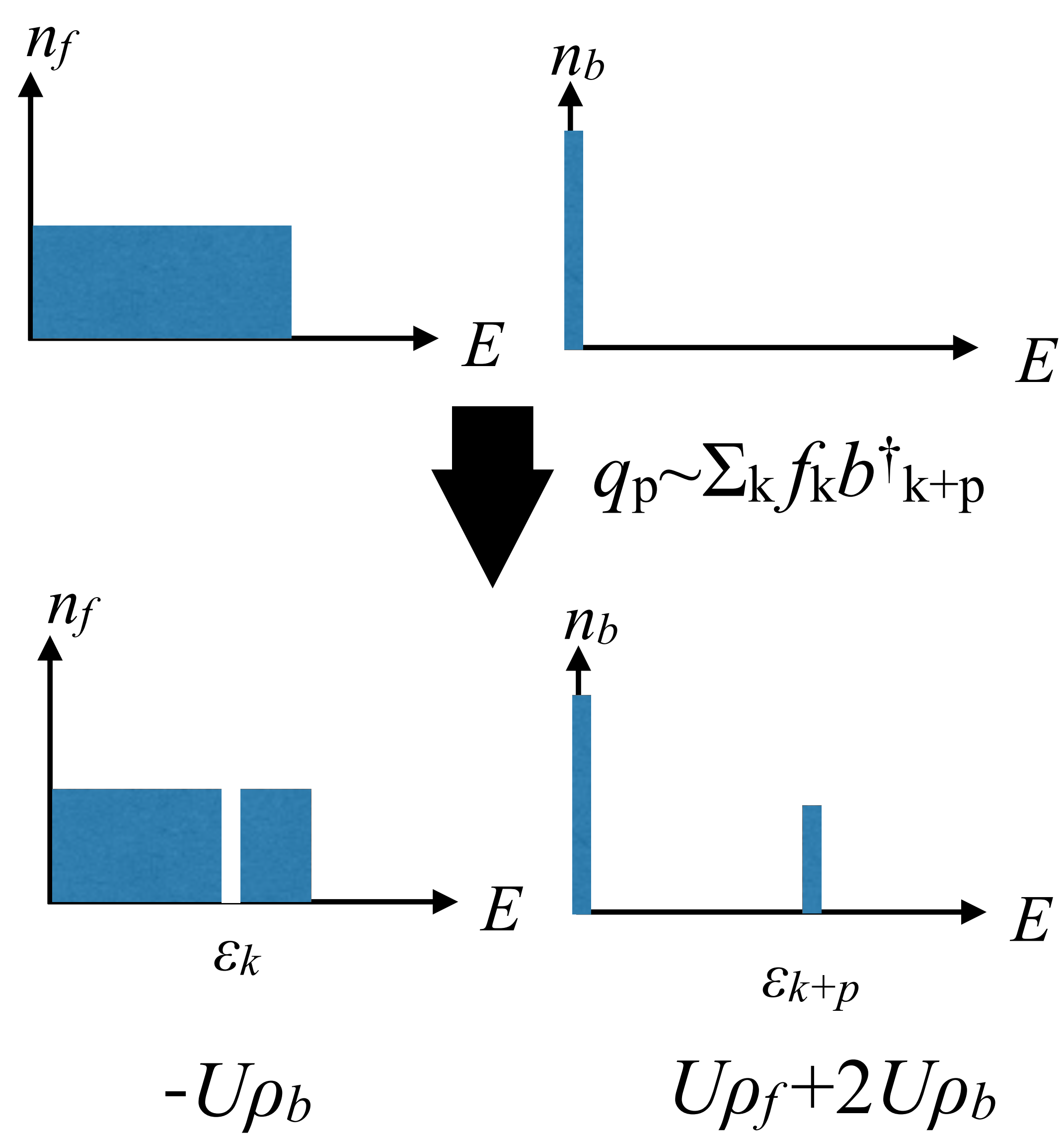
In free case, no difference. Let us consider interacting case.

Mean field approximation

Fermion particle-Boson hole excitation (Continuum)

At p=0

$$G^0(p) = -\frac{\rho_f}{\overline{\omega} + U\rho}$$

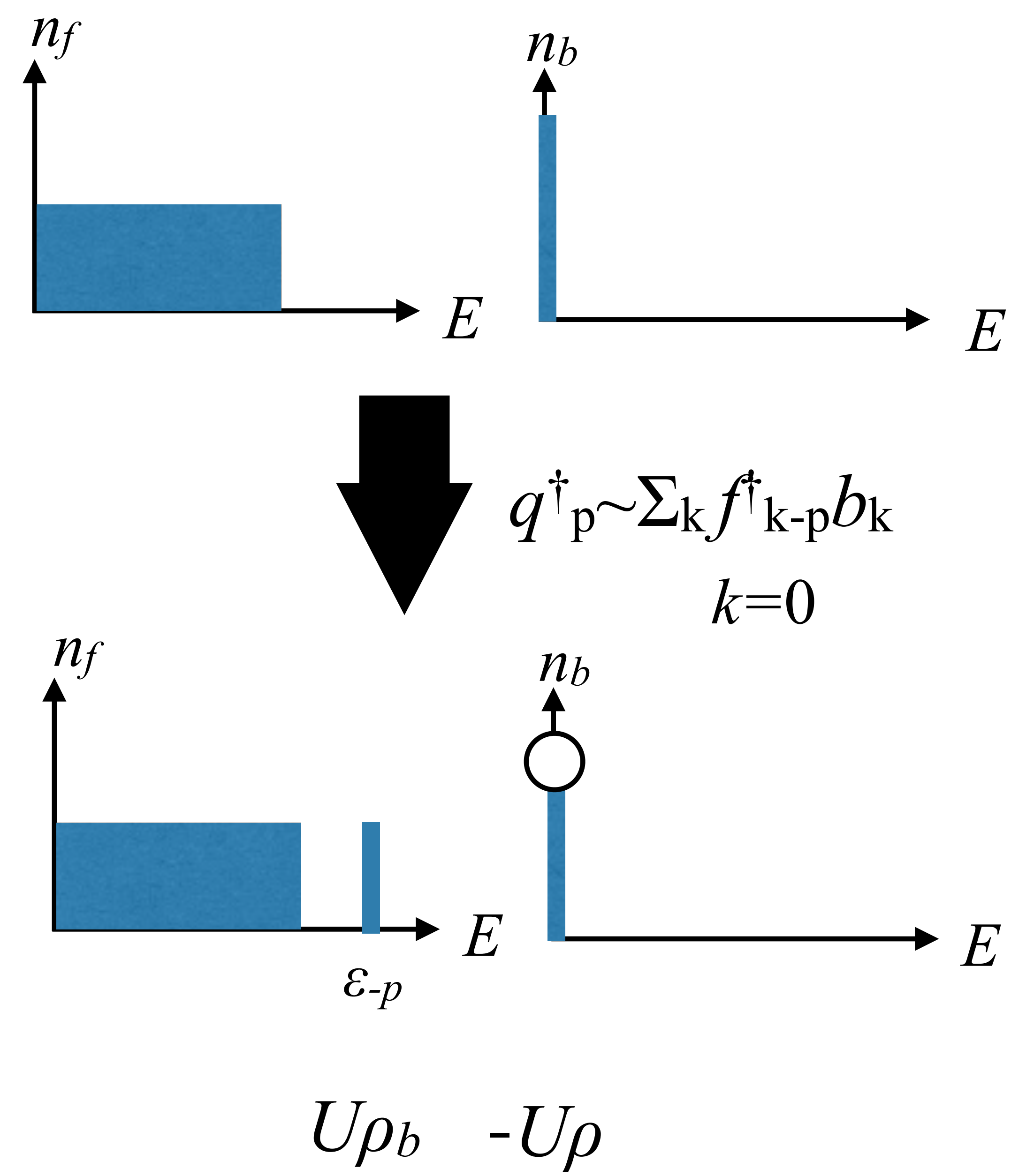


Goldstino spectrum (interacting case)

Fermion hole-Boson particle excitation (Pole)

At p=0

$$G^{\mathrm{MF}}_{\mathrm{pole}}(p) = -\frac{\rho_b}{\overline{\omega} + U\rho_f}$$



Goldstino spectrum (interacting case)

RPA

$$G_{\mathrm{RPA}}(p) = \frac{1}{[G^0(p)]^{-1} + U}$$

At p=0

$$G^0(p) = -\frac{\rho_f}{\overline{\omega} + U\rho}$$

$$G_{\mathrm{RPA}}(p) = -\frac{\rho_f}{\overline{\omega} + U\rho_b}$$

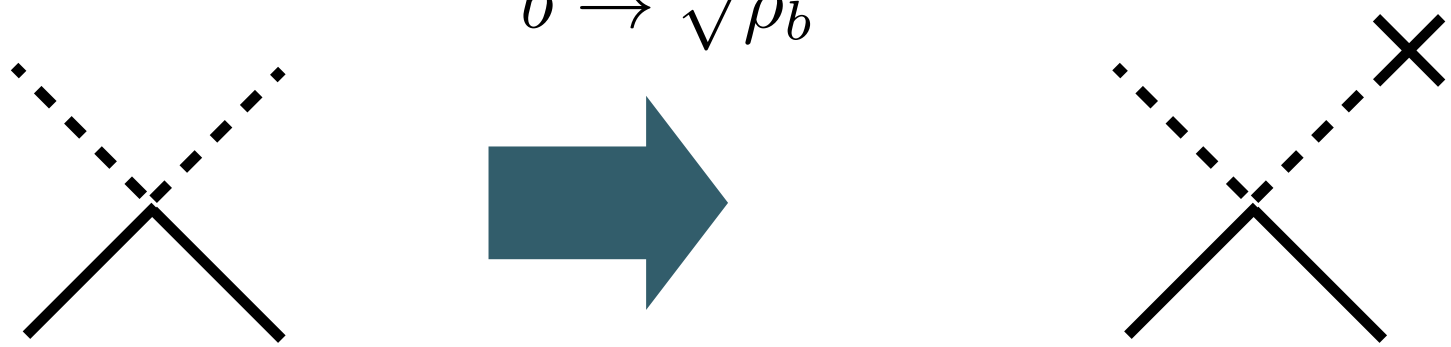
The gap remains!

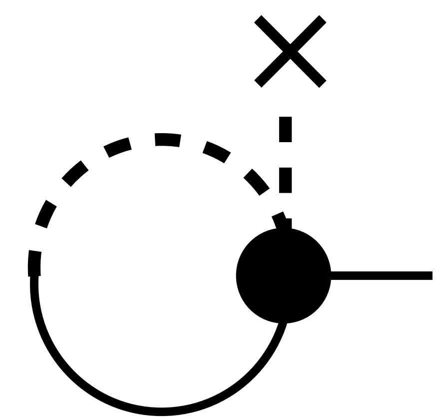
Inconsistent with the NG theorem,
so we should have missed something again…

Goldstino spectrum (interacting case)

Three-point coupling due to BEC

$$b \to \sqrt{\rho_b}$$

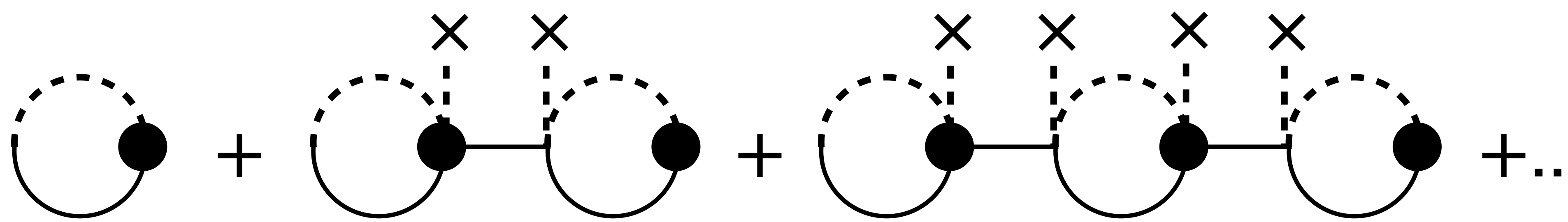




Mixing between
Fermion particle-Boson hole excitation (Continuum)
and Fermion hole-Boson particle excitation (Pole)

Goldstino spectrum (interacting case)

<u>Taking into account the mixing</u>



$$\tilde{G}(p) = \frac{1}{[G^{\mathrm{RPA}}(p)]^{-1} - U^2 G^{\mathrm{MF}}_{\mathrm{pole}}(p)}$$

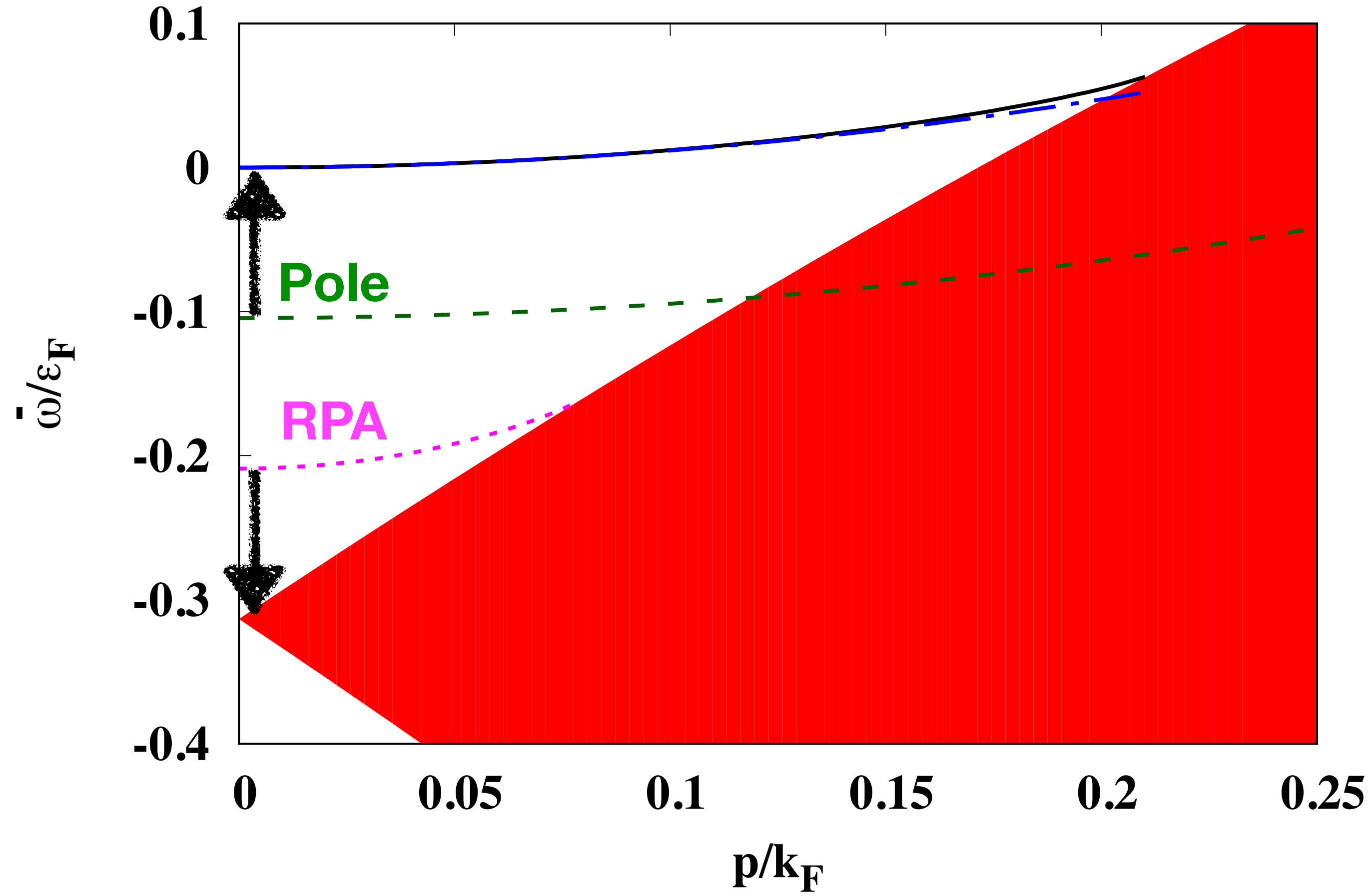
At p=0 $\quad G_{\mathrm{RPA}}(p) = -\frac{\rho_f}{\overline{\omega} + U\rho_b} \qquad G^{\mathrm{MF}}_{\mathrm{pole}}(p) = -\frac{\rho_b}{\overline{\omega} + U\rho_f}$

$$\tilde{G}(p) \simeq -\frac{1}{\rho}\frac{\rho_f^2}{\overline{\omega}}$$

Gap disappears!

Goldstino spectrum (interacting case)

Result at finite p



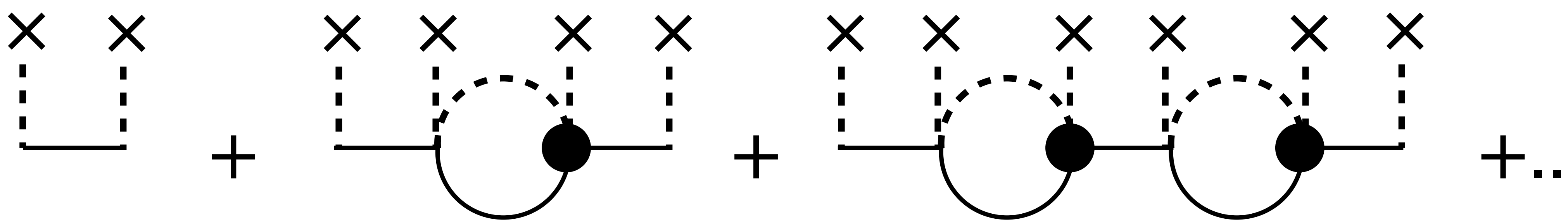
Level Repulsion

At small p, 2-peak structure due to level repulsion

(Goldstino+continuum)

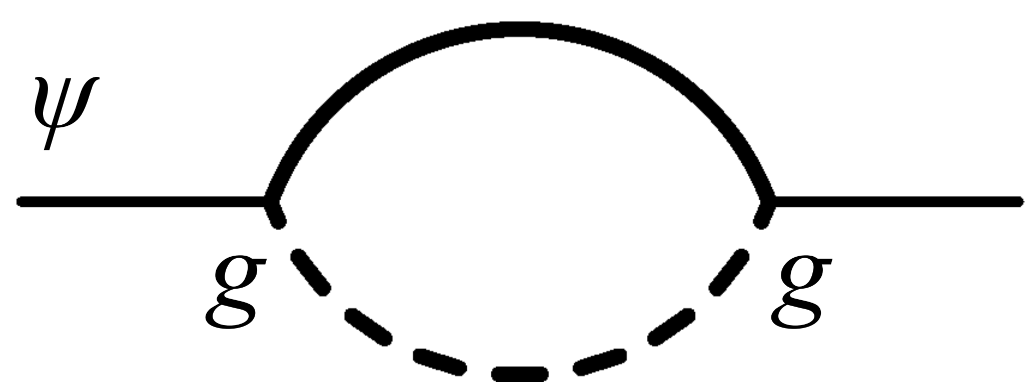
Goldstino spectrum (interacting case)

Mixing from the other point of view

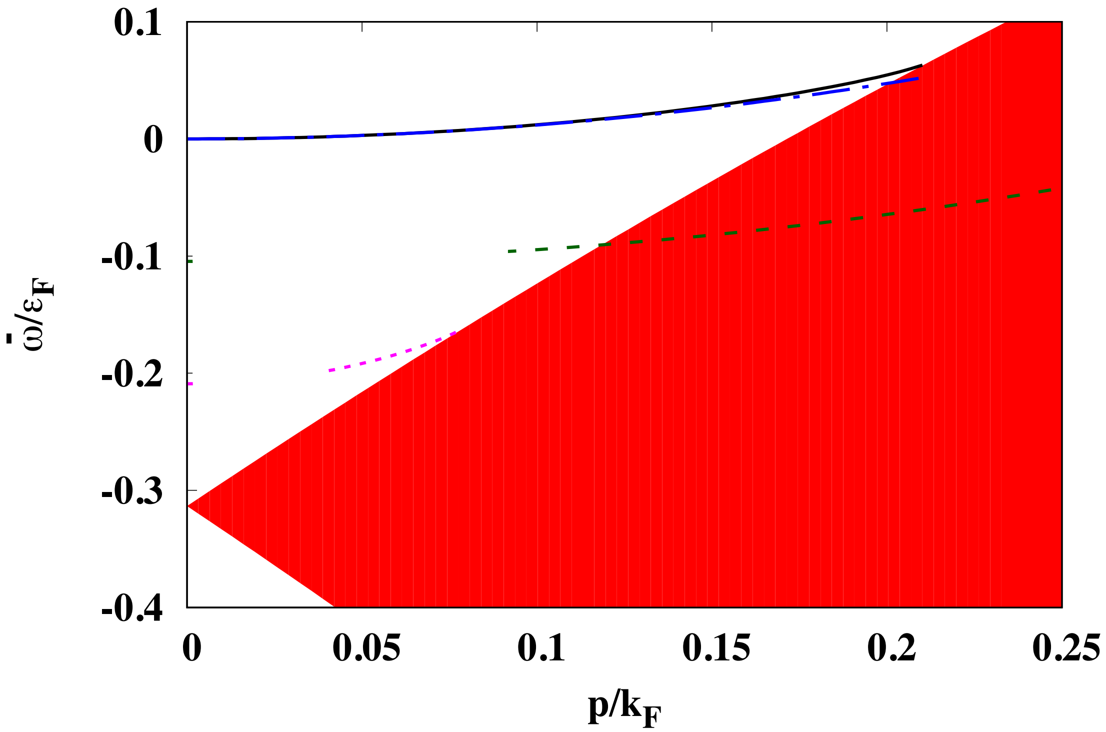


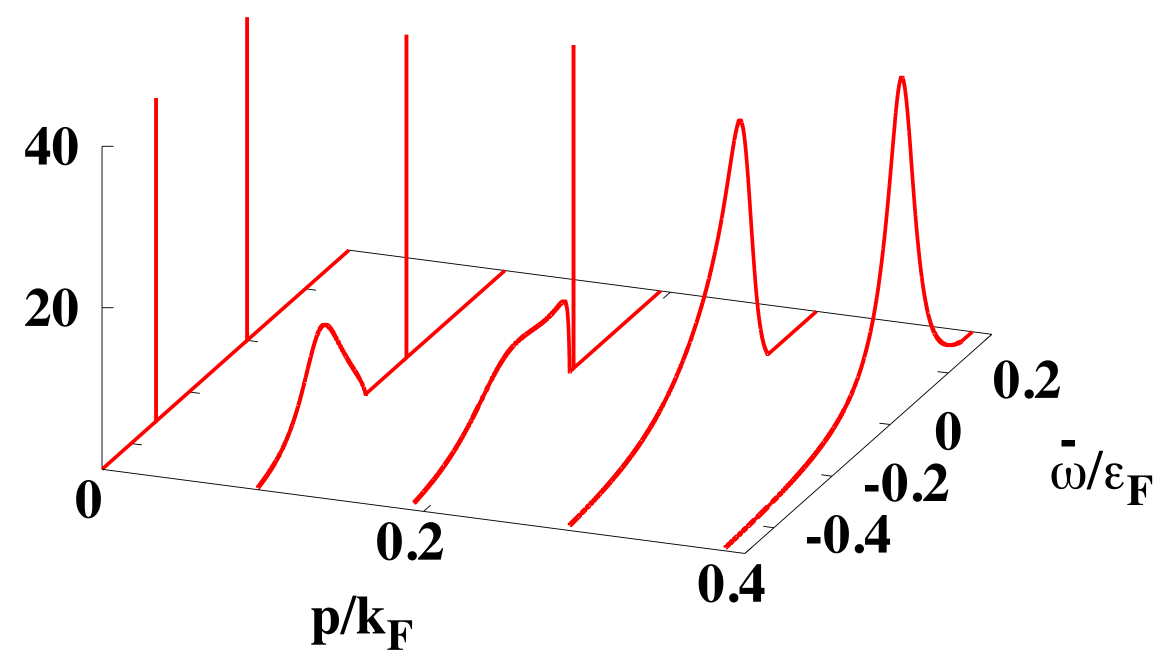
Fermion spectrum is also significantly affected by the mixing with the supercharge! Novel feature in BEC phase.

Very similar to QCD!



Goldstino spectrum (interacting case)





Similar result to the goldstino spectrum

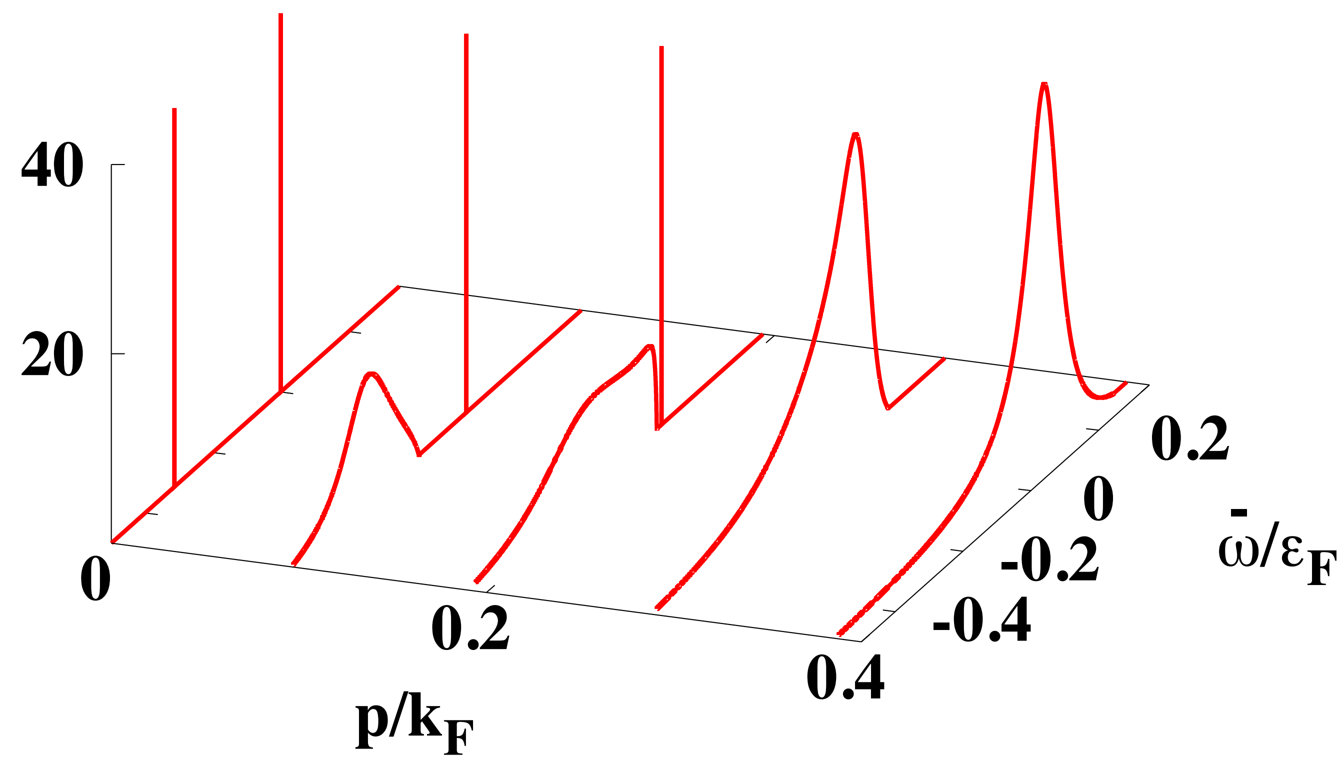
Small p: Goldstino pole + Continuum

$$\varpi = \alpha \frac{p^2}{2m} \qquad Z = \frac{\rho_b}{\rho} \qquad\qquad Z = \frac{\rho_f}{\rho}$$

Large p: Free particle pole $\quad \varpi = \frac{p^2}{2m} - \mu_b \qquad Z = 1$

Possible Experimental Detection

1. Fermion spectrum



At small *p*, it is quite different from the free result.
(2-peak structure: Goldstino+continuum)

It can be detected via the spectroscopy?

Possible Experimental Detection

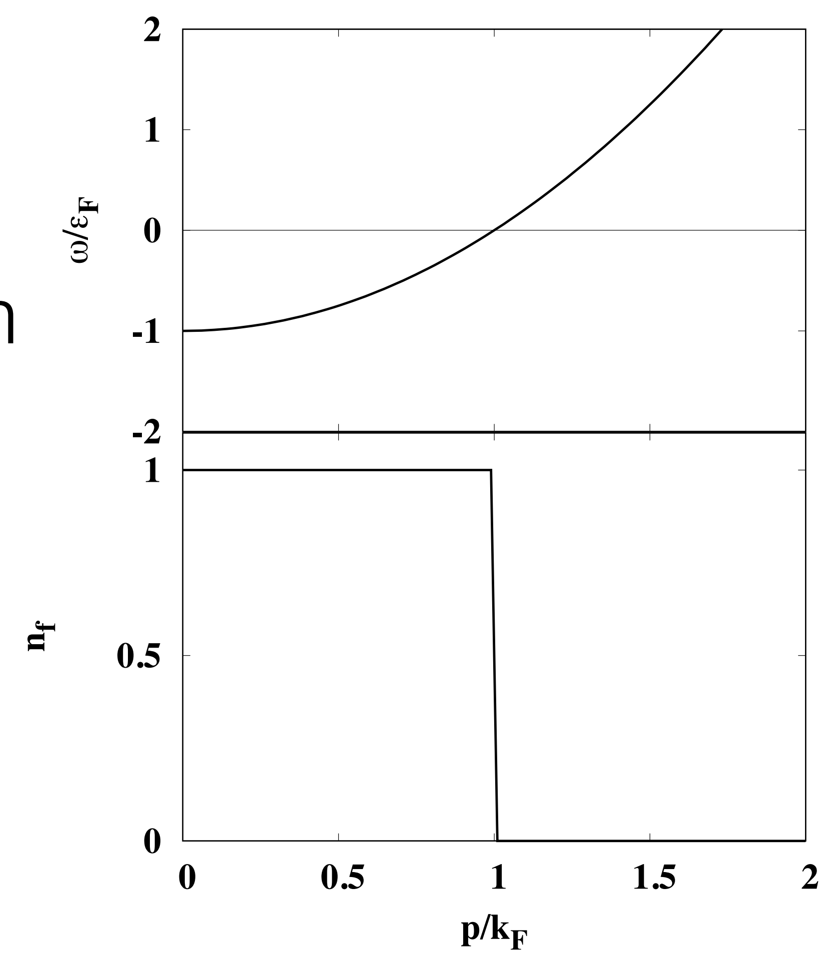
2. Fermion distribution

Because the fermion spectrum is modified, the fermion distribution in momentum space is also changed.

Free case

Only one branch in the spectrum

$\omega<0$ states are occupied ($p<k_F$).



Possible Experimental Detection

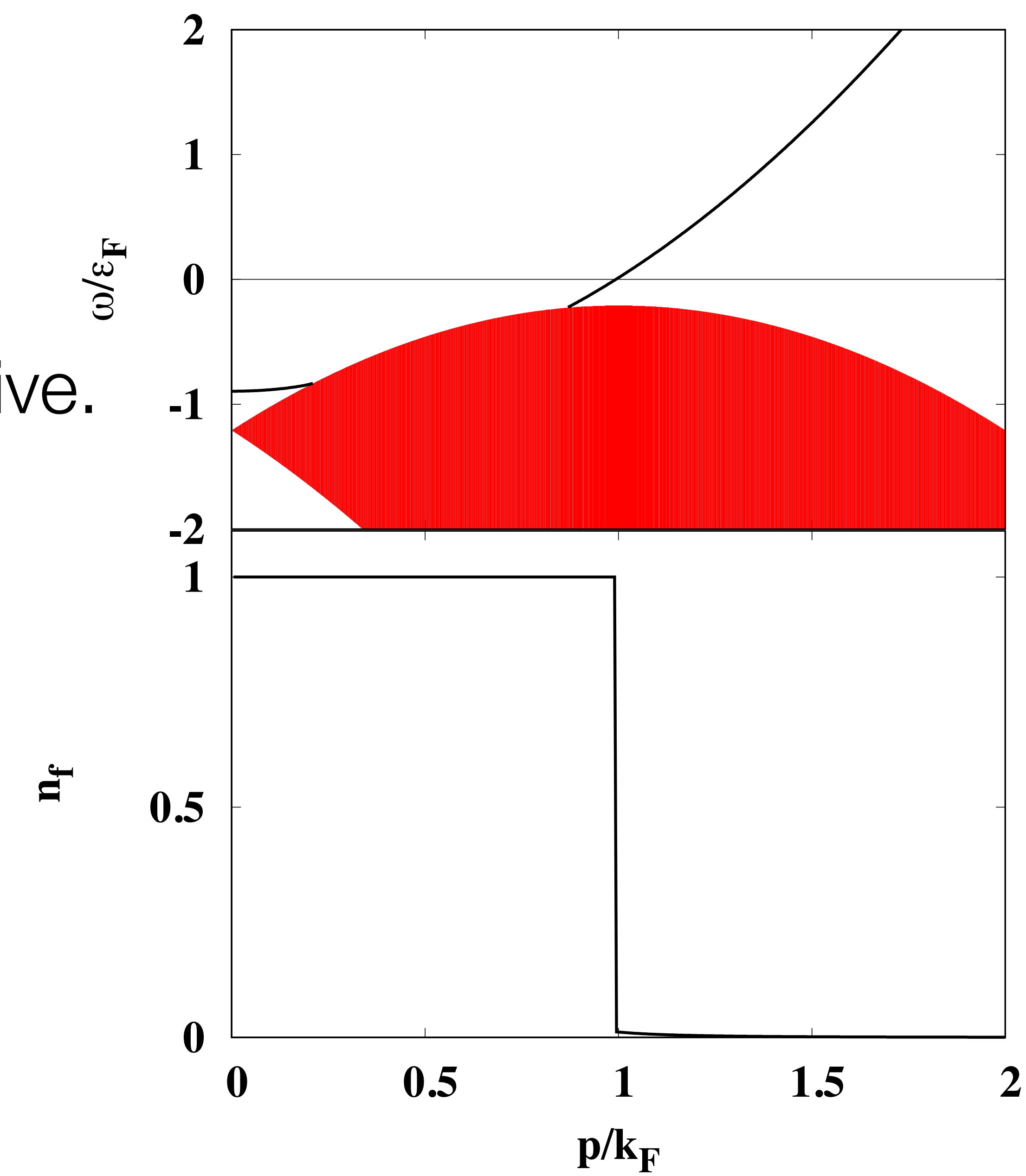
Weak coupling case

Goldstino pole+Continuum

Near k_F, the pole energy becomes positive.

Almost same as free case,
because the weight of the pole near k_F
is almost one.

$U\rho_f/\epsilon_F = 0.1$



Possible Experimental Detection

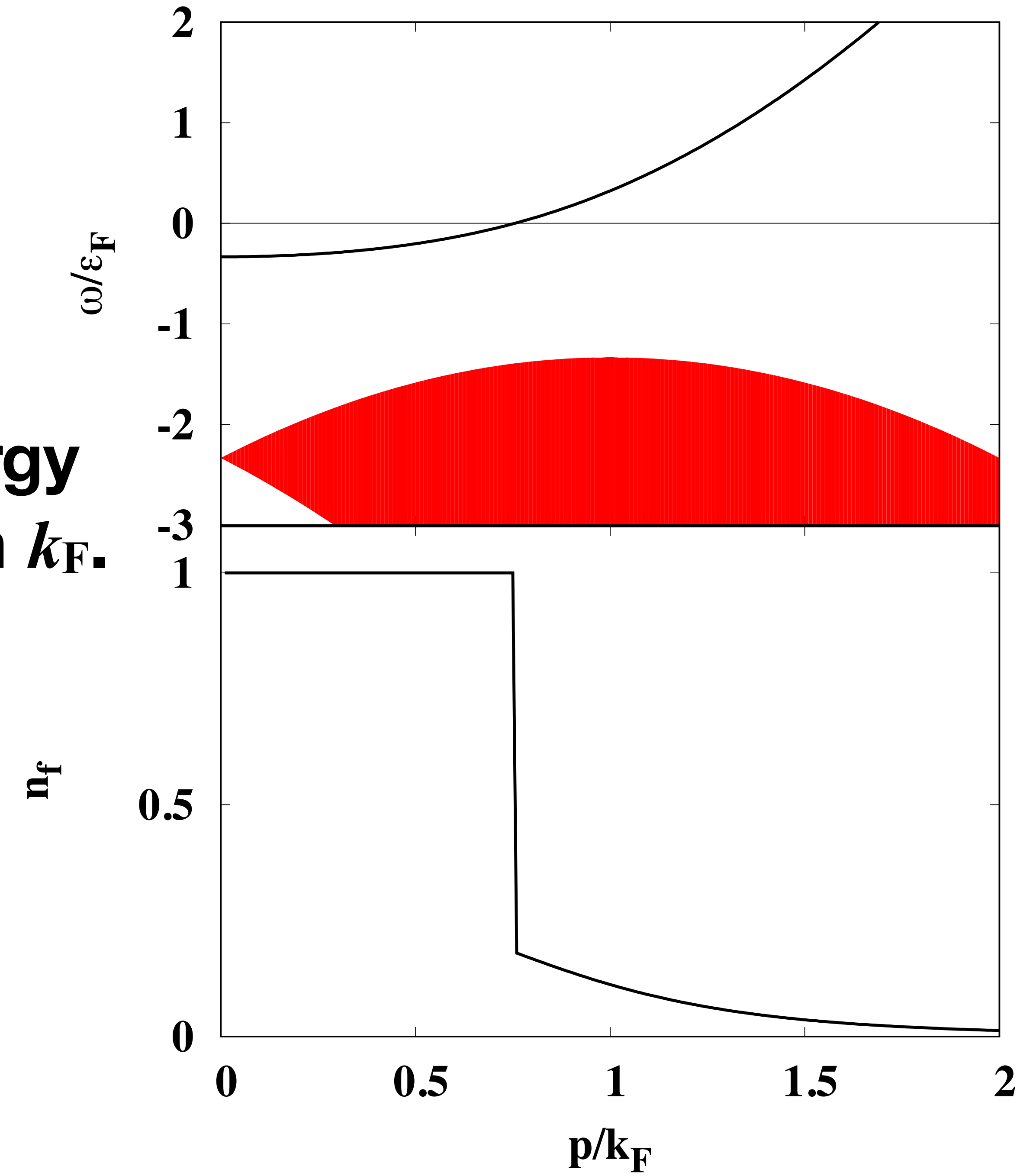
Strong coupling case

Goldstino pole and Continuum is separated, since the distance ($U\rho$) becomes large in strong coupling.

The energy at which the pole energy becomes positive is different from k_F.

Fermi sea is distorted.

$U\rho_f/\epsilon_F = 2/3$



Summary

- We analyzed **the spectral properties of the goldstino** in the absence/presence of interaction **with RPA.**

- We observed the **crossover from small p to large p region (from interaction dominant to free case)**.

- In BEC phase, the importance of the **mixing process between Fermion particle-Boson hole excitation and the Fermion hole-Boson particle excitation** was emphasized.

- We discussed the possibility for **experimental detection of the goldstino.**