



Transport coefficients of QGP in strong magnetic fields

Daisuke Satow (Frankfurt U. 

Collaborators: Koichi Hattori, Xu-Guang Huang (Fudan U, Shanghai 

Shiyong Li, Ho-Ung Yee (Uni. Illinois at Chicago 

Dirk Rischke (Frankfurt U. 

K. Hattori and D. S.,

Phys. Rev. D **94**, 114032 (2016).

K. Hattori, S. Li, D. S., H. U. Yee,

Phys. Rev. D **95**, 076008 (2017).

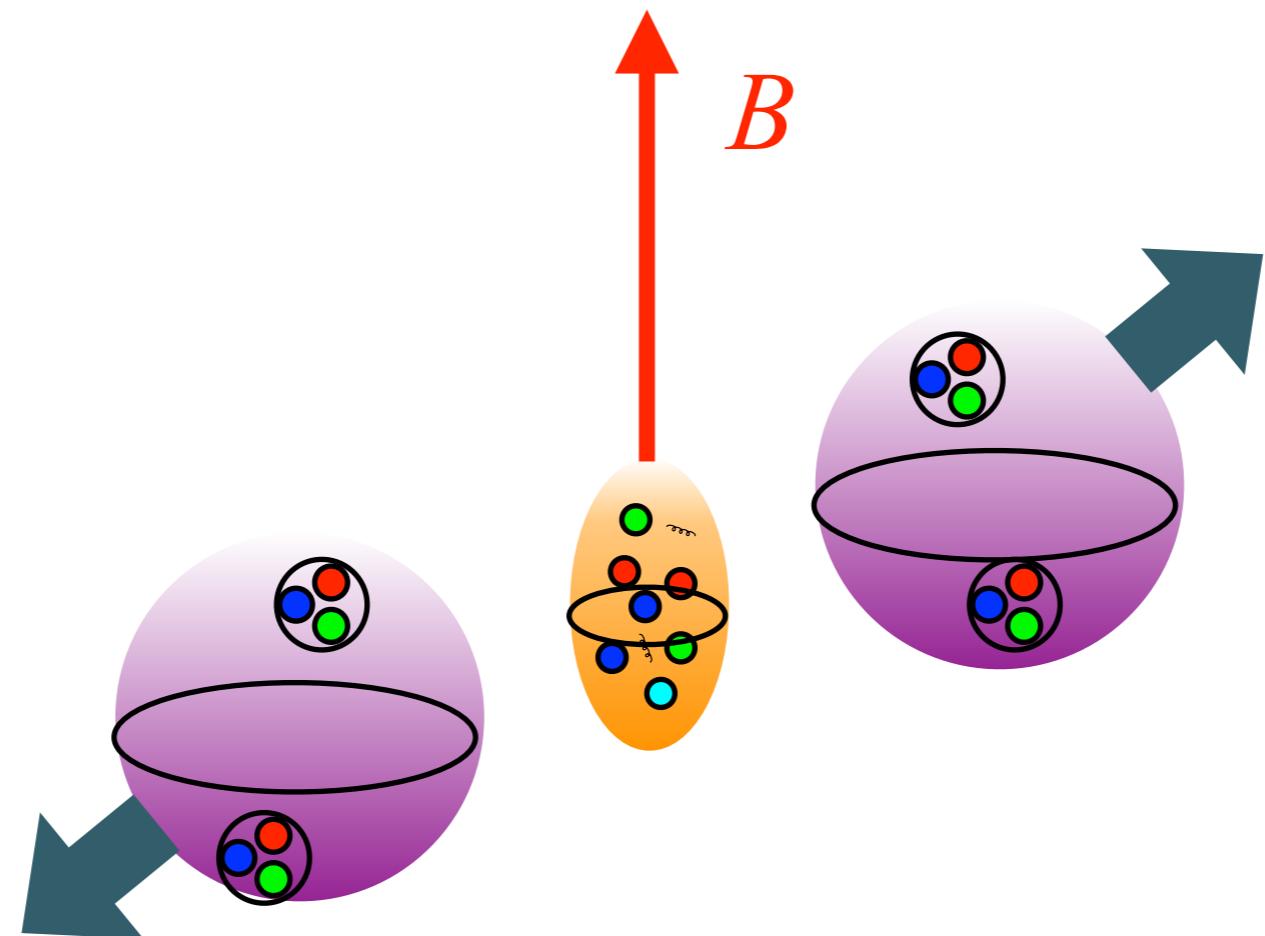


Outline

- (Long) Introduction
 - quarks and gluons in strong B
- Electrical Conductivity
- Bulk Viscosity
- Shear Viscosity (only power counting...)
- Summary

Introduction

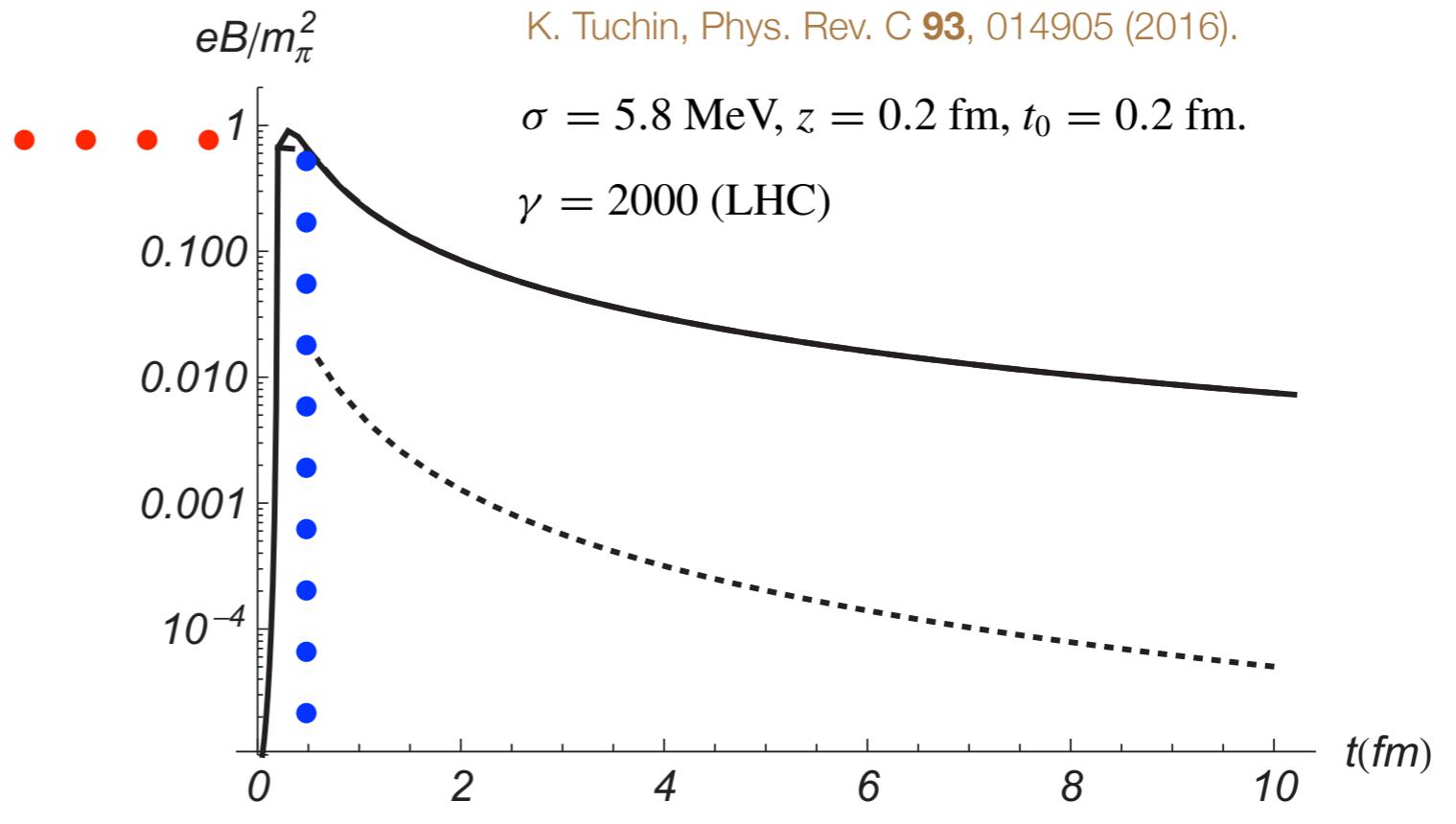
Strong magnetic field (B) may be generated in heavy ion collision due to Ampere's law.



Introduction

strength:

$\sqrt{eB} \sim 100$ [MeV]
(Comparable with T)



lifetime: ~ 0.3 fm

At thermalization time (~ 0.5 fm), there still may be strong B .

Heavy ion collision may give a chance to investigate QCD matter at **finite temperature** in **strong magnetic field**.

Introduction

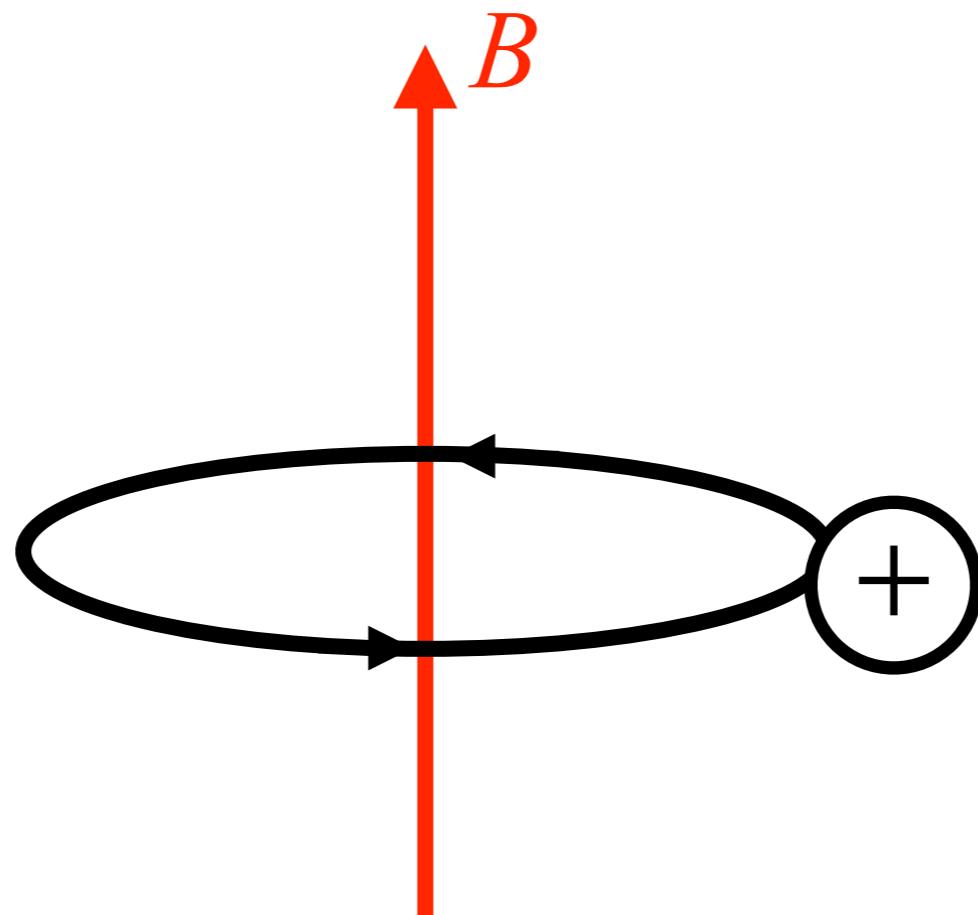
**How the particles behaves when
 \sqrt{eB} is much larger than the other
energy scales of the system?**

$(\sqrt{eB} \gg T, m, \Lambda_{\text{QCD}}\dots)$

Quark in Strong B

One-particle state of quark in magnetic field

Classical: Cyclotron motion due to Lorentz force



Quark in Strong B

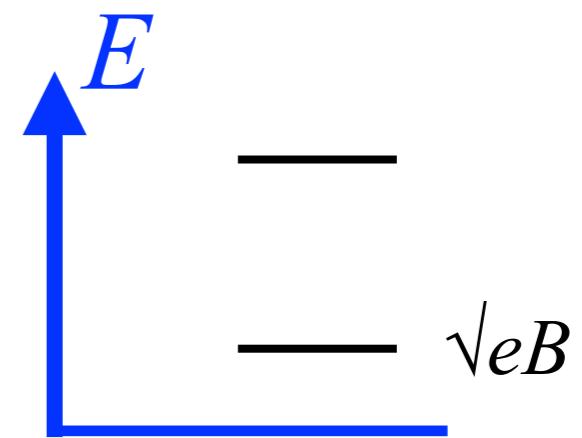
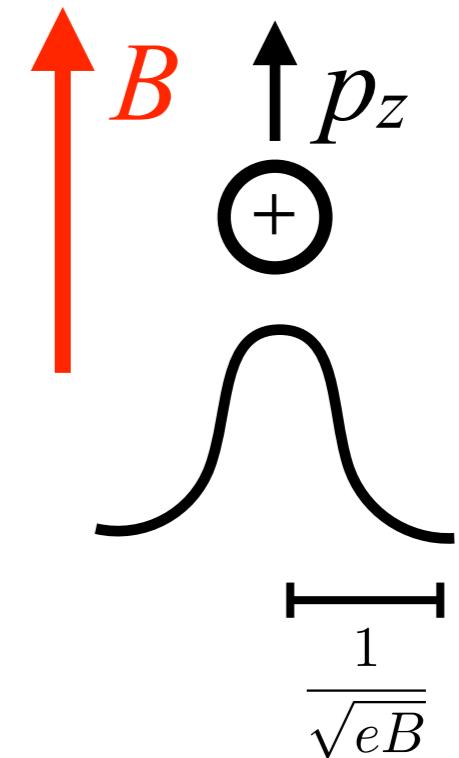
Quantum: Landau Quantization

Longitudinal: Plane wave

Transverse: Gaussian

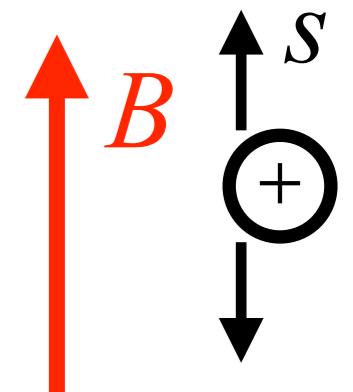
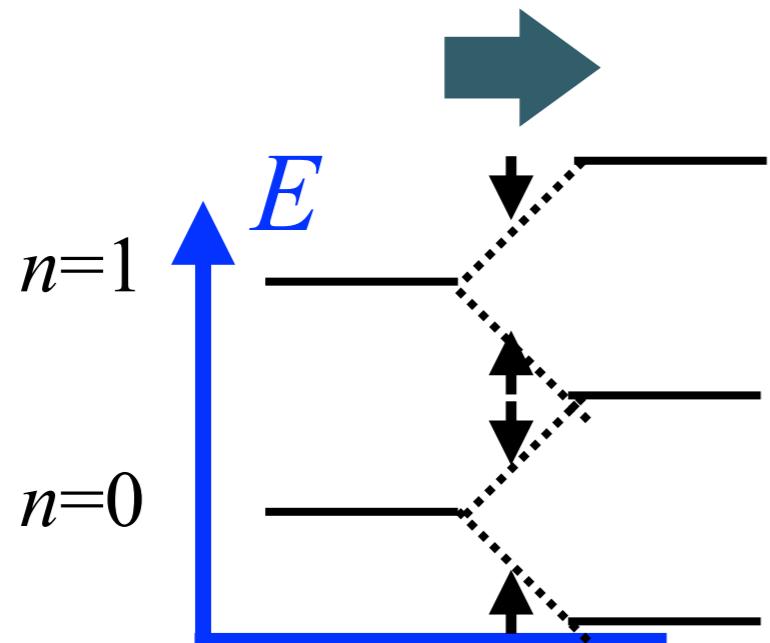
$$E_n = \sqrt{(p_z)^2 + m^2 + 2eB \left(n + \frac{1}{2} \right)}$$

The gap ($\sim \sqrt{eB}$) is generated by zero-point oscillation.



Quark in Strong B

For spin-1/2 particle, we have Zeeman effect:

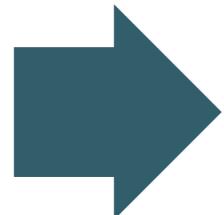


$$E_n = \sqrt{(p_z)^2 + m^2 + 2eB \left(n + \frac{1}{2} \mp \frac{1}{2} \right)}$$

When $n=0$ (LLL), gap is small ($m \sim 1\text{MeV}$).
When $n>0$, gap is large ($\sim \sqrt{eB} \sim 100\text{MeV}$)

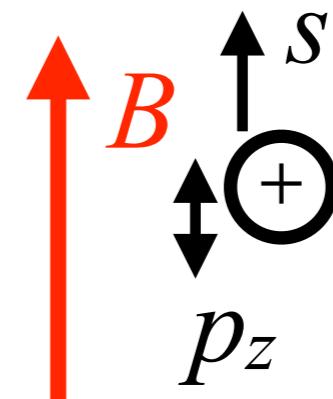
Lowest Landau Level (LLL) Approximation

When the typical energy of particle (T) is much smaller than gap (\sqrt{eB}),
the higher LL does not contribute ($\sim \exp(-\sqrt{eB}/T)$),
so **we can focus on the LLL.**



**One-dimension dispersion,
no spin degrees of freedom.**

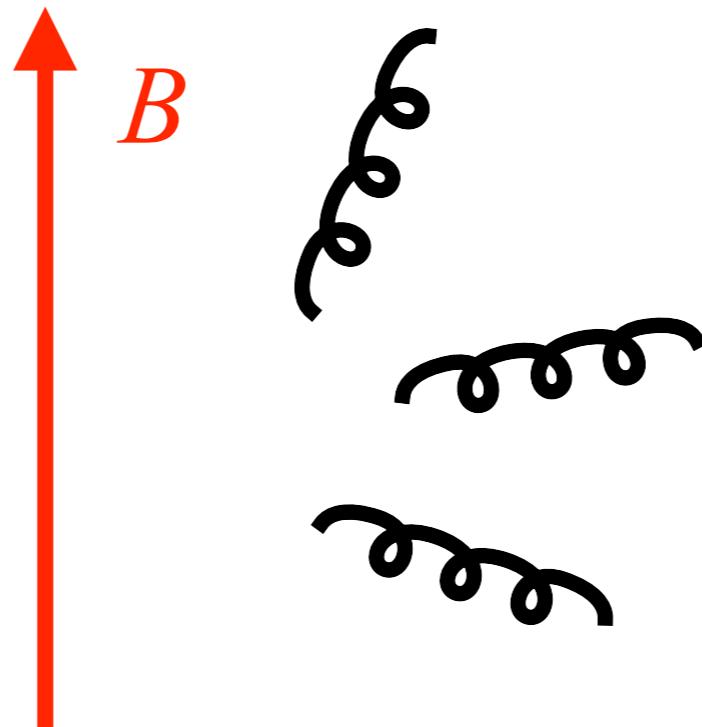
$$E_n = \sqrt{(p_z)^2 + m^2}$$



In heavy-ion collision, this condition can be marginally realized ($T \sim \sqrt{eB} \sim 100\text{MeV}$).
But in Weyl semi-metal, it is already realized ($T \sim 1\text{meV}$, $\sqrt{eB} \sim 10\text{eV}$).

Gluon in Strong B

Gluon does not have charge, so it does not feel B in the zeroth approximation.

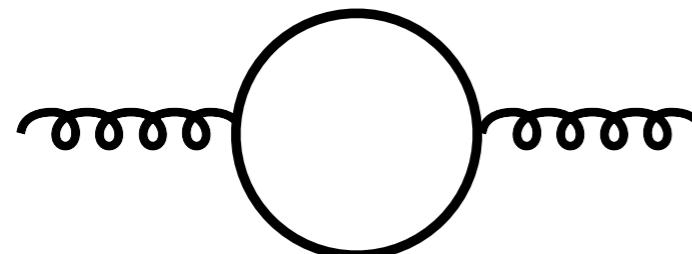


Massless boson in 3D

Gluon in Strong B

K. Fukushima, Phys. Rev. D **83**, 111501 (2011).

Coupling with (1+1)D quarks generates gluon mass.
(Schwinger mass generation)



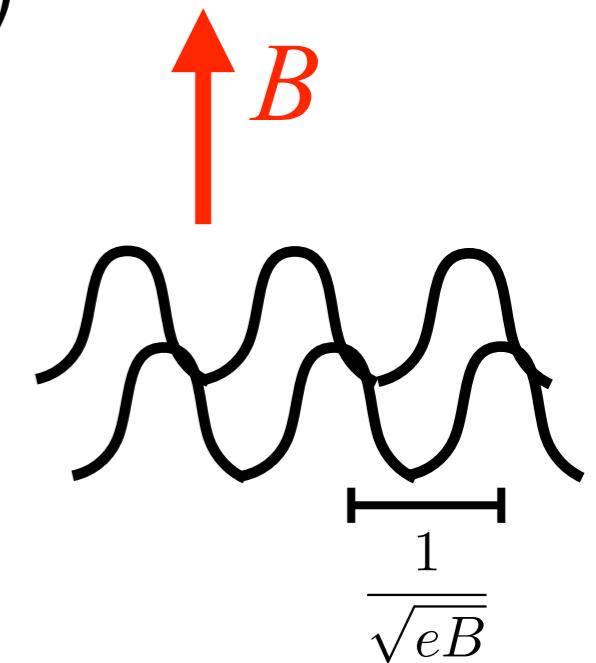
(surface density)
 $\sim (\text{average distance})^{-2} \sim eB$

Color factor

$$M^2 \equiv \frac{1}{2} \times \frac{g^2}{\pi} \sum_f \frac{|eB|}{2\pi} \sim g^2 eB$$

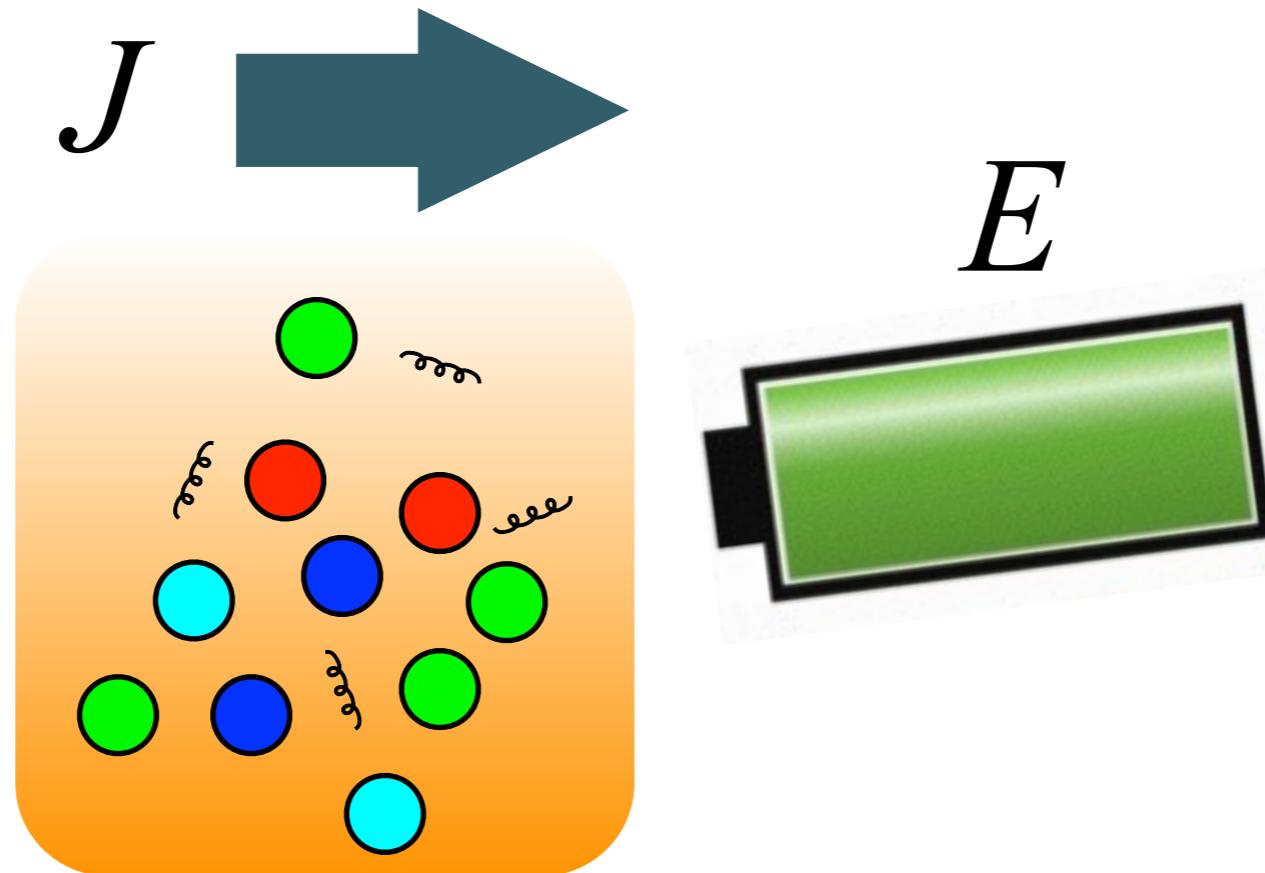
**Schwinger
mass**

Landau degeneracy



Electrical Conductivity

$$J = \sigma E$$



Motivation to Discuss Electrical Conductivity

Electrical conductivity is phenomenologically important because

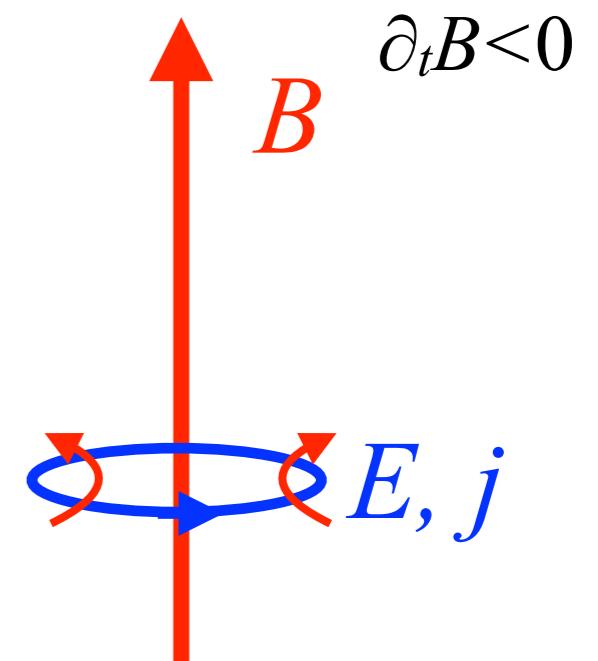
- Input parameter of magnetohydrodynamics (transport coefficient)

- May increase lifetime of \mathbf{B} (Lenz's law)

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$$

$$\cancel{\partial_t \mathbf{E}} = \nabla \times \mathbf{B} - \mathbf{j}$$

When σ is large

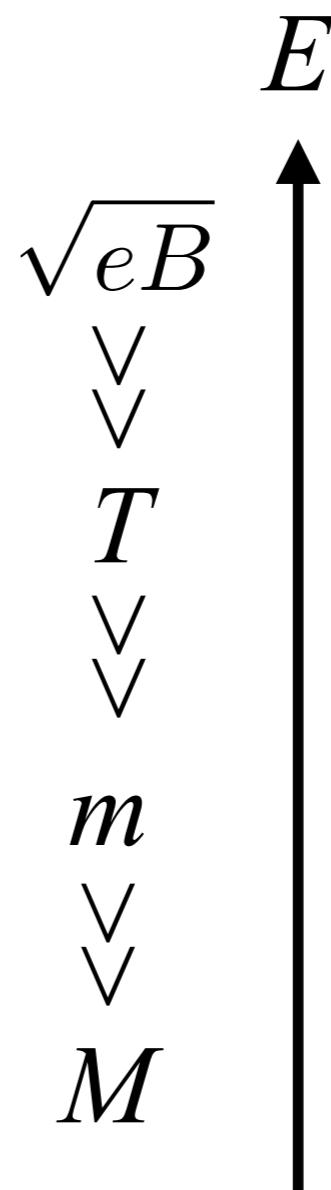
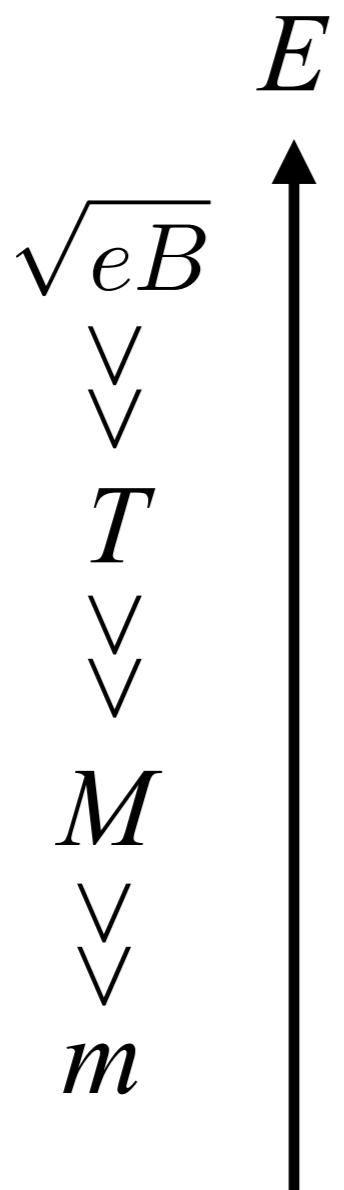


Hierarchy of Energy Scale at LLL

For ordering of m and M , we consider the both cases.

$$(m \ll M \text{ and } m \gg M)$$

$$(M \sim g\sqrt{eB})$$



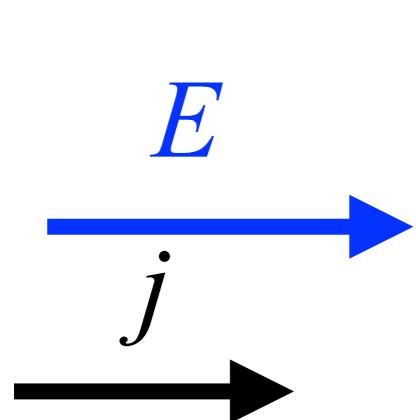
Electrical Conductivity

D. S., Phys. Rev. D, **90**, 034018 (2014).

Conductivity at weak B ($\sqrt{eB} \ll T$)

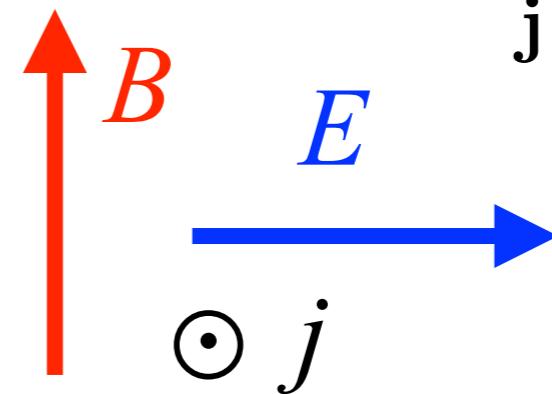
$B=0$

$$j^i = \sigma^{ij} E^j$$



weak B

$$\mathbf{j} = \sigma_1 \mathbf{E} \times \mathbf{B}$$



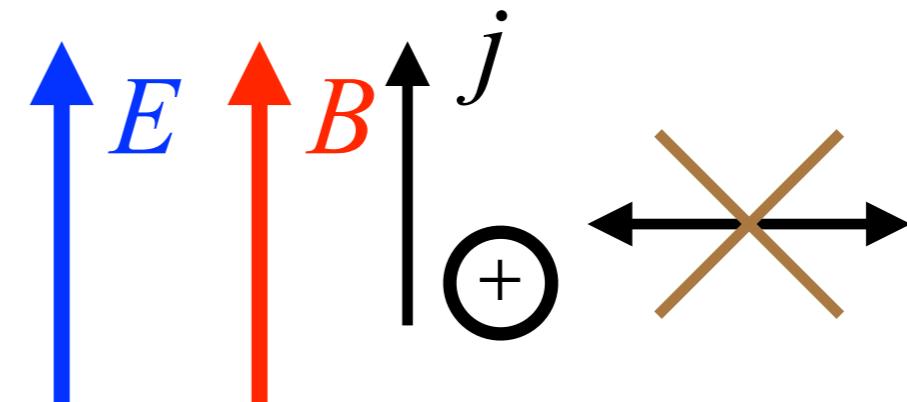
$$\sigma^{ij} = \begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}$$

$$\sigma^{ij} = \begin{bmatrix} \sigma_0 & \sigma_1 & 0 \\ -\sigma_1 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}$$

Electrical Conductivity

Strong B (LLL)

Quarks are confined in the direction of B , so there is no current in other directions.

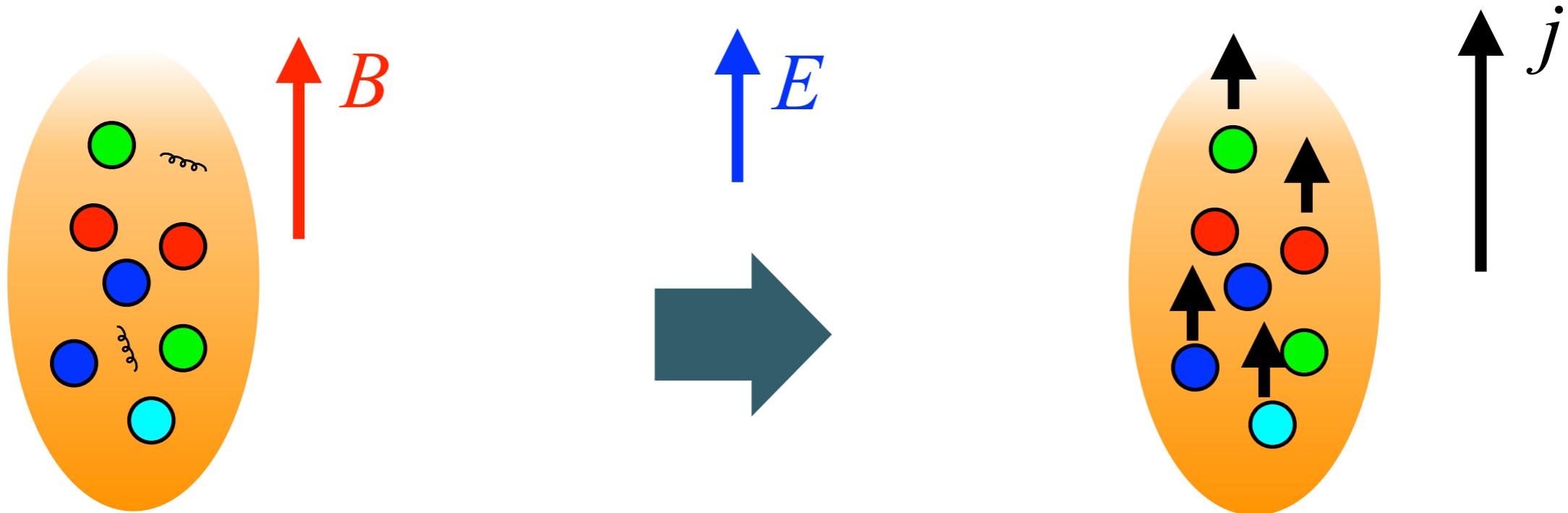


→ **σ^{33} is finite, other components are zero.
(Very different from weak B case)**

$$j^i = \sigma^{ij} E^j$$

$$\sigma^{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

Calculation of conductivity



Thermal equilibrium
in strong B

Linear response
against E

Slightly non-equilibrium,
finite j

$$n^f(k^3, T, Z) = n_F(\epsilon_k^L)$$

$$\epsilon_k^L \equiv \sqrt{(k^3)^2 + m^2}$$

$$n^f(k^3, T, Z) = n_F(\epsilon_k^L) + \delta n^f(k^3, T, Z)$$

$$j^3(T, Z) = 2e \sum_f q_f N_c \frac{|eq_f B|}{2\pi} \int \frac{dk^3}{2\pi} v^3 \delta n^f(k^3, T, Z) = \sigma^{33} E^3$$

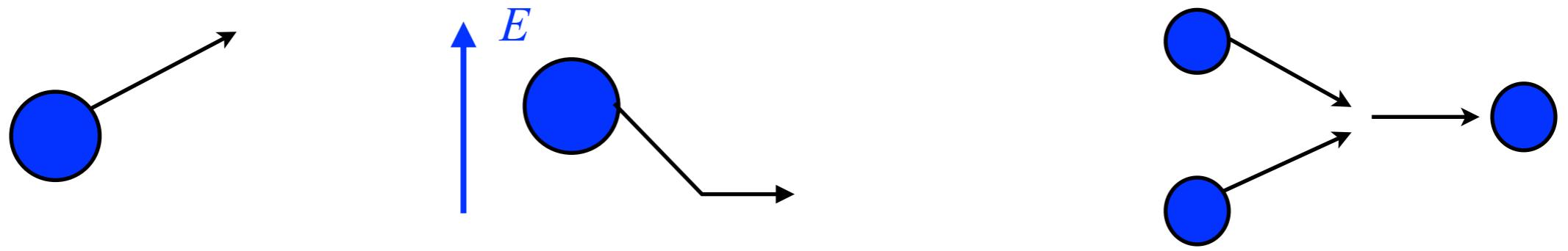
Landau degeneracy

$$v^3 \equiv \partial \epsilon_k^L / (\partial k^3) = k^3 / \epsilon_k^L$$

Evaluation of δn_F is necessary.

Calculation of conductivity

Evaluate n_F with **(1+1)D Boltzmann equation**



$$[\partial_T + v^3 \partial_Z + eq_f E^3(T, Z) \partial_{k^3}] n^f(k^3, T, Z) = C[n]$$

Constant and homogeneous E

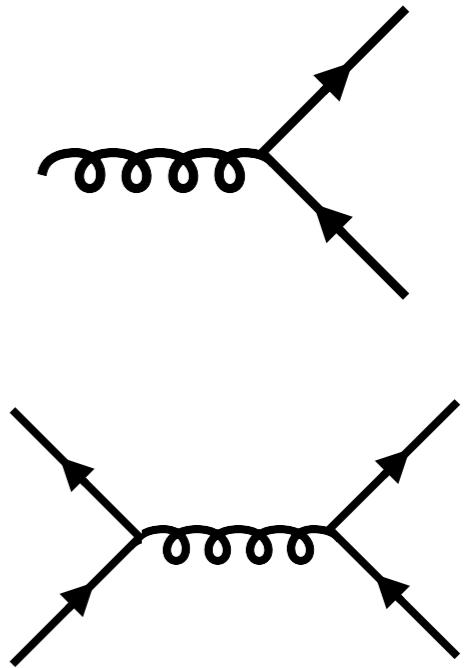
Possible Scattering Process for Conductivity

The scattering process is very different from that in $B=0\dots$

$B=0$

1 to 2 scattering is kinematically forbidden;
one massless particle can not decay to two
massless particles.

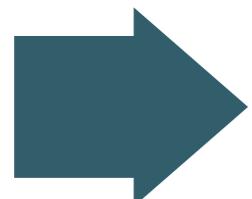
2 to 2 is leading process.



strong B

Gluon is effectively massive in (1+1)D

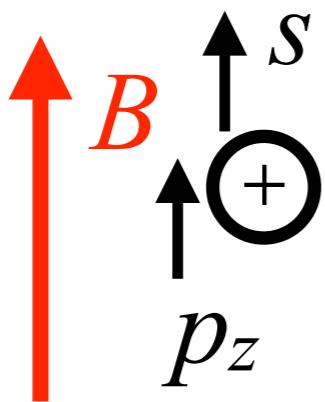
$$E = \sqrt{p_z^2 + p_{\perp}^2 + M^2}$$



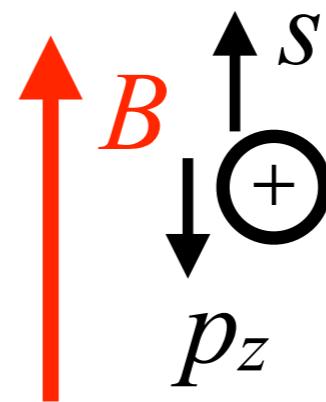
**Decay of a gluon into quark and anti-quark becomes kinematically possible.
(1 to 2)**

Chirality in (1+1)D

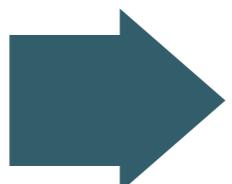
Spin is always up.



$$\chi = +1$$



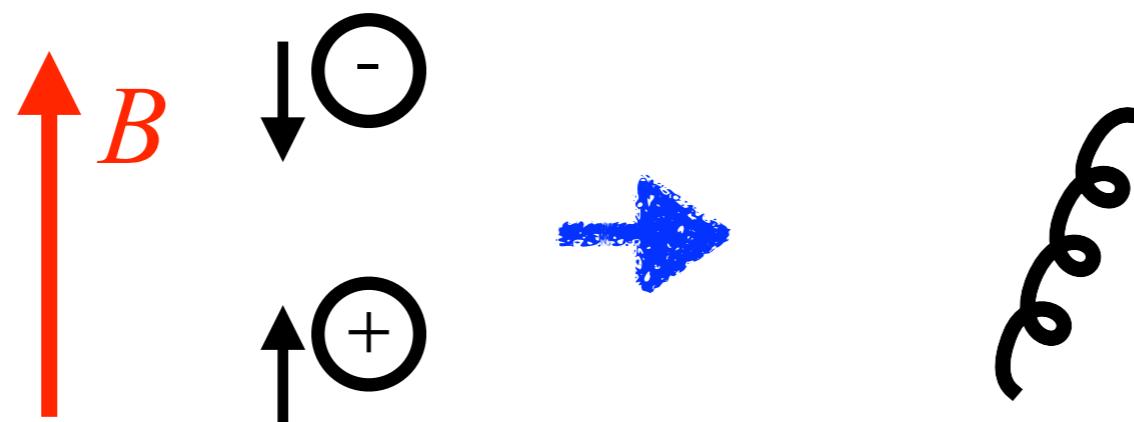
$$\chi = -1$$



When $m=0$, the direction of p_z determines chirality.

Chirality in (1+1)D

Chirality is conserved at $m=0$:



$$\chi = +1 + 1 = 2$$

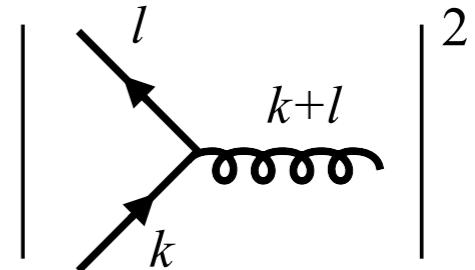
$$\chi = 0$$

→ 1 to 2 scattering is forbidden at $m=0$.

Possible Scattering Process for Conductivity

Collision term for 1 to 2:

$$C[n] = \frac{1}{2\epsilon_k^L} \int_l |M|^2 [n_B^{k+l}(1 - n_F^k)(1 - n_F^l) - (1 + n_B^{k+l})n_F^k n_F^l]$$

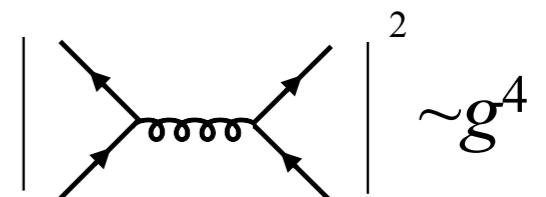


$$|M|^2 = 4g^2 C_f m^2$$

Vanishes at $m=0!$

(chirality conservation)

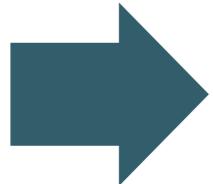
cf: 2 to 2



Calculation of conductivity

$$eq_f E^3(T, Z) \partial_{k^3} n^f(k^3, T, Z) = C[n]$$

linearize $n^f(k^3, T, Z) = n_F(\epsilon_k^L) + \delta n^f(k^3, T, Z)$



$$eq_f E^3 \beta v^3 n_F(\epsilon_k^L) [1 - n_F(\epsilon_k^L)] = C[\delta n^f(k^3, T, Z)]$$

$(\beta=1/T)$

$$C[n] = \frac{1}{2\epsilon_k^L} \int_l |M|^2 [n_B^{k+l} (1 - n_F^k) (1 - n_F^l) - (1 + n_B^{k+l}) n_F^k n_F^l]$$

linearize

$$C[\delta n] = -\frac{1}{2\epsilon_k^L} \int_l |M|^2 [\delta n_F^k (n_B^{k+l} + n_F^l) - \delta n_F^l (n_B^{k+l} + n_F^k)]$$

damping rate of quark ($= -2\xi_k \delta n^k F$)

Calculation of conductivity

Solution for δn^F with damping rate ξ_k

$$\delta n_F^k = -\frac{1}{2\xi_k} eq_f E^3 \beta v^3 n_F(\epsilon_k^L) [1 - n_F(\epsilon_k^L)]$$

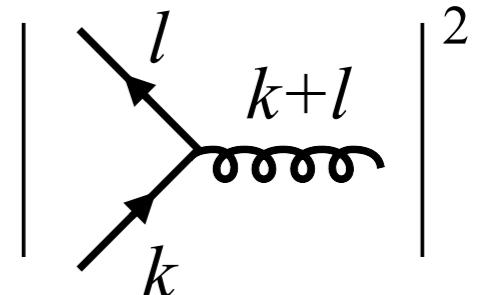
$$j^3(T, Z) = 2e \sum_f q_f N_c \frac{|B_f|}{2\pi} \int \frac{dk^3}{2\pi} v^3 \delta n^f(k^3, T, Z)$$

$$\rightarrow j^3 = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} 4\beta \int \frac{dk^3}{2\pi} (v^3)^2 \frac{1}{2\xi_k} n_F(\epsilon_k^L) [1 - n_F(\epsilon_k^L)] E^3$$

$$\sigma^{33}$$

Quark Damping Rate

$$\epsilon_k^L \xi_k = \frac{g^2 C_F m^2}{4\pi} \int_m^\infty dl^0 \frac{n_F(l^0) + n_B(l^0 + \epsilon_k^L)}{\sqrt{(l^0)^2 - m^2}}$$



leading-log approximation ($\ln[T/m] \gg 1$)

$l^0 \ll T$ dominates

$$\rightarrow \epsilon_k^L \xi_k \simeq \frac{g^2 C_F m^2}{4\pi} \left[\frac{1}{2} + n_B(\epsilon_k^L) \right] \int_m^\infty dl^0 \frac{1}{\sqrt{(l^0)^2 - m^2}}$$

$$\simeq \frac{g^2 C_F m^2}{4\pi} \left[\frac{1}{2} + n_B(\epsilon_k^L) \right] \ln \left(\frac{T}{m} \right)$$

matrix element

soft fermion and hard boson

$$n_F(1+n_B) + (1-n_F)n_B = n_F + n_B$$

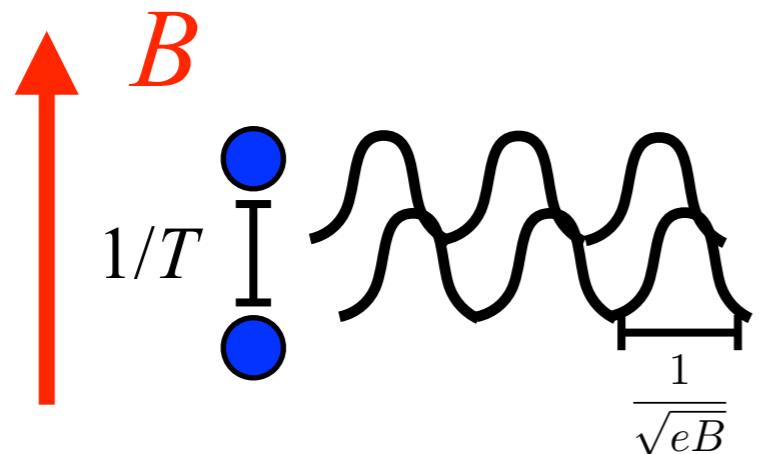
**log divergence
in phase space**

integral

UV cutoff: T

IR cutoff: m

Results



(average distance among quarks) $\sim 1/T$
 \rightarrow (quark density in 1D) $\sim T$

Quark density in 1D

$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/m)}$$

Landau
degeneracy

Quark damping
rate

Due to chirality conservation, collision is forbidden when $m=0$. Thus, $\sigma \sim 1/m^2$.

Despite $T \gg m$, it is very sensitive to m !!

When $M \gg m$, $\ln(T/m) \rightarrow \ln(T/M)$.

Other Term Does Not Contribute

$$C[\delta n] = -\frac{2g^2 C_F m^2}{\epsilon_k^L} \int_l [\delta n_F^k (n_B^{k+l} + n_F^l) - \underline{\delta n_F^l (n_B^{k+l} + n_F^k)}]$$

Other Term

$$\delta n_F^l = -\frac{eq_f}{2\xi_l} E^3 \partial_{l^3} n_F(\epsilon_l^L) : \text{odd in } l^3$$

function of $(\epsilon_k^L + \epsilon_l^L)$

$$(\text{Other term}) \sim \int_l \underbrace{(n_B^{k+l} + n_F^k)}_{\text{even in } l^3} \delta n_F^l \rightarrow 0$$

Our result (only retaining quark damping rate term)
is correct.

Equivalent Diagrams

Our calculation is based on (unestablished)
(1+1)D kinetic theory,

but actually **we can reproduce the same
result by field theory calculation.**

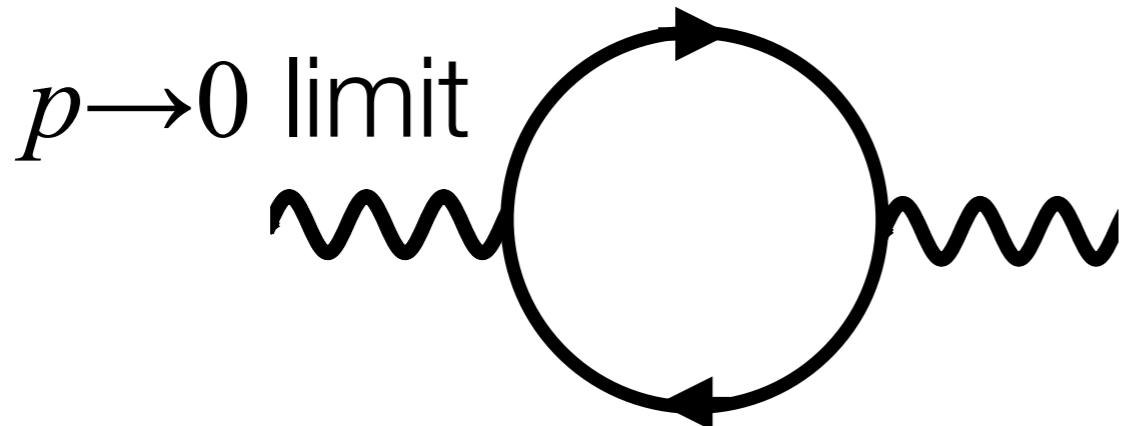
J. -S. Gagnon and S. Jeon, Phys. Rev. D **75**, 025014 (2007); **76**, 105019 (2007).

Kubo formula: $\sigma^{ij} \equiv \lim_{\omega \rightarrow 0} \frac{\Pi^{Rij}(\omega)}{i\omega}$

$$j^\mu \equiv e \sum_f q_f \bar{\psi}_f \gamma^\mu \psi_f$$

f : flavor index, q_f : electric charge

$$\Pi^{R\mu\nu}(x) \equiv i\theta(x^0) \langle [j^\mu(x), j^\nu(0)] \rangle$$



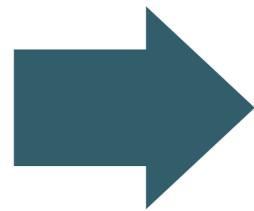
Possible Phenomenological Implications

1. Order Estimate

$$\alpha_s = \frac{g^2}{4\pi} = 0.3,$$

$m = 3\text{MeV}(u, d), 100\text{MeV}(s),$

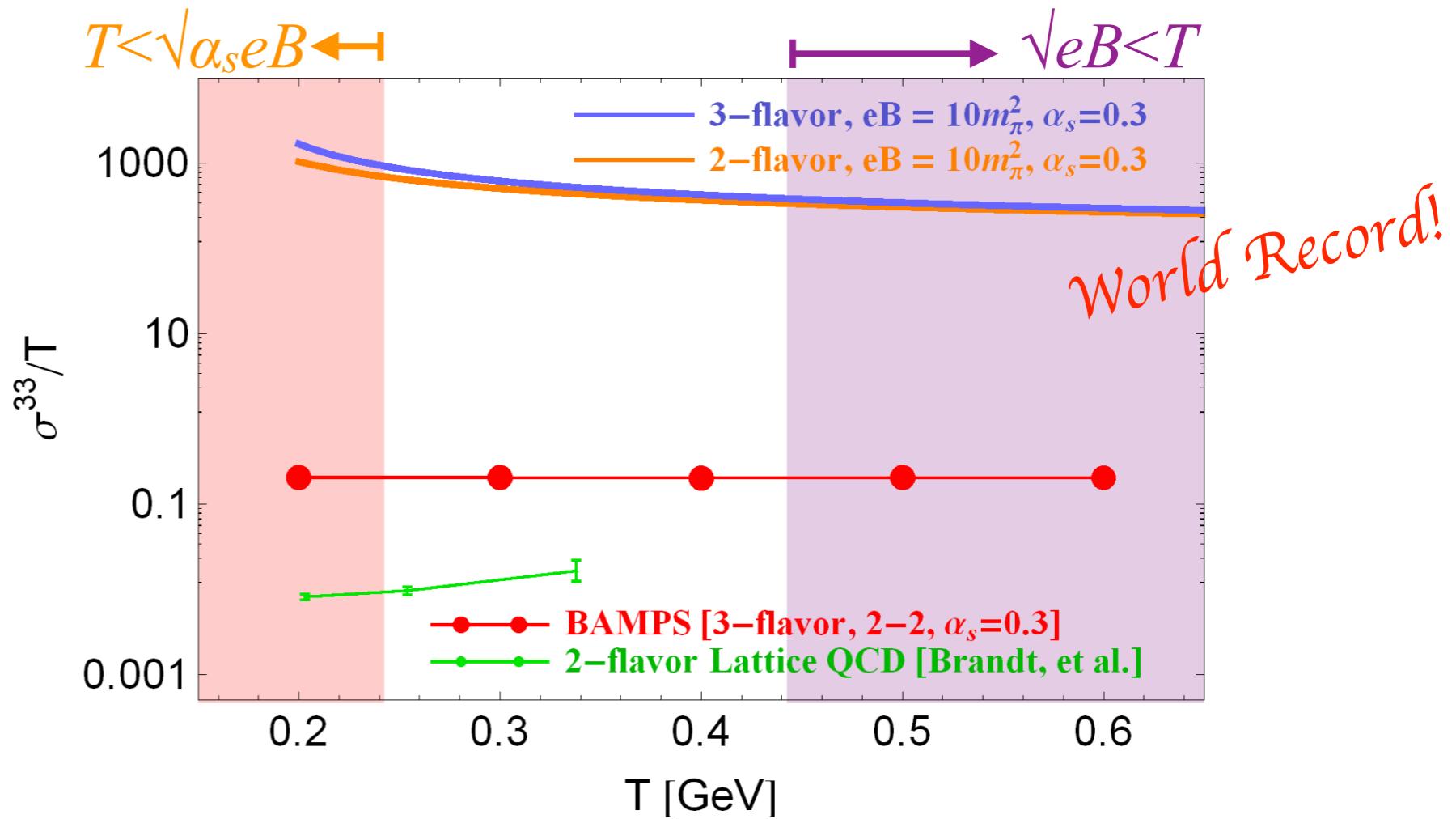
$eB = 10m_\pi^2 = (440\text{MeV})^2.$



$M=160\text{MeV} \gg m$

$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/M)}$$

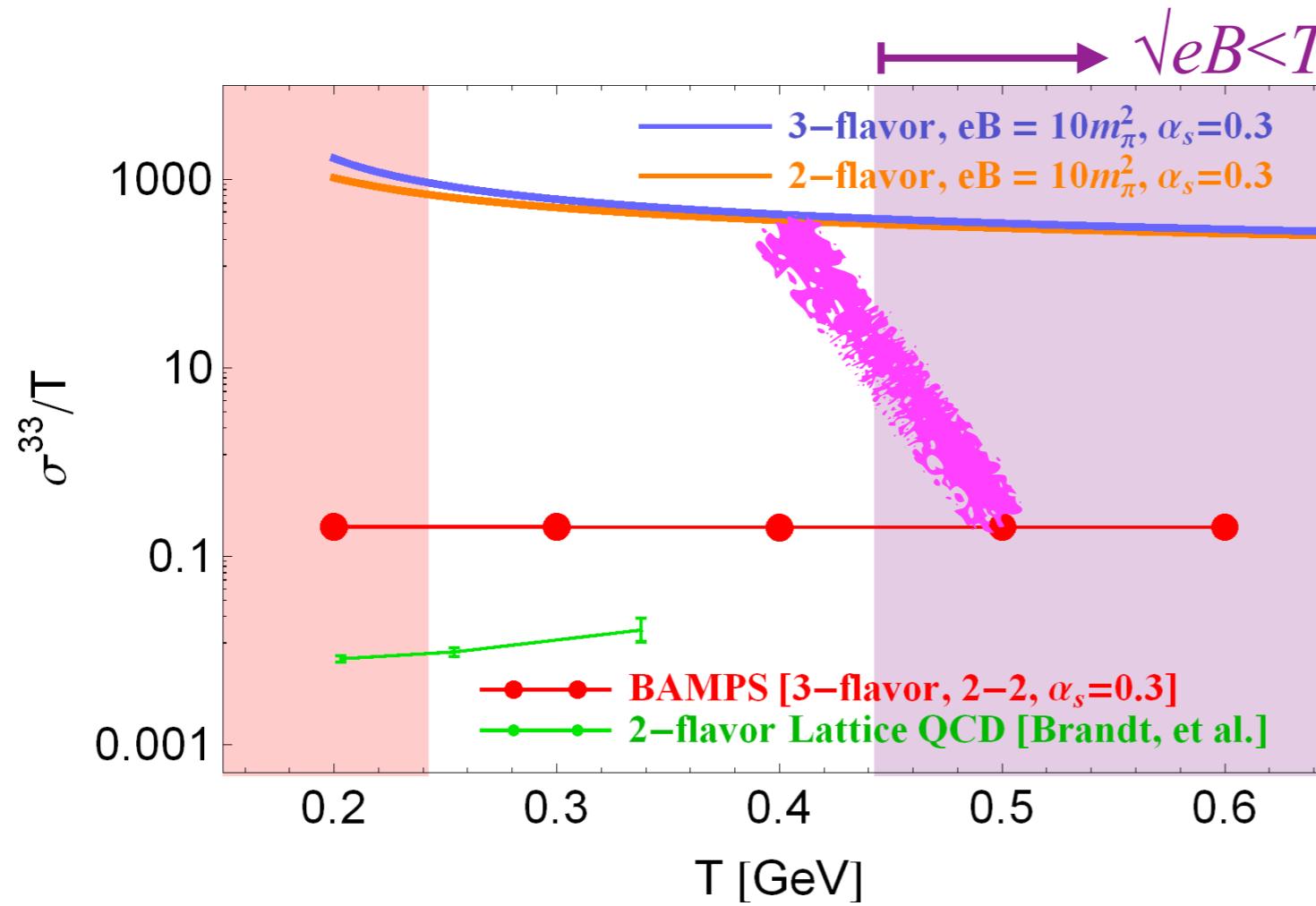
Because of m^{-2} dependence, s contribution is very small.



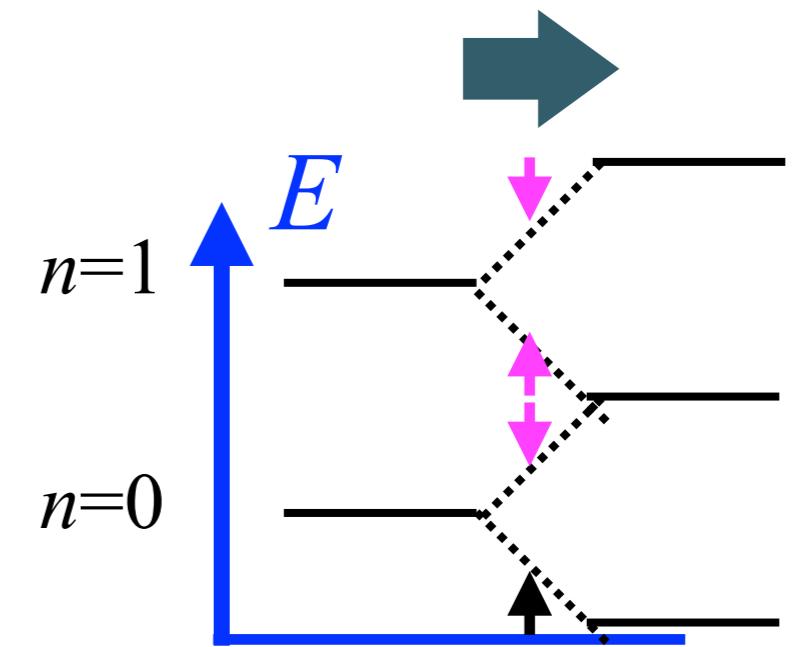
BAMPS: M. Greif, I. Bouras, C. Greiner and Z. Xu, Phys. Rev. D **90**, 094014 (2014).

Lattice: B. B. Brandt, A. Francis, B. Jaeger and H. B. Meyer, Phys. Rev. D **93**, 054510 (2016).

Possible Phenomenological Implications



$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/M)}$$



Beyond LLL approximation, there are also spin down particles, so the scattering is not suppressed by m^2 .

→ **σ^{33} is expected to be smaller at large T , so that it smoothly connects with $B=0$ result.**

Possible Phenomenological Implications

2. Soft Dilepton Production

G. D. Moore and J. -M. Robert, hep-ph/0607172.

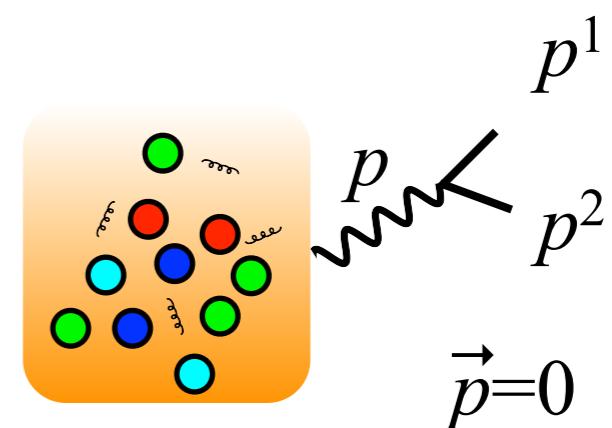
$$\frac{d\Gamma}{d^4 p} = \frac{\alpha}{12\pi^4 \omega^2} T \sigma^{33}$$

\therefore (virtual photon emission rate) $\sim n_B(\omega) \text{Im} \Pi^\mu_\mu \sim T \sigma^{33}$

σ^{33} is large

(photon interaction
energy w leptons)

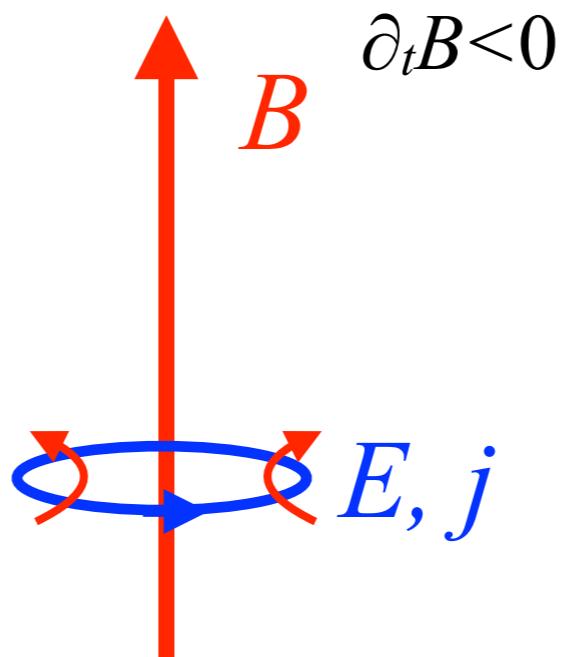
$$e\sqrt{eB} \ll \omega \ll \frac{g^2 m^2}{T} \ln \left(\frac{T}{M} \right)$$



→ Soft dilepton production is enhanced by B ?

Possible Phenomenological Implications

3. Back Reaction to EM Fields



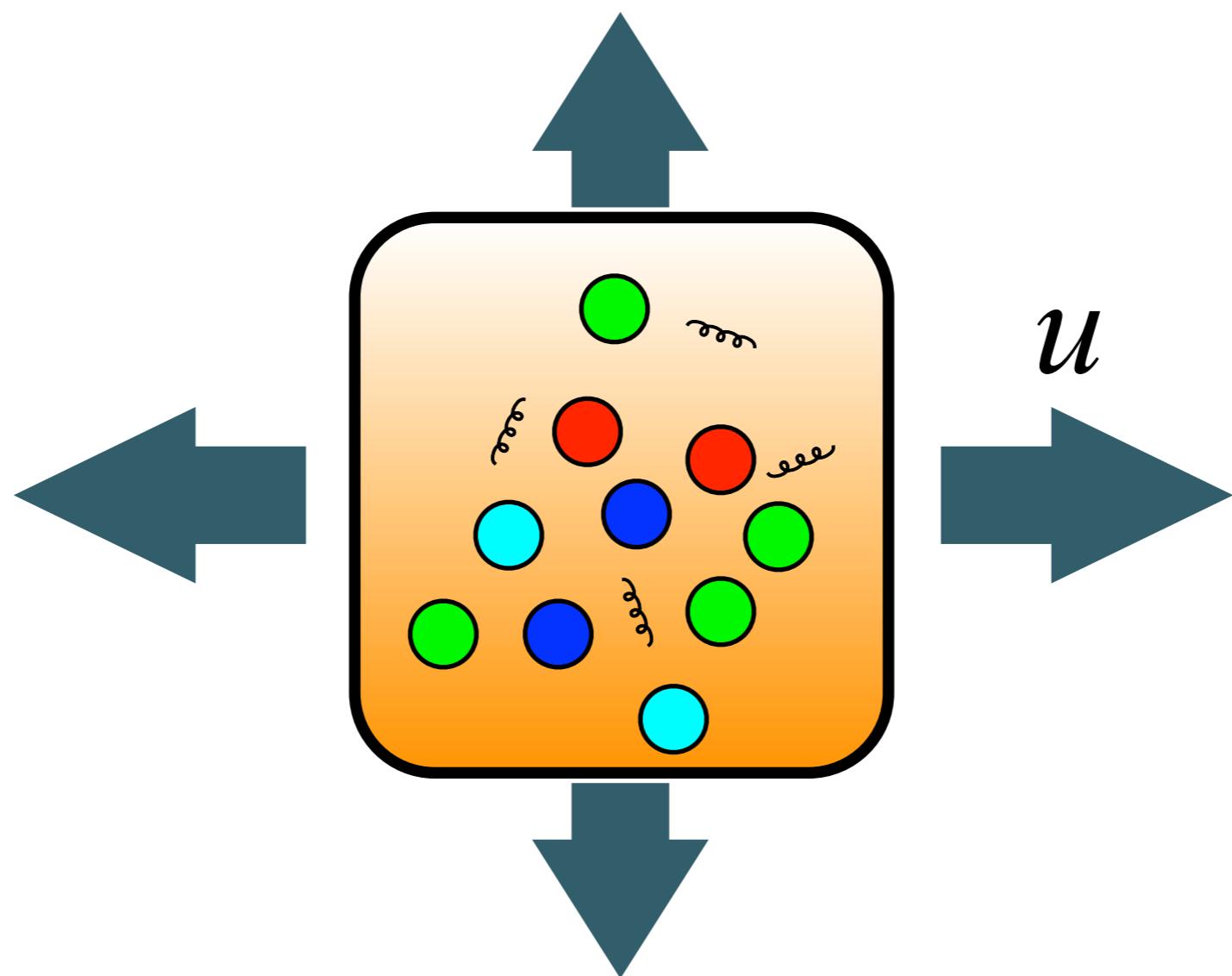
Bad news:

In LLL approximation, we have **no current in transverse plane**, so **Lenz's law does NOT work!**

The lifetime of B does not increase...

Bulk Viscosity

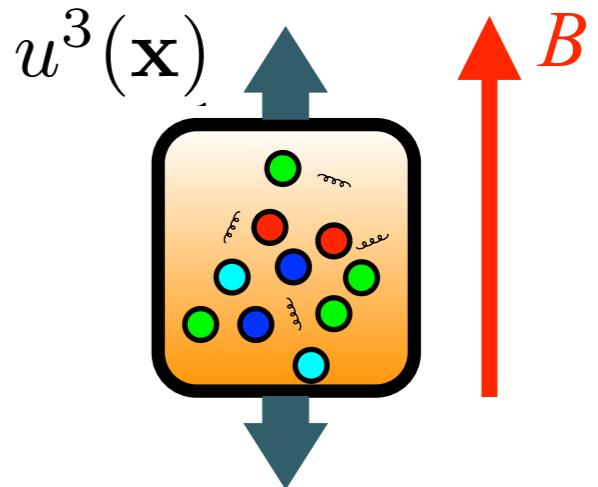
$$\Delta P = -\zeta \nabla \cdot u$$



Linearized Boltzmann equation

Boltzmann eq. without E $(\partial_t + v^3 \partial_z) f(\mathbf{k}, \mathbf{x}, t) = C[f]$

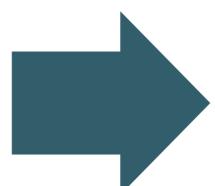
Expansion in direction of B :



$$f_{\text{eq}}(\mathbf{k}, \mathbf{x}, t) + \delta f(\mathbf{k}, \mathbf{x}, t) \quad f_{\text{eq}}(\mathbf{k}, \mathbf{x}, t) \equiv [\exp\{\beta(t)\gamma_u(\epsilon_k^L - k^3 u^3(\mathbf{x}))\} + 1]^{-1}$$

nonequilibrium deviation (responsible for viscosity)

linearize



$$-\beta n_F(\mathbf{k})[1 - n_F(\mathbf{k})]X(\mathbf{x}) [\Theta_\beta \epsilon_k^L - v^3 k^3] \simeq -\tau_k^{-1} \delta f(\mathbf{k}, \mathbf{x}, t)$$

expansion decreases T

$X(\mathbf{x}) \equiv \partial_z u^3(\mathbf{x})$: expansion (source term)

$$\Theta_\beta \equiv \left(\frac{\partial P_\parallel}{\partial \varepsilon} \right)_{n, B} : (\text{speed of sound})^2$$

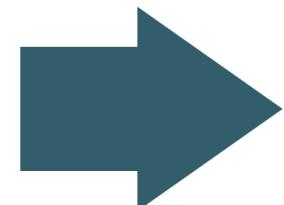
Linearized Boltzmann equation

$$\text{solution: } \delta f(\mathbf{k}, \mathbf{x}, t) \simeq \beta \tau_k n_F(\mathbf{k}) [1 - n_F(\mathbf{k})] X(\mathbf{x}) [\Theta_\beta \epsilon_k^L - v^3 k^3]$$

Conformal case ($m=0$):

1 (not 1/3, since the quarks
live in one-dimension)

$$\begin{array}{c} \nearrow \\ \Theta_\beta \epsilon_k^L \end{array} \quad \begin{array}{c} \nearrow \\ v^3 \end{array} \quad \begin{array}{c} \nearrow \\ k^3 \end{array} \quad \begin{array}{c} \nearrow \\ 1 \end{array}$$


$$\delta f = 0.$$

**In conformal case, the system is at equilibrium
even after the expansion.**

No nonequilibrium deviation, no bulk viscosity.

Linearized Boltzmann equation

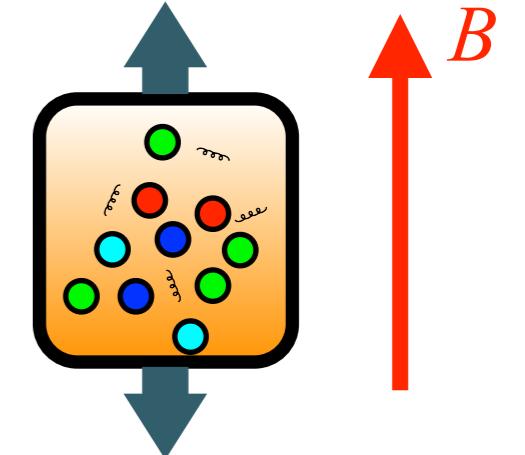
solution: $\delta f(\mathbf{k}, \mathbf{x}, t) \simeq \beta \tau_k n_F(\mathbf{k}) [1 - n_F(\mathbf{k})] X(\mathbf{x}) \frac{[\Theta_\beta \epsilon_k^L - v^3 k^3]}{[(k^3)^2 - \Theta_\beta (\epsilon_k^L)^2] / \epsilon_k^L}$

$$\Delta(P_{\parallel} - \Theta_{\beta} \epsilon) = -3 \zeta_{\parallel} X(x)$$

X-G. Huang, A. Sedrakian, D. Rischke,
Annals Phys. **326** 3075 (2011).

expansion decreases T .

So even in no-dissipative case, the pressure changes, and thus this contribution needs to be subtracted.



$$\delta[P_{\parallel} - \Theta_{\beta} \epsilon] = N_c \frac{|eq_f B|}{2\pi} \frac{1}{\pi} \int_{-\infty}^{\infty} dk^3 \frac{(k^3)^2 - \Theta_\beta (\epsilon_k^L)^2}{\epsilon_k^L} \delta f(\mathbf{k}, \mathbf{x}, t)$$

$[(k^3)^2 - \Theta_\beta (\epsilon_k^L)^2]$

Two conformal breaking factor $[(k^3)^2 - \Theta_\beta (\epsilon_k^L)^2]^2 \sim (m^2)^2$

Results

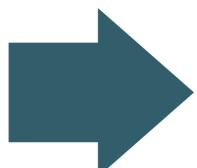
$$\zeta_{||} = N_c \frac{|eq_f B|}{2\pi} \frac{9.6m^2}{g^2 \pi^2 C_f T \ln(T/m)}$$

Conformal breaking

$$\frac{(m^2)^2}{m^2} = m^2$$

Chirality non-conservation

Same as the conductivity, except for
the **extra m^4 dependence,**
due to the conformal breaking
factor.



**s quark contribution would dominates over u/d contribution,
in contrast to the electrical conductivity.**

(Same as $B=0$ case)

$$\zeta \sim \frac{m^4}{g^4 T \ln(g^{-1})}$$

P. Arnold, C. Dogan, G. Moore,
Phys. Rev. D **74** 085021 (2006).

Possible Phenomenological Implications

1. Order Estimate

Contribution from s quark

$$\zeta_{\parallel} = N_c \frac{|eq_f B|}{2\pi} \frac{9.6m_s^2}{g^2\pi^2 C_f T \ln(T/m)}$$

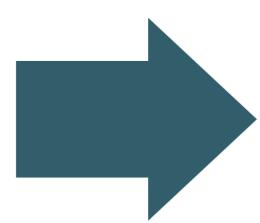
$$\alpha_s = \frac{g^2}{4\pi} = 0.3,$$
$$m = 3\text{MeV}(u,d), 100\text{MeV}(s),$$
$$eB = 10m_\pi^2 = (440\text{MeV})^2.$$

$M=160\text{MeV}>>m$

($B=0$)

$$\zeta_{\parallel} = \zeta_{\perp} \simeq 0.13 \frac{m_s^4}{T}$$

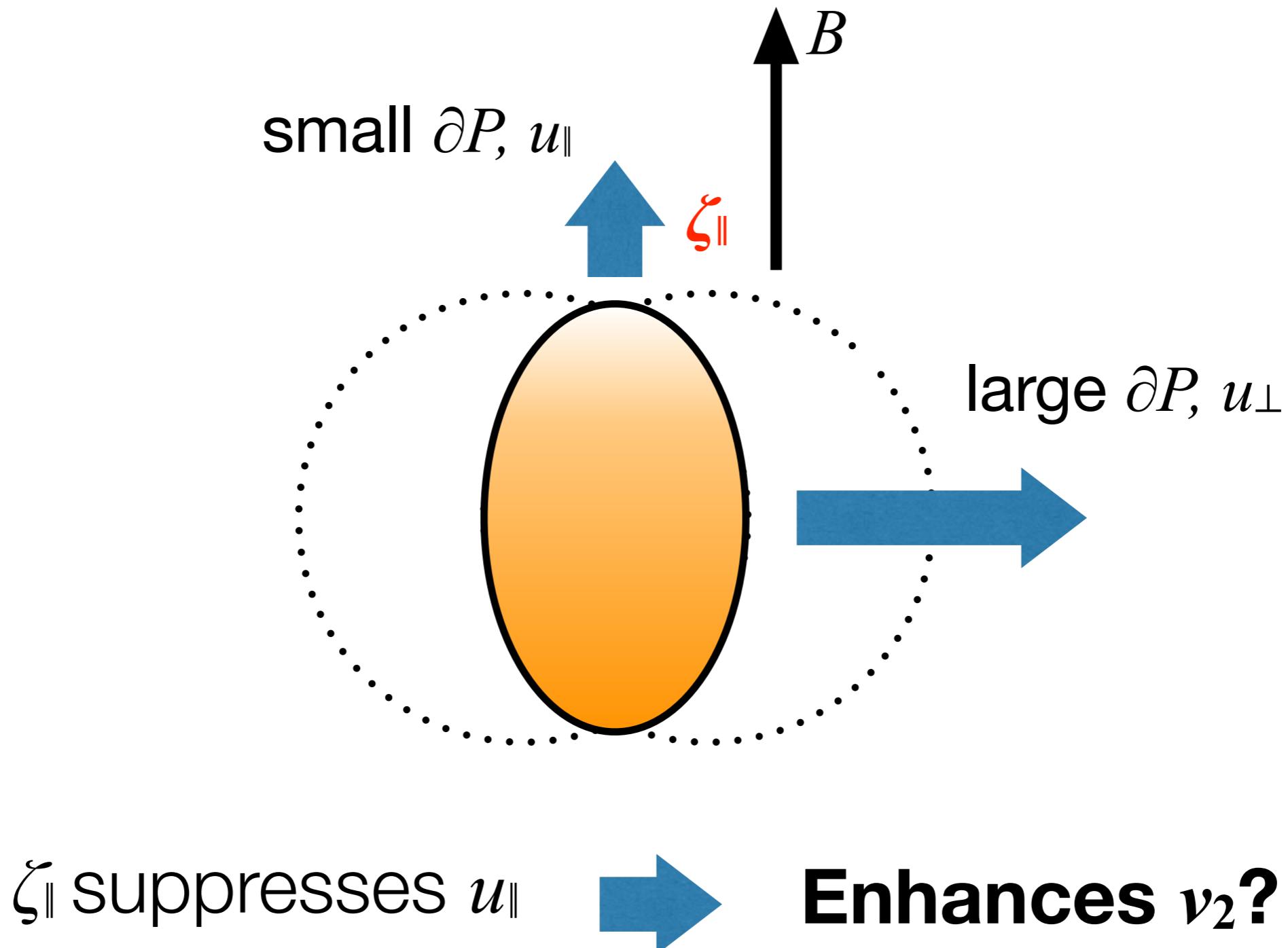
P. Arnold, C. Dogan, G. Moore, Phys. Rev. D **74** 085021 (2006).


$$\frac{\zeta_{\parallel}}{\zeta_{B=0}^s} \simeq 4.7 \frac{1}{\ln(T/M)}$$

B enhances ζ_{\parallel} .

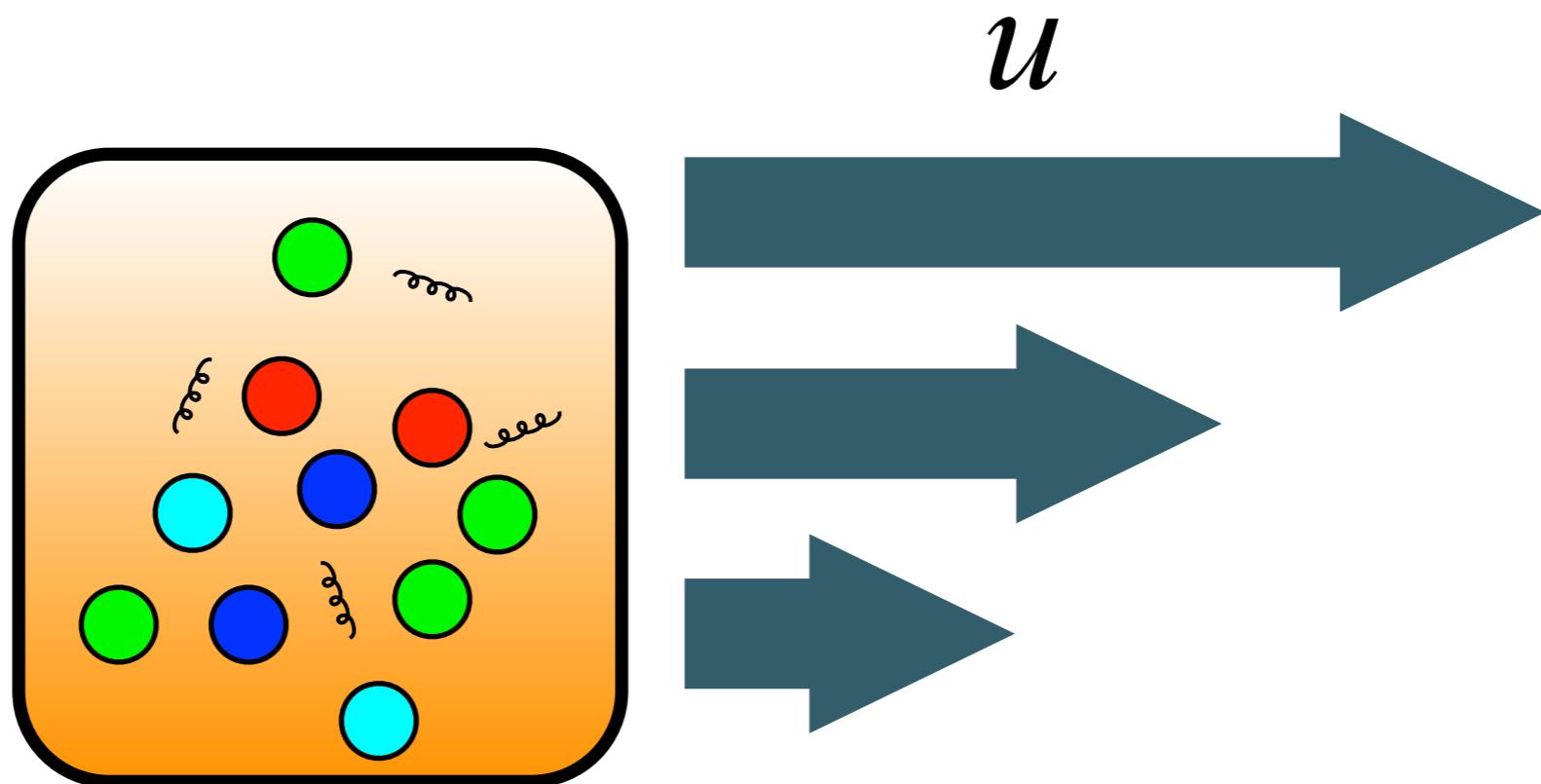
Possible Phenomenological Implications

2. possible effect on flow



Shear Viscosity

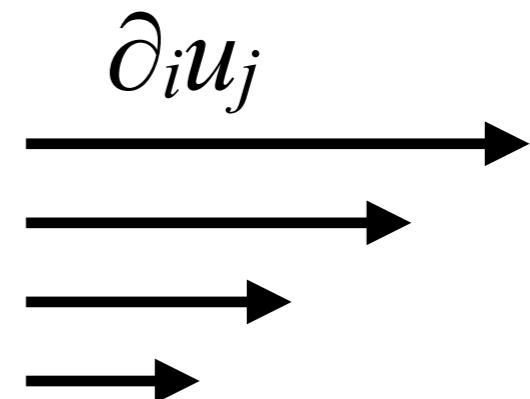
$$\Delta T_{ij} \sim -\eta \partial_i u_j$$



Shear viscosity

$$\Delta T_{ij} \sim -\eta \partial_i u_j$$

Flow of momentum i in j -direction



This flow needs at least 2 spatial dimensions.

→ **LLL quark lives in 1-dimension,
so it does not contribute to the shear viscosity.**

Shear viscosity

Only the gluon contributes to the shear viscosity.

The power counting is the same as $B=0$ case:

Contribution to momentum flow	Gluon density
$\eta \sim \frac{T \times T^3}{g^4 T \ln(g^{-1})} = \frac{T^3}{g^4 \ln(g^{-1})}$	

Gluon damping rate

Summary (electrical conductivity)

$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/m)}$$

Landau
degeneracy

Quark density in 1D

Quark damping rate

When $M \gg m$, $\ln(T/m) \rightarrow \ln(T/M)$.

The conductivity is enhanced by large B , and small m . The sensitivity to m was explained in terms of chirality conservation.

Summary (bulk viscosity)

$$\zeta_{\parallel} = N_c \frac{|eq_f B|}{2\pi} \frac{9.6m^2}{g^2 \pi^2 C_f T \ln(T/m)}$$

When $M \gg m$, $\ln(T/m) \rightarrow \ln(T/M)$.

Conformal breaking

$$\frac{(m^2)^2}{m^2} = m^2$$

Chirality non-conservation

The bulk viscosity is proportional to m^2 , due to the conformal-breaking effect.

Future Perspective

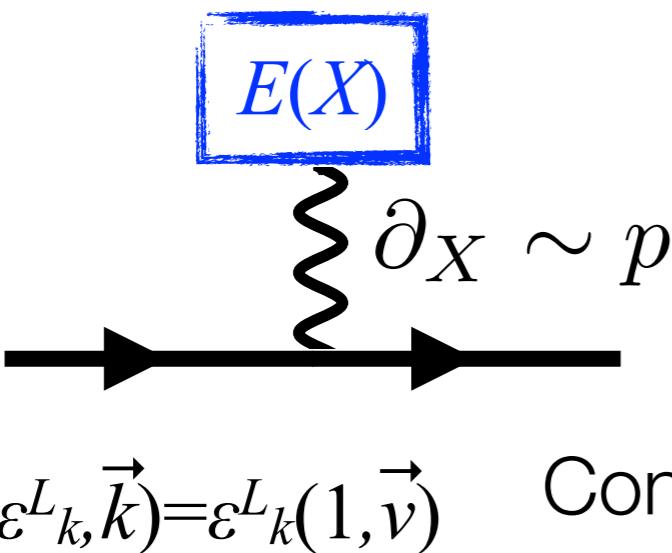
- Go beyond LLL approximation... (more realistic B)
- Ask hydro guys to simulate MHD with our transport coefficients

Back Up

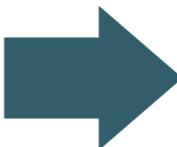
Equivalent Diagrams

J. -P. Blaizot and E. Iancu, Nucl. Phys. B **557**, 183 (1999).

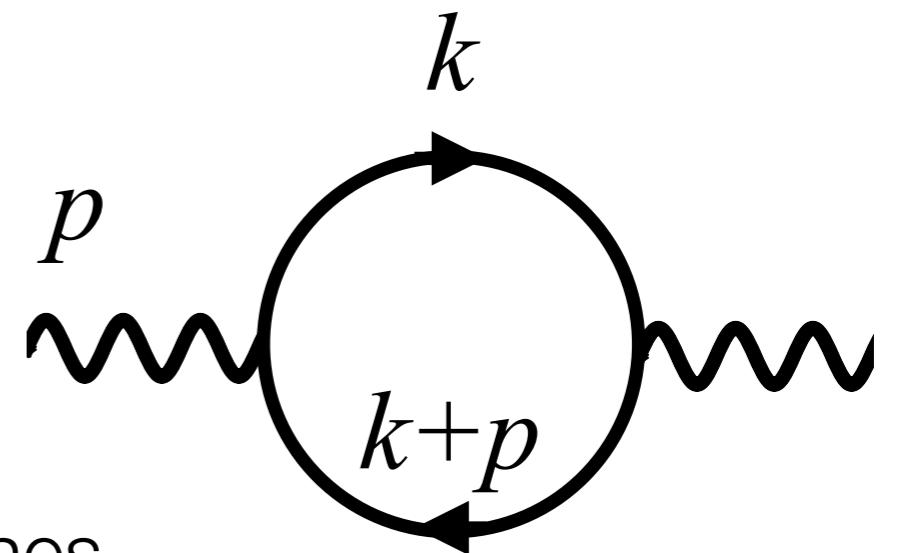
Kinetic eq.



$$k = (\varepsilon^L_k, \vec{k}) = \varepsilon^L_k(1, \vec{v})$$



Field Theory



Connect the ends of lines

$$\frac{1}{(k + p)^2 - m^2 + 2i\xi_{k+p}(k^0 + p^0)}$$

$$(v \cdot \partial_X + 2\xi_k) \delta n^f = -eq_f E^3 \partial_k n^f(k)$$

on-shell condition: $k^2 - m^2 = 2i\xi_k k^0$,
 $p \rightarrow 0$ limit

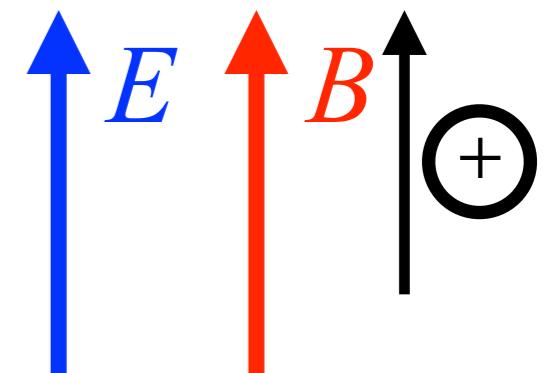


$$\frac{1}{2(k \cdot p + 2i\xi_k k^0)}$$

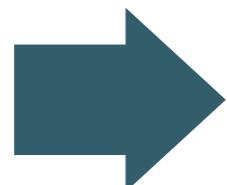
Electrical Conductivity

Axial anomaly: $\partial_t n_A = \frac{e^2 N_c N_F}{2\pi^2} \mathbf{E} \cdot \mathbf{B} - \frac{1}{\tau_R} n_A$

Stationary solution: $n_A = \frac{e^2 N_c N_F}{2\pi^2} \mathbf{E} \cdot \mathbf{B} \tau_R$



Chiral magnetic effect: $\mathbf{J} = \frac{e^2 N_c N_F}{2\pi^2} \mu_A \mathbf{B} = \frac{e^2 N_c N_F}{2\pi^2 \chi} n_A \mathbf{B}$



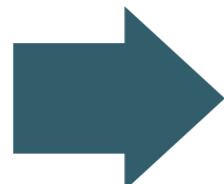
$$\sigma_{zz} = \frac{e^4 (N_c N_F)^2 B^2}{4\pi^4 \chi} \tau_R \quad \chi = N_c \frac{1}{2\pi} \left(\frac{eB}{2\pi} \right)$$

1. Sphaleron

$$\frac{1}{\tau_{R,s}} = \frac{(2N_F)^2 \Gamma_s}{2\chi T} \quad \Gamma_s \sim \alpha_s^5 \log(1/\alpha_s) T^4$$

2. Current quark mass

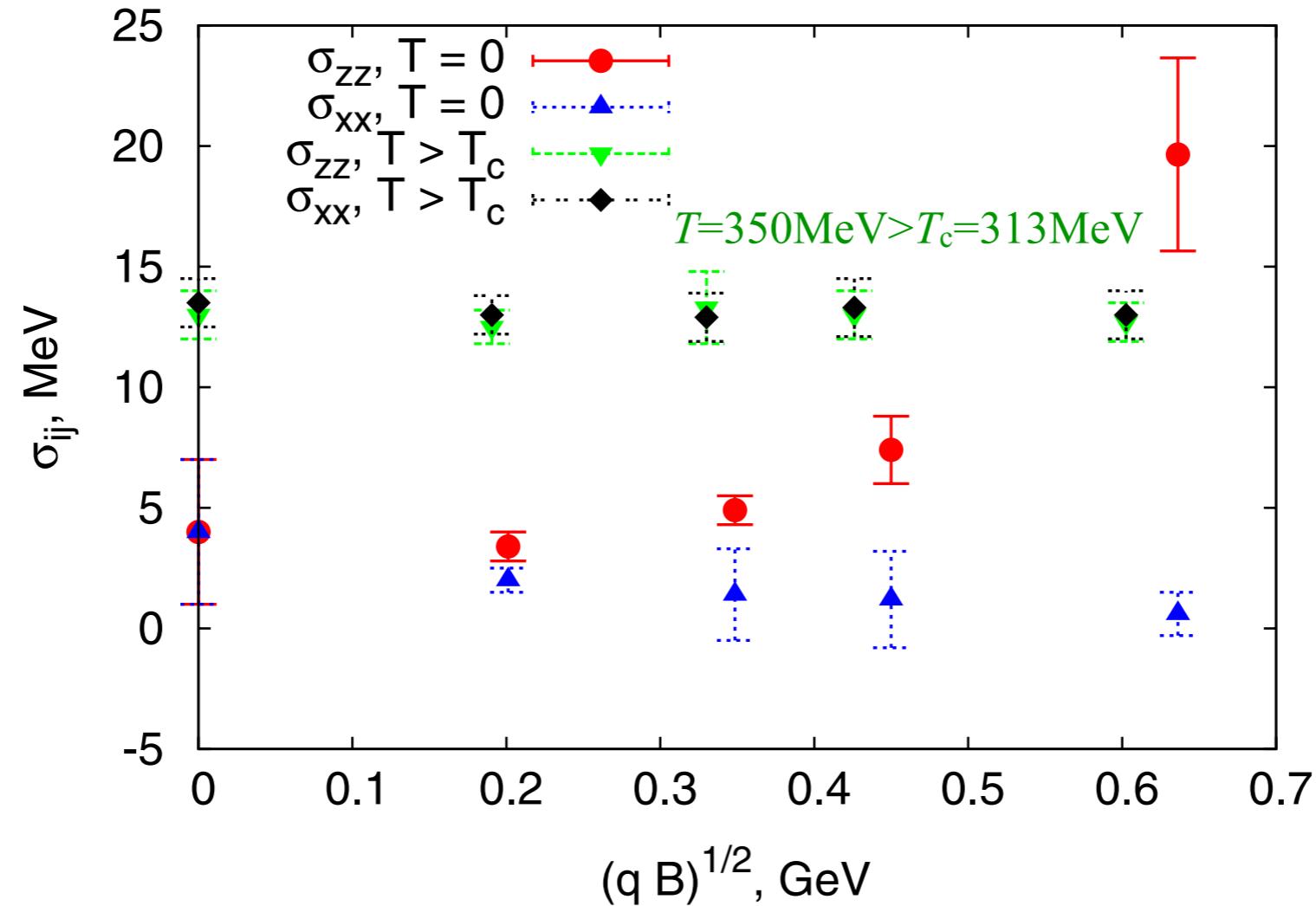
$$\frac{1}{\tau_{R,m}} \sim \alpha_s m_q^2 / T$$



$$\sigma_{zz} \sim e^2 N_c (eB) T \frac{1}{\alpha_s m_q^2}$$

Comparison with Other Result

Lattice ($N_c=2$, **quench**, MEM): P. Buividovich et. al., Phys. Rev. Lett. **105**, 132001 (2010).



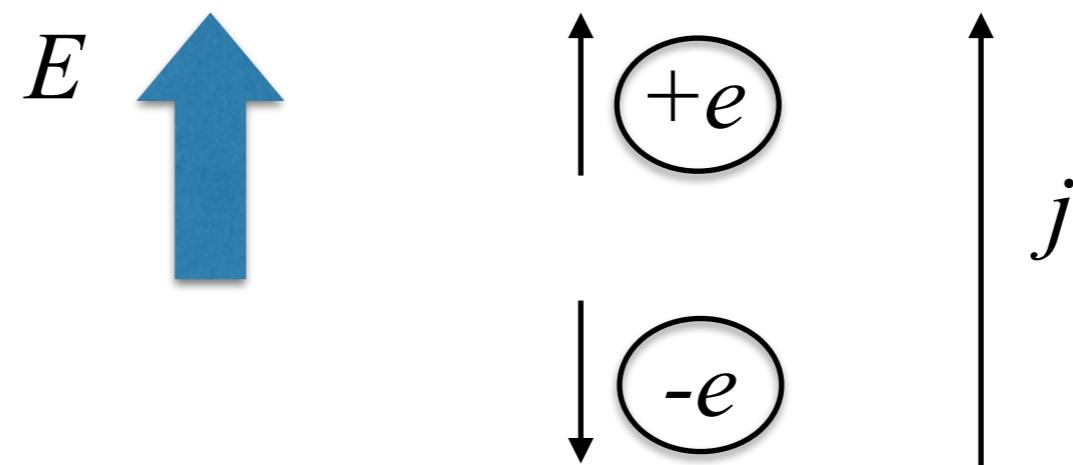
Constant in B .

It contradicts with our result (\sqrt{B}).

They should agree in larger $T, B\dots$

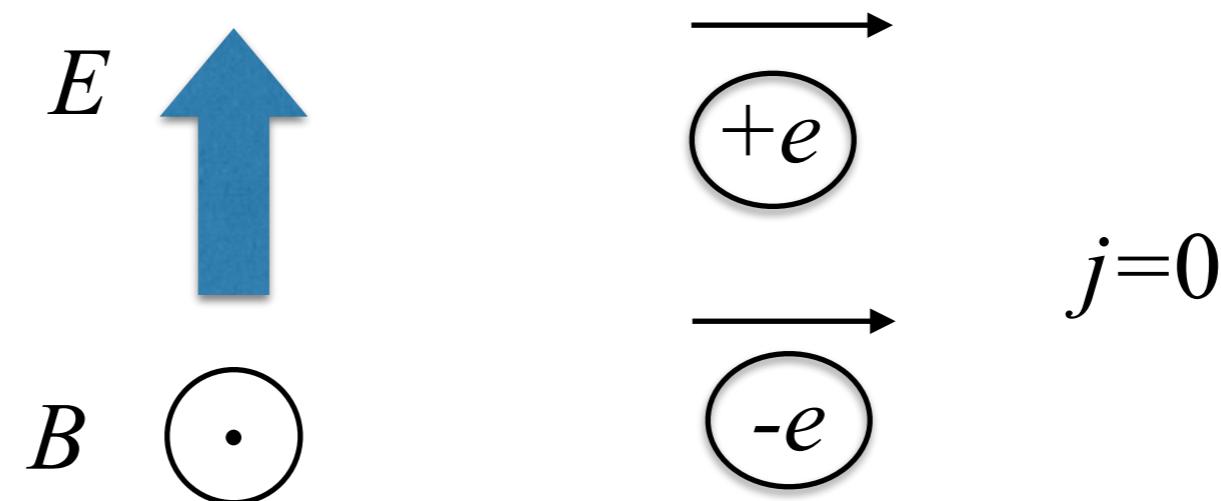
Collision-dominant case – Boltzmann equation

Ohmic current



exists even when $\mu=0$.

Hall current

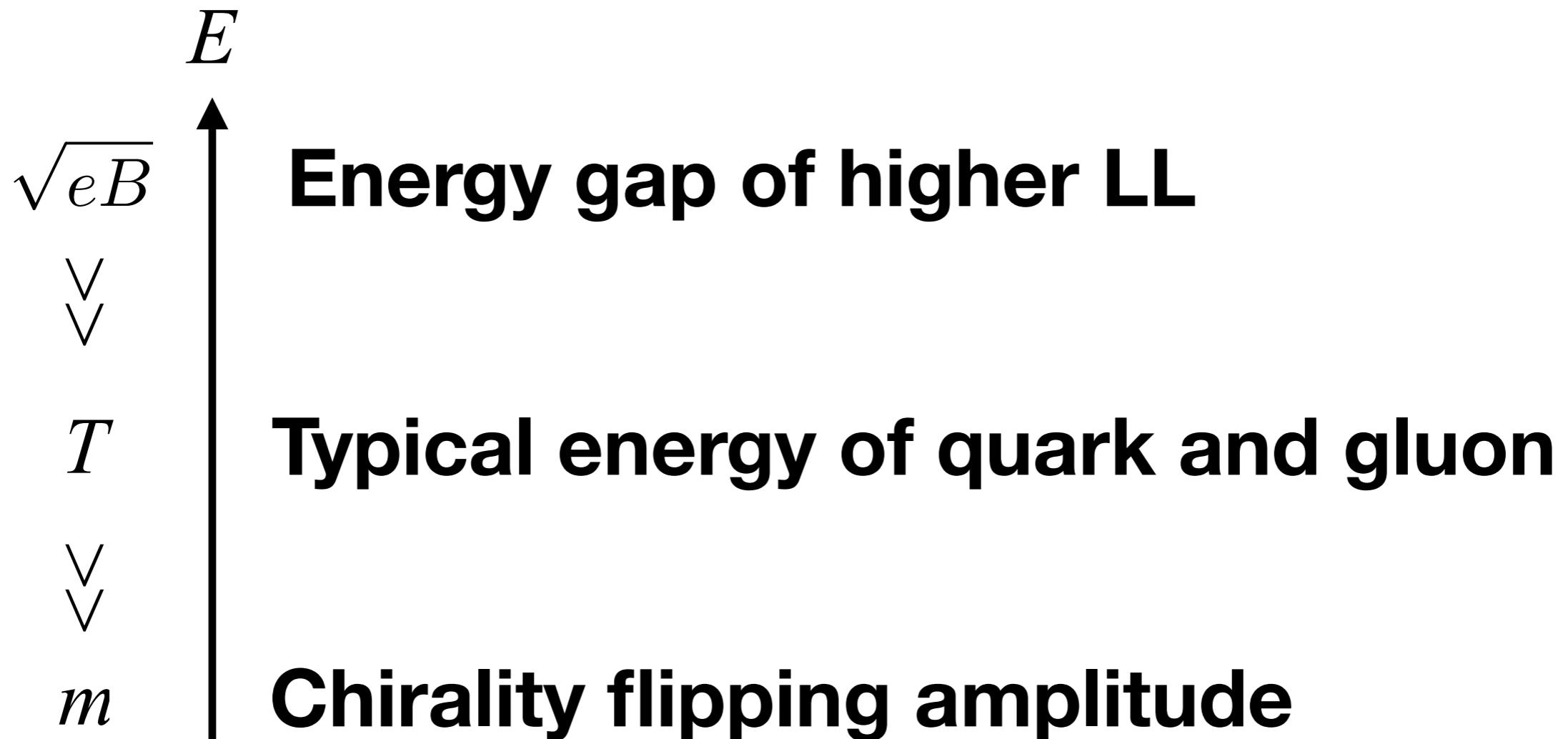


cancels when $\mu=0$.

Motivation

→ **Compute electrical conductivity in strong B limit!**

Hierarchy of Energy Scale at LLL



We also assume that **the gluon screening mass ($M \sim g\sqrt{eB}$) is much smaller than T** , so that the gluon is thermally excited.

Summary

- We calculated electrical conductivity and bulk viscosity in strong B using the LLL approximation, for $m \ll M$ and $m \gg M$ cases

$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/m)}$$

Landau
degeneracy

Quark damping rate

When $M \gg m$, $\ln(T/m) \rightarrow \ln(T/M)$.

- We found that the conductivity is enhanced by large B , and small m . The sensitivity to m was explained in terms of chirality conservation.