MATLAB for SMS

HW 3: Curve Fitting

1. Consider a stationary photon emission process. Such a process is a stochastic one, in that the amount of time that will elapse between successive emissions depends only on the time since the last emission, and not on any past or future events. Since the process is stationary, the mean number of emitted photons, lambda, within some discrete, uniform time period, is constant. Variation in the number of emitted photons, k, across a set of such time periods is described by a Poisson distribution:

$$P(k, \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$$

   The variance associated with a Poisson distribution is equivalent to the mean number of events, i.e., $\sigma^2 = \lambda$.

   a. Generate the distribution, assuming $k \geq 0$, associated with a mean number of emitted photons lambda = 5. (Use something like k = 0:30 and use it as your x-axis. Look up 'factorial' in the help files.)
   b. As a consequence of the central limit theorem, it is often assumed that Poisson-distributed variables are well-approximated by a normal distribution whose mean and variance are equivalent. Is this the case for the distribution generated in part a? (Hint: fit the generated curve to a normal distribution with *k* as the x-axis. Does mean = variance = 5? Does it *look* like a good fit? Does the sum of squared residuals indicate a good fit?)
   c. In the context of the central limit theorem, does the result in part b make sense? Explain. (For instance, what would be the result if lambda = 100?)

The following equations arise often. For each equation:

   a. Generate the curve given the supplied parameters.
   b. Fit the curve to the parametric model function (the same equation with variable parameters) with least squares.
   c. Optimize your least squares fit with unconstrained optimization. Ensure the parameters returned are equivalent to those that were input.

2. Lorentzian (Cauchy) distribution with x0 = 550 and gamma = 50.

$$L(x, x_0, \gamma) = \frac{\gamma}{\pi[(x - x_0)^2 + \gamma^2]}$$

3. Generate an exponential decay curve with a rate $k = 1 \times 10^{-4}$ s. Fit the curve with the FCS function $G(\tau)$, with $R_z = 0.01$.

$$f(\tau) = e^{-k\tau}$$

$$G(\tau) = \frac{1}{(1 + \tau/\tau_D)\sqrt{1 + R_z(\tau/\tau_D)}}$$

Display your generated curves overlaid with their respective fits (as pdfs arranged in whatever manner you like). Calculate and display the sum of squared residuals on each plot. Email them to jntaylor@rice.edu by 10 am, Monday, July 18.