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Comparison of inverse filter real-time equalization methods for non-minimum phase loudspeaker systems

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ABSTRACT

Three time domain digital inverse filter design techniques are considered for non-minimum phase loudspeaker systems equalization, namely: FIR filter obtained with adjustable modeling delay, IIR filter followed by “excess-phase” compensation and warped filter also followed by “excess-phase” compensation. Off-line inverse filtering results using real measured impulse responses of loudspeaker systems are compared and discussed for each design technique on the basis of the time equalization error, similar response’s magnitude flatness, phase linearity and filter order. Real-time inverse filter implementations requirements on a real set-up, using a digital signal processor of the Texas Instruments TMS320 family are also compared based on computational load and memory needs. Results show that loudspeaker equalization with an inverse IIR filter followed by “excess-phase” compensation appears as a good compromise solution

1. INTRODUCTION

Loudspeaker systems still remain one of the weakest links of the audio chain. Even with careful construction commercial loudspeaker systems are characterized by linear and nonlinear distortions that can degrade and introduce "colour" in the reproduced sound.

The compensation of this behaviour by pre-processing the audio signal with the inverse model (inverse filter) of the loudspeaker, known as equalization, is not a new idea and is the objective of the present work.

Loudspeaker equalization is a difficult research topic because, among other reasons, loudspeaker systems are in general non-minimum phase systems, so only approximated inverse filters are possible.

Moreover, the approximated inverse filter for a target flat magnitude and linear phase responses can be quite high order, demanding large computational requirements to be posed on the hardware for real-time processing.

In this paper the inverse filter design problem for non-minimum phase loudspeaker systems' equalization of the linear distortion, in real-time working conditions is considered. A comparative study between several state-of-the-art approaches is outlined and presented in the following.

Three time domain digital inverse filter design techniques are developed for non-minimum phase loudspeaker systems equalization, namely: FIR (Finite Impulse Response) inverse filter obtained with adjustable modeling delay [1][2], IIR (Infinite Impulse Response) [3] inverse filter followed by excess-phase compensation and warped FIR inverse filter [4] [5] also followed by excess-phase compensation.

Off-line simulations using real measured impulse response of two loudspeaker systems are compared and discussed for each design technique on the basis of the time equalization error, approximated equal magnitude flatness of the frequency response, phase linearity and filter order.

Real-time inverse filter implementations requirements on a real set-up, using a digital signal processor of the Texas Instruments TMS320 family are also compared based on computational load and memory needs.

2. MINIMUM AND NON-MINIMUM PHASE SYSTEMS. APPROXIMATED INVERSE OF NON-MINIMUM PHASE SYSTEMS

The techniques used and presented in the paper for real-time digital equalization of loudspeakers are best understood remembering the properties and the theory of discrete-time linear invariant systems.

A discrete-time linear time invariant system is minimum phase if its zeros are inside the unit circle and is maximum phase if its zeros are outside the unit circle. A stable non-minimum phase system has zeros and poles inside the unit circle and also zeros outside the unit circle

Also, a general discrete-time rational non-minimum phase system $H(z)=B(z)/A(z)$ can be expressed as [6 - chap. 5 - pp 240-250]

$$H(z) = H_{eq\_min}(z) H_{ap}(z) \tag{2}$$

where $H_{eq\_min}(z)$ is the equivalent minimum phase system of $H(z)$ and $H_{ap}(z)$ is the remaining an allpass system.

$H_{eq\_min}(z)$ is formed by reflection of all the zeros outside the unit circle to their reciprocal locations inside the unit circle, so $H_{eq\_min}(z)$ is named the equivalent minimum phase system of $H(z)$ because it has exactly the same magnitude response and is a minimum phase system.

$H_{ap}(z)$ comprises all the zeros that lie outside the unit circle together with poles to cancel the reflected reciprocal zeros in $H_{eq\_min}(z)$. The allpass's phase response is sometimes named the "excess phase" part of the system.

The decomposition of a system $H(z)$ in its $H_{eq\_min}(z)$, $H_{ap}(z)$ parts or in its minimum phase and maximum phase parts can de done using cepstral analysis [6 - chap. 12 - pp 768-815] or, if known, based on the pole-zero map of the system.

For digital frequency response compensation purposes based on preprocessing the input to undo the undesirable (non-ideal) characteristic of the system $H(z)$ an inverse system $F(z)$ is desire such that $H(z)F(z)=1$. In the time domain this is equivalent to the convolution operation $h(n)*f(n)=\delta(n)$ where the result of convolving the system's impulse response $h(n)$, with the inverse system's impulse response f(n), is the Dirac impulse sequence δ(n).

The inverse system F(z)=1/H(z) is a stable one if H(z) is a minimum phase system and an unstable causal one if H(z) is a non-minimum phase system.

However if H(z) is a non-minimum phase system a stable noncausal inverse system is possible, that can be made causal by adding an appropriate delay. So, only approximated delayed inverse systems are possible for non-minimum phase systems.

The necessary delay for making causal the noncausal stable infinite long inverse system's impulse response depends on the proximity of the non-minimum phase zeros of the system H(z) to the unit circle, as will be exemplified below.

Consider for example the 2º order system,

$$H(z) = \frac{(1-0.4z^{-1})(1-2z^{-1})}{1-0.8z^{-1}+0.52z^{-2}}$$

with one real minimum phase system and one real non-minimum phase zero.

A causal recursive approximated inverse system of order 8/1 has been design truncating the infinite long noncausal impulse response of the inverse system of H(z) and delayed it by 7 samples.

The discrete-time impulse responses of the system, the inverse filter and the equalized system are presented in figures 1a), 1b) and 1c). From figure 1c) is seen that the equalized system's response is a time delayed one as expected.

In figure 2 the pole-zero map of the approximated inverse filter is presented, where six zeros have been placed to cancel the non-minimum phase zero of the system; the other two zeros inside the unit circle cancel the two complex conjugate poles of the system H(z); the pole cancel the minimum phase zero of H (z).

If the non-minimum phase zero is closer to the unit circle - a real zero with modulo 1.2 instead of 2 - a long delay (24 samples) and a higher order (25/1) recursive inverse system will be necessary as can be seen in figure 3 in a similar representation as in figure 1.

That is, as the non-minimum phase zero is closer to the unit circle longer will be the noncausal part of the inverse's impulse response and consequently longer will be the necessary delay to make it causal for the same truncation error. Obviously higher order approximated inverse systems will be required for time-domain equalization purposes

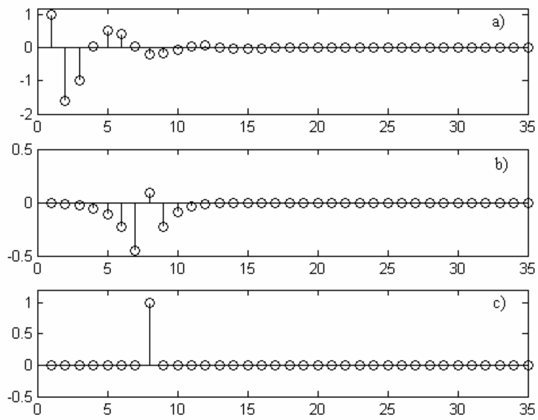

Figure 1 a) system b) inverse filter and c) equalized impulse responses for the case of the non-minimum phase real zero with modulo 2

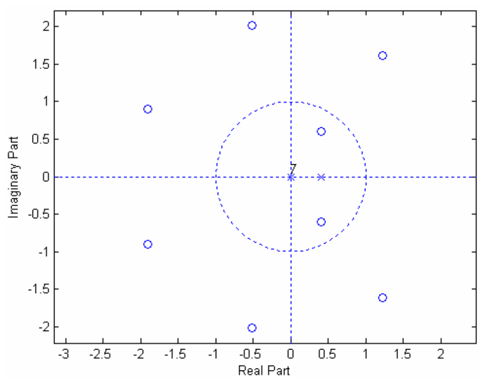

Figure 2 Pole-zero map of the inverse filter for the case of the non-minimum phase real zero with modulo 2

The inverse system approach outlined so far tries to equalize the magnitude and phase responses of the system. However if the phase response of the non-minimum phase system does not need to be equalized an inverse system only for magnitude equalization purposes can be of lower order using for that the equivalent minimum phase part ($H_{eq\_min}(z)$ or $h_{eq\_min}(n)$) in the inverse filter design process.

For non-minimum phase systems it is also possible to do separately magnitude equalization and phase equalization. In this case after magnitude equalization the remaining non equalized phase response can be compensated with a phase equalizer filter using for that allpass's phase response available in $H_{ap}(z)$ or in $h_{ap}(n)$.

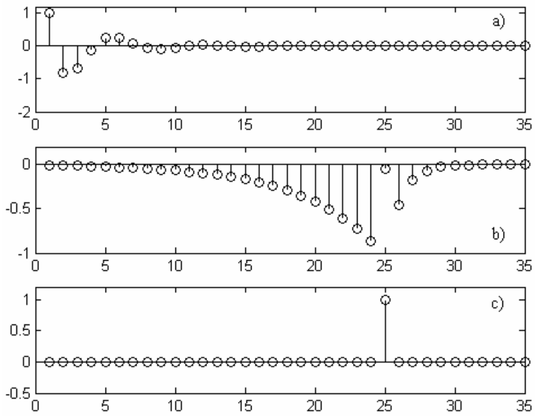

Figure 3 a) system b) inverse filter and c) equalized impulse responses for the case of the non-minimum phase real zero with modulo 1.2

The digital equalization techniques developed in this paper for magnitude and phase equalization of non-minimum phase loudspeaker systems are based on the design of approximate delayed inverse filters for pre-processing the loudspeaker's audio signal input.

3. INVERSE FILTER DESIGN TECHNIQUES

Three digital inverse filter, time-domain, design techniques are considered, namely based on FIR (Finite Impulse Response), on IIR (Infinite Impulse Response) and on warped filters for loudspeaker equalization purposes.

These inverse filter design techniques are based on the minimization of the error (in time domain) between the loudspeaker on-axis impulse response h(n) and the desired impulse response ($\delta(n)$) for which a flat spectrum with linear phase is expected.

For the loudspeaker equalization purposes presented in this paper the on-axis impulse response of the loudspeaker is available and is assumed that it perfectly describes the loudspeaker on-axis frequency behaviour.

Each one design inverse filter design technique is briefly outlined and described in this section and will be complemented in section 4 with the results of each one application to the loudspeaker's impulse response equalization.

3.1. Least squares FIR inverse filter

This technique of time inverse FIR filter design is based on the deterministic inverse modeling configuration [1][7][8] depicted in figure 1.

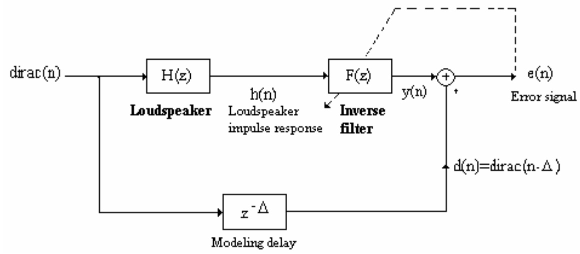

Figure 4 Inverse modeling configuration for least squares inverse FIR filter design

In this method the FIR inverse filter design is obtained in the least squares sense by minimization of the sum of squares of the error signal e(n).

Let h(n) be the loudspeaker's impulse response of length L and f(n) the inverse filter impulse response of length M. The inverse filter f(n) would be such that $h(n)*f(n)=\delta(n)$ where $\delta(n)$ is the Dirac impulse.

The modeling delay, Δ, is necessary for the causality of the inversion of non-minimum phase systems as has been explained in the previous section 2 and as stated in [1][2].

If $y(n) = h(n) * f(n) = \sum_{i=0}^{L-1} f(n-i)h(i)$ with L+M-1 terms, then we intend to minimize the total squared error

$$\sum_{n=0}^{M+L-2} e^2(n) = \sum_{n=0}^{M+L-2}\left[\delta(n-\Delta) - \sum_{i=0}^{L-1} f(n-i)h(i)\right]^2 \qquad (2)$$

The coefficients of the inverse filter are the solution [7][8] of the system of equations,

$$\mathbf{R.f} = \mathbf{c} \Leftrightarrow \mathbf{f} = \mathbf{R.}^{-1}\mathbf{c} \qquad (3)$$

where **R** is the autocorrelation matrix of the impulse response h(n) and **c** is the crosscorrelation between h(n) and the desire impulse response δ(n) for each value of the modeling delay Δ.

For each value of the modeling delay Δ will be an inverse filter solution. The inverse modeling error attains a minimum for values of the modeling delay close to half of the length of the FIR inverse filter as stated in [1][8].

3.2. IIR inverse filter based on loudspeaker's pole-zero model followed by FIR "excess phase" equalizer

As has been previously stated the on-axis impulse response of the loudspeaker is available. However a compact description based on a parametric transfer function model [9] will be also useful as opposed to the long impulse response description.

The inverse filter design that will be explained and that follows the reference [3][10][11] is based on the knowledge of a parametric model (pole-zero model) obtained from the loudspeaker impulse response.

The loudspeaker modeling problem can be stated as follows. Given h(n), the loudspeaker's impulse response, a parametric model, with p poles and q zeros, of the type

$$G(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + ... + b_q z^{-q}}{1 + b_1 z^{-1} + ... + b_p z^{-p}} \tag{4}$$

with impulse response g(n) is desired that best approximates h(n).

The estimation of the parameters b_i and a_i can be stated as the minimization of the sum of the squared error sequence between the loudspeaker's impulse response h(n) and model's impulse response g(n),

$$\min_{\mathbf{a},\mathbf{b}} \|e\|^2 = \min_{\mathbf{a},\mathbf{b}} \sum_{n=0}^{N-1} [h(n) - g(n)]^2 \tag{5}$$

where

$\mathbf{b} = [b_0 \quad b_1 \quad \cdots \quad b_q]^T$ and $\mathbf{a} = [a_0 \quad a_1 \quad \cdots \quad a_p]^T$.

The multidimensional optimization problem on **b** and **a** of this "output-error" formulation is highly nonlinear on **a** and can be solved using an iterative search procedure like the classic Gauss-Newton or like the Levenberg-Marquardt method. In this work it was used the iterative minimization algorithm proposed by Steiglitz-McBride [12] that behaves quite well to find a parametric model G(z) of the loudspeaker.

With this non-minimum phase model with transfer function, G(z), an equivalent minimum-phase model, $G_{eq\_min}(z)$, is constructed by simply reflecting the non-minimum phase zeros of the model, as has been explained in section 2. The direct inverse of this equivalent minimum phase system is therefore stable and can be applied for loudspeaker magnitude equalization.

The remaining all-pass component $G_{ap}(z)$ ("the excess phase") of the model G(z) whose inverse is unstable can be left uncompensated or if desire can be compensated (equalized) by means of a stable approximation.

In reference [3] an FIR filter approximation to the unstable inverse of the allpass component $G_{ap}(z)$ has been proposed.

This FIR phase equalizer is design as the truncated, time-reversed and time-shifted version of the impulse response of the all-pass component $G_{ap}(z)$ of the loudspeaker parametric model G(z). In this way a delayed inverse FIR that approximates the inverse of the all-pass is obtained [13].

This simple design technique will also be used for the compensation (equalization) of the "excess phase" component of non-minimum phase loudspeaker systems.

3.3. Inverse warped FIR filter followed by FIR "excess phase" equalizer

The uniform frequency resolution that characterizes the FIR filters normally implies high order filters for a satisfactory equalization of the low frequency region. Warped FIR (WFIR) filters are characterized by a non-uniform frequency resolution as opposed to FIR filters [4][5].

These warped FIR filters are obtained from FIR filters replacing the unit delays with first-order all-pass sections of the form

$$D(z) = \frac{z^{-1} - \lambda}{1 - \lambda z^{-1}} \tag{6}$$

where λ is a warping parameter.

The phase response of the first–order all-pass sections imposes the non-uniform frequency resolution of the warped filter – frequency warping effect.

The new "warped frequency" (f_{warp}) is related to the original frequency (f) by the expression [4]

$$f_{warp} = f + \frac{f_s}{\pi}\,\mathrm{arctg}\left(\frac{\lambda \mathrm{sen}(2\pi f/f_s)}{1-\lambda\cos(2\pi f/f_s)}\right) \quad (7)$$

The warping parameter λ allows the adjustment of the warping on the frequency axis; for positive values of λ the frequency resolution is increased at low frequencies and for negative values of λ the frequency resolution is increased at high frequencies.

The warping parameter λ can be selected with auditory perception criteria. Smith *et al* [14] have stated a formula, afterwards rectified by Härmä [4], to compute λ to match the amount of frequency warping with the psycho-acoustical perception given by the Bark scale, for a given sampling rate (f_s),

$$\lambda = 1.0674\left(\frac{2}{\pi}\,\mathrm{arctg}\left(0.06583 * {f_s}/{1000}\right)\right)^{1/2} - 0.1916 \quad (8)$$

Figure 5 shows this frequency warping effect for a sampling rate of 48 kHz for different values of the warping parameter λ.

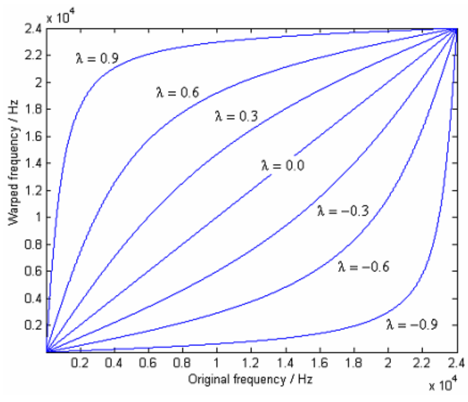

Figure 5 Frequency warping effect (equation 6 with a sampling frequency of 48 kHz)

The design of WFIR filters that has been followed is the one suggested in [4] and [5] that applies warping to the equivalent minimum-phase component $h_{eq\_min}(n)$ of the loudspeaker's impulse response and then applies an autoregressive (AR) modeling technique like Linear Prediction Coding. The coefficients of the polynomial of the AR model are the coefficients of the warped FIR inverse filter.

With this warped filter design process based on the equivalent minimum phase component only magnitude equalization is applied.

However in a similar way to the previous filter design technique the "excess phase" compensation will also be done using the remaining allpass component $h_{ap}(n)$ of the loudspeaker impulse response h(n).

4. OFF-LINE LOUDSPEAKER EQUALIZATION

Using the inverse filter design techniques presented in the previous section a comparative off-line test with two loudspeakers has been carried on with the purpose of evaluate the three design techniques based on the error of equalization and the inverse filter order.

4.1. Loudspeaker systems

Two loudspeaker units have been used in the comparative evaluation test.

The loudspeaker 1 is a small-sized two way vented-box with a 127 mm low-frequency element, a 14 mm dome tweeter and a passive crossover. The loudspeaker's anechoic impulse response was measured at a sampling rate of 48 kHz [15][5].

The loudspeaker 2 is a monitor (220x360x270mm) from Tannoy® with a dual concentric driver of 165 mm and a passive crossover for the frequency range of 52 – 20000 Hz. The loudspeaker's impulse response was measured at a sampling rate of 44.1 kHz [16].

4.2. Error criterion for objective evaluation

The inverse filter design techniques under evaluation minimize an error signal defined in time domain, namely the error difference between the loudspeaker impulse response and the desired impulse response (a Dirac one, with flat spectrum). For this reason the criterion that has been chosen for comparison purposes of the equalization results is the quadratic norm of the error sequence, defined as

$$\|e(n)\|_2 = \left(\sum_{n=0}^{M+L-2} e^2(n) \right)^{1/2} \qquad (9)$$

where

$$e(n) = \delta(n-\Delta) - \sum_{i=0}^{L-1} f(n-i)h(i)$$

4.3. Loudspeaker equalization results

A comparative evaluation of the three inverse filter design techniques based on the final design error should be done assuming the same number of free parameters in each design technique.

For that purpose it was used as reference the number of coefficients (the length) of the FIR inverse filter. With this criterion, for example, in the case of the IIR inverse filter for magnitude equalization followed by an FIR "excess phase" equalizer the total number of coefficients should be equal to the length of the FIR inverse filter.

Least squares FIR inverse filter

The length of the FIR inverse filter used as reference for loudspeaker equalization is of 256.

In figures 6a) and 6b) the measured impulse responses and in figures 6c) and 6d) the equalized impulse response of the loudspeaker 1 are presented (figures b) and d) are presented in a logarithmic scale). Discrete-time responses are represented as continuous just for representation.

The modeling delay used in the design of the FIR inverse filter for loudspeaker 1 was 115 samples.

Figure 7 presents, in the frequency domain, the results of the loudspeaker's equalization. Figure 7a) shows the measured anechoic frequency response (FR) (magnitude and unwrapped phase), 7b) the inverse FIR filter's FR and 7c) the equalized loudspeaker's FR.

Figures 8 and 9 presents the equalization results for loudspeaker 2 with a 256 FIR inverse filter designed with a modeling delay of 154 samples.

IIR inverse filter followed by FIR equalizer

This equalization technique is based on loudspeaker's parametric model. Figure 10 shows a comparison of the FRs of the loudspeaker 1 and the model obtained with 24 zeros and 24 poles using the Steiglitz-McBride algorithm. The norm of the approximation error is 0.007 (-43.1 dB).

Figure 11 presents in the frequency domain the results of the loudspeaker 1 equalization based on 24^{th} order model of the loudspeaker's impulse response. These curves are shifted in magnitude for representation purposes.

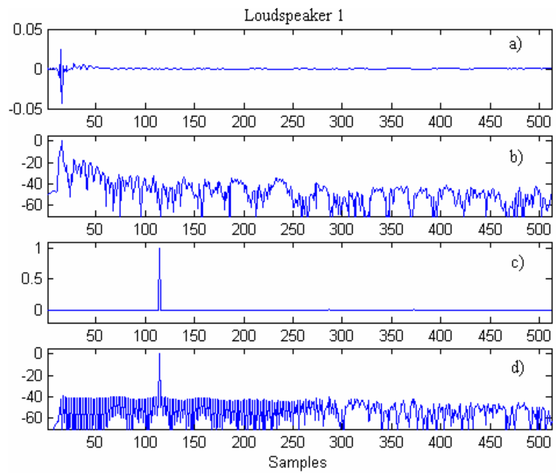

Figure 6 – a) and b) measured impulse response; c) and d) equalized impulse responses with FIR of length 256 with Δ=115 (b) and d) in log scale)

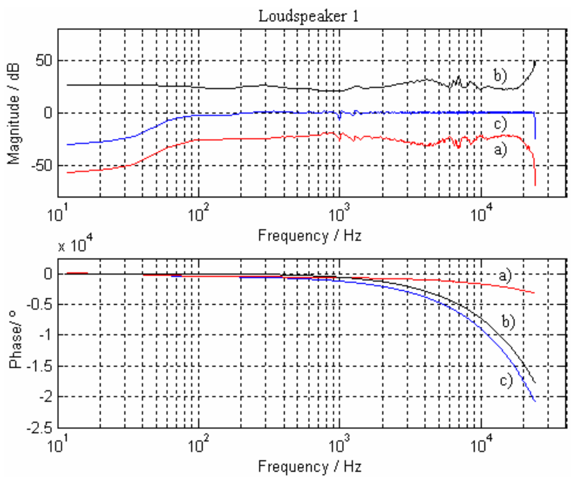

Figure 7 – FR's (magnitude and unwrapped phase) of the a) loudspeaker b) inverse FIR filter of length 256 c) equalized loudspeaker's response

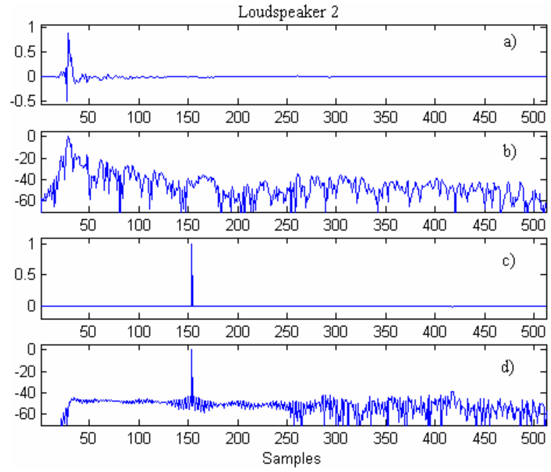

Figure 8 - a) and b) measured impulse response; c) and d) equalized impulse responses with FIR of length 256 with Δ=154 (b) and d) in log scale)

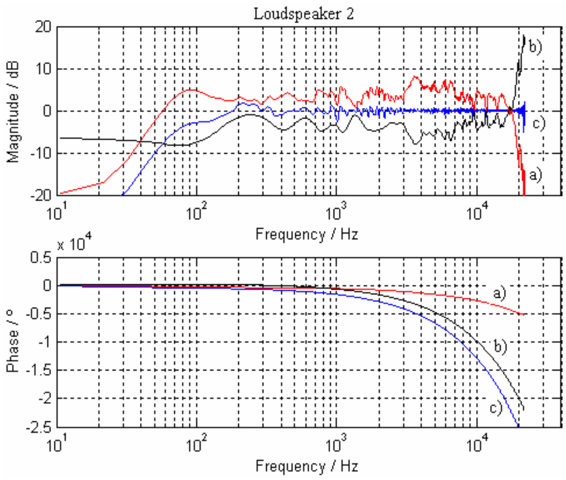

Figure 9 – FR's (magnitude and unwrapped phase) of the a) loudspeaker b) inverse FIR filter of length 256 c) equalized loudspeaker

In figure 11b) only magnitude equalization is applied by an IIR inverse filter of 24th order; this is obtained as the direct inverse of the equivalent minimum-phase system of the model, as stated in 3.2. The phase responses of the non equalized and of the equalized loudspeaker are approximately the same.

The excess phase compensation is done with an FIR filter of length 256 - 2x24 = 208. In figure 11c), in addition to the magnitude equalization of 11b), excess-phase compensation has been done. The truncation of the time-reversed-shifted impulse response of the all-pass component $g_{ap}(n)$ of loudspeaker's model results in worse magnitude equalization in the low frequency region [3][13]. This can be seen in a small fashion by comparing responses 11b) and 11c).

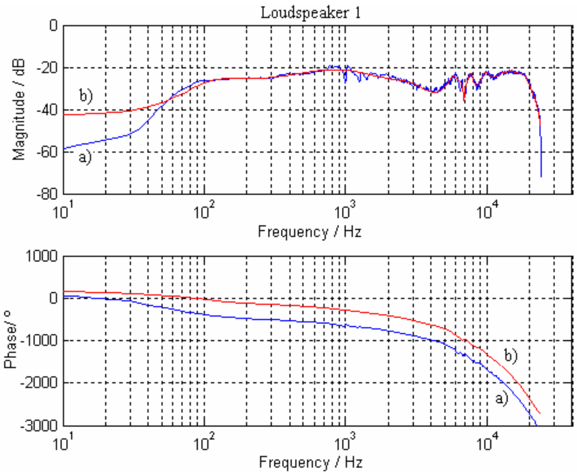

Figure 10 – a) measured loudspeaker's FR b) FR of 24th order IIR pole-zero model

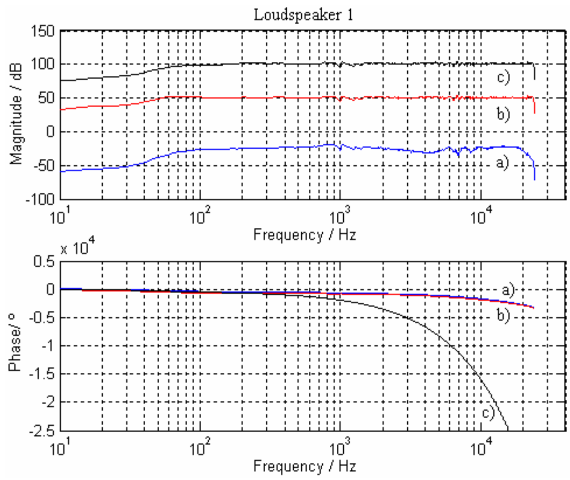

Figure 11 – FR's (magnitude and unwrapped phase) of the a) loudspeaker b) magnitude equalized loudspeaker (IIR filter of 24th order c) equalized loudspeaker (with "excess phase" compensation by a 208 FIR filter)

Figure 12 presents in the same way as in figures 6 and 8 the time impulse responses for the case of the IIR inverse filter with an FIR equalizer for loudspeaker 1.

In a similar way to figures 10 – 12, figures 13 - 15 presents the equalization results for loudspeaker 2 based the 48th order model obtained with an approximation error of 0.093 (-20.6 dB)). The FIR "excess phase" equalizer is of length 160.

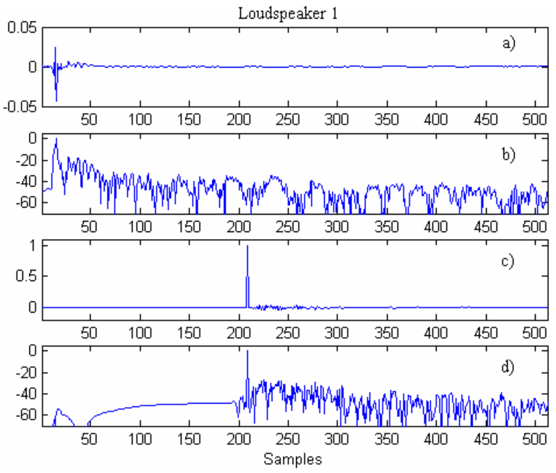

Figure 12 - a) and b) measured impulse response; c) and d) equalized impulse responses with a 24th IIR inverse filter followed by a 208 FIR inverse phase equalizer (b) and d) in log scale)

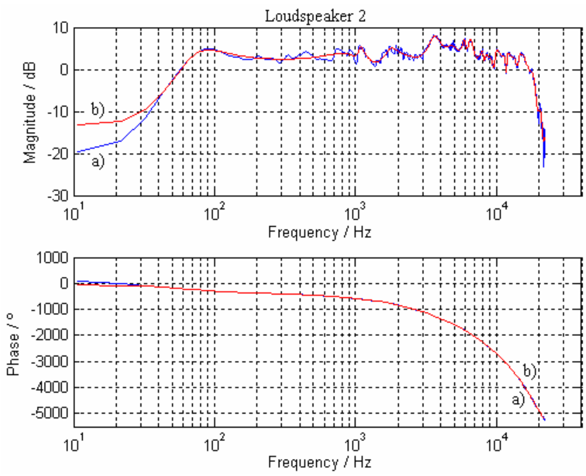

Figure 13 – a) measured loudspeaker's FR b) FR of 48th order IIR pole-zero model

Inverse warped FIR filter followed by FIR "excess phase" equalizer

This loudspeaker equalization technique is based on the inverse warped FIR design technique outlined in 3.3 for magnitude equalization. For "excess phase" equalization an FIR phase equalizer is design.

Figure 16 shows loudspeaker 1 equalization results with a warped FIR (WFIR) filter.

Figure 16b) shows the result of the application of a WFIR filter of length 128 design with a warping parameter $\lambda = 0.766$ for the loudspeaker's FR (16a)) magnitude equalization. The low-frequency region of the equalized FR is almost flat as opposed to the equalized FR with an inverse FIR filter of higher order (figure 7).

Figure 16c) shows the equalized loudspeaker's FR with "excess phase" compensation done with an FIR filter of length 128. Note that the magnitude equalization is worse in the low frequency region in figure 16c) than in figure 16b) due to the truncation of the impulse response of the remaining allpass component in the phase equalizer design step.

Figure 17 presents in the time domain the equalization results for loudspeaker 1.

In a similar way to figures 16 and 17, figures 18 and 19 presents the equalization results for loudspeaker 2 also with a 128 WFIR ($\lambda = 0.7564$) followed by a 128 FIR equalizer.

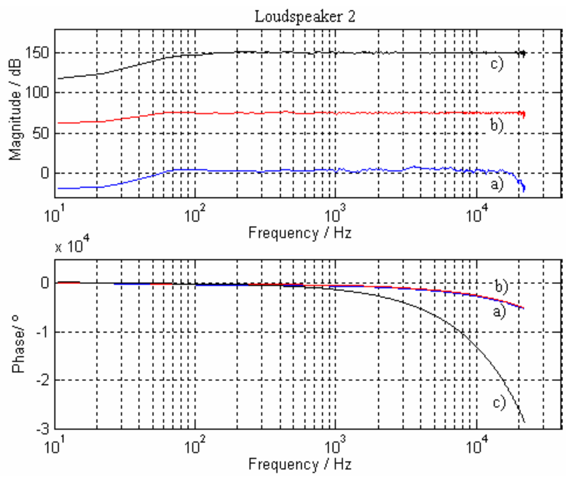

Figure 14 – FR's (magnitude and unwrapped phase) of the a) loudspeaker b) magnitude equalized loudspeaker (IIR filter of 48th order c) equalized loudspeaker (with "excess phase" compensation by a 160 FIR filter)

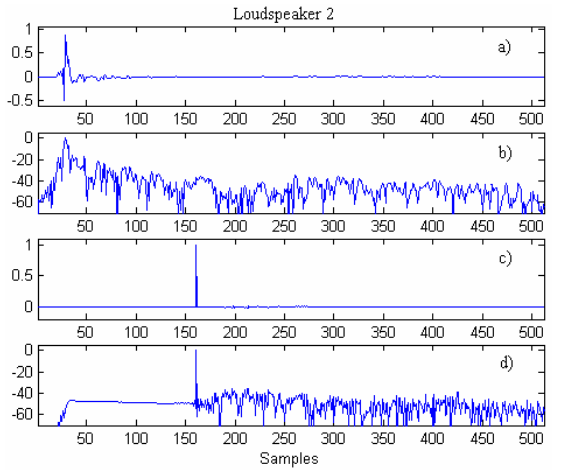

Figure 15 - a) and b) measured impulse response; c) and d) equalized impulse responses with a 24th IIR inverse filter followed by a 160 FIR phase equalizer (b) and d) in log scale)

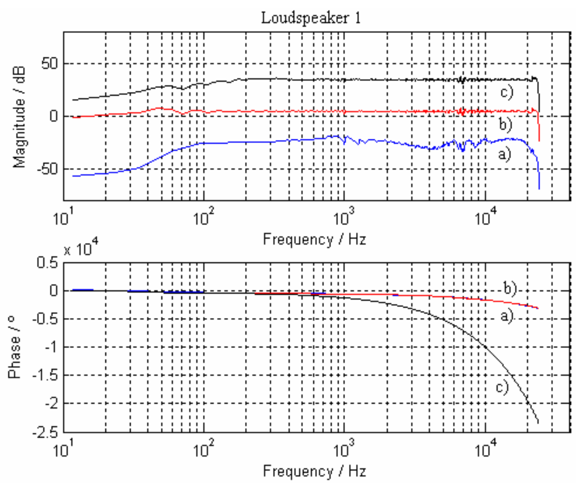

Figure 16 – FR's (magnitude and unwrapped phase) of the a) loudspeaker b) magnitude equalized loudspeaker (WFIR filter of length 128) c) equalized loudspeaker (with "excess phase" compensation by a 128 FIR filter)

Comparison

The comparative test between the three equalization techniques is based on the same number (256) of coefficients of the inverse filters. The norm of the final error (equation 9) of each one equalization technique is presented in table 1 for loudspeaker 1 and in table 2 for loudspeaker 2.

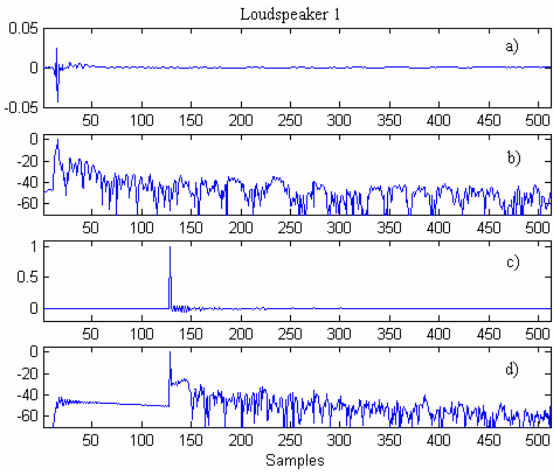

Figure 17 - a) and b) measured impulse response; c) and d) equalized impulse responses with a WFIR of length 128 followed by a 128 FIR phase equalizer (b) and d) in log scale)

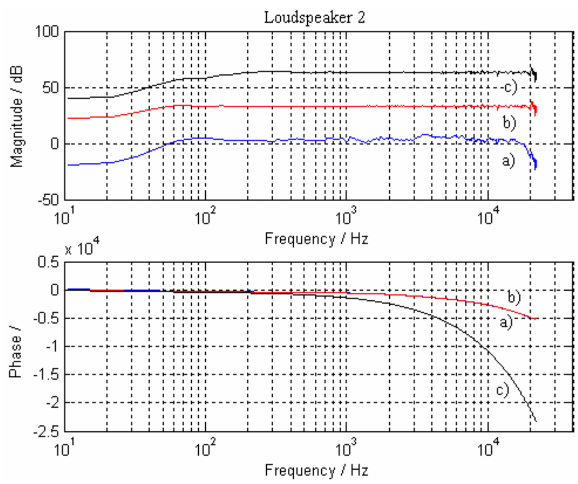

Figure 18 – FR's (magnitude and unwrapped phase) of the a) loudspeaker b) magnitude equalized loudspeaker (WFIR filter of length 128) c) equalized loudspeaker (with "excess phase" compensation by a 128 FIR filter)

Loudspeaker time domain equalization results using the least squares FIR inverse filter have the minimum error. This it is not unexpected as the 256 filter parameters in the FIR inverse filter design are optimized simultaneously as opposed to the other two equalization solutions that are based on a non-optimized "two-step" design process.

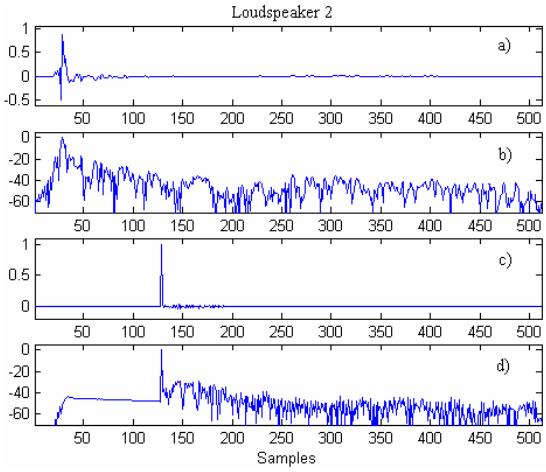

Figure 19 - a) and b) measured impulse response; c) and d) equalized impulse responses with a WFIR of length 128 followed by a 128 FIR phase equalizer (b) and d) in log scale)

Table 1 – Inverse filter design error – loudspeaker 1

Inverse filter type	**Order**	**Error**
FIR	256	0.11
IIR **FIR phase equalizer**	24 208	0.1965
WFIR **FIR phase equalizer**	128 128	0.2120

Table 2 – Inverse filter design error – loudspeaker 2

Inverse filter type	**Order**	**Error**
FIR	256	0.0893
IIR **FIR phase equalizer**	48 160	0.1096
WFIR **FIR phase equalizer**	128 128	0.1617

Time domain equalization with FIR inverse filters seems effective for both loudspeakers (figures 8 and 12). The low frequency region is less equalized than the high frequency region of the loudspeaker; for loudspeaker 2 this is clearly seen in figure 9c).

With WFIR filter the low-frequency region is well equalized in magnitude for both loudspeakers with a lower order filter than the FIR inverse filter one, as is expected. However the FIR phase equalizer of length 128 is not satisfactory. Even with other length combinations - instead of 128 WFIR and 128 FIR - with 256 as reference does not leave to a different conclusion. In this test the equalization with warped filters has the highest error for both loudspeakers.

Time domain equalization with IIR inverse followed by an FIR phase equalizer seems effective for both loudspeakers. The equalization error for loudspeaker 1 is quite high but for loudspeaker 2 the error is close to the minimum attained by the FIR inverse filter. Also the equalized low region of the frequency band of loudspeaker 2 is better that with the FIR inverse filter.

For loudspeaker 1 the quite high equalization error is perhaps due to the lower order model used in the IIR inverse filter design. The equalization can be reduced with a high order loudspeaker model. Just as an example with a model of 48th order the equalization error is reduced to 0.1469.

A further analysis of the equalization error in the case of the IIR inverse filter followed by an FIR equalizer also reveal that the length of the FIR phase equalizer can be reduced without appreciable effects on the equalization error for both loudspeakers.

The previous observation was used to find out some improvements concerning filter order and equalization results.

For loudspeaker 1 with the same 24^{th} order model instead of an FIR phase equalizer of length 208 a length of 104 was design with a minor change of the error from 0,1965 (-14,1 dB) to 0.1988 (-14,03 dB). The results for this case are presented in frequency and in time domain in figures 20 and 21. The magnitude equalization is not affected due to a shorter FIR phase equalizer.

With loudspeaker 2 for comparison and exemplification purposes a similar shorter phase equalizer of length 80 was done with an error of 0,1229 (-18,2 dB) instead of 0,1096 (-19,2 dB). The equalization results are presented in figures 22. For this phase equalizer length reduction the magnitude equalization is clearly worse in low frequency region comparing figures 22c) and 14c).

Nevertheless a shorter FIR phase equalizer of length 128 for example provides approximately the same magnitude equalization in the low frequency region with an error of 0,1133 (-18.9 dB).

The warped filter (WFIR) based equalization technique developed in this paper also uses an FIR phase equalizer. However filter order reductions regarding the length of phase equalizer are not easily attainable as they have great impact on the magnitude equalization as was already stated.

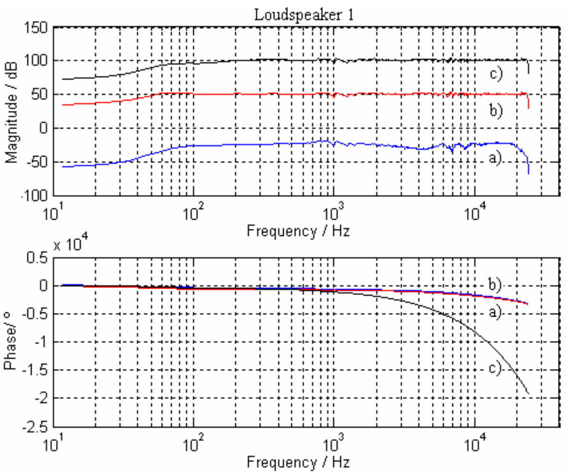

Figure 20 – FR's (magnitude and unwrapped phase) of the a) loudspeaker b) magnitude equalized loudspeaker (IIR filter of 24th order c) equalized loudspeaker (with "excess phase" compensation by a 104 FIR filter)

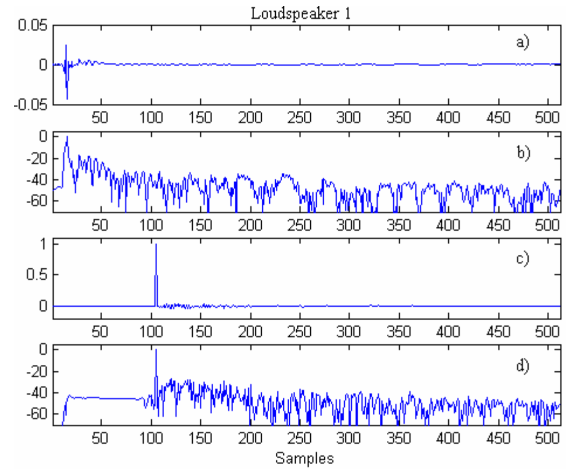

Figure 21 - a) and b) measured impulse response; c) and d) equalized impulse responses with a 24th IIR inverse filter followed by a 104 FIR inverse phase equalizer (b) and d) in log scale)

As ending note of this section a general conclusion concerning the equalization of non-minimum loudspeaker systems using approximated delayed inverse filters is:

- least squares FIR inverse filter design is a effective technique normally leading to filters of high order;

- IIR inverse filter using loudspeaker's pole-zero model followed by an FIR phase equalizer is a valuable equalization design tool; hand-tune optimization of the equalization error and the filter order is easily as magnitude and phase equalization are separated;

- WFIR filters allow better low frequency region magnitude equalization with lower order filters than with FIR inverse filters;

- the loudspeaker frequency response under equalization is a key factor condition which is the best equalization solution.

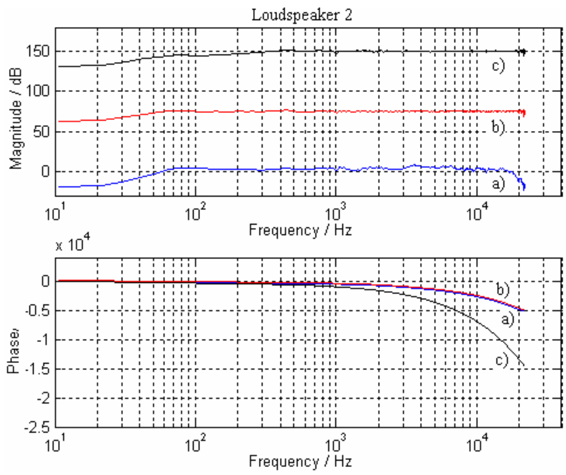

Figure 22 – FR's (magnitude and unwrapped phase) of the a) loudspeaker b) magnitude equalized loudspeaker (IIR filter of 48th order c) equalized loudspeaker (with "excess phase" compensation by a 80 FIR filter)

5. IMPLEMENTATION REQUIREMENTS IN A DSP FOR REAL-TIME PROCESSING

Digital signal processors (DSP) are today a common tailored hardware for real-time applications demanding powerful computational capabilities.

With comparative purposes the computational implementation requirements on a common floating processor were evaluated for each one of the three equalizations solutions developed in this paper.

The hardware supporting this comparison is an EVMC30 evaluation card with the floating point DSP TMS320C30 from Texas Instruments that the authors are familiar with from previous digital filter applications [12].

For comparison purposes table 3 and table 4 shows for loudspeaker 1 and loudspeaker 2 the numbers of cycles of the digital signal processor required per output sample for each one of the inverse filters for the loudspeaker equalization task. The IIR implementation requirements are outlined for second-order series.

Table 3 – DSP requirements - loudspeaker 1

Inverse filter type	Order	DSP inst. cycles
FIR	256	9+N=265
IIR	24	8+3,5(N-1)=92
FIR phase equalizer	104	9+N=113
WFIR	128	9+4*N=521
FIR phase equalizer	128	9+N=137

Table 4 – DSP requirements - loudspeaker 2

Inverse filter type	Order	DSP inst. cycles
FIR	256	9+N=265
IIR	48	8+3,5(N-1)=176
FIR phase equalizer	128	9+N=137
WFIR	128	9+4*N=521
FIR phase equalizer	128	9+N=137

The FIR inverse filter implementation is the easiest one as it does not demand the implementation of two types of filters as the other two equalization solutions.

From table 3 and 4 the loudspeaker's FR equalization with an FIR inverse filter appear as a computational efficient solution for loudspeaker 2.

From table 3 and 4 the loudspeaker's FR equalization with an inverse IIR filter followed by an FIR filter for "excess-phase" compensation appear to be a good solution for loudspeaker 1; even with a higher loudspeaker' model (48th) and a 104 FIR phase equalizer the computational requirements of the IIR inverse filter solution place it also as a good option. For loudspeaker 2 the implementation requirements are lightly higher than with the FIR inverse filter.

Even with lower order - can be 5 times lower than an FIR as stated in [5] for magnitude equalization - the warped FIR filter computational implementation requirements are 4 times higher than the FIR filter [4][5].

Data memory requirements of the DSP for the an FIR filter is 2N and for a IIR filter is 3,5(N-1) [12], leaving still an advantage factor for the IIR solution.

Within the examples outlined in this section the inverse FIR inverse filter and IIR inverse filter followed by an FIR "excess-phase" equalizer appears to be good solutions compromising between filter order and computational requirements of the floating-point digital signal processor for both loudspeakers.

6. CONCLUSIONS

In this paper the inverse filter design problem for real-time equalization of non-minimum phase loudspeaker systems' equalization was considered. A comparative study between three state-of-the-art approaches applied to two loudspeakers units was outlined and presented.

A comparative test between the inverse filter design techniques based on the final time equalization error indicate as the best the FIR inverse filter. The IIR inverse filter followed by an FIR phase equalizer appears also as good equalization option.

The comparison of the DSP computational requirements for real-time equalization has also been done. The equalization solution based on the IIR inverse filter appears as a good option for the implementation in a DSP.

Although needing a large set of loudspeakers under test and needing more objective (and subjective) evaluation tests with DSP implementations in real-time working conditions, the non-minimum phase loudspeaker equalization with a IIR inverse filter followed by an FIR filter for "excess-phase" compensation appears as a good compromise solution between filter order, computational requirements of a floating-point digital signal processor, time equalization error and flatness of the final loudspeaker's response.

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