An Efficient Approach to Minimum Phase Prefiltering of Short Length Filters

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Abstract—*Motivations for performing prefiltering based on root finding are presented, for an interference canceling receiver employing a reduced-state equalizer such as DFSE or RSSE. Since the interference canceling filter (ICF) has a MMSE-DFE structure, that shortens the channel impulse response (CIR), a low complexity minimum phase prefilter can be applied before equalization. Root-finding based prefiltering for second order filters are of particular interest, since closed form solutions can be obtained with less computations. For such second order filters, CIR can be classified as minimum, mixed or maximum phase, based on few inequalities which directly use the complex-valued channel coefficients. Proposed inequalities help in retaining maximum accuracy with low complexity, by avoiding some approximation algorithms involved in root identification on DSP. While samples corresponding to minimum and maximum phase channels are processed directly, root-finding is employed only to transform the mixed phase channels to their minimum phase equivalents.*

Keywords—*Minimum phase prefiltering, Interference cancellation, Reduced state equalization.*

I. INTRODUCTION

Interference cancellation in mobile receivers has gained significant interest due to possibility of efficient DSP implementations and network capacity benefits. While the inter symbol interference (ISI) caused by multipath fading is cancelled by Viterbi equalizers or its variants, space-time filters are typically employed for interference cancellation. Dual antenna receivers are expected to gradually replace conventional receivers in 3GPP GERAN evolution, due to their high performance in interference-limited scenarios. Though computational complexity in such receivers can be significantly reduced using various approaches known in literature, processing two streams of samples still remains complex and some algorithms require considerable focus.

In this paper, an approach for prefiltering in dual antenna receivers is discussed, for usage with reduced state equalizers. It is well-known that performance loss when using reduced state equalizers can be minimized by transforming the CIR to their minimum phase equivalent, which has their energy concentrated in the leading taps. Prefiltering methods based on Prediction error filter and Decision feedback equalizer with minimum mean squared error criterion (MMSE-DFE) have been discussed in [1,2,3], and prediction error filters are typically preferred due to their less computational complexity while providing significant performance gains.

Dual antenna receivers for higher order modulation schemes involve an interference canceling filter with a DFE structure employing feed forward and feedback filters of short length as discussed in [5,6]. While the ICF does reduce length of CIR favorably, further performance gains can be achieved if the channel is strictly minimum phase. Since feedback filter length is short, direct root-finding based minimum phase prefilter can be employed after interference cancellation, which would involve low computational complexity.

In section II, dual antenna receiver structure is presented, and motivations for prefiltering are developed in section III. Method for filter classification is discussed in section IV, and an efficient prefiltering method is proposed in section V. Performance and complexity analyses are presented in section VI.

II. SYSTEM MODEL

Interference cancellation using antenna arrays has been a well-known technique in combating interference, on same or adjacent frequencies. DARP Phase II of GERAN considers Mobile Station Receiver Diversity (MSRD) as one of the key features for improving receiver performance for both GMSK and 8PSK modulation schemes. Receiver structure for dual antenna EDGE 8PSK equalization involves an interference canceling filter, followed by a reduced state equalizer. Reduced state equalization techniques such as DFSE or RSSE, are normally employed to cancel ISI for higher order modulation schemes like 8PSK of EDGE in order to reduce computational complexity, as discussed in [16, 17]. While these reduced state equalizers would suffer performance loss compared to full-state MLSE, prefiltering for such reduced state equalizers have been shown in [3], to have minimized such losses significantly.

ICF can be designed using an adaptive filter that provides suitable weights to signals from both antennas and combines them [5]. 26 symbol training sequence is typically used for weight estimation, and samples are filtered with appropriate feed-forward filters, leaving residual ISI to be cancelled by subsequent reduced state equalizer. Minimization of MMSE cost function as discussed in [6] can be chosen for efficient interference cancellation. Since the interference cancellation technique uses a MMSE-DFE structure, beneficial channel shortening results [7].

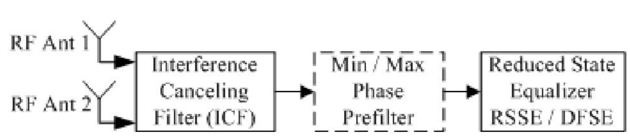

Fig. 1. Dual antenna receiver structure

For this filter design, decisions on length of feed-forward and feed-back filters involve common trade-off analysis over performance and complexity. Feed-forward filters of length 3 or 4 are suitable for efficient DSP implementation. In [4], optimum feedback filter length when performing impulse response truncation is addressed and a choice of 3 has been favored for ensuring good performance in all GSM channel profiles, with less complexity. Performance analysis of dual antenna receiver also indicates that 3 would be the optimal choice for feedback filter of the ICF.

III. MOTIVATIONS FOR PREFILTERING

Prefiltering involves minimum phase transformation, wherein zeros of the CIR located outside the unit circle are cancelled and reflected onto their conjugate reciprocal locations. It is well- known [12] that a minimum phase filter has their energies concentrated on the leading taps compared to all other CIR with same magnitude response. In conventional 8PSK equalizers with single antenna, prefiltering could be based on identification of prediction error filter, by employing the Levinson-Durbin algorithm, or a MMSE-DFE could serve as the channel shortening filter before reduced state equalization, as discussed in [1,2]. Linear prediction based method is more suitable in terms of performance and complexity, for usage in conventional EDGE 8PSK equalization.

For dual antenna receivers, one alternate to minimum phase prefiltering would be to increase the number of states in ISI canceling Viterbi equalizer, which for this system, would mean an increase from 8 states to 64 states, so as to perform a full-state MLSE for 8PSK modulation scheme, with CIR length of 3. This increases complexity significantly, which would not be affordable.

Another alternate could be to employ the prediction error filter based minimum phase prefiltering as in case of conventional equalizers. For example, a prediction error filter could be identified based on the one-sided autocorrelation sequence of the CIR. Minimum phase property of prediction error filter has been discussed in [15]. All-pass filters could then be derived using an efficient method as described in [1]. While this option would ensure that channel as seen by the equalizer is minimum phase to a large extent, complexity increases dramatically here as well. A 12-tap prediction error filter identification and subsequent prefiltering would take around 10K cycles on a 16-bit Dual-MAC DSP. Spectral factorization is another option but is known to be complex, when considered for an efficient dual antenna EDGE receiver implementation.

Since the ICF has a MMSE-DFE structure, additional minimum phase prefiltering could be avoided in order to reduce complexity. MMSE-DFE has been analyzed in [7, 8] and later discussed in [1] wherein the FIR feed forward filter has been noted to provide a good approximation of the all pass filter that result in minimum phase CIR. However, decision of feed forward filter length of ICF primarily depends on interference cancellation performance with minimal computational load on matrix inversion technique. Due to such complexity considerations, short feed forward filters are chosen and such filters would not always result in minimum phase channels, indicating that further improvements could be achieved by adding a suitable filter.

Hence, it is preferable to employ a low complexity prefiltering algorithm based on root-finding after the ICF. Such a root-finding based prefiltering can be adaptive and minimum phase CIR can be ensured with high accuracy. Feedback filter length of 3 is chosen for ICF and implications of a second order root-finding based approach for prefiltering are discussed in following sections.

Root finding method can be discussed as two separate functionalities, first being the filter type determination wherein the feedback filter of ICF is classified as minimum, maximum or mixed phase. This classification helps to employ an adaptive prefiltering strategy in which additional all-pass prefiltering is employed only for mixed phase channels.

IV. FILTER TYPE DETERMINATION

Performance of a fixed-point DSP implementation of a root-finding based prefiltering method primarily depends on accurate channel zero location identification. This, in turn, depends on usage of approximations in square root (SQRT) and division (DIV) algorithms. Square root of a complex number h = h_r+jh_i can be obtained using the following relations, which involves three real-valued square root operations.

$$s_r = \frac{1}{\sqrt{2}}\sqrt{\sqrt{h_r^2 + h_i^2} + h_r};$$
$$s_i = \frac{sgn(h_i)}{\sqrt{2}}\sqrt{\sqrt{h_r^2 + h_i^2} - h_r}$$

Number of square root operations involved in a DSP algorithm is often minimized, since apart from finite word length effects, typical DSP implementation of identifying square root involves approximation methods in order to reduce inherent complexity. One such method is given in [11], which provides a reasonable approximation of square root of numbers within range of 0.5 to 1, based on a series expansion. Another well-known and efficient method involves a lookup table and interpolation, wherein length and width of the lookup table can be tuned to improve accuracy, at the expense of memory. Here, normalization to the range of 0.25 to 1 is initially performed, followed by table-lookup and interpolation and finally denormalization is performed to the original range. It is well-known that both methods could provide inaccurate results due to the approximations, apart from losses due to word length limitations. It can be inferred that when coefficients are complex-valued, loss would accrue over 3 SQRT operations.

Stability triangle is typically employed to assess the stability of a real-valued second order filter, without explicit computation of pole locations. In order to determine stability, or in other terms, for identifying whether both poles are inside the unit circle, inequalities using filter coefficients can be employed, as given in [11]. For a real-valued filter H with two poles, stability triangle has been defined by two inequalities, wherein h_0, h_1, h_2 ϵ R, and K corresponds to filter gain.

$$\text{For H} = \frac{K}{h_0+h_1z^{-1}+h_2z^{-2}}$$
$$|h_1| < h_0 + h_2.....(R_{nm})$$
$$|h_2| < |h_0|..........(R_{min})$$

If both inequalities are satisfied, it can be deduced that the filter is stable without having to compute the roots of the polynomial. Similarly, for determining the minimum-phase nature of a real-valued filter with two zeros, same inequalities can be used. Inequality R_{nm} (non-mixed) can be used to differentiate mixed and non-mixed phase filters and inequality R_{min} can then be used to differentiate minimum and maximum phase filters. Above inequalities are valid as long as channel coefficients are real while the roots may be real or complex. For second order filters with complex-valued channel coefficients, above inequalities cannot be employed.

Schur-Cohn stability test that analyzes the reflection coefficients of a second order polynomial can be considered for such problems. Following inequalities based on Schur-Cohn stability test [9] can be formulated and used, that does not involve any SQRT operations, wherein h_0, h_1, h_2 ϵ C

$$|g_1 - g_2g_1^*|^2 < (1 - |g_2|^2)^2..(SC_{nm})$$
$$|g_2|^2 < 1......(SC_{min})$$

where $g_1 = \frac{h_1}{h_0}; g_2 = \frac{h_2}{h_0}$ and * denotes complex conjugate.

When the leading tap is 1 or some constant value, division would not be required. When leading coefficient of CIR is not a constant, additional division arithmetic would be involved which raises similar concerns like square root algorithm on accuracy loss vs. complexity. Hence, instead of inequality SC_{nm}, modified inequalities are identified which when used, would not suffer any loss due to SQRT or DIV approximations. When both zeros (z_1, z_2) are inside or outside unit circle, or in other terms, if filter is non-mixed phase, following relation holds good.

$$(1 - |z_1|^2)(1 - |z_2|^2) < 0$$

Following inequality C_{nm} can be derived using above relation,

$$2(|h_0|^2 + |h_2|^2) - |h_1|^2 > |d|....(C_{nm})$$
$$\text{where } d = h_1^2 - 4h_0h_2$$

Above inequality can also be reformulated so as to avoid square root operations as,

$$x > y..........(C_{nm1})$$
$$(x - y)^2 > |d|^2..(C_{nm2})$$
$$\text{where } x = 2(|h_0|^2 + |h_2|^2); y = |h_1|^2$$

Inequalities proposed above can be used to determine if CIR is non-mixed phase, i.e., C_{nm1}, C_{nm2} together serves as the counterpart of R_{nm} for complex-valued coefficients. For such a non-mixed phase filter, minimum phase nature can then be identified using squared version of inequality R_{min} specified for stability triangle.

$$|h_2|^2 < |h_0|^2.....(C_{min})$$

If inequality C_{min} does not satisfy, the filter can be classified as maximum phase. Stability analysis of second order filters with complex-valued coefficients can also employ the proposed inequalities to avoid similar accuracy loss due to optimized square root and division operations.

V. ADAPTIVE PREFILTERING APPROACH

Filter type classification based on proposed inequalities helps in performing the additional minimum phase prefiltering adaptively as illustrated in Fig 2.

```
If (C_nm1 and C_nm2 satisfies)
    % Minimum or maximum phase
    If (C_min satisfies)
        % Minimum phase
    Else
        % Maximum phase
        [Optional] All-pass prefiltering of data
    End
Else
    % Mixed phase
    Find the root outside unit circle
    All-pass prefiltering of data
End
```

Fig. 2. Prefiltering strategy

Proposed prefiltering strategy helps in performing root identification only for mixed phase channels, thereby minimizing the misdetection probability of minimum phase nature of CIR. After root identification, required all pass filter for filtering data would be unstable, and approach in [2] can be employed. Since below relation holds good, stable all pass filter on the right side can be used for prefiltering of data, as proposed by Gerstacker et al.

$$\frac{H_{min}(z)}{H(z)} = \frac{H^*(1/z^*)}{H^*_{min}(1/z^*)}$$

It can be noted that no additional operations are performed on minimum and maximum phase channels. If required, maximum phase channels can be transformed to their minimum phase equivalent without any root identification. Since all-pass filter can be directly determined based on channel coefficients, root finding is avoided and data can be filtered through the combined filter that considers both roots. Proposed inequalities also facilitate significant complexity reduction compared to other methods, when minimum and maximum phase CIR are processed, that are more frequent in many channel profiles.

Time reversed processing analyzed by Ariyavisitakul [10] facilitates gains similar to that of time forward processing. In conventional 8PSK equalizers, both time forward and time reversed processing are employed starting from middle of the burst as illustrated in Fig 3. In conventional equalizers, CIR truncation does not take place, and it is beneficial to perform Viterbi path metric initialization based on training sequence.

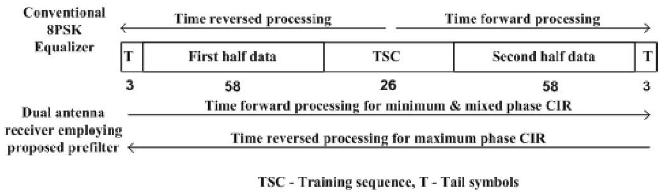

Fig. 3. Equalization Flow

In dual antenna receivers that employ proposed method for prefiltering, instead of performing both time forward and time reversed processing for each burst, either of them can be selected depending upon whether filter is maximum phase. In all cases, tail symbols present in either end of the burst can be used for Viterbi path metric initialization due to short length of selected CIR. It can be noted that if this strategy could not be adopted, additional all pass filtering would be required for maximum phase filters, during prefiltering.

VI. Simulation Results

Different channel profiles and interference conditions have been considered for performance analysis of MCS-5 logical channel in an EDGE 8PSK mobile receiver.DARP Phase II dual antenna system model is employed for the transmission and channel model as described in [13,14]. A front-end channel filter of 110 kHz is used and in the digital baseband system, symbol rate sampling is performed on both antennas, yielding 2 samples per symbol together. ICF as described in [6] is used which adaptively weighs the 2 samples and yields filtered output at the rate of 1 sample per symbol. ICF employs a feed-forward filter of length 4 and feedback filter of length 3. Eight state DFSE equalizer is employed with Euclidean distance between received and estimated symbol as branch metric. Antenna correlation of 0 and antenna gain imbalance of 0 dB are considered, as indicated in [13] for different DARP test scenarios (DTS). No frequency hopping scenarios are considered. SNR indicates the signal to noise ratio as normally defined for AWGN modeling, and CI1R refers to the carrier to dominant interferer ratio as described in DARP specifications. 24000 bursts in single slot configuration for transmission of 6000 blocks of data, are considered for each SNR or CI1R.

In Fig 4, Raw BER performance in DTS-1 scenario is compared for a dual antenna receiver employing proposed root-finding based minimum phase prefilter against similar receiver without additional prefilter. While small gains are observed for Raw BER in range of 10%, Gain of 1.5 dB is observed for low Raw BER values in range of 1%. In Fig 5, similar gains are observed in AWGN scenario for EQ-50.

In linear prediction (LP) based prefiltering, achievable performance improvement depends on zero locations and length of prediction-error filter, which could be chosen in the range

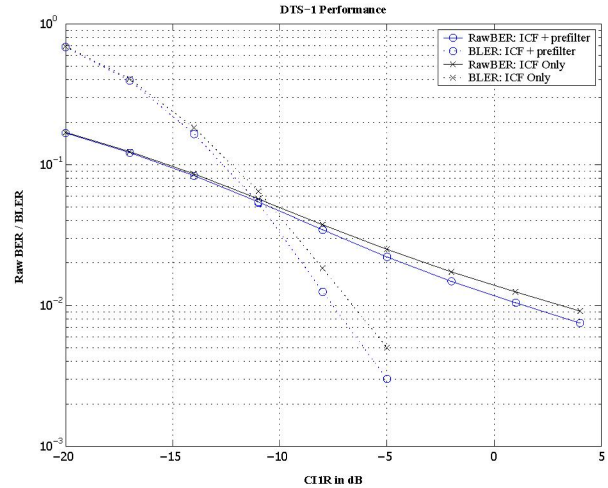

Fig. 4. Performance comparison in DTS-1

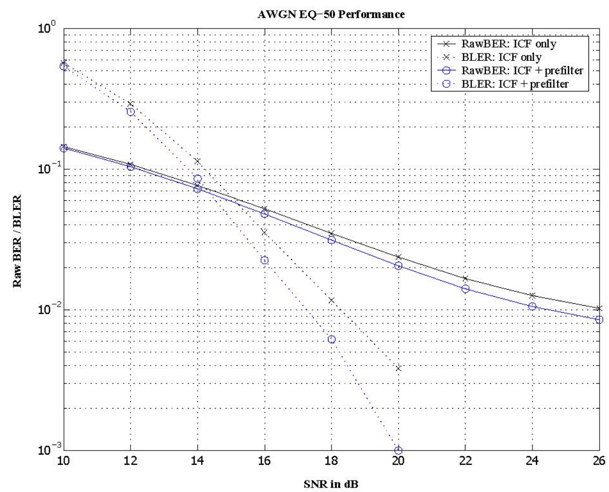

Fig. 5. Performance comparison in EQ-50

9-12 for reasonable performance with less complexity. On the other hand, performance improvement of proposed scheme depends primarily on the location of zeros of the CIR after filtering through ICF. The locations of the zeros in turn, depend on transmit filter, power delay profile of multipath channel, receive filter as discussed in [4], and also on ICF. Since ICF also plays a role in location of zeros, ratio of zeros outside unit circle to total after ICF is considered for analysis. In table 1, ratio of mixed or maximum phase CIR to total number of realizations is tabulated in terms of percentages, for 24000 bursts. SNR or CI1R considered for this analysis correspond to Raw BER in the range 0.01% to 1%.

For DTS-1 scenario, table 1 indicates that for roughly 80% of time, ICF is able to shorten CIR to a minimum phase equivalent. If additional minimum phase prefiltering is not performed, performance loss would correspond to the remaining 20% non-minimum phase channel realizations. Non-minimum phase characteristic is observed in less number of realizations for DTS-2,5 and AWGN only scenarios like TU-50, TU-3 and HT-100, and corresponding dB gains are expectedly low, in range of 0.1-0.3 dB. It should be noted that performance gains also depend on distance between zero location and unit circle. Equalizer test channel EQ50 is characterized by equal gain on all taps, and [4] indicates that distribution of number of zeros inside or outside unit circle would be identical. Table 1 indicates that after interference cancellation, performance of channel profiles such as EQ-50 could be significantly improved due to high percentage of non-minimum phase channels.

Table 1. Zeros outside unit circle after ICF

Interference	Scenario	Mixed %	Max %
CCI	DTS-1,1b	17.7	1.9
	DTS-2	5.9	0.1
	DTS-5	5.9	0.14
AWGN	TU-50	6.4	<0.1
	TU-3	6.9	<0.1
	HT-100	7.4	0.76
	EQ-50	24.2	12.3

Fixed point implementation of the proposed prefiltering strategy, on a 16-bit Dual-MAC DSP, indicates that when peak MIPS is considered, worst case scenario of mixed phase CIR would consume roughly 2K cycles. This is quite low compared to 10K cycles of linear prediction based prefiltering method. If time reversed processing is not performed for maximum phase CIR, around 3K cycles would be required since required all pass filter is longer for maximum phase CIR. Also, average MIPS can be reduced considerably since no additional processing is performed for minimum phase CIR.

In scenarios where ICF provides non-minimum phase CIR frequently, good performance gains can be achieved with complexity considerably less than LP method. In scenarios where ICF provides minimum phase CIR frequently, significant reduction in average & peak MIPS can be realized, compared to LP method, due to adaptive nature of prefiltering.

VII. Conclusions

Efficient prefiltering is discussed for dual antenna receivers employing reduced state equalizers, and an additional root-finding based prefilter is recommended. Method is computationally less intensive, and provides small performance gains in many CCI/AWGN scenarios and considerable gains in few scenarios. Method to identify minimum phase nature of second order filter with complex coefficients is proposed, that minimizes accuracy loss due to square root and division operations, and helps in performing an efficient root-finding based all-pass prefiltering. While focus has been on dual antenna receiver for EDGE 8PSK modulation scheme, receivers for higher order modulation schemes (analyzed in 3GPP GERAN) employing DFSE/RSSE with an ICF, can also use the method discussed.

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