UGM 2007 - Rayleigh Damping

1 INTRODUCTION

OrcaFlex version 9.1 permits Rayleigh damping coefficients to be defined for lines (when using the implicit integration scheme). We have examined how different values of Rayleigh damping influence tension and curvature results for a simple catenary system. We consider both a flexible riser and a steel pipe.

Note that version 9.1a does not permit hysteretic bending to be analysed with the implicit integrator. The latest in-house release does permit this, allowing us to study the influence of Rayleigh damping coupled with hysteretic bending. This capability will be included in version 9.1b which is planned for release in October.

2 DAMPING ON LINES

There are many different forms of damping, or energy dissipation, experienced by a line in a subsea environment. Typical sources of damping are:

1. Hydrodynamic drag.
2. Seabed friction.
3. Friction between layers in a composite structure.
4. Internal material damping.

The first three terms above generally give rise to the largest energy losses for such a subsea line. The material damping losses would be expected to be smaller in comparison. The same may not be true of a jumper line in air but we restrict ourselves to a discussion of subsea lines here.

Hydrodynamic drag and seabed friction are modelled using specific data items in OrcaFlex. Rayleigh damping and hysteretic bending can be used to model the remaining two sources of energy loss (items 3 and 4) as discussed below.

2.1 RAYLEIGH DAMPING IN ORCAFLEX

Rayleigh damping defines the damping matrix of the system from the following equation

$$C = \mu M + \lambda K$$

where

M = mass matrix

K = stiffness matrix

μ, λ = Rayleigh damping constants

The equation combines a mass proportional term (μM) with a stiffness proportional term (λK).

Rayleigh damping results in damping ratios that vary with the frequency of the response. Stiffness proportional damping gives damping ratios that increase linearly with *frequency*. Mass proportional damping gives damping ratios that increase linearly with *period*. The relationship between response frequency ω (rad/s) and damping ratio ξ (a value of 1 corresponds to critical damping) is:

$$\xi = 0.5(\mu/\omega + \lambda\omega) \tag{1}$$

Using a single coefficient in isolation (stiffness proportional only or mass proportional only) produces damping that is linear with either period or frequency. The damping coefficient can be adjusted to produce a particular damping value at a particular response frequency. A typical application of this would be to specify the critical damping at the wave period.

Caution should be exercised when using only mass proportional damping if there is any significant low frequency response. This is because mass proportional damping will apply a large damping to these low frequency responses.

Many sources discourage the use of mass proportional damping for this reason. For example, DNV-OS-F201 recommends using only stiffness proportional damping for "compliant structures undergoing large rigid body motions".

Clearly the same caution should be exercised when using stiffness proportional damping if there are significant high frequency responses because the stiffness proportional damping will apply a large damping effect to these high frequency responses.

Using both mass and stiffness proportional coefficients results in a damping curve as defined by formula (1). The shape of this curve can be adjusted, by varying the coefficients, to produce a flatter response over a particular period range. A typical application of this would be in irregular waves where there is a range of possible response periods.

2.2 Rayleigh Damping In Riser Analysis

Because we will be considering only regular waves, we initially choose to define a single damping response period. As is common practice we choose to use stiffness proportional damping only.

If we relate the stiffness proportional damping to a particular response period, OrcaFlex will apply the same damping in all modes (axial, bending and torsional). For a steel pipe, this is correct, as the damping is due purely to material damping, which will be the same in each direction.

However, for a flexible, we can consider two sources of internal damping: structural damping due to internal friction and slippage between the layers in the flexible and material damping from the different components in the flexible. These correspond to items 3 and 4 repectively in the list of energy losses given at the start of this section. In a typical flexible, we would expect the internal damping to lead to greater damping of the bending modes than of the axial and torsional modes.

One approach would be to define different damping coefficients in different directions, including the slippage and internal friction effects through an increase in the bending damping value. OrcaFlex does allow this approach but it is not to be recommended since the effects of hysteretic bending damping due to internal friction are best modelled using an amplitude proportional damping model. Accordingly we use OrcaFlex's built-in hysteretic bending model to capture this effect and combine it with Rayleigh damping to model the material damping of the flexible components. Using this approach we can apply the same damping coefficients for axial, bending and torsional modes.

3 FLEXIBLE PIPE STUDY

3.1 FLEXIBLE PIPE DATA

The flexible pipe is modelled in a simple catenary configuration in a water depth of 100m. The horizontal distance from hang-off to the nominal touchdown point is 81m with a touchdown arc length of 131m.

The following table summarises the line type properties for the flexible line. Note that these properties are not based on a specific flexible construction but are generic estimated values.

Outer diameter	Inner diameter	Mass per unit length	Bend stiffness	Axial stiffness
350mm	250mm	180kg/m	Hysteretic	700×10^3 kN

Table 3-1 : Summary Of Line Properties

Where "Hysteretic" bending stiffness is specified, this is applied through a hysteretic non-linear bend moment – curvature relationship for the line. This relationship is shown in the following figure. Again, please note that this is a purely general relationship that is not intended to replicate any particular flexible design or construction.

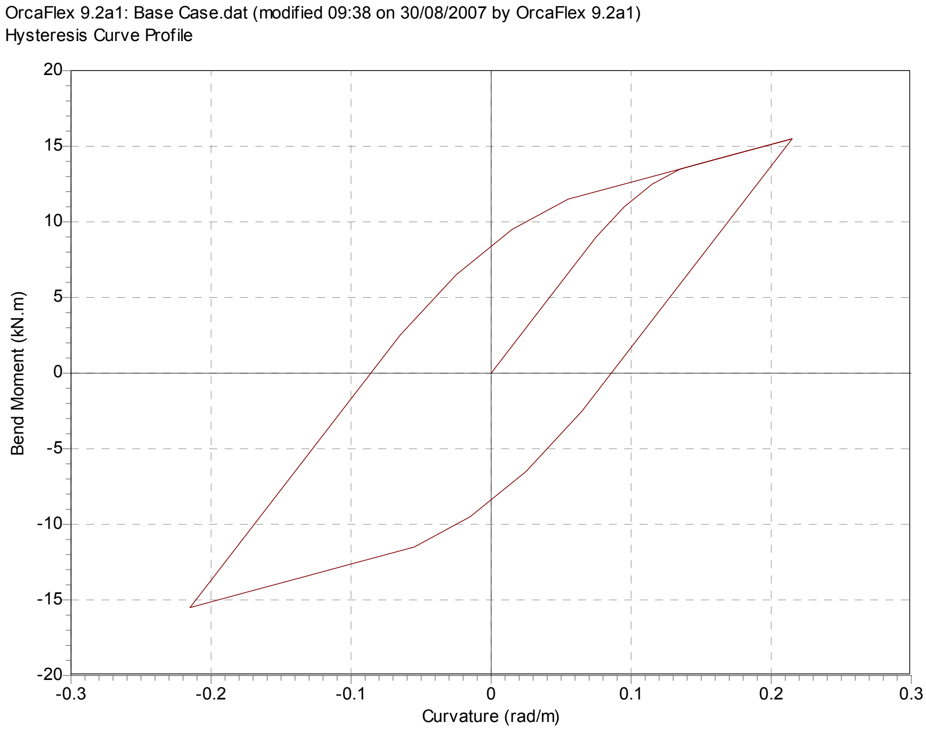

Figure 3-1 : Bend Moment – Curvature Relationship For Hysteretic Bending

The flexible was considered both with and without damping. Where damping was applied, a material damping coefficient of 3% was assumed. This was initially applied as stiffness proportional Rayleigh damping, configured to achieve 3% of critical damping at the wave period.

The flexible was analysed in four regular wave load cases applied as pure near cases (weather from the hang-off towards the touchdown). These are summarised in the following table.

Wave Height (m)	Wave Period (s)
2	4
4	6
6	8
8	9

Table 3-2 : Regular Load Cases For Flexible Riser

3.2 Tension and Curvature Results

The following three plots show the variation with wave height of maximum tension, minimum tension and curvature. Note that 'Damped Flexible' refers to 3% stiffness proportional damping in this case.

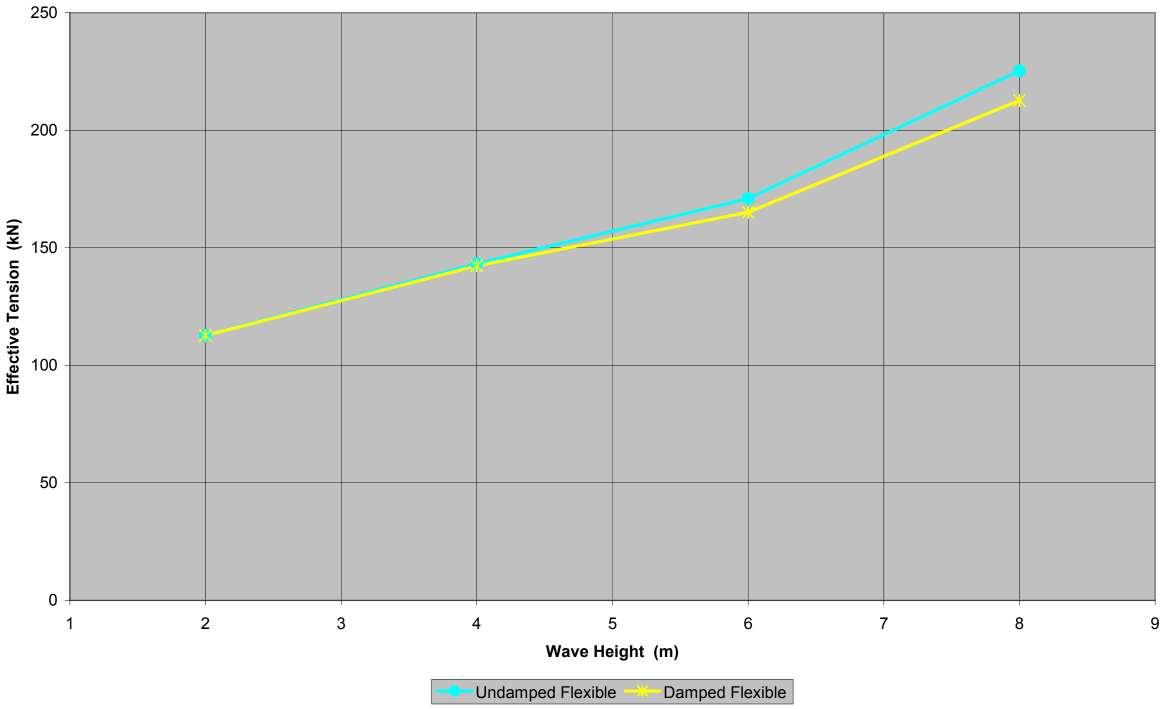

Figure 3-2 : Variation Of Maximum Tension With Wave Height

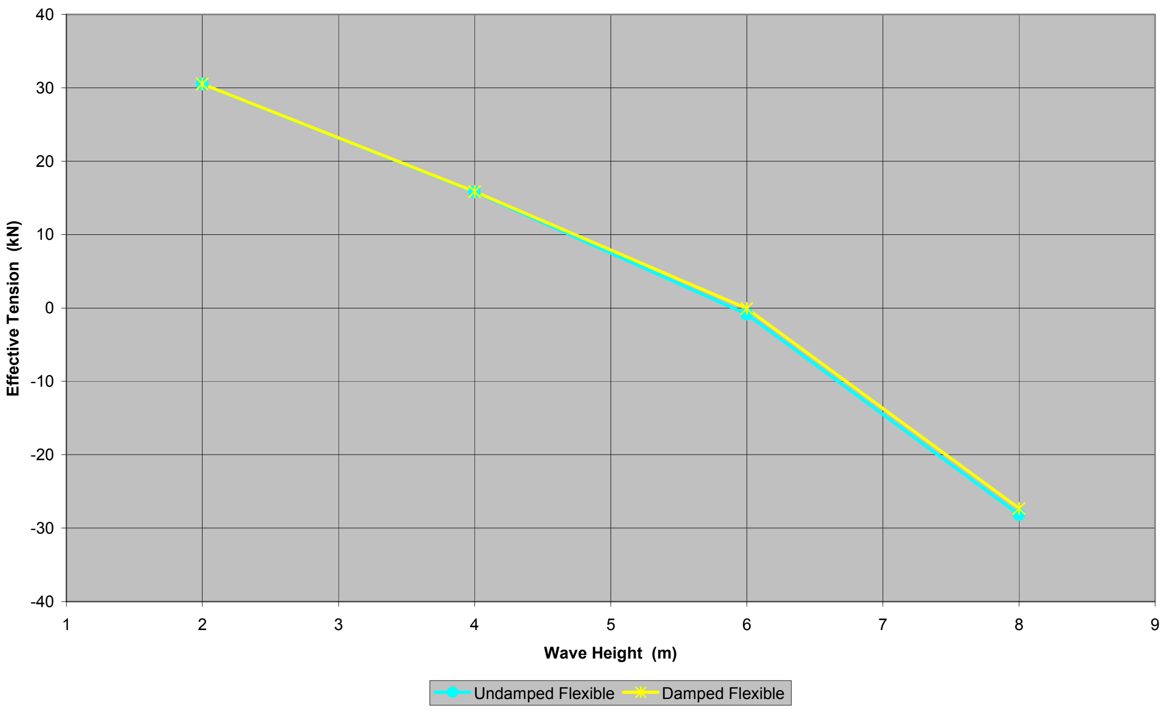

Figure 3-3 : Variation Of Minimum Tension With Wave Height

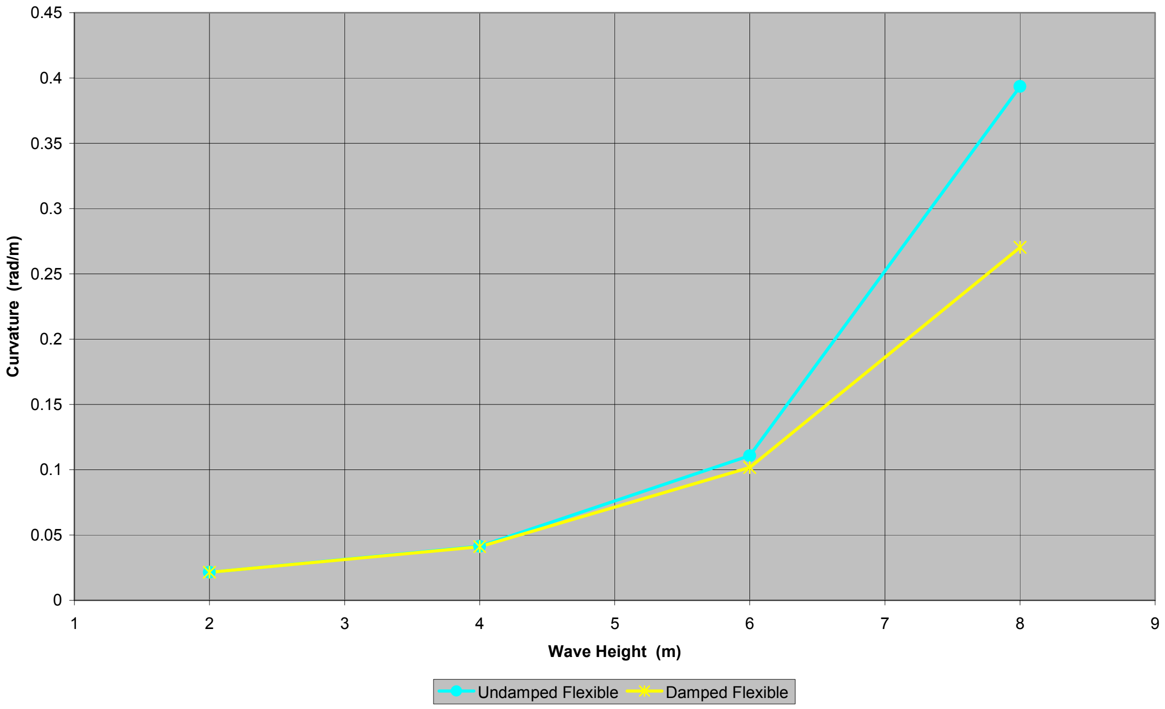

Figure 3-4 : Variation Of Maximum Curvature With Wave Height

The maximum tension plot indicates that the damping has a minor effect on maximum tension at the highest wave height (Figure 3-2) but negligible effect at the lower three wave heights. Where a difference is shown, the maximum tension is slightly greater in the undamped case.

The minimum tension shows no significant effect of damping for any wave height.

The curvature plot shows a more marked effect of damping at the highest wave height but negligible effect over the lower three wave heights. The results indicate that the undamped line has a greater curvature than the damped line where a difference is shown.

These results show that the 3% damping has no significant influence on the results except at the highest wave height. The following section discusses the system response at the highest wave height in more detail.

3.3 Discussion Of Behaviour

As would be expected, the maximum tension tends to increase with wave height while the minimum tension reduces with increasing wave height. Figure 3-3 shows that significant levels of compression are encountered in the 8m wave height load case. In fact, this case shows the formation of a large compressive wave near the touchdown point.

The following screen snap-shot shows the a close-up of the touchdown region of the undamped flexible at the time of maximum curvature.

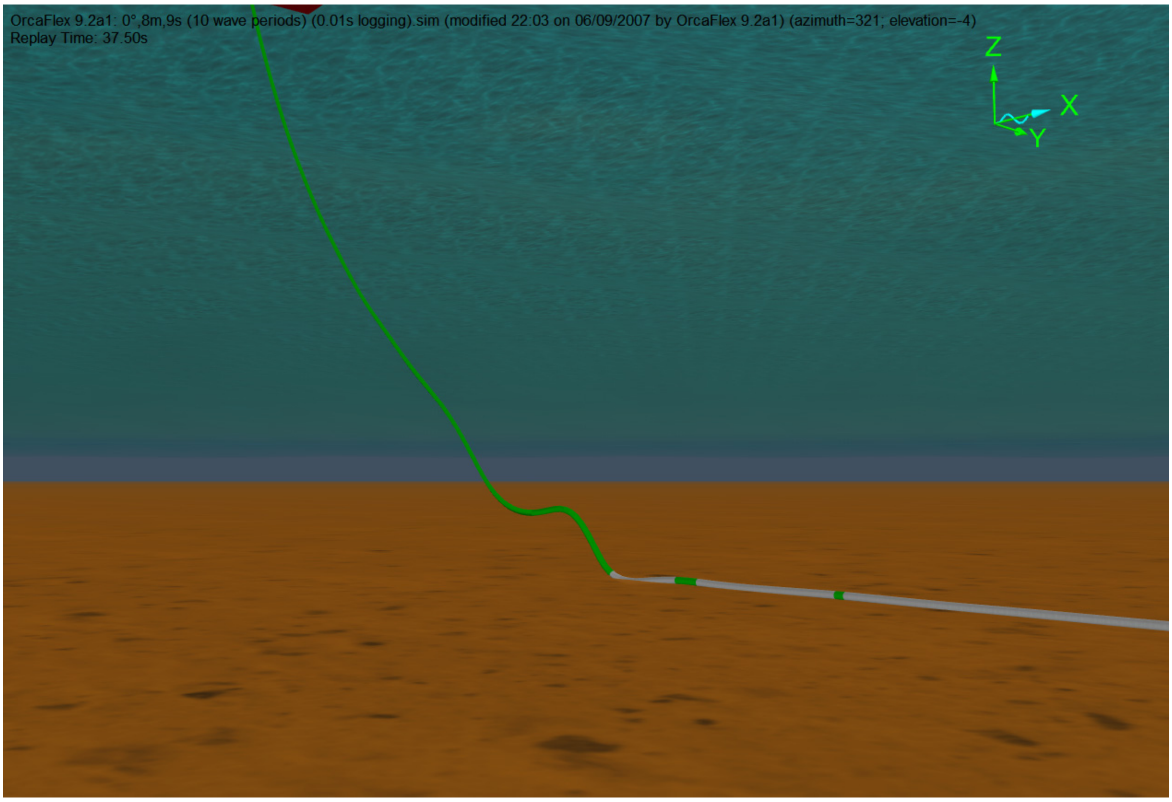

Figure 3-5 : Snap-Shot Of Flexible Touchdown Region At Time Of Maximum Curvature

In reality, such a compressive wave would be designed out of a riser system, as it is an unacceptable behaviour. However, for the purposes of the present study, we choose to examine the response in more detail to understand why the inclusion of Rayleigh damping produces a significant reduction of curvature for the compressive wave case.

The following time histories show the variation of y-curvature (at the arc length of maximum curvature) for the two flexible cases in the 8m wave height. These results are taken over 5 regular wave periods.

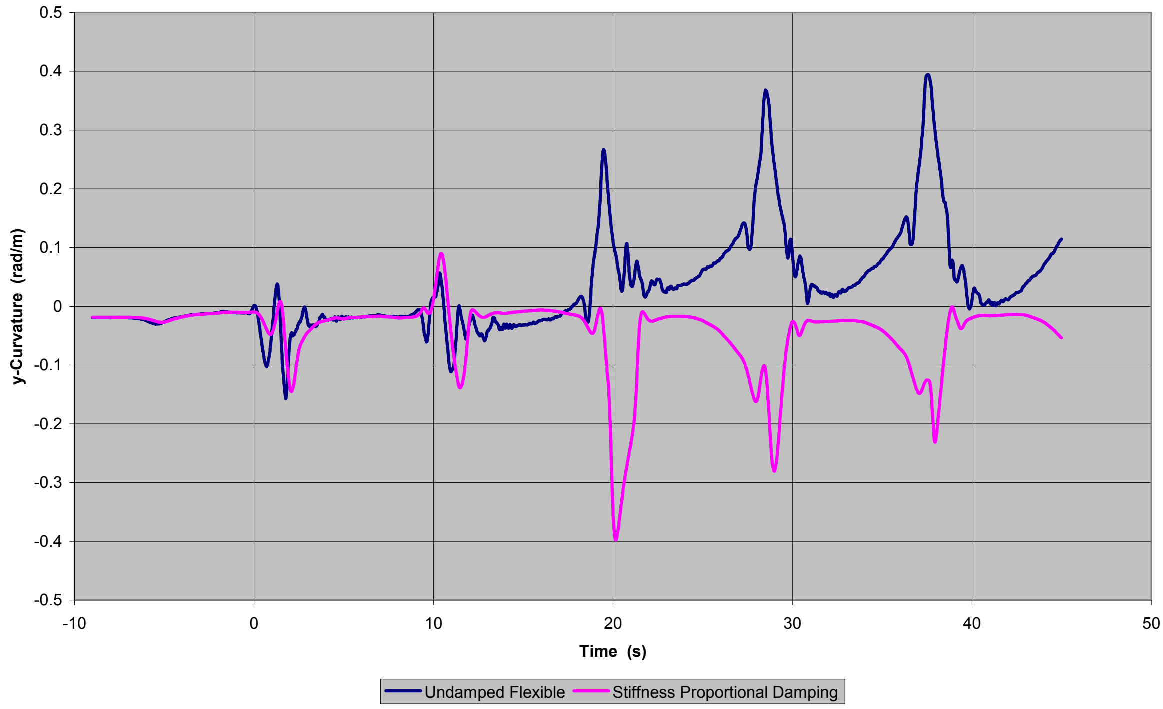

Figure 3-6 : Time Histories Of y-Curvature For Undamped And 3% Stiffness Proportional Damped Flexible In 8m Wave Height

It is clear from a comparison of the time histories that the system is not producing a settled response. Extending the simulation for an additional 5 regular wave periods eventually resulted in a settled response for the damped flexible. However, the undamped flexible did not show a settled response within 10 regular waves. This indicates the inherently 'chaotic' nature of the compressive wave response and may partially explain the differences in curvature seen in Figure 3-4.

However, the system damping may also have an effect.

The time history plots for the 8m wave height show that there is significant response at frequencies greater than the wave period due to the compressive wave. This is particularly notable in the undamped case. The following plot shows a close-up of the time history shown in Figure 3-6 for the undamped flexible.

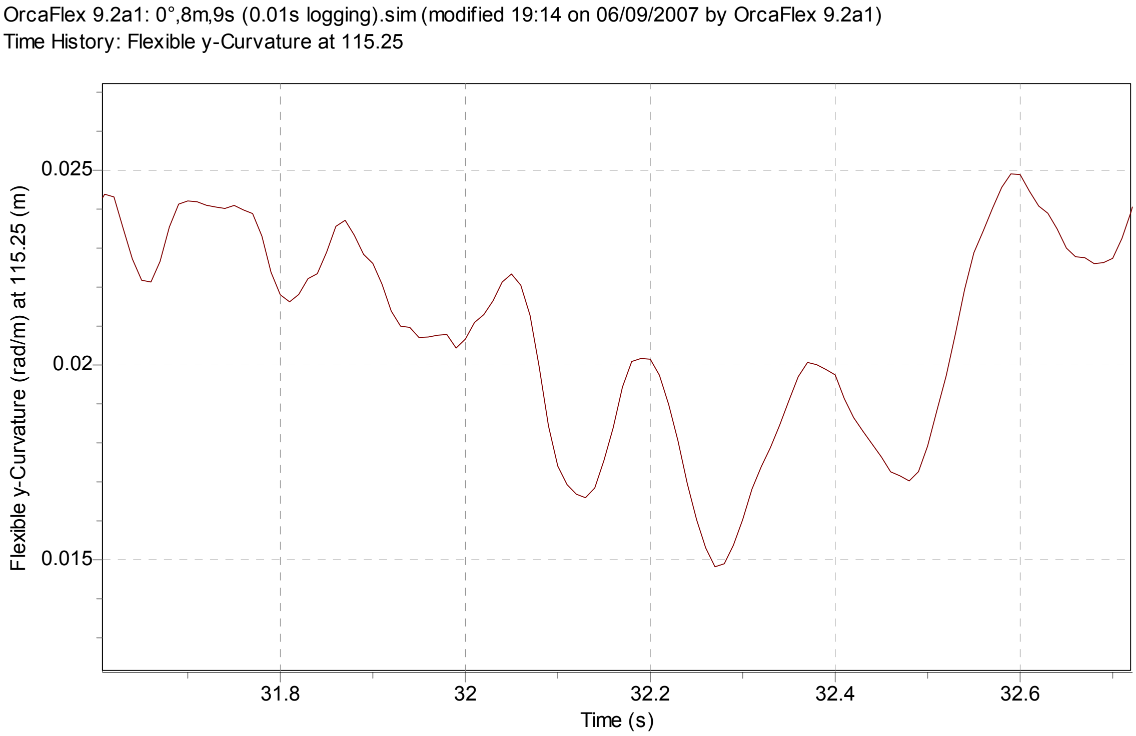

Figure 3-7 : Close-Up Of Time History From 8m Wave Height For Undamped Flexible

Closer examination of the high frequency responses shown in Figure 3-7 suggests that they have a period of approximately 0.2s, i.e. a frequency of 5Hz. This is much higher than the wave response frequency (which is approximately 0.1Hz).

The following plot shows the variation of damping with response frequency for the flexible with stiffness proportional damping applied.

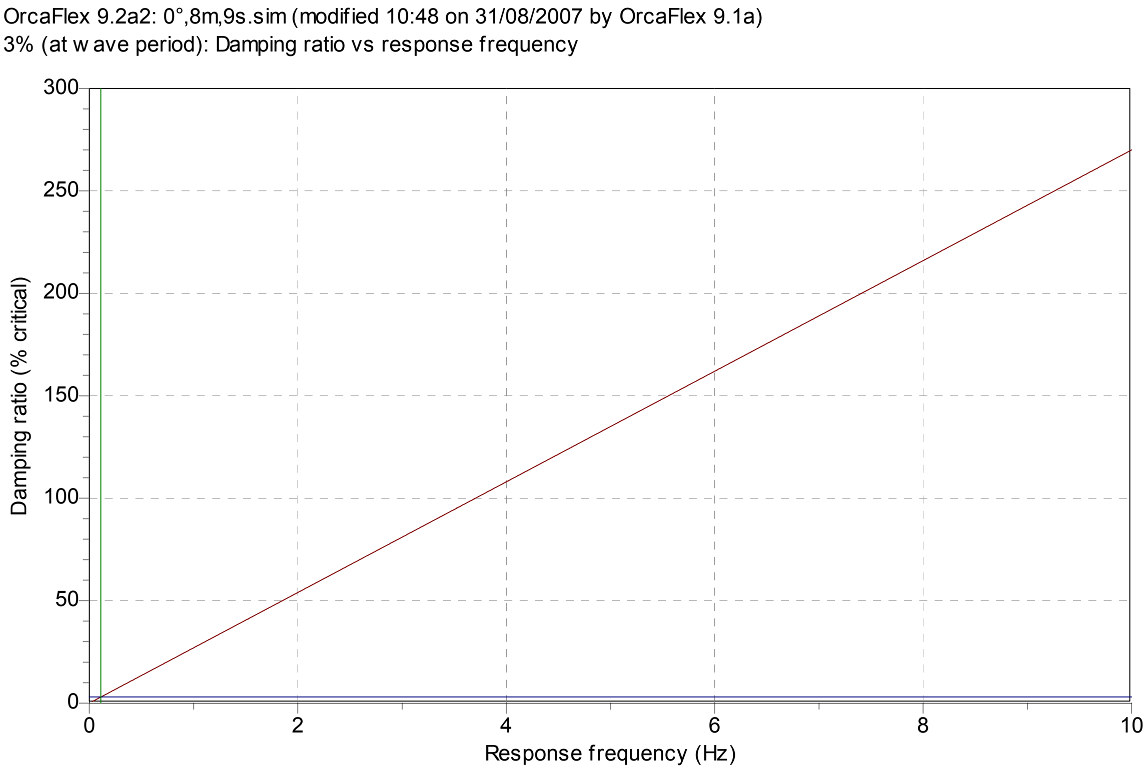

Figure 3-8 : Stiffness Proportional Damping With 3% Critical Applied At 9s Response Period

As the plot shows, stiffness proportional damping leads to a linear increase in damping with response frequency. This means that frequencies greater than the selected response period will be damped more than lower frequencies. In our case, we have set the damping response period equal to the wave period (9s in the case of the 8m wave height).

Thus, all responses with a frequency greater than approximately 0.1Hz experience damping greater than 3% critical. As the damping varies linearly with frequency, we can see from the plot that a frequency of 5Hz would experience damping of 135% critical. This means that the cases discussed above where only stiffness proportional damping was applied have unrealistically over-damped the high frequency responses.

Recall that Rayleigh damping is intended to represent the material damping in this case (as the structural damping is modelled by the hysteretic bend stiffness) and so we desire to damp all responses with 3% critical damping. As discussed in section 2.1, using a combination of stiffness and mass proportional damping permits us to produce a flatter variation in damping with response period. The following plot shows the effect of selecting 0.1s (10Hz) and 9s (wave period) as the periods at which 3% damping will be achieved.

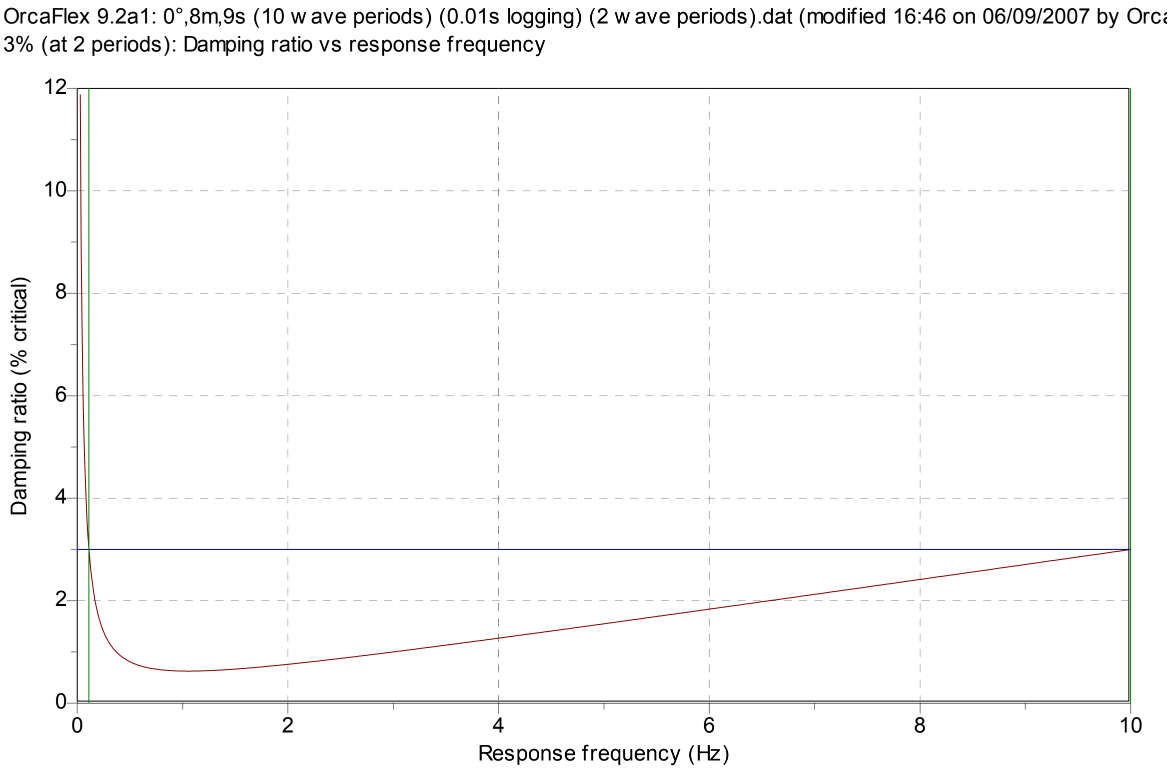

Figure 3-9 : Mass And Stiffness Proportional Damping With 3% Critical Applied At 0.1s And 9s Response Periods

The 8m wave height case for the flexible with damping applied was re-analysed using the damping curve presented in Figure 3-9. The following plot compares the time history of y-curvature at the maximum curvature arc length for the mass and stiffness proportional case against the undamped flexible response.

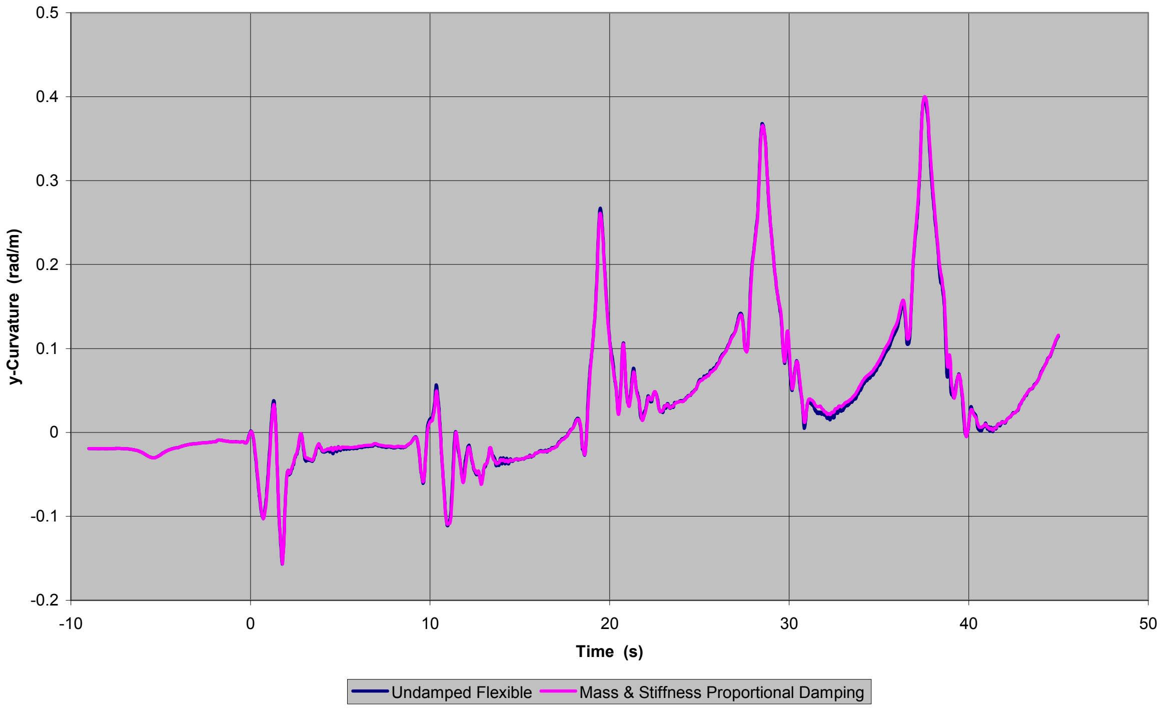

Figure 3-10 : Comparison Between Undamped And Mass And Stiffness Proportional Damped Flexible

It is clear from the plot that the two time histories are virtually identical with no significant differences (even for the relatively high frequency responses). This indicates that the application of 3% critical damping has no effect on the response of the flexible, even in a system exhibiting a significant compressive wave.

Therefore, we can see that the differences in response noted between the damped and undamped cases in Figure 3-4 and Figure 3-6 were due to incorrect application of the Rayleigh damping coefficients. Using stiffness proportional damping only with a response period equal to the wave period resulted in severe over-damping of the high frequency responses.

4 STEEL CATENARY RISER STUDY

4.1 STEEL PIPE DATA

The steel catenary riser (SCR) with outer diameter of 350mm and wall thickness of 25mm was analysed in a water depth of 1800m. The horizontal distance from hang-off to nominal touchdown was 1083m with a touchdown arc length of 2210m.

The regular wave load cases used in the SCR study are given below. These were applied in the pure far direction with the weather heading from the touchdown point towards the hang-off.

Wave Height (m)	Wave Period (s)
10	11
15	13
20	15
25	17

Table 4-1 : Regular Load Cases For SCR

Again, both damped and undamped cases were analysed. For the damped cases, a Rayleigh damping ratio of 0.3% critical was used. Following the results from the flexible study, we applied the damping in two ways. Firstly, as stiffness proportional damping only but also as mass and stiffness proportional damping. The period range for the mass and stiffness proportional damping was selected to span the response periods observed in the undamped case.

4.2 TENSION AND CURVATURE RESULTS

The following plots show the tension time histories at the topside hang-off for the lowest and highest wave heights considered.

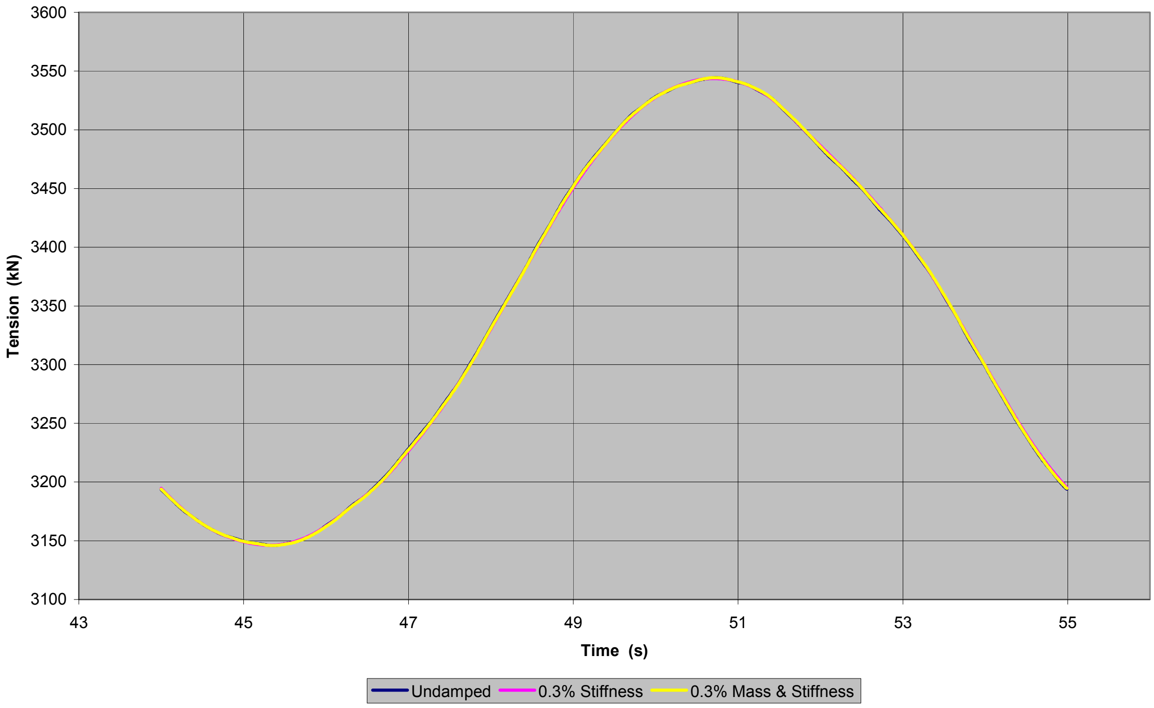

Figure 4-1 : Topside Tension Time History For 10m Wave

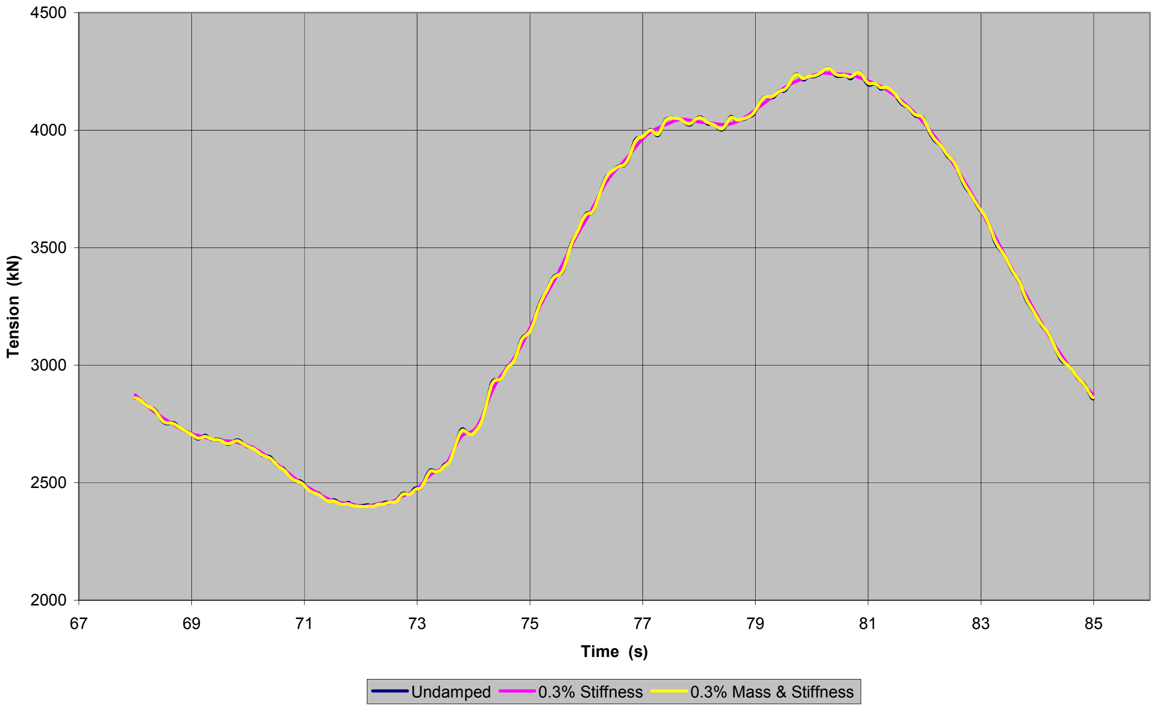

Figure 4-2 : Topside Tension Time History For 25m Wave

Figure 4-1 shows that all three responses (undamped and both damped) are identical where there are no high frequency components observed in the response. However, high frequency responses are evident in Figure 4-2 where we can see a response with approximately 0.5s period. The following figure shows a close-up of one section of the tension time history.

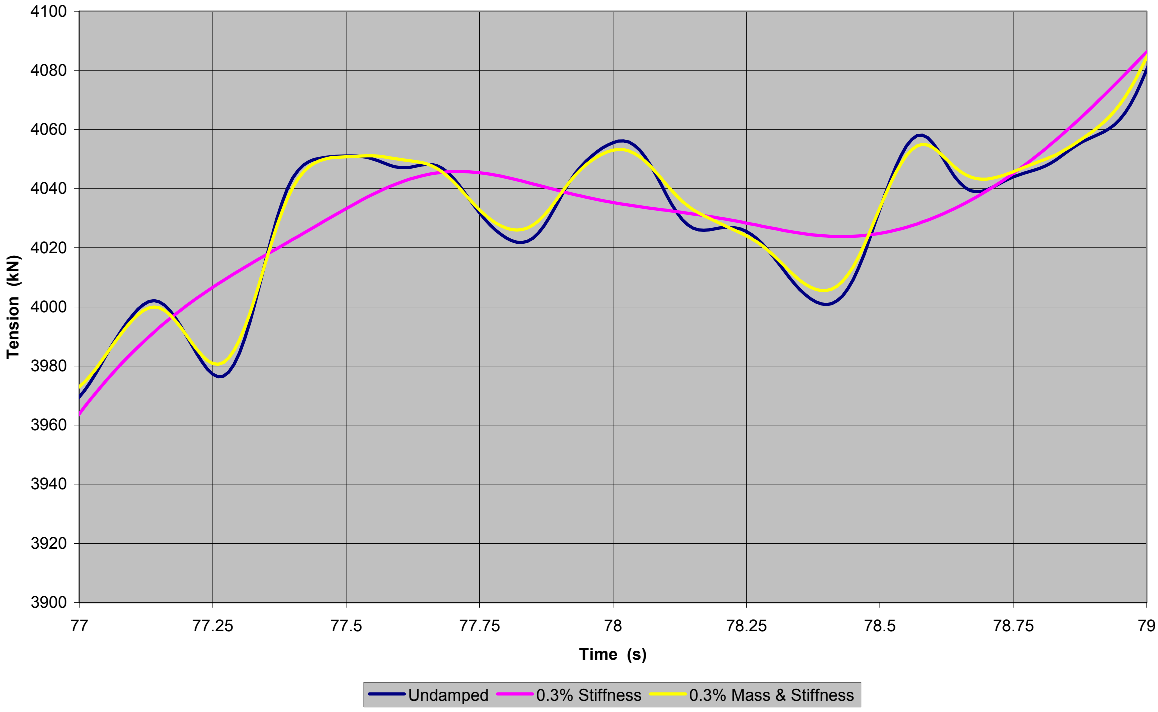

Figure 4-3 : Close-Up Of Topside Tension Time History For 25m Wave

The high frequency response is observed for the undamped case and the case with mass and stiffness proportional damping applied (albeit with slightly reduced amplitude). However, the case with stiffness proportional damping only does not show the high frequency response.

Comparing the results of the mass and stiffness proportionally damped cases shows that they are virtually indistinguishable. There is a very slight decrease in response amplitude for the damped case but this is negligible.

The following plot shows the curvature time history for a point near touchdown for the highest wave height case. Note that we report y-curvature here rather than absolute curvature, hence the negative values of curvature shown.

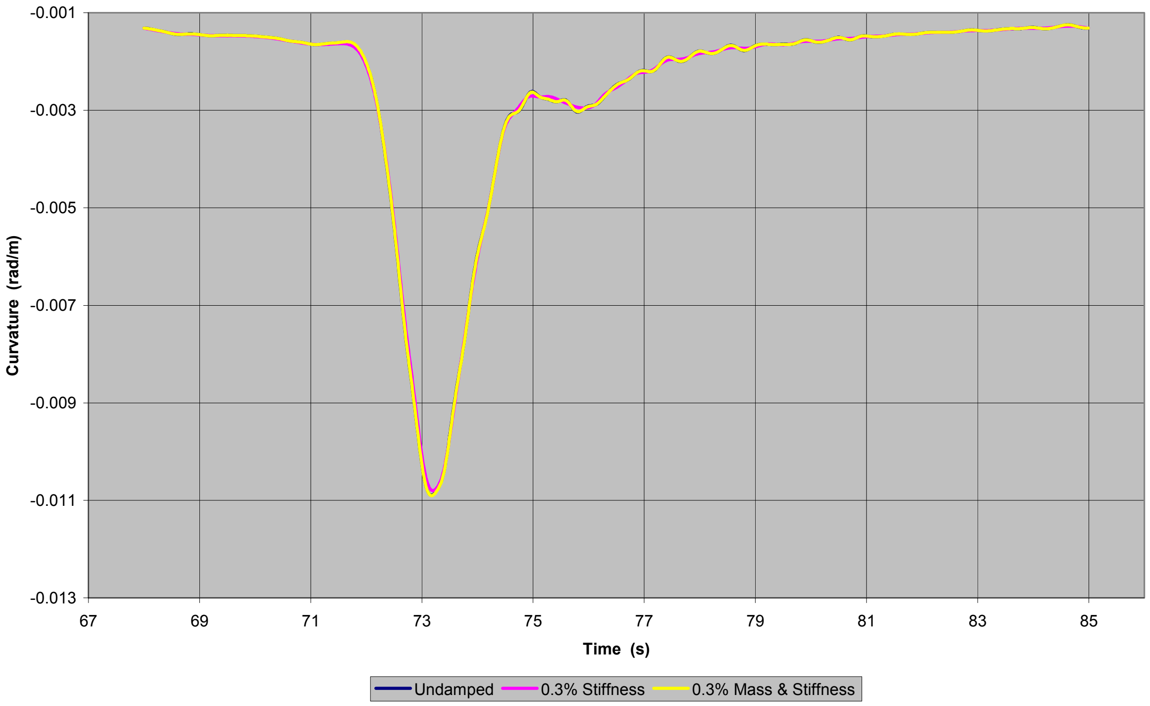

Figure 4-4 : Curvature Time History For 25m Wave Height

A close-up of the curvature time history is shown below.

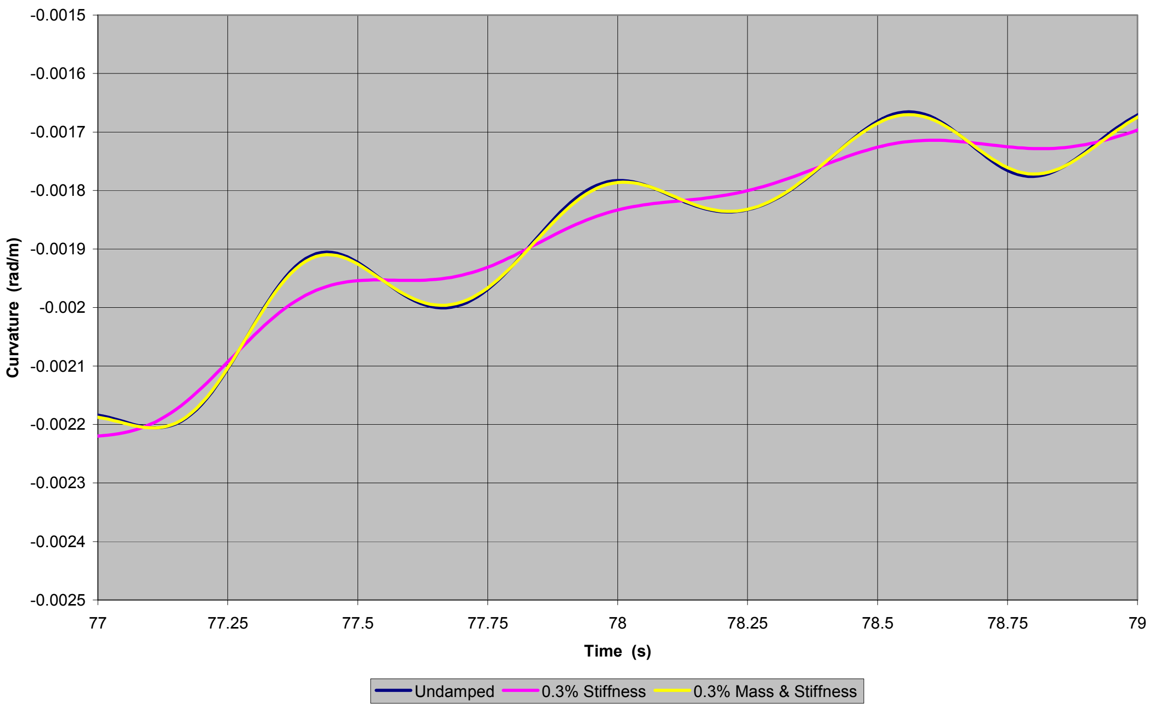

Figure 4-5 : Close-Up Of Curvature Time History For 25m Wave Height

Again, we see a high frequency response in the undamped case and the mass and stiffness proportional damping case that is not evident in the case with stiffness proportional damping only applied.

Overall, the results for the SCR agree with the behavioural trends observed in the flexible analysis. Where higher frequency responses are present, using stiffness proportional damping only can result in over-damping of the high frequencies. In such cases, damping needs to be correctly applied over the whole range of response periods. When damping is correctly applied, it has no significant affect on results.

5 IMPLICIT TIME STEPS

In order to accurately model the higher frequency responses quite a short time step is required. For the flexible case we actually used 0.001s. This is probably shorter than strictly necessary but we were aiming to be conservative in our modelling.

In practice, as noted earlier, the type of compressive waves noted in the flexible response should be designed out of the system and so final productions runs should not require such short time steps.

6 SUMMARY AND CONCLUSIONS

We have considered the behaviour of a steel catenary riser (SCR) and a flexible laid in a simple catenary configuration using the new Rayleigh damping feature in OrcaFlex. We have considered both damped and undamped line models. For the flexible riser we have also included hysteretic bending.

Where applied, the damping used for the SCR was 0.3% critical while the damping used for the flexible was 3% critical. In both cases, we considered applying the damping as stiffness proportional only and as mass and stiffness proportional. Equal damping was applied in axial, torsional and bending modes, as we used the damping to model material damping only (structural damping in the flexible was modelled through the hysteretic bend stiffness).

The different line models were considered with a series of different wave heights and periods.

For the flexible, we see the formation of compressive waves at the highest wave heights. This leads to high frequency responses in the system and an unsettled, chaotic behaviour. Normally, we would re-design the system to remove such behaviour but here we have studied it further to understand how damping might affect the response.

Stiffness proportional damping of 3% at wave period results in unrealistically high damping at higher frequencies and so the higher frequency responses of the compressive wave are over-damped. This causes significant differences in the behaviour observed for the undamped and stiffness proportionally damped systems.

The same compressive wave load case was re-analysed with both stiffness and mass proportional damping applied. Response periods were set based on the wave period and the observed high frequency response. This removed the unrealistic over-damping of the higher frequency responses.

Comparison of the curvature results for the flexible with mass and stiffness proportional damping applied against the undamped response showed no significant differences. This indicates that the application of 3% damping has no affect on the flexible response, even in a system exhibiting compressive waves, as long as the damping is correctly applied.

For the SCR, we observed similar behaviour. For lower wave heights, no high frequency responses were observed. In these cases, we obtained identical results in terms of tension and curvature for all three models (undamped and both damped cases). At the higher wave heights, we again observed high frequency responses. These were damped out by the case with stiffness proportional damping only applied. However, the undamped and mass and stiffness proportionally damped cases gave virtually identical results even when high frequency responses were evident. Again, this indicates that correctly applied damping has no significant effect on results.

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5th October 2007