0.1 Def

We call integral operator on S any $\mathbb{R}$-linear mapping $I : S \to \mathbb{R}$ that satisfies the following conditions:

(1) If $f \in S$ is such that $\forall \omega \in \Omega, f(\omega) \geq 0$ then $I(f) \geq 0$

(2) If $(f_n)_{n\in\mathbb{N}}$ is a decreasing sequence of elements in S such that $\forall \omega \in \Omega \lim\limits_{n\to+\infty} f_n(\omega) = 0$ then

$$\lim_{n\to+\infty} I(f_n) = 0$$

$(\forall \omega \in \Omega, n \in \mathbb{N}, f_n(\omega) \geq f_{n+1}(\omega))$

0.2 Def

Let Ω be a set. We call semialgebra on Ω any $\mathcal{C} \subsetneqq \wp(\Omega)$ that verifies:

- $\varnothing \in \mathcal{C}$
- $\forall (A, B) \in \mathcal{C}^2, A \cap B \in \mathcal{C}$
- $\forall (A, B) \in \mathcal{C}^2, \exists (C_i)_{i=1}^n$ a finite family of elements in $\mathcal{C}$ such that $B \setminus A = \bigsqcup\limits_{i=1}^{n} C_i$

0.3 Def

Let $\mathcal{C}$ be a semialgebra on Ω. The set

$$\{A \in \wp(\Omega) \mid \exists n \in \mathbb{N}, \exists (A_i)_{i=1}^n \in \mathcal{C}^n, A = \bigsqcup_{i=1}^{n} A_i\}$$

is called the algebra generated by $\mathcal{C}$

0.4 Def

Let $\mathcal{C} \subseteq \wp(\Omega)$. We denote by $\sigma(\mathcal{C})$ the intersection of all σ-algebras on Ω containing $\mathcal{C}$. It's the smallest σ-algebra containing $\mathcal{C}$

0.5 Def

Let $f : X \to Y$ be a mapping of sets.

- For any $\mathcal{C}_Y \subseteq \wp(Y)$ we denote by

$$f^{-1}(\mathcal{C}_Y) := \{f^{-1}(B) \mid B \in \mathcal{C}_Y\}$$

- For any $\mathcal{C}_X \subseteq \wp(X)$ we denote by

$$f_*(\mathcal{C}_X) := \{B \subseteq Y \mid f^{-1}(B) \in \mathcal{C}_X\}$$

0.6 Def

Let $(X, \mathcal{G}_X)$ and (Y, g_Y) be measurable spaces, $f : X \to Y$ be a mapping. If $f^{-1}(g_Y) \subseteq \mathcal{G}_X$ or equivalently $g_Y \subseteq f_8(\mathcal{G}_X)$ (or $\forall B \in g_Y, f^{-1}(B) \in \mathcal{G}_X$) then we say that f is measurable.

0.7 Def

Let Ω be a set $((E_i, \mathcal{E}))_{i\in I}$ be a family of measurable spaces. $f = (f_i)_{i\in I}$ where $f_i : \Omega \to E_i$ is a mapping. We denote by $\sigma(f)$ the σ-algebra $\sigma(\bigcup\limits_{i\in I} f_i^{-1}(\mathcal{E}_i))$
It's the smallest σ-algebra on Ω making all f_i measurable.

If I_μ is an integral operator, we say that μ is σ-additive.

0.8 Def

If $\exists(A_n)_{n\in\mathbb{N}}$ such that $\Omega = \bigcup\limits_{n\in\mathbb{N}} A_n$ and $\mu(A_n) < +\infty$ then μ is said to be σ-finite.

0.9 Def

We fix a measure space $(\Omega, \mathcal{G}, \mu)$ the set of measurable mappings $f : \Omega \to \mathbb{R}$ such that

$$f_{L^p} := (\int_\Omega |f(\omega)|^p \, \mu(dx))^{\frac{1}{p}} < +\infty$$