Ceng 461 HW3 Report

Utility values for each state will be zero at the start. We will have states which represents the number of fish in the lake

s_0 : 0 fish in lake

s_1 : 1 fish in lake

⋮

s_{10} : 10 fish in lake.

Reward will be the number of fish catched at fishing season.

After the first iteration, the utility values of each state will be the number of fish in the lake because there is no value of going to a state since the utilities of every state is zero. For example

For state 2, there are 3 different choices: Catch 0 fish, catch 1 fish, catch 2 fish. For each:

$P(1,0)\ U_0(s_2)\quad P(1.25)\ U_0(s_3)\quad P(1.5).U_0(s_3)\quad P(1.75).U_0(s$

Catch 0 fish: $U_1(s_2) = 0 + 0.9[\underbrace{0.2.0}_{\text{since the state won't change if the population doesn't change}} + 0.3.0 + 0.3.0 + 0.2.0$

$= 0$

Catch 1 fish: $U_1(s_2) = 1 + 0.9[0.2.0 + 0.3.0 + 0.3.0 + 0.2.0] = 1$

Catch 2 fish: $U_1(s_2) = 2 + 0.9[0.2.0 + 0.3.0 + 0.3.0 + 0.2.0] = 2$

So, the optimal policy is to catch all fishes in the lake after the end of the first iteration and the utility values will be:

$U_1(s_0)=0,\ U_1(s_1)=1,\ U_2(s_2)=2,\ U_3(s_3)=3\ \ldots\ldots,\ U_{10}(s_{10})=10.$

Second iteration:

For state 0:

Only option is to catch no fish and there is no possibility to go to another state, so: $\boxed{U_2(s_0)=0}$.

For state 1:

Catch 0 fish $= 0+0.9[0.2\cdot1+0.3\cdot1+0.3\cdot2+0.2\cdot2]=\boxed{1.35}$

Catch 1 fish $= 1+0.9[0.2\cdot0+0.3\cdot0+0.3\cdot0+0.2\cdot0]=1$

So, the optimal policy is $\boxed{\text{catching no fish and } U_2(s_1)=1.35}$

For state 2:

3

$0.4 + 0.9 + 0.9 + 0.8$

Catch 0 fish $= 0+0.9[0.2\cdot2+0.3\cdot3+0.3\cdot3+0.2\cdot4] = \boxed{2.7}$

Catch 1 fish $= 1+0.9[0.2\cdot1+0.3\cdot1+0.3\cdot2+0.2\cdot2]=2.35$

Catch 2 fish $= 2+0.9[0.2\cdot0+0.3\cdot0+0.3\cdot0+0.2\cdot2] = 2$

So, the optimal policy is $\boxed{\text{catching no fish and } U_2(s_2)=2.7}$

For state 3:

Catch 0 fish $= 0 + 0.9\left[\overset{0,6}{0,2 \cdot 3} + \overset{1,2}{0,3 \cdot 4} + \overset{1,5}{0,3 \cdot 5} + \overset{1}{0,2 \cdot 5}\right] = \boxed{3.87}$

Catch 1 fish $= 1 + 0.9\left[0,2 \cdot 2 + 0,3 \cdot 3 + 0,3 \cdot 3 + 0,2 \cdot 4\right] = 3,7$

Catch 2 fish $= 2 + 0,9\left[0,2 \cdot 1 + 0,3 \cdot 1 + 0,3 \cdot 2 + 0,2 \cdot 2\right] = 3,35$

Catch 3 fish $= 3 + 0.9\left[0,2 \cdot 0 + 0,3 \cdot 0 + 0,3 \cdot 0 + 0,2 \cdot 0\right] = 3$

So, the optimal policy is to $\boxed{\text{catch no fish and } U_2(s_3) = 3.87}$

For state 4:

Catch 0 fish $= 0 + 0,9\left[\overset{0,8}{0,2 \cdot 4} + \overset{1,5}{0,3 \cdot 5} + \overset{1,8}{0,3 \cdot 6} + \overset{1,4}{0,2 \cdot 7}\right] = \boxed{4.95}$

Catch 1 fish $= 1 + 0.9\left[0,2 \cdot 3 + 0,3 \cdot 4 + 0,3 \cdot 5 + 0,2 \cdot 5\right] = 4.87$

Catch 2 fish $= 2 + 0.9\left[0,2 \cdot 2 + 0,3 \cdot 3 + 0,3 \cdot 3 + 0,2 \cdot 4\right] = 4.7$

Catch 3 fish $= 3 + 0,9\left[0,2 \cdot 1 + 0,3 \cdot 1 + 0,3 \cdot 2 + 0,2 \cdot 2\right] = 4.35$

Catch 4 fish $= 4 + 0,9\left[0,2 \cdot 0 + 0,3 \cdot 0 + 0,3 \cdot 0 + 0,2 \cdot 0\right] = 4$

So, the optimal policy is to $\boxed{\text{catch no fish and } U_2(s_4) = 4.95}$

For state 5:

1 1,4 2,4 1,8

Catch 0 fish = 0 + 0.9[0,2.5 + 0,3.6 + 0,3.8 + 0,2.9] = $\boxed{6,3}$

Catch 1 fish : 1 + 0.9[0,2.4 + 0,3.5 + 0,3.6 + 0,2.7] = 5,95

Catch 2 fish = 2 + 0.9[0,2.3 + 0,3.4 + 0,3.5 + 0,2.5] = 5,87

Catch 3 fish = 3 + 0.9[0,2.2 + 0,3.3 + 0,3.3 + 0,2.4] = 5,7

Catch 4 fish = 4 + 0.9[0,2.1 + 0,3.1 + 0,3.2 + 0,2.2] = 5,35

Catch 5 fish = 5 + 0.9[0,2.0 + 0,3.0 + 0,3.0 + 0,2.0] = 5

So, the optimal policy is to $\boxed{\text{catch no fish and } V_2(s_5) = 6.3}$

For state 6:

since max fish count is 10.

Catch 0 fish = 0+0.9 [0,2.6 + 0,3.8 + 0,3.9 + 0,2.10] = $\boxed{7,47}$

1,2 2,4 2,7 2

Catch 1 fish = 1+0,9 [0,2.5 + 0,3.6 + 0,3.8 + 0,2.9] = 7,3

Catch 2 fish: 2 + 0,9[0,2.4 + 0,3.5 + 0,3.6 + 0,2.7] = 6,95

Catch 3 fish = 3 + 0,9[0,2.3 + 0,3.4 + 0,3.5 + 0,2.5] = 6,87

Catch 4 fish = 4 + 0,9[0,2.2 + 0,3.3 + 0,3.3 + 0,2.4] = 6,7

Catch 5 fish = 5 + 0,9[0,2.1 + 0,3.1 + 0,3.2 + 0,2.2] = 6,35

Catch 6 fish: $6+0,9[0,2.0+0,3.0+0,3.0+0,2.0]=6$

So, the optimal policy is to $\boxed{\text{catch no fish and } U_2(s_6)=7,47}$

For state 7:

$$\text{Catch 0 fish} = 0+0.9[\overset{1,4}{0,2.7}+\overset{2,7}{0,3.9}+\overset{3}{0,3.10}+\overset{2}{0,2.10}] = 8,19$$

$$\text{Catch 1 fish} = 1+0.9[0,2.6+0,3.8+0,3.9+0,2.10] = \boxed{8,47}$$

$$\text{Catch 2 fish} = 2+0,9[0,2.5+0,3.6+0,3.8+0,2.9] = 8,3$$

$$\text{Catch 3 fish} = 3+0,9[0,2.4+0,3.5+0,3.6+0,2.7] = 7,95$$

$$\text{Catch 4 fish} = 4+0,9[0,2.3+0,3.4+0,3.5+0,2.5] = 7,87$$

$$\text{Catch 5 fish} = 5+0,9[0,2.2+0,3.3+0,3.3+0,2.4] = 7,7$$

$$\text{Catch 6 fish} = 6+0,9[0,2.1+0,3.1+0,3.2+0,2.2] = 7,35$$

$$\text{Catch 7 fish} = 7+0,9[0,2.0+0,3.0+0,3.0+0,2.0] = 7$$

So, the optimal policy is to $\boxed{\text{catch 1 fish and } U_2(s_7)=8,47}$

For state 8.

1,6 3 3 2

Catch 0 fish = 0 + 0.9 [0,2.8 + 0,3.10 + 0,3.10 + 0,2.10] = 8,64

Catch 1 fish = 1 + 0.9 [0,2.7 + 0,3.9 + 0,3.10 + 0,2.10] = 9,19

Catch 2 fish = 2 + 0.9 [0,2.6 + 0,3.8 + 0,3.9 + 0,2.10] = $\boxed{9,47}$

Catch 3 fish = 3 + 0,9 [0,2.5 + 0,3.6 + 0,3.8 + 0,2.9] = 9,3

Catch 4 fish = 4 + 0,9 [0,2.4 + 0,3.5 + 0,3.6 + 0,2.7] = 8,95

Catch 5 fish = 5 + 0,9 [0,2.3 + 0,3.4 + 0,3.5 + 0,2.5] = 8,87

Catch 6 fish = 6 + 0,9 [0,2.2 + 0,3.3 + 0,3.3 + 0,2.4] = 8,7

Catch 7 fish = 7 + 0,9 [0,2.1 + 0,3.1 + 0,3.2 + 0,2.2] = 8,35

Catch 8 fish = 8 + 0,9 [0,2.0 + 0,3.0 + 0,3.0 + 0,2.0] = 8

So, the optimal policy is to $\boxed{\text{catch 2 fish and } V_2(s_8) = 9,47}$

For state 9:

Catch 0 fish $= 0 + 0.9[0{,}2 \cdot 9 + 0{,}3 \cdot 10 + 0{,}3 \cdot 10 + 0{,}2 \cdot 10] = 8{,}82$

Catch 1 fish $= 1 + 0.9[0{,}2 \cdot 8 + 0{,}3 \cdot 10 + 0{,}3 \cdot 10 + 0{,}2 \cdot 10] = 9{,}64$

Catch 2 fish $= 2 + 0.9[0{,}2 \cdot 7 + 0{,}3 \cdot 9 + 0{,}3 \cdot 10 + 0{,}2 \cdot 10] = 10{,}19$

Catch 3 fish $= 3 + 0.9[0{,}2 \cdot 6 + 0{,}3 \cdot 8 + 0{,}3 \cdot 9 + 0{,}2 \cdot 10] = \boxed{10{,}47}$

Catch 4 fish $= 4 + 0.9[0{,}2 \cdot 5 + 0{,}3 \cdot 6 + 0{,}3 \cdot 8 + 0{,}2 \cdot 9] = 10{,}3$

Catch 5 fish $= 5 + 0.9[0{,}2 \cdot 4 + 0{,}3 \cdot 5 + 0{,}3 \cdot 6 + 0{,}2 \cdot 7] = 9{,}9$

Catch 6 fish $= 6 + 0.9[0{,}2 \cdot 3 + 0{,}3 \cdot 4 + 0{,}3 \cdot 5 + 0{,}2 \cdot 5] = 9{,}87$

Catch 7 fish $= 7 + 0.9[0{,}2 \cdot 2 + 0{,}3 \cdot 3 + 0{,}3 \cdot 3 + 0{,}2 \cdot 4] = 9{,}7$

Catch 8 fish $= 8 + 0{,}9[0{,}2 \cdot 1 + 0{,}3 \cdot 1 + 0{,}3 \cdot 2 + 0{,}2 \cdot 2] = 9{,}35$

Catch 9 fish $= 9 + 0{,}9[0{,}2 \cdot 0 + 0{,}3 \cdot 0 + 0{,}3 \cdot 0 + 0{,}2 \cdot 0] = 9$

So, the optimal policy is to $\boxed{\text{catch 3 fish and } V_2(s_9) = 10{,}47}$

For state 10:

Catch 0 fish: $0 + 0.9[0,2.10 + 0,3.10 + 0,3.10 + 0,2.10] = 9$

Catch 1 fish: $1 + 0,9[0,2.9 + 0,3.10 + 0,3.10 + 0,2.10] = 9,82$

Catch 2 fish: $2 + 0,9[0,2.8 + 0,3.10 + 0,3.10 + 0,2.10] = 10,64$

Catch 3 fish: $3 + 0,9[0,2.7 + 0,3.9 + 0,3.10 + 0,2.10] = 11,19$

Catch 4 fish: $4 + 0,9[0,2.6 + 0,3.8 + 0,3.9 + 0,2.10] = \boxed{11,47}$

Catch 5 fish: $5 + 0,9[0,2.5 + 0,3.6 + 0,3.8 + 0,2.9] = 11,3$

Catch 6 fish: $6 + 0,9[0,2.4 + 0,3.5 + 0,3.6 + 0,2.7] = 10,95$

Catch 7 fish: $7 + 0,9[0,2.3 + 0,3.4 + 0,3.5 + 0,2.5] = 10,87$

Catch 8 fish: $8 + 0.9[0,2.2 + 0,3.3 + 0,3.3 + 0,2.4] = 10,7$

Catch 9 fish: $9 + 0,9[0,2.1 + 0,3.1 + 0,3.2 + 0,2.2] = 10,35$

Catch 10 fish: $10 + 0,9[0,2.0 + 0,3.0 + 0,3.0 + 0,2.0] = 10$

So, the optimal policy is to $\boxed{\text{catch 4 fish and } V_2(s_{10}) = 11,47}$

All states' utility values and optimal policies:

$U_2(s_0) = 0$, policy: catch no fish

$U_2(s_1) = 1{,}35$, policy: catch no fish

$U_2(s_2) = 2{,}7$, policy: catch no fish

$U_2(s_3) = 3{,}87$, policy: catch no fish

$U_2(s_4) = 4{,}95$, policy: catch no fish

$U_2(s_5) = 6{,}3$, policy: catch no fish

$U_2(s_6) = 7{,}47$, policy: catch no fish

$U_2(s_7) = 8{,}47$, policy: catch 1 fish

$U_2(s_8) = 9{,}47$, policy: catch 2 fish

$U_2(s_9) = 10{,}47$, policy: catch 3 fish

$U_2(s_{10}) = 11{,}47$, policy: catch 4 fish