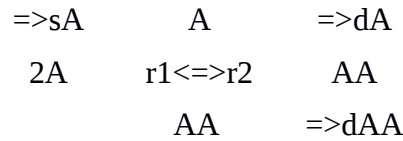


Two molecules of A form the dimer AA. Concentrations are at a steady state determined by the law of mass action according to this reaction network :



where s_A is the synthesis rate of A (units of concentration*time⁻¹), r_1 is the dissociation rate of AA (units of concentration⁻¹*time⁻¹), r_2 is the association rate of A, and d_A, d_{AA} are degradation rates (units of time⁻¹).

Let c_A, c_{AA} denote the concentrations of the two molecular species :

Because at equilibrium the concentration of A is constant :

Equation 1 : $s_A + 2r_1 c_{AA} = d_A c_A + 2r_2 c_A^2$

Because at equilibrium the concentration of AA is constant :

Equation 2 : $r_2 c_A^2 = d_{AA} c_{AA} + r_1 c_{AA}$

Starting from equation 2, we get an expression for c_{AA} :

$$r_2 c_A^2 = d_{AA} c_{AA} + r_1 c_{AA}$$

$$0 = (d_{AA} + r_1) c_{AA} - r_2 c_A^2$$

$$c_{AA} = \frac{r_2 c_A^2}{d_{AA} + r_1}$$

With k_{AA} defined as $\frac{r_2}{d_{AA} + r_1}$:

$$c_{AA} = k_{AA} c_A^2$$

Starting from equation 1 :

$$s_A + 2r_1 c_{AA} = d_A c_A + 2r_2 c_A^2$$

$$0 = 2r_2 c_A^2 + d_A c_A - s_A - 2r_1 c_{AA}$$

By substituting c_{AA} :

$$0 = 2r_2 c_A^2 + d_A c_A - s_A - 2r_1 k_{AA} c_A^2$$

From the definition $k_{AA} = \frac{r_2}{d_{AA} + r_1}$, we get $r_2 - r_1 k_{AA} = d_{AA} k_{AA}$ and therefore :

$$0 = 2r_2 c_A^2 + d_A c_A - s_A - 2r_1 c_A^2 k_{AA}$$

$$0 = 2r_2 c_A^2 + d_A c_A - s_A - 2r_1 c_A^2 k_{AA}$$

$$0 = 2c_A^2 (d_{AA} k_{AA}) + d_A c_A - s_A$$

Applying the quadratic formula :

$$c_A = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-d_A \pm \sqrt{d_A^2 - 4(2d_{AA}k_{AA})(-s_A)}}{2(2d_{AA})(k_{AA})}$$

$$c_A = \frac{-d_A \pm \sqrt{d_A^2 + 8d_{AA}k_{AA}s_A}}{4d_{AA}k_{AA}}$$

In the formula above, the coefficients are such that $a > 0$, $b \geq 0$, $c \leq 0$ and thus $b \leq \sqrt{b^2 - 4ac}$. It follows that $-b + \sqrt{b^2 - 4ac}$ is non-negative, while $-b - \sqrt{b^2 - 4ac}$ is non-positive. We therefore choose $-b + \sqrt{b^2 - 4ac}$ to obtain a non-negative value of c_B .

$$c_A = \frac{-d_A + \sqrt{d_A^2 + 8d_{AA}k_{AA}s_A}}{4d_{AA}k_{AA}}$$