Post-Inhibitory Rebound-like Behavior in Networks of Pulse-Coupled Integrate-and-Fire Neurons

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ABSTRACT

A fundamental challenge in computational neuroscience is to build biologically realistic models while controlling the model's complexity and dimensionality so that it remains tractable for analysis. Many large-scale computational models rely on networks of integrate-and-fire model neurons (IF). Unlike Hodgkin-Huxley-type model neurons, an IF neuron alone cannot produce rich nonlinear effects such as post-inhibitory rebound (PIR) and bursting. Here we show that a pair of pulse-coupled IF neurons can give rise to a PIR-like effect, which we define as a brief increase in a neuron's firing rate following the termination of a strong inhibition. We derive the conditions under which such PIR-like effect occurs. In addition, we investigate the population dynamics of a larger network of IF neurons in response to inhibitory inputs. This work suggests that a careful choice of the IF model parameters may help preserve some biologically-relevant nonlinear behaviors.

PROBLEM DESCRIPTION

Post-inhibitory rebound is a computational property found in some biological neurons where they are able to emit a spike or burst of spikes directly following the release of a strong inhibitory current. We characterize a PIR-like event as an increase in the instantaneous firing rate of a neuron above it's average firing rate following the release of an inhibitory current.

A pair of pulse-coupled leaky integrate and fire neurons with a global inhibition term are described by the following system:

$$\tau^{(1)}\frac{dV^{(1)}}{dt} = g(t) + I^{(1)} - V^{(1)} + w_{21}\sum_{t_n^{(2)}\in S^{(2)}} \delta(t - t_n^{(2)}) \qquad S^{(p)} = \{t \mid V^{(p)}(t) = 1\}$$

$$\tau^{(2)}\frac{dV^{(2)}}{dt} = g(t) + I^{(2)} - V^{(2)} + w_{12}\sum_{t_n^{(1)}\in S^{(1)}} \delta(t - t_n^{(1)}) \qquad \lim_{t\to t_n^{(p)}+} V^{(p)}(t^{(p)}) = 0$$

Where we define the instantaneous firing rate as:

$$r^{(p)}(t_n^{(p)}) = \frac{1}{t_n^{(p)} - t_{n-1}^{(p)}}$$

And the average firing rate as:

$$\lim_{t\to\infty}\left(\frac{\mathrm{card}(\{t_n^{(p)} \mid t_n^{(p)} < t\})}{t}\right)$$

SYSTEM PROPERTIES

Before analyzing a PIR-like effect directly, we derived several general descriptors of the system. Mainly, the existence of an neuron's ability to emit spikes while its paired neuron is releasing spikes at regular intervals, the voltage at these times, and the system's asymptotic bounds.

We found the existence of spiking if:

$$V_\infty^{(p)} > 1$$

Where:

$$V_\infty^{(p)} = w_{qp}(1 - I^{(q)}) - I^{(p)}$$

And that the voltage of a neuron at the instance of the k[th] spike is given by:

$$V_k^{(p)} = m^k(V_0^{(p)} - V_\infty^{(p)}) + V_\infty^{(p)}$$

Where

$$m = \left(\frac{1 - I^{(q)}}{-I^{(q)}}\right)$$

In addition, we found that the asymptotic voltage bounds of the system are:

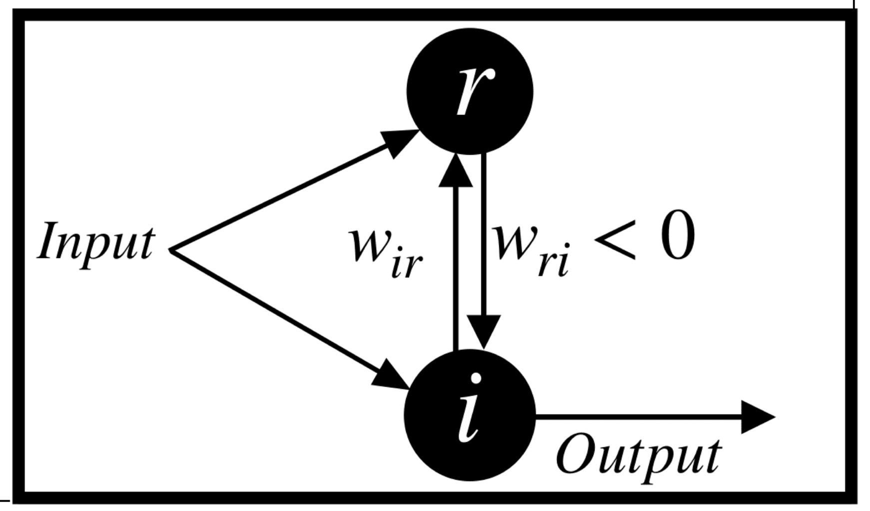
If $V_\infty^{(p)} < 1$ and $V_\infty^{(q)} > 1$

$$V_{\mathrm{asym}}^{(p)} = [V_\infty^{(p)}, V_\infty^{(p)} + w_{qp}], V_{\mathrm{asym}}^{(q)} = (1, 0]$$

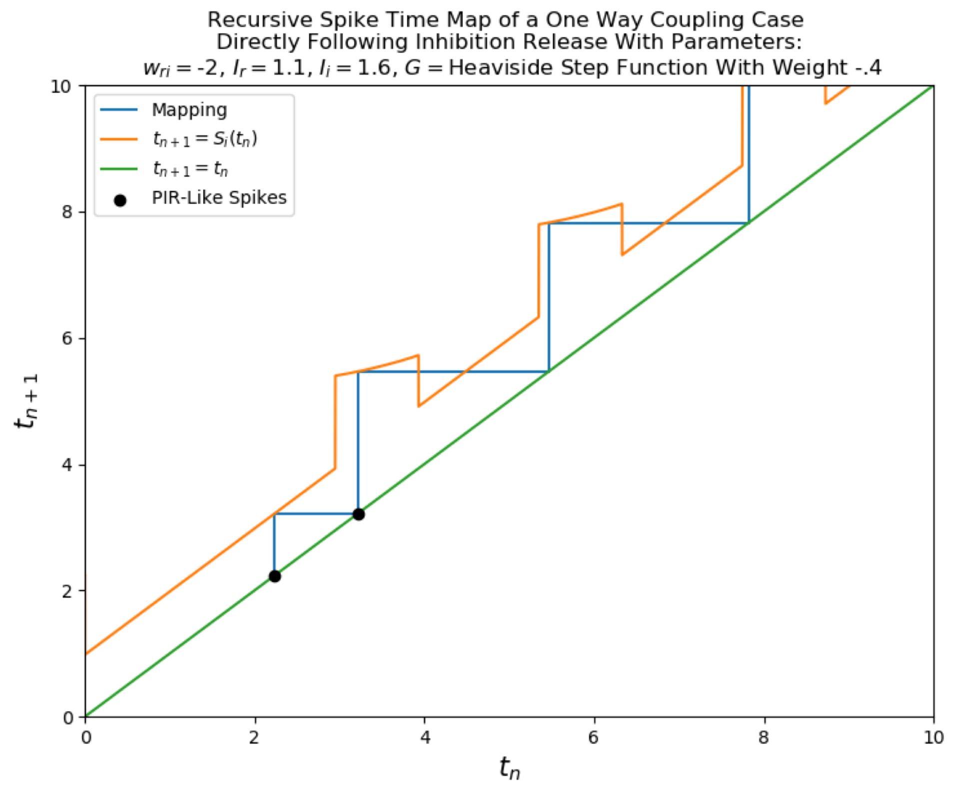
If $V_\infty^{(p)} > 1$ and $V_\infty^{(q)} > 1$

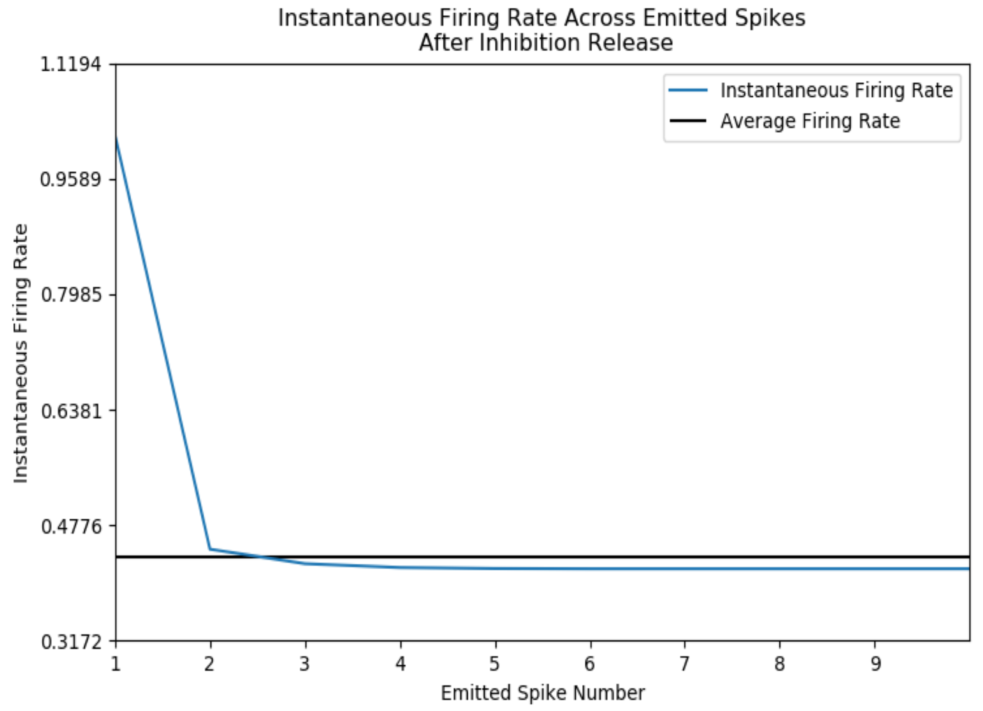
$$V_{\mathrm{asym}}^{(p)} = (1, \min\{w_{pq}, 0\}], V_{\mathrm{asym}}^{(q)} = (1, \min\{0, w_{pq}\}]$$

We then chose a specific neural motif to study, depicted below. This motif was chosen simply because analyzing an individual neuron's firing rate is far more tractable than analyzing both at once.



PRIMARY RESULT





Under the parameters,

$$m^k(1 - I^{(i)}) < a < 0 \ , \ V_\infty^{(r)} > 1 \ , \quad G < 0 \quad , \quad \tau^{(r)} = \tau^{(i)}$$

Where:

$$a = (-I^{(r)} + V_\infty^{(r)})m^k - V_\infty^{(r)} + 1 \quad , \quad G = e^{\frac{t_{\mathrm{on}}}{\tau}} \int_{t_{\mathrm{off}}}^{t_{\mathrm{off}}} e^t g(t)dt$$

We derived an upper bound for G such that k+1 spikes would be emitted by neuron i before neuron r could fire directly following the inhibition release.

$$G = \frac{-a(V_{\mathrm{on}}^{(i)} - I^{(i)}) + m^k(1 - I^{(i)})(V_{\mathrm{on}}^{(r)} - I^{(r)})}{a - m^k(1 - I^{(i)})}$$

And that:

$$\frac{\partial G}{\partial V_{\mathrm{on}}^{(i)}} = \frac{-a}{a - m^k(1 - I^{(i)})} > 0$$

$$\frac{\partial G}{\partial V_{\mathrm{on}}^{(r)}} = \frac{m^k(1 - I^{(i)})}{a - m^k(1 - I^{(i)})} < 0$$

Which means that

$$G < \frac{-a(\inf(V_{\mathrm{asym}}^{(i)}) - I^{(i)}) + m^k(1 - I^{(i)})(\sup(V_{\mathrm{asym}}^{(r)}) - I^{(r)})}{a - m^k(1 - I^{(i)})}$$

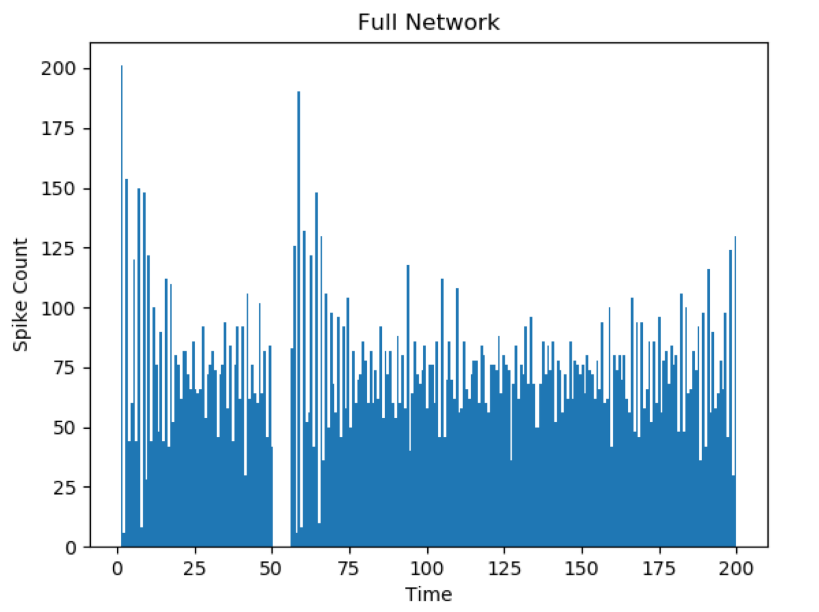
shows that any for any resting voltage in the asymptotic bounds, neuron i will emit a series of k+1 spikes directly following inhibition release.

Given our parameters, neuron r is then guaranteed to fire an infinite amount of times across t and, in doing so, will lower the average firing rate of neuron i from what it was during the k+1 spikes directly following the release of inhibition. Thus we derived a sufficient condition for a PIR-like effect in this motif.

SECONDARY RESULTS

We also showed that a PIR-like effect can be present in larger networks of leaky integrate and fire neurons via simulation.

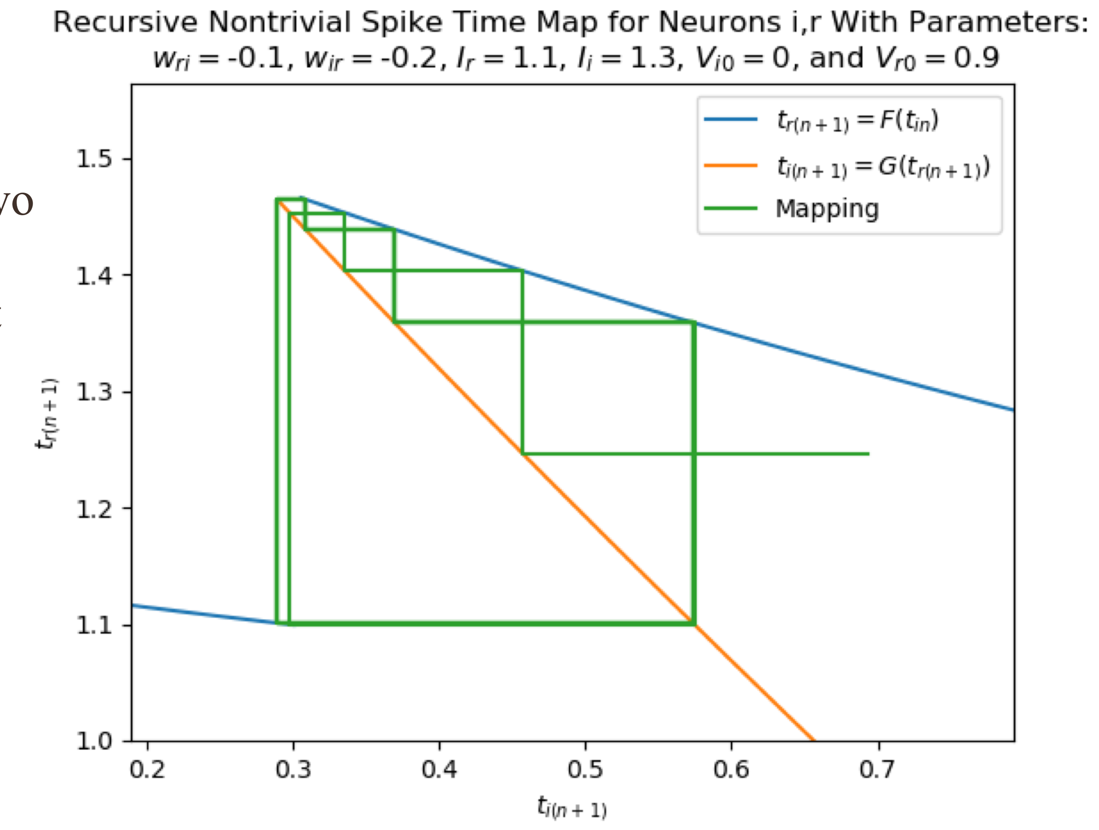
In addition, we derived a more general sufficient condition for bursting-like effects in the two neuron system. This considers the case when a pulse function is applied to both neurons simultaneously with differing signs.



FUTURE WORK

In all the cases we looked at in this research, we focused on the firing output of a single neural motif where the output is read from a single neuron. An interesting but more complicated case would be to derive conditions under which the combined firing rate of the two neurons exhibits a PIR-like effect. Another future avenue of study would be to derive conditions under which a pair of neurons natively exhibit bursting-like effects without external input.

However, both cases involve directly using the behavior of the two neuron system which can exhibit very analytically difficult limit cycles.



AWKNOWLEDGEMENTS

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