Properties of Exponential

Samuel Dearman

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$$x^1 = x$$

[1]

$x^n = x_1 * x_2 * \cdots * x_n$ where $x_1 = x_2 = \cdots = x_n$

Proof.

$$x^2 = x * x$$

$$\text{and}$$

$$x^1 = x \implies \frac{x^2}{x^1} = x^1 \implies x^{2-1} = \frac{x^2}{x^1} \implies x^0 = x^{1-1} = \frac{x}{x} = 1$$

$\square$

Conclusion: $x^0 = 1$

[2]

Given the proof above, we can say that $x^{-1} = \frac{x^0}{x}$ which is $\frac{1}{x}$

in the same sense, $x^{-2} = \frac{x^0}{x^2} = \frac{1}{x^2}$

What About $\mathbb{Q}$ numbers as exponential? How can we find them?

Proof.

$$16^1 = 4^2 = 2^4 = a^8 \ldots$$

$$2^2 = 4^1 = 16^?$$

You see that exponents divided by 2 when bottoms increases.

$$16^{1/2} = 4^1 = 2^2 = a^4$$

We can understand $1^n = 1$ whatever n might be. So $2^{1/2}$ is somewhere between 2 and 1. $\square$

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[1]Identity law
[2]inverse law