BITTIGER

DS 501 Data scientist express bootcamp

[Ella]

BITTIGER

版权声明

What is statistics? Why is it important?

Statistics

- All about quantifying uncertainty (which subject is studying certainty?)
- Why uncertainty? **imperfect** data
    - Psychology
    - Geology
    - Economics
- Why imperfect data?
    - Only observe a small fraction, e.g., candidate poll
    - Only observe indirect signs, e.g., neuro signal
    - Data always contain noise, e.g, measurement error

Probability

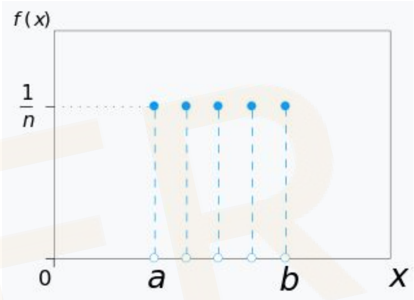
- Probability is central to statistics
  - Probability = Statistics?
- Basic concepts
  - **Experiment**
  - **Sample space**: all possible outcomes of an **experiment**
  - **Event:** subset of sample space

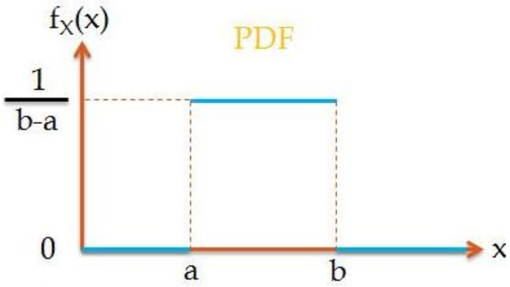
Random variable and types

- Random variable (X)
  - Enumerate sample space to real value
- Categories
  - Discrete
  - Continuous
- Examples
  - Flip a coin; Daily traffic; Number of people click on an ad; BMI
- R
  - bar chart, pie chart
  - Probability density

How to describe probability of random variable

- Probability density function (PDF)
  - PDF: x ~ P(X = x)
  - For X, frequency of possible outcome x's occurrences of the total possible occurrences
    - Example: rolling dice, P(X = 1) = ⅙
  - Discrete (pmf) v.s continuous (pdf)
  - Non-negative, sum up to 1
- Rules
  - 0 ≤ P(X = x) ≤ 1, P(X = Ω) = 1, P(X = ∅) = 0, P(X = x) + P(X != x) = 1, P(X = x or Y = y) = P(X = x) + P(Y = y) − P(X = x and Y =y)



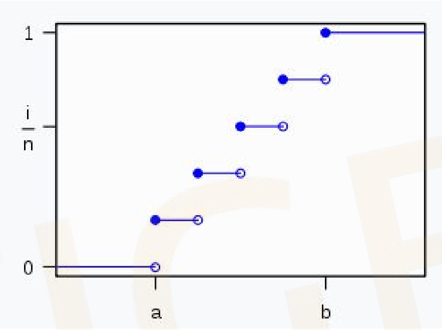


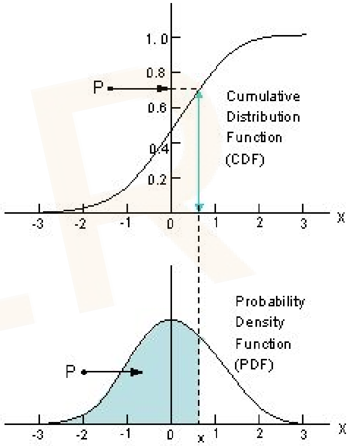
Example interview questions

- 2 dices, probability of at least one 4 shows up
- 2 dices, probability of the sum is 6
- …

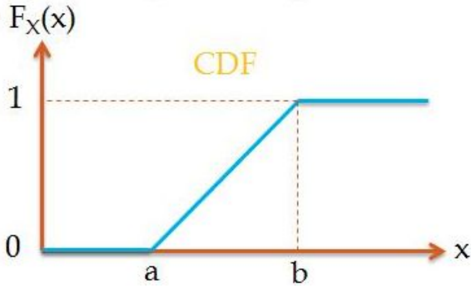
How to describe probability of random variable(cont’d)

- Cumulative density function (CDF)
  - CDF: x ~ P(X <= x)
  - Discrete v.s continuous
  - Non-decreasing, up to 1
- Examples
  - Roll dice
  - Uniform distribution





Relations Between Two Different Typical Representations of a Population

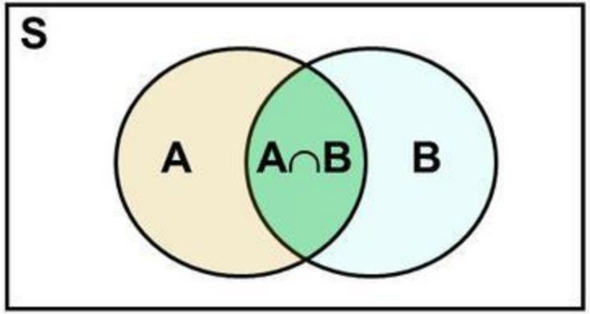


Conditional probability

- How to update uncertainty after getting new information?
    - P(dice = 1) = ⅙, what if we know it's an odd number?
- Definition, Probability of A given B

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

- Intuition
    - Only consider B
    - Renormalize
- Example question



Independence

- Def: $P(A \cap B) = P(A) \times P(B)$
- Considering conditional probability $P(A \mid B) = \frac{\boxed{P(A \cap B)}}{P(B)}$

$$P(A \mid B) = \frac{P(A)P(B)}{P(B)} = P(A)$$

- A and B are conditionally independent given C

$$P(AB \mid C) = P(A|C)\,P(B|C)$$

- Example question

A and B are independent <->
A and B are independent given C ?

Bayes' theorem

- Reverse the conditional probability

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A \mid B)P(B) + P(A \mid B^c)P(B^c)}$$

- Prove it
  - Hint:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

- Fair coin and unfair coin problem

Expectation

- Measure what's the center of a random variable
  - Discrete $E[X] = \sum_x xp(x)$
  - Continuous $E(X) = \int_x x\,P(X = x)$
  - Roll dice, flip coin, uniform distribution
- Population mean v.s. sample mean
  - Sample mean **itself** is random variable, estimator of population mean
  - Unbiased: E(sample mean) = population mean
- Why always larger sample is better?
  - Biased when fewer data?
  - Proof. Hint: E(X + Y) = E(X) + E(Y)

Variance

- Variance (σ^2) measure how random variable spreads

$$Var(X) = E[(X-\mu)^2] = E[X^2] - E[X]^2$$

  - Calculate variance from rolling dice
- Standard deviation (σ)
- Sample variance (s^2) and standard deviation (s)

$$S^2 = \frac{\sum_{i=1}(X_i - \bar{X})^2}{n-1}$$

  - Why (n-1): Unbiased estimator of σ^2
  - Proof (HW), Hint Var(X+Y) = Var(X) + Var(Y) if X, Y independent

Important discrete distribution

- Bernoulli
  - $P(X = x) = p^x (1-p)^{1-x}$
- Binomial
  - Sum of N Bernoulli variables
  - $$\mathrm{P}(X = x) \equiv \binom{N}{x} p^x (1-p)^{N-x}$$

E(X) = p, Var(X) = p (1-p)

What's the mean and variance?

E(X) = N*p, Var(X) = N*p(1-p)

More details about other distributions

- Geometric distribution
  - Prob of seeing first success with k independent trials, each with success probability p.

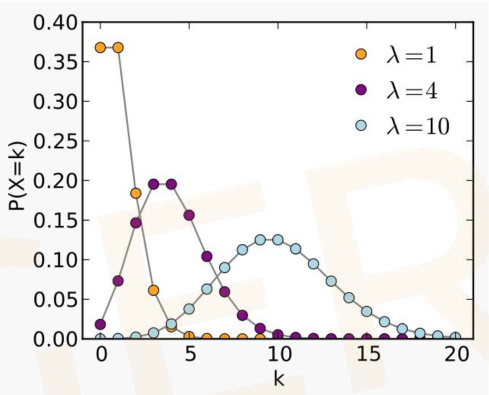
$$\Pr(X = k) = (1-p)^{k-1}\, p$$

- Negative geometric distribution
  - Prob of seeing r failures with k success trials , each with success probability p.

$$\Pr(X = k) = \binom{k+r-1}{k} p^k (1-p)^r$$

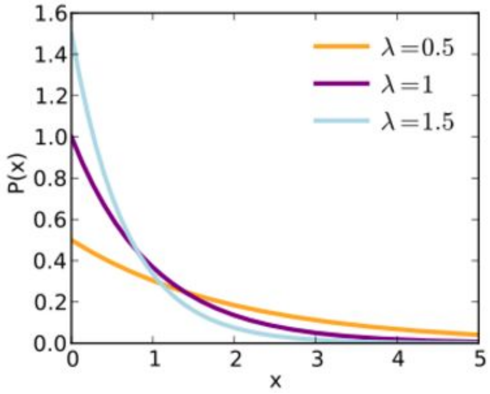
Important distribution

- Poisson, count data per time unit
  - $P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$
  - Examples



- Exponential distribution
  - In a poisson distribution, inter-arrival time follows exponential distribution
  - PDF: $\lambda e^{-\lambda x}$ CDF: $1 - e^{-\lambda x}$
  - Expectation $\mathrm{E}[X] = \frac{1}{\lambda}$
  - Variance $\mathrm{Var}[X] = \frac{1}{\lambda^2}$
  - “Memory-less”

$P(X > s+t \mid X > s) = P(X > t)$



Important continuous distribution

- Normal distribution

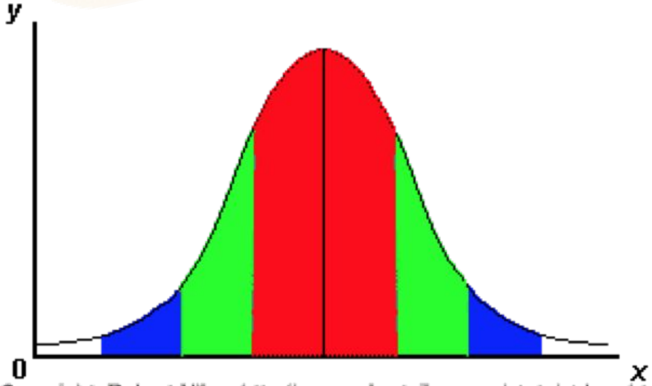
$$P(X = x) = (2\pi\sigma^2)^{-1/2} e^{-(x-\mu)^2/2\sigma^2}$$

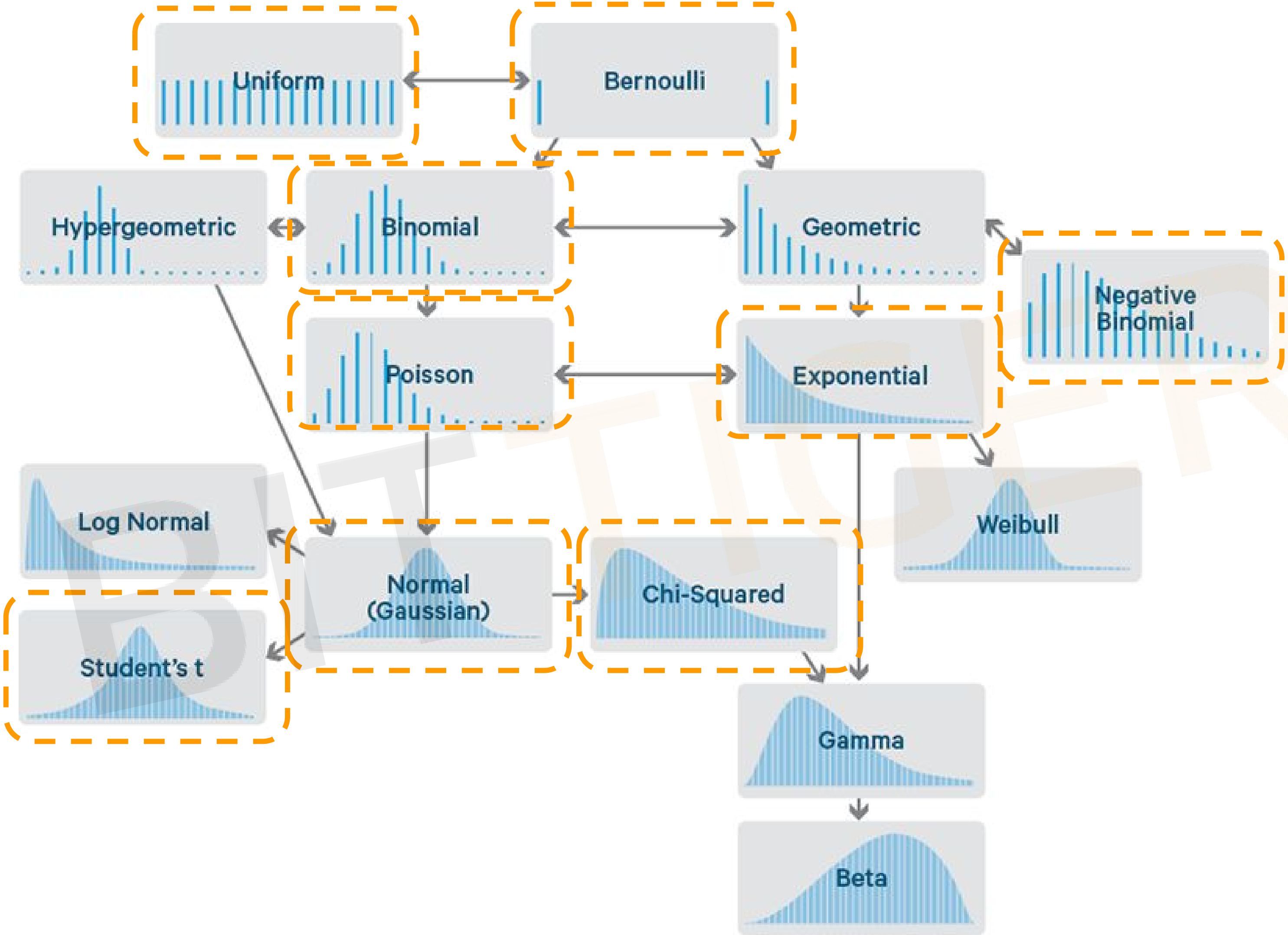
$E[X] = \mu$ and $Var(X) = \sigma^2$

- Standard normal distribution (Z) $\mu = 0$ and $\sigma = 1$

$$X = \mu + \sigma Z \sim \mathrm{N}(\mu, \sigma^2)$$

- Beauty of normal curve (6σ)
  - [μ - 3σ, μ + 3σ] covers 99.7%
  - [μ - 2σ, μ + 2σ] covers 95%
  - [μ - σ, μ + σ] covers 68%




Uniform
Bernoulli
Hypergeometric
Binomial
Geometric
Negative Binomial
Poisson
Exponential
Log Normal
Weibull
Normal (Gaussian)
Chi-Squared
Student's t
Gamma
Beta

Example interview questions

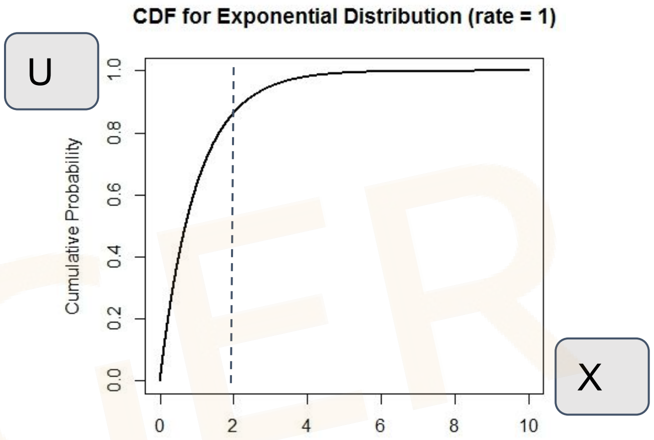
- In a country in which people only want boys every family continues to have children until they have a boy. If they have a girl, they have another child. If they have a boy, they stop. What is the proportion of boys to girls in the country?
  - What distribution is this?
  - What's expected number of kids (X) each family has?
  - What the ratio of boys to girls?

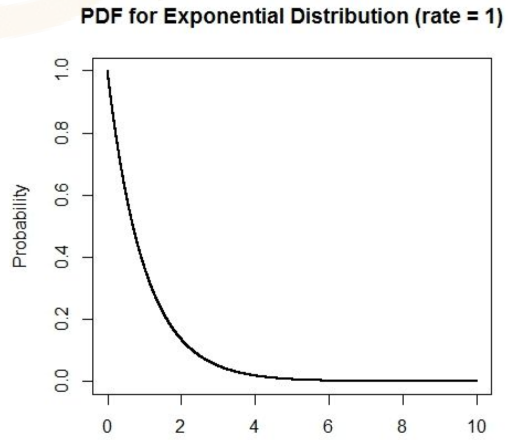
Another type of distribution Interview question

- How can we generate certain distribution?
    - Q1: Generate exponential distribution from uniform distribution
    - Q2: Generate normal distribution from uniform distribution

Inverse transform sampling

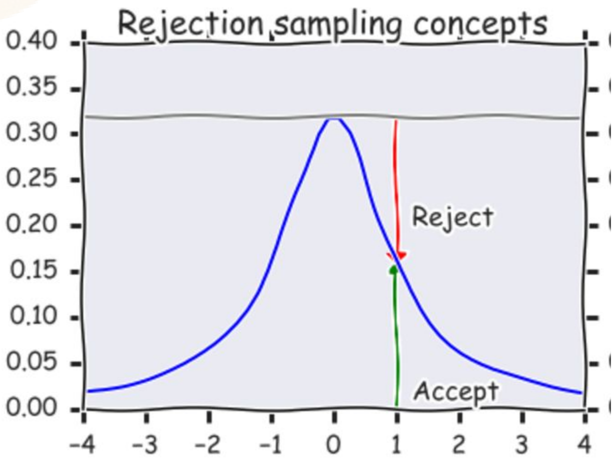
- To generate X with PDF: P(X)
  - Calculate CDF of X: F(X) and inverse $F^{-1}(.)$
  - Generate U with uniform distribution, say u
  - Calculate $F^{-1}(u)$
  - Repeat n times
- Intuition
- Let’s solve Q1
- Can we solve Q2 as well?





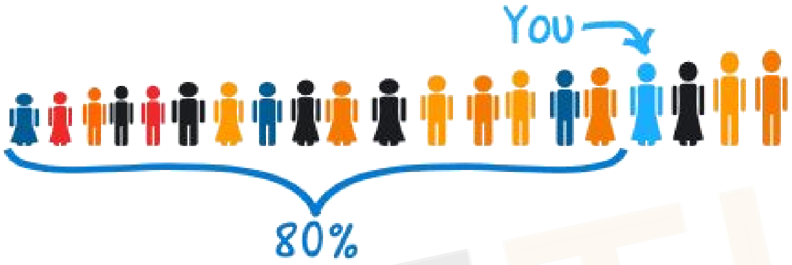
Acceptance rejection sampling

- Simple version: if good, accept; else, reject.
  - Ex1: Generate Y with uniform [¼, 1] given X ~ [0, 1]
  - Ex2: Generate random integer Y in [1, 7] given random integer X in [1, 5] 5*(X-1) + X
  - Ex3: Generate normal distribution given uniform distribution
    - Sample a point, x, calculate P(x)
    - Generate random number u from uniform distribution
    - If u < P(x), accept x, else reject, back to step 1
- Check out this video



Percentile

- Definition (c%, N)



- Special percentiles
  - Median
  - 1st quantile
  - 3rd quantile
- Median, quantiles in normal distribution?
  - Look up **z score**, alpha: area outside [-z, z]

Why normal distribution is so important?

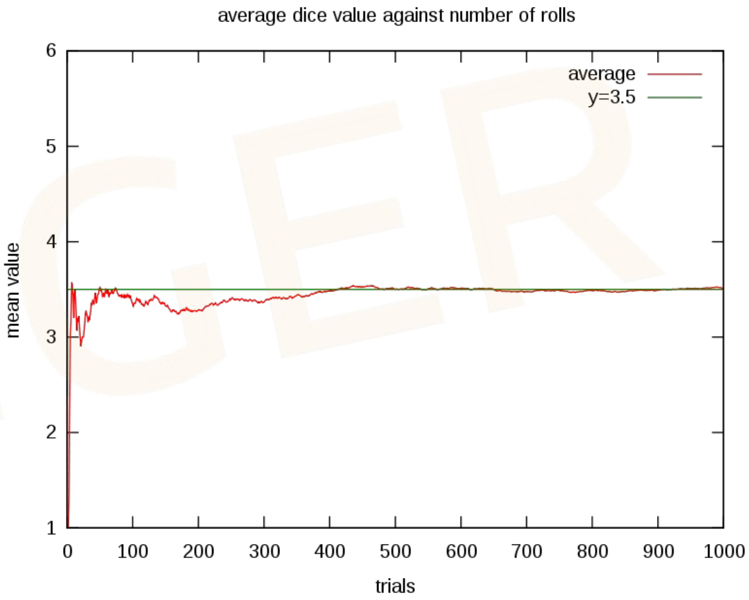
Law of large numbers

$X_1, X_2 \ldots X_n \ldots$ are independent, identically - distributed (IID) random variables, X has finite mean μ.

- Sample mean $\bar{X}_n \equiv \frac{1}{n}\sum_{i=1}^{n} X_i$ converge to the true mean μ

  as n increases:

$$\bar{X}_n \xrightarrow[n \to \infty]{} \mu$$



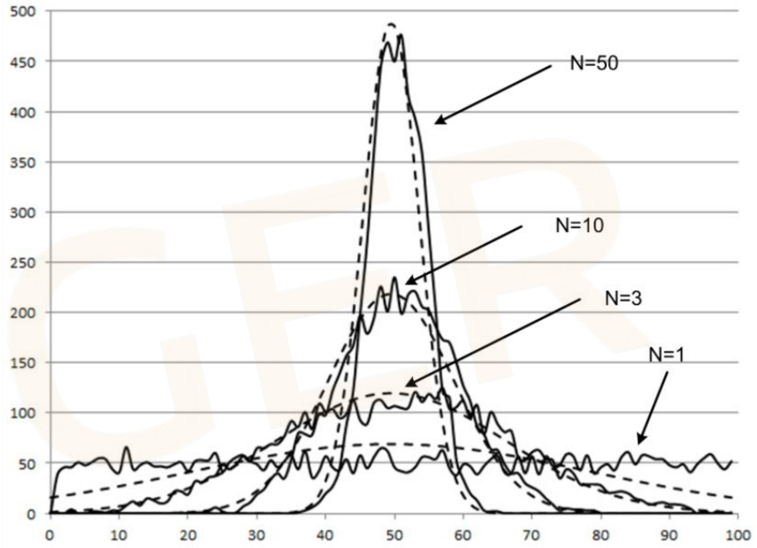
Central limit theorem

- $X_1, X_2 \ldots X_n \ldots$ are independent, identically - distributed (IID) random variables, X has finite mean μ and variance σ^2

$$\bar{X}_n \equiv \frac{1}{n}\sum_{i=1}^{n} X_i \sim N(\mu, \sigma^2/n)$$

(replacing σ by sample standard error, CLT still holds)

- Application
  - Binomial distribution
    - Flip coin example

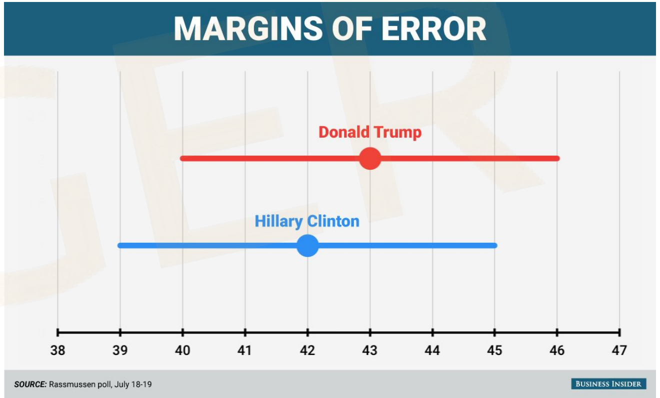


Confidence intervals

- Recall normal curve
  - What range covers 95% of possibility? 99%?
- Confidence interval (CI)
  - Derive 95% CI of **p** in Binomial distribution

$$\hat{p} \pm z_{1-\alpha/2}\sqrt{\frac{p(1-p)}{n}} \qquad \hat{p} \pm \frac{1}{\sqrt{n}}$$ approx.

  - How many voters in the candidate poll?
  - Can Trump relax? How many are enough?



When CLT doesn't hold?

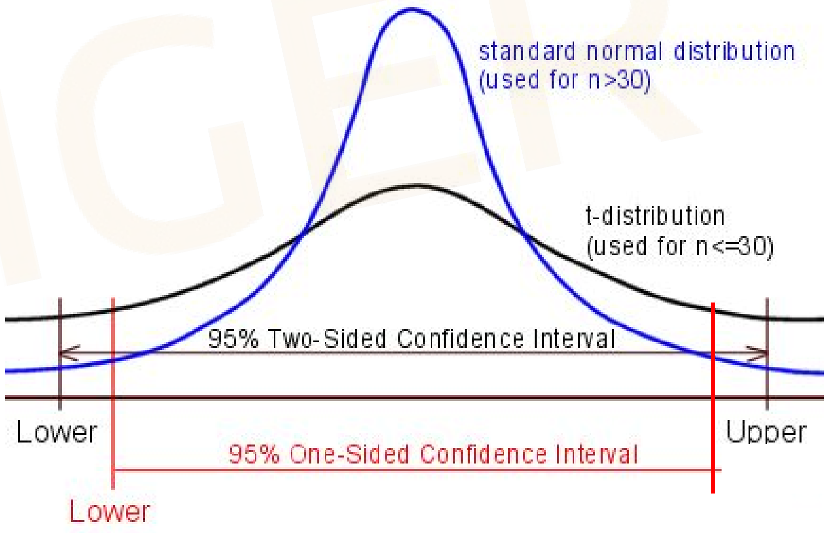
- Normal distribution -> T distribution when N < 30
  - T distribution has only one parameter: degree of freedom (df = N-1)
  - Approximate normal as df increases
  - CI under normal distribution

$$Mean_{estimate} \pm z_{1-\alpha/2} * StdErr_{estimate}$$

  - CI under t distribution

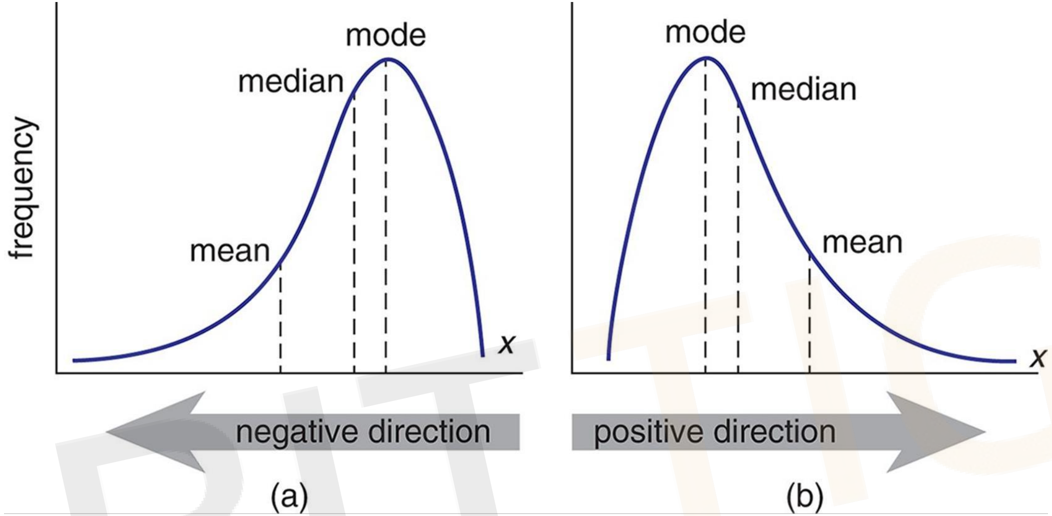
$$Mean_{estimate} \pm t_{n-1} * StdErr_{estimate}$$

  - z or t?



What about CI of not normal distribution?

Skewed distribution



- What caused skewness
  - Bounded: lower bound or upper bound
- Can we still use normal distribution to estimate mean of skewed variable?

How to treat skewed variable

- Log transformation
- Use median, if so, what's CI of median?
    - Non-parametric method (v.s. parametric method)
        - **Bootstrap**
        - Jackknife

Covariance and correlation

- Definition
  - $cov(X, Y) = E[(X - E[X])(Y - E[Y])]$
  - $\varrho_{X,Y} = corr(X, Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$
- What's sample covariance and correlation?
  - $cov(X, Y) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \frac{1}{n-1}(\sum x_i y_i - n\bar{x}\bar{y})$
  - $cor(X, Y) = \frac{cov(X,Y)}{s_x s_y}$

Properties of correlation

- Properties
  - cor*(X, Y)* = cor*(Y, X)*
  - -1 <= cor*(X, Y)* <= 1
  - cor*(X, Y)* = 1 or -1: *X, Y* aligned on a line perfectly
  - cor*(X, Y)* = 0: *X, Y* are independent
- Geometric interpretation
  - $$\varrho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}} = \frac{\sum_{i=1}^{n} x_i' y_i'}{\|x_i'\| \|y_i'\|} = cos(x', y')$$
  - cor(*X, Y*) = 0, cor(*Y, Z*) = 0 => cor(*X, Z*) = 0?