Lab –AC circuits combining resistors, inductors and capacitors (RLC) – Part 1

A. Objectives

The main objectives of this lab are to become familiar with reactive elements (capacitors and inductors) in AC circuits and to use a function generator and oscilloscope to measure the capacitive reactance in a RC circuit, inductive reactance in a LR circuit, and the resonant frequency and quality factor of an RLC circuit.

B. Equipment (TBC)

C. Introduction

At the end of the last experiment, you applied a square-wave voltage to charge and discharge a capacitor (in series with a resistor) with an applied period that was several times longer than the time constant $\tau = RC$ of the circuit. A more usual application of AC voltage is sinusoidal and the period being typically much shorter than τ. In this experiment, we will study the behavior of the circuit for different AC at frequencies *f* (or period). We will begin with the resistor-capacitor (RC) circuit. Then we will introduce inductors, which has a property called inductance denoted by the symbol L, and form an inductor-resistor (RL) circuit. Finally, we will form an RLC circuit, which exhibit unique resonance behaviors.

D. Capacitive Reactance X_C & RC circuit

First, we describe <u>the voltage across a resistor and the current flowing through a resistor</u>. The sinusoidal emf from the AC generator can be written as:

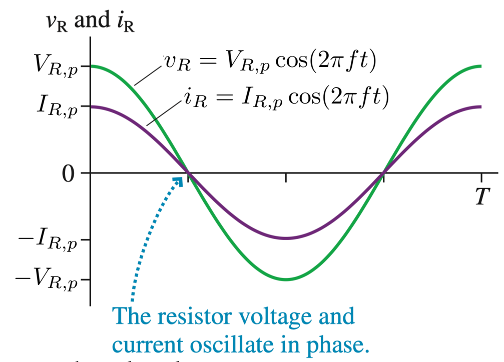
$$\varepsilon(t) = \varepsilon_p \cos(2\pi f t)$$

where the p-subscript denotes the peak value and without the p-subscript is the instantaneous value. The voltage across the resistor is then:

$$v_R(t) = V_{p,R} \cos(2\pi f t) \; .$$

The relationship $\sum \Delta V = 0$ still holds. The value of $V_{p,R}$ depends on other elements in the circuit. Resistors are simple devices that do not store charge (which capacitors do) and do not oppose change in current (which inductors do). In this case, the current flowing through a resistor is simply given by:

$$i_R(t) = I_{p,R} \cos(2\pi f t)$$



The fundamental voltage-to-current relationship also holds for both instantaneous and peak values:

$$v_R = i_R R \quad \text{and.} \quad V_{p,R} = I_{p,R} R$$

where R is the resistance of the resistor. For AC circuits, the instantaneous current (and thus the peak currents) flowing through all the components are the same still.

The above relationships are more complicated for a capacitor. Because the current flowing through a capacitor is frequency dependent. For a DC voltage applied on a capacitor, the steady-state current (i.e. after waiting several time constants) is zero. To capture the behavior of a capacitor, we define a quantity called the capacitive reactance X_C given by:

$$X_C = \frac{1}{2\pi f C}$$

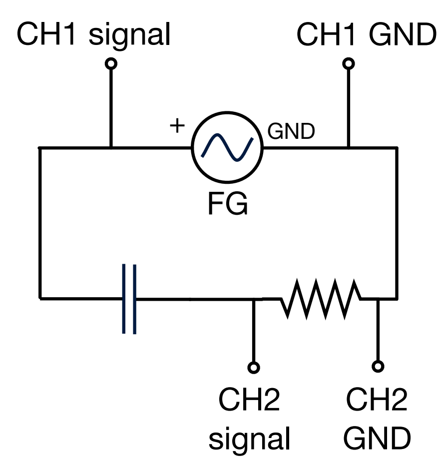
where C is the capacitance of the capacitor (in Farads). This quantity is useful because we can now have voltage-to-current relationship for a capacitor given by:

$$V_{p,C} = I_{p,C} X_C$$

This is only valid for the peak values.

Question: Determine the SI units for X_C. Recall that for a capacitor, the charge stored $Q = C(\Delta V)$, and $\Delta V = IR$, where the I is the current, which is the flow of charge $I = dQ/dt$.

In this part of the lab, you will study the impact of varying the *f* of the AC voltage applied across a capacitor. You will predict the frequency-dependent X_C value for the capacitor using the manufacturer's capacitance value. Then you will determine it using the voltage-to-current relationship for a capacitor. We will build the circuit as shown to the right. To determine the current flowing in the circuit, and thus the current flowing through the capacitor, we will use a (small) R = 100 Ohm resistor. We will measure the voltage drop across the resistor to determine the current. We must do this because an oscilloscope does not have a "ammeter" mode.



There are several subtle things to note about this circuit:

(1) this is a voltage divider circuit but, as you shall see from your calculations, the X_C of the capacitor is much larger than R, so having R in the circuit, which we use to make our "ammeter", does not disturb the current and voltage in the circuit much at our frequencies of interest.

(2) If you wish to check the above, you can connect CH2 of the oscilloscope across the capacitor (but still leaving in the resistor)

(3) Inside the oscilloscope, the grounds of the two channels are shorted together. And the function generator shares the same ground as the oscilloscope via the wall outlet. (This is called having a "common ground", as opposed to having "floating grounds".) Therefore, you must connect the circuit with the resistor connecting directly to the GND of the frequency generator AND the CH1 ground.

We will be using a C = 0.1 μF capacitor and a R = 100 Ω resistor. Calculate the capacitive reactance X_C for the frequencies in the table below using the capacitance values above. Then determine the expected peak-to-peak current that should flow in the circuit.

Apply a sinusoidal signal from the FG V_{pp} = 5V and frequency starting at 100 Hz.

Display the voltage from the FG in CH1 of the scope, and display voltage across the resistor on CH2.

Remember, the resistor is used to measure the current in the circuit. Use the peak voltage across the resistor to determine the current flowing in the circuit via the relationship $V_{p,R} = I_{p,R}R$

Frequency f (Hz)	Calculate X_C	Calculate Current expected in the circuit: $I_{pp}=V_{pp}/X_C$ (mA)	Measured peak voltage across resistor $V_{p,R,meas}$	Use the measured voltage to determine current flowing in the circuit $I_{pp.R,\ meas}$
100	(complete in labbook)	(complete in labbook)	(complete in labbook)	(complete in labbook)
200	(complete in labbook)	(complete in labbook)	(complete in labbook)	(complete in labbook)
400	(complete in labbook)	(complete in labbook)	(complete in labbook)	(complete in labbook)

Question: The peak current in your circuit is not constant with frequency. Why is this based on the physics of a capacitor?

E. RC phase shift

Connect a resistor with R = 1 kΩ to 2 kΩ and a capacitor with C = 0.1 μF in series to the function generator.
Connect CH1 of the scope across the resistor and CH2 across the entire circuit. Make the three grounds (function generator and two scope channels) are all connected together.
Set the AC amplitude from the function generator to $V_{p\text{-}p}$ = 8 V and a frequency of 1000 Hz.

Draw a phasor diagram for the current in the circuit, resistor voltage V_R , capacitor voltage V_C, and the emf.
What are you measuring with the scope in this experiment?

Calculate the following:

a) Capacitive reactance X_C
b) The expected phase between the two signals based on theory: $\phi = \text{atan}(X_C/R)$.
c) Determine the phase in your circuit by displaying the two signals on the oscilloscope and measuring the time shift between the traces as shown in the picture:

The phase is determined from your measured time shift as follows by: ϕ (in deg) = 360º x Time Shift/Period of Signal

Estimate your measurement uncertainties in the table (and justify and describe your estimate in your labbook and lab report). Propagate the errors to find the uncertainty in the phase. Properly report the phase, and also calculate the discrepancy between the expected phase and your directly measured pahse.

Frequency f (Hz)	Calculate X_C	Phase (deg) calculate	Time shift (measured)	Time shift uncertainty	Period of Signal T_0	Phase (deg) with uncertainty from measurement	Discrepancy between phases with uncertainty
100	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*
500	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*
1000	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*
2000	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*
5000	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*
10,000	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*	*(labbook)*