PHYS 210 Homework #4

[45pts+7EC] **Instructions:** Read and attempt all of them. Remember to write neatly, BOX your final answer, and list your collaborators (or list "none"). Show all your steps explicitly. (No credit if you line of reasoning is not clear.) Promptly check and rework the problems after solutions are released.

[10pts] **Problem 1:** *Romeo & Juliet on a boat*
(Time estimate: 20 minutes)
Romeo and Juliet are sitting at opposite ends of a symmetric 300 lb rowboat that is at rest on a calm lake. The rowboat is 4 m long and it's center of mass is precisely in the middle of the boat when it is empty. Romeo weights 150 lbs and Juliet weights 100 lbs.

(a) [5pts] Romeo walks from the west end of the boat over to Juliet on the east end and asks for a kiss. How far did the boat move and in which direction?

(b) [5pts] Juliet smacks Romeo in the face. Offended and embarrassed, Romeo runs to the west end of the boat and jumps in the lake with a horizontal speed of 4 m/s relative to the ground. What is the boat's velocity?

[10pts] **Problem 2:** *Elastic, equal-mass collision of nuclei or billiard balls*
(Time estimate: 20 minutes)
Consider two billiard balls each with mass m. The second ball is initially at rest, while the first moves towards the second with velocity $\boldsymbol{v}_1$. The first ball strikes the second off-center and both balls then move (after the collision) with new velocities $\boldsymbol{v}_1'$ and $\boldsymbol{v}_2'$, neither of which lie along the initial velocity direction $\boldsymbol{v}_1$. [If the collision was dead center, both balls would move only in 1D before and after the collision; because the first ball strikes the second off-center, that is not the case here.] Assume that the collision is perfectly elastic. Show that after the collision, the two billiard balls move on paths that make a 90° angle with respect to each other. [Hint: this problem can be very easy or very hard depending on how you go about it. First recognize the condition on the final velocity vectors that you want to show. Do not attempt to do the problem in terms of vector components. Rather, write down momentum and energy conservation in vector form; i.e., you can write a kinetic energy as $\frac{1}{2}m\boldsymbol{v}^2$, which is the same as $\frac{1}{2}mv^2$ since $v^2 = \boldsymbol{v}^2 \equiv \boldsymbol{v}\cdot\boldsymbol{v}$. Then make smart use of the dot product to manipulate the two equations that you have. Resist the temptation to write out separate equations for the x and y components.] This result is useful in the analysis of nuclear collisions. If a target nucleus at rest is bombarded by an impacting nucleus, and the impactor and target depart on perpendicular trajectories, it is an indicator that the nuclei have similar mass and undergo an elastic collision.

[10pts] **Problem 3:** *proton + nucleus collision*
(Time estimate: 20 minutes)
A proton with mass m_p and initial speed v_0 collides with a stationary nucleus with *unknown mass* M. The collision is elastic and head-on; the proton rebounds straight back towards its origin with $\frac{4}{9}$ of its initial kinetic energy. Find the mass ratio of the nucleus to the proton, M/m_p.

Do one of the two problems below. If you do both, you can get up to 7 points extra credit for correctly completing the second problem (with no regrade points on the extra credit problem.) Your lowest score of the two will be considered the extra credit problem.

[15pts] **Problem 4:** *COM of a triangular plate*
(Time estimate: 30 minutes +)
Consider a right triangular plate with uniform density and thickness t. One side (with length a) rests on the positive x-axis. The hypotenuse extends from the origin to $(x, y) = (a, b)$, with $b < a$; both a and b are positive. Find the coordinates of the center of mass of the triangle. Express $X_{\rm CM}$ as a fraction of the base length a and $Y_{\rm CM}$ as a fraction of the triangle height b. Your answer should not include the mass, density, or thickness—only a or b as described above. [Hint: There are different ways to solve this problem. It will be helpful to write out the equation of the line $y(x)$ formed by the hypotenuse. Then consider very thin rectangular strips that are oriented vertically (with thickness dx to find $X_{\rm CM}$) or horizontally (with thickness dy to find $Y_{\rm CM}$). Check that your answer agrees with your intuition for where the COM should be.]

[15pts] **Problem 5:** *COM of a uniform hemisphere*
(Time estimate: 40 minutes)
Find the center of mass of a uniform density half-sphere with radius R. [Hint: orient the hemisphere symmetrically on the positive z-axis with the equator in the x-y plane. There are two ways to solve this problem. The first splits the hemisphere into thin disks perpendicular to the z-axis with radii $r(z)$ and thickness dz. An alternative approach is to do the integrals in spherical coordinates (r, θ, φ) using the appropriate expressions for the volume element dV and z. Unless you are familiar with spherical coordinates already, I suggest the former approach.] Remember that you will need to relate the mass and density. Your answer should be a fraction of the radius R.