Miscellanea

$$\left|\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}\right| = r^2\sin\theta$$

$$\left|\frac{\partial(x,y)}{\partial(r,\theta)}\right| = \left|\frac{\partial(x,y,z)}{\partial(r,\phi,z)}\right| = r$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\sin\theta \xrightarrow{\frac{\mathrm{d}}{\mathrm{d}\theta}} \cos\theta \to -\sin\theta$$

Trigonometry

$$\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C} \quad \text{(law of sines)}$$

$$C = \sqrt{A^2 + B^2 - 2AB\cos(c)} \quad \text{(law of cosines)}$$

Derivatives

$$\frac{\mathrm{d}(uv)}{\mathrm{d}x} = u'v + uv'$$

$$\frac{\mathrm{d}(u/v)}{\mathrm{d}x} = \frac{u'v - uv'}{v^2}$$

$$\frac{\mathrm{d}(u(v))}{\mathrm{d}x} = u'(v)v'$$

Kinematics

$$\vec{v}(t) = \vec{v}_0 + \vec{a}t \quad \text{(velocity)}$$

$$\vec{\omega}(t) = \vec{\omega}_0 + \vec{\alpha}t \quad \text{(rot. form)}$$

$$\vec{x}(t) = \vec{x}_0 + \vec{v}t + \frac{1}{2}\vec{a}t^2 \quad \text{(displacement)}$$

$$\vec{\theta}(t) = \vec{\theta}_0 + \vec{\omega}t + \frac{1}{2}\alpha t^2 \quad \text{(rot. form)}$$

$$v_f = \sqrt{{v_i}^2 + 2a\Delta x} \quad (v_f)$$

Momentum

$$\vec{p} = m\vec{v} \quad \text{(momentum)}$$

$$\vec{\ell} = I\vec{\omega} = \vec{r} \times \vec{p} \quad \text{(rot. form)}$$

$$\dot{\vec{P}} = \vec{F}_{\text{ext}} \quad \text{(3rd law)}$$

$$\dot{\vec{\ell}} = \vec{\tau} = I\vec{\alpha} = \vec{r} \times \vec{F} \quad \text{(rot. form)}$$

$$\begin{aligned}\Delta\vec{P} &= \vec{P}_f - \vec{P}_i = \vec{J} \\ &= \int \vec{F}\mathrm{d}t \quad \text{(impulse)} \\ &= \vec{F}_{\text{avg}}\Delta t\end{aligned}$$

Total Angular Momentum

$$\begin{aligned}\vec{L} &= \sum_{i=1}^{N} \vec{\ell}_i \\ &= \vec{R}_{\text{CM}} \times \vec{P}_{\text{CM}} \quad (\vec{L}_{\text{orb}} \text{ of CM about axis}) \\ &+ \sum_{i=1}^{N} r_i' \times \vec{p}_i{}' \quad (\vec{L}_{\text{spin}} \text{ of body about CM})\end{aligned}$$

Collisions

An *elastic* collision conserves kinetic energy.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$$

An *inelastic* collision does not.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m\vec{v}\,'$$

Changing Mass

Rockets

$$\vec{F}_{\text{th}} = -\dot{m}\vec{v}_{\text{ex}} \quad \text{(thrust)}$$

$$\vec{v}(t) = \vec{v}_0 + \vec{v}_{\text{ex}}\ln\left(\frac{m_0}{m(t)}\right) \quad (\Delta V)$$

Center of Mass

$$\vec{R} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i \quad \text{(CM summation)}$$

$$M = \int \rho \mathrm{d}V \quad \text{(total mass)}$$

$$\vec{R} = \frac{1}{M}\int \vec{r}\mathrm{d}m \quad \text{(CM integral)}$$

$\mathrm{d}m$ can usually be rewritten in terms of $\mathrm{d}V$

$$\vec{R} = \frac{1}{M}\int \vec{r}\rho(\vec{r})\mathrm{d}V$$

Look for symmetries by which you can substitute $\vec{R}$ and $\vec{r}$ for scalars.

Moment of Inertia

$$\sum_{i=1}^{N} = m_i r_\perp^2 \quad \text{(summation)}$$

$$I = \int r_\perp^2 \mathrm{d}m \quad \text{(integral)}$$

Parallel Axis Theorem

$$I = I_{\text{CM}} + Md^2$$

where $I \parallel I_{\text{CM}}$ and d is the distance between I & I_{CM}

Perpendicular Axis Theorem

For *planar lamina* (flat, plate-like objects)

$$I_z = I_x + I_y$$