Miscellanea

$$\left|\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}\right| = r^2\sin\theta$$

$$\left|\frac{\partial(x,y)}{\partial(r,\theta)}\right| = \left|\frac{\partial(x,y,z)}{\partial(r,\phi,z)}\right| = r$$

Trigonometry

$$\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C} \quad \text{(law of sines)}$$

$$C = \sqrt{A^2 + B^2 - 2AB\cos(c)} \quad \text{(law of cosines)}$$

Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin(\theta \pm \frac{\pi}{2}) = \pm\cos\theta$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta = 1/\cos\theta$$

$$\csc\theta = 1/\sin\theta$$

Calculus

Derivatives

$$\frac{\mathrm{d}(uv)}{\mathrm{d}x} = u'v + uv'$$

$$\frac{\mathrm{d}(u/v)}{\mathrm{d}x} = \frac{u'v - uv'}{v^2}$$

$$\frac{\mathrm{d}(u(v))}{\mathrm{d}x} = u'(v)v'$$

$$\sin\theta \xrightarrow{\frac{\mathrm{d}}{\mathrm{d}\theta}} \cos\theta \to -\sin\theta$$

Integrals

$$\int\frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^{ax}\mathrm{d}x = \frac{1}{a}e^{ax} + C$$

Kinematics

$$\vec{v}(t) = \vec{v}_0 + \vec{a}t \quad \text{(velocity)}$$

$$\vec{\omega}(t) = \vec{\omega}_0 + \vec{\alpha}t \quad \text{(rot. form)}$$

$$\vec{x}(t) = \vec{x}_0 + \vec{v}t + \frac{1}{2}\vec{a}t^2 \quad \text{(displacement)}$$

$$\vec{\theta}(t) = \vec{\theta}_0 + \vec{\omega}t + \frac{1}{2}\alpha t^2 \quad \text{(rot. form)}$$

$$v_f = \sqrt{{v_i}^2 + 2a\Delta x} \quad (v_f)$$

Momentum

$$\vec{p} = m\vec{v} \quad \text{(momentum)}$$

$$\vec{\ell} = I\vec{\omega} = \vec{r}\times\vec{p} \quad \text{(rot. form)}$$

$$\dot{\vec{P}} = \vec{F}_{\text{ext}} \quad \text{(2nd law)}$$

$$\dot{\vec{\ell}} = \vec{\tau} = I\vec{\alpha} = \vec{r}\times\vec{F} \quad \text{(rot. form)}$$

$$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = \vec{J} = \int\vec{F}\mathrm{d}t = \vec{F}_{\text{avg}}\Delta t \quad \text{(impulse)}$$

Total Angular Momentum

$$\vec{L} = \sum_{i=1}^{N}\vec{\ell}_i = \vec{R}_{\text{CM}}\times\vec{P}_{\text{CM}} \quad (\vec{L}_{\text{orb}} \text{ of CM about axis}) + \sum_{i=1}^{N} r_i' \times \vec{p_i}' \quad (\vec{L}_{\text{spin}} \text{ of body about CM})$$

Collisions

An *elastic* collision conserves kinetic energy.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$$

An *inelastic* collision does not.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m\vec{v}'$$

Changing Mass

Rockets

$$\vec{F}_{\text{th}} = -\dot{m}\vec{v}_{\text{ex}} \quad \text{(thrust)}$$

$$\vec{v}(t) = \vec{v}_0 + \vec{v}_{\text{ex}}\ln\left(\frac{m_0}{m(t)}\right) \quad (\Delta V)$$

Center of Mass

$$\vec{R} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i \quad \text{(CM summation)}$$

$$M = \int\rho\mathrm{d}V \quad \text{(total mass)}$$

$$\vec{R} = \frac{1}{M}\int\vec{r}\mathrm{d}m \quad \text{(CM integral)}$$

$\mathrm{d}m$ can usually be rewritten in terms of $\mathrm{d}V$

$$\vec{R} = \frac{1}{M}\int\vec{r}\rho(\vec{r})\mathrm{d}V$$

Look for symmetries by which you can substitute $\vec{R}$ and $\vec{r}$ for scalars.

Moment of Inertia

$$\sum_{i=1}^{N} = m_i r_\perp^2 \quad \text{(summation)}$$

$$I = \int r_\perp^2\mathrm{d}m \quad \text{(integral)}$$

Parallel Axis Theorem

$$I = I_{\text{CM}} + Md^2$$

where $I \parallel I_{\text{CM}}$ and d is the distance between I & I_{CM}

Perpendicular Axis Theorem

For *planar lamina* (flat, plate-like objects)

$$I_z = I_x + I_y$$

Work & Energy

$$W = \int_C \vec{F}\cdot\mathrm{d}\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}'(t)\mathrm{d}t = \int_{x,y}\langle F_x, F_y\rangle\cdot\langle\mathrm{d}x,\mathrm{d}y\rangle \quad \text{(work integral)}$$

$$W = \Delta T \quad \text{(Work-Energy Theorem)}$$

$$\vec{F} = -\nabla U \quad \text{(potential from force)}$$

$$U_s = \frac{1}{2}kx^2 \quad \text{(spring potential energy)}$$

$$U_g = mgh \quad \text{(gravitational potential energy)}$$

Conservative Forces

A *conservative force* is one with a corresponding potential energy. This means:

- the force depends *only* on position
- the work done by that force is the same for *any* path $\implies \nabla\times\vec{F} = 0$

Lagrangian Mechanics

For a d-dimensional system of N particles & M constraints, it is said there are $dN - M$ *degrees of freedom* or generalized coordinates (q_i) that describe the positions of those particles. For each coordinate, a *Lagrange equation*

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\frac{\partial\mathcal{L}}{\partial\dot{q}_i}}_{\text{gnr. } p} = \overbrace{\frac{\partial\mathcal{L}}{\partial q_i}}^{\text{gnr. } F}$$

gives the equation of motion.

cyclic coordinates
Coordinates upon which the Lagrangian *does not* depend on.

constraint Relation between coordinates, e.g. $x^2 + y^2 + z^2 = R^2$ for a spherical pendulum. *Reduces* the number of coordinates needed to describe the system.