double-projection.h Documentation

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Section 1

Introduction

This header file provides a method to update the velocity and pressure gradient separately, mitigating error accumulation in the pressure gradient due to a non-strictly divergence-free cell-centered velocity. This approach requires an additional projection step.

As discussed in the documentation for `centered.h`, the cell-centered velocity $\mathbf{u}^n$ is not strictly divergence-free. This is because it is computed by interpolating the source term (comprising acceleration and pressure gradient), which complements the strictly divergence-free face-centered velocity. Consequently, when updating the pressure gradient ∇p in the next time step by solving the Poisson equation for $\mathbf{u}^{***}$, defined as

$$\mathbf{u}^{***} = \mathbf{u}^n - \Delta t \nabla \cdot (\mathbf{u} \otimes \mathbf{u})^{n+1/2} + \frac{\Delta t}{\rho^{n+1/2}} \nabla \cdot \left(2\mu_f^{n+1/2} \mathbf{D}^{**} \right), \tag{1}$$

the non-divergence-free error in $\mathbf{u}^n$ accumulates, as $\mathbf{u}^{***}$ includes $\mathbf{u}^n$. As reported on the `Basilisk` website, this issue can be significant under certain conditions.

Section 2

Solution Approach

To address this issue, we exclude $\mathbf{u}^n$ from $\mathbf{u}^{***}$ as therotically it satisfies $\nabla \cdot \mathbf{u}^n = 0$. The pressure gradient ∇p is then computed by solving

$$\nabla^2 p^{n+1} = \nabla \cdot \left[-\Delta t \nabla \cdot (\mathbf{u} \otimes \mathbf{u})^{n+1/2} + \frac{\Delta t}{\rho^{n+1/2}} \nabla \cdot \left(2\mu_f^{n+1/2} \mathbf{D}^n \right) \right]. \tag{2}$$

To maintain a divergence-free velocity field, an additional projection iteration is required. The velocity update process is now defined as follows:

$$\frac{\rho^{n+1/2}}{\Delta t} (\mathbf{u}^{**} - \mathbf{u}^n) + \rho^{n+1/2} \left[\nabla \cdot (\mathbf{u} \otimes \mathbf{u})^{n+1/2} + \frac{\nabla p^n}{\rho^{n-1/2}} \right] = \nabla \cdot \left(2\mu_f^{n+1/2} \mathbf{D}^{n+1/2} \right), \tag{3}$$

$$\nabla^2 \delta p = \nabla \cdot \mathbf{u}^{**}, \quad \mathbf{u}^{n+1/2} = \mathbf{u}, \tag{4}$$

$$\mathbf{u}^{n+1} = \mathbf{u}^{**} - \frac{\Delta t}{\rho^{n+1/2}} \nabla \delta p^n, \tag{5}$$

where δp is the temporary pressure gradient to correct the velocity field.