$$G(s) = \begin{bmatrix} \dfrac{4}{(s+1)(s+2)} & \dfrac{-1}{(s+1)} \\ \dfrac{2}{(s+1)} & \dfrac{-1}{2(s+1)(s+2)} \end{bmatrix}$$

(2×2)

$r = \text{rank}\,[G(s)] = 2$

$$G(s) = \frac{1}{d(s)} \cdot P(s) = \frac{1}{(s+1)(s+2)} \begin{bmatrix} 4 & -(s+2) \\ 2(s+2) & -0{,}5 \end{bmatrix}$$

$$P(s) = \begin{bmatrix} 4 & -(s+2) \\ 2(s+2) & -0{,}5 \end{bmatrix} \sim S(s) = \begin{bmatrix} \varepsilon_1'(s) & 0 \\ 0 & \varepsilon_2'(s) \end{bmatrix}$$

ALGORITMO

$D_0(s) = 1$

$D_1(s) = \text{MCD}\,\{4, -(s+2), 2(s+2), -0{,}5\} = 1$

$$D_2(s) = \text{MCD}\left\{ \begin{vmatrix} 4 & -(s+2) \\ 2(s+2) & -0{,}5 \end{vmatrix} \right\} = -2 + 2\,(s+2)^2$$

determinante

$= 2\,(s^2+4s+4) - 2 = 2s^2 + 8s + 6 = 2\,(s^2+4s+3)$

$= \underbrace{2}_{\text{si trascura}}\,(s+1)(s+3)$

$$\varepsilon_1'(s) = \frac{D_1(s)}{D_0(s)} = 1$$

$$\varepsilon_2'(s) = \frac{D_2(s)}{D_1(s)} = (s+1)(s+3)$$

$$G(s) \sim M(s) = \frac{1}{(s+1)(s+2)} \begin{bmatrix} 1 & 0 \\ 0 & (s+1)(s+3) \end{bmatrix} = \begin{bmatrix} \frac{1}{(s+1)(s+2)} & 0 \\ & \frac{s+3}{(s+2)} \end{bmatrix}$$

$p(s) = (s+1)(s+2)^2$ polinomio dei poli

$z(s) = (s+3)$ polinomio degli zeri

grado di Smith McMillan pari a 3 (grado di $p(s)$)

3 poli $s=-1$ $s=-2$ $s=-2$

1 zero $s=-3$

Gli zeri non sono individuabili dalla $G(s)$

I poli sono individuabili dalla $G(s)$

ma non ne conosciamo la molteplicità

COROLLARIO

$G(s)$ quadrata (a meno di cancellazioni)

$$\det[G(s)] = c \, \frac{z(s)}{p(s)} \qquad c = \text{costante}$$

$$\det[G(s)] = \frac{2(s+3)}{(s+1)(s+2)^2}$$

$z(s) = (s+3)$

$p(s) = (s+1)(s+2)^2$

```
>> syms s
>> g11 = 4/((s+1)*(s+2))
>> g12 = -1/(s+1)
>> g21 = 2/(s+1)
>> g22 = -0.5/((s+1)*(s+2))
>> G = [g11 g12 ; g21 g22]
>> det(G)
```

Matlab

```
Gss = ss(G)

z = tzero(minreal(Gss))
p = pole(minreal(Gss))
```

$$G(s) = \begin{bmatrix} \frac{1}{(s+1)} & \frac{1}{(s+1)(s+2)} \\ \frac{s}{(s+1)(s+2)} & \frac{2s+1}{(s+1)(s+2)} \end{bmatrix}$$

$$= \frac{1}{(s+1)(s+2)} \begin{bmatrix} s+2 & 1 \\ s & 2s+1 \end{bmatrix}$$

$$P(s) = \begin{bmatrix} s+2 & 1 \\ s & 2s+1 \end{bmatrix} \sim S(s) = \begin{bmatrix} \varepsilon_1'(s) & 0 \\ 0 & \varepsilon_2'(s) \end{bmatrix}$$

$$D_0(s) = 1$$

$$D_1(s) = \text{MCD} \{ s+2, 1, s, 2s+1 \} = 1$$

$$D_2(s) = \text{MCD} \left\{ \begin{vmatrix} s+2 & 1 \\ s & 2s+1 \end{vmatrix} \right\} = (s+2)(2s+1) - s = 2s^2 + 4s + 2$$

$$= 2(s^2 + 2s + 1) = \underbrace{2}_{\text{si trascura}} (s+1)^2$$

$$\varepsilon_1'(s) = \frac{D_1(s)}{D_0(s)} = 1 \qquad \varepsilon_2'(s) = \frac{D_2(s)}{D_1(s)} = (s+1)^2$$

$$S(s) = \begin{bmatrix} 1 & 0 \\ 0 & (s+1)^2 \end{bmatrix}$$

$$G(s) \sim M(s) = \frac{1}{(s+1)(s+2)} \begin{bmatrix} 1 & 0 \\ 0 & (s+1)^2 \end{bmatrix} = \begin{bmatrix} \frac{1}{(s+1)(s+2)} & 0 \\ 0 & \frac{s+1}{s+2} \end{bmatrix}$$

$z(s) = (s+1) \qquad z = -1$

$p(s) = (s+1)(s+2)^2 \qquad p_1 = -1 \quad p_2 = -2 \quad p_3 = -2$

grado di Smith McMillan è 3

N.B. polo e zero coincidenti in $s = -1$

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{1}{(s+1)(s+2)} \\ \frac{s}{(s+1)(s+2)} & \frac{2s+1}{(s+1)(s+2)} \end{bmatrix}$$

(A, B, C, D)

$x \in \mathbb{R}^n \quad n=5$

$$\det(G(s)) = \frac{\ell}{(s+2)^2}$$

cancellazione
polo e zero in $s=-1$

G costruita con il comando tf

Z= tzero (minreal (ss(G))).
P= pole (minreal (ss(G)))

$s_0=-1$ zero di trasmissione

$$\text{rank} \begin{bmatrix} s_0 I - A & -B \\ C & D \end{bmatrix} = 6 < n + \min(p,m) = 7$$

G $m \times p$

$$\underset{m}{y} = \underset{m \times p}{G} \; \underset{p}{u}$$