学院____ 专业____ 班级____ 学号____ 姓名____ 任课教师____

装 订 线

扬 州 大 学 试 题 纸

(2021 －2022 学年第 二 学期)

数学科学 学院 数学 21 级、信科 21 级 班(年)级课程

初等数论 自测题三

考试形式：开卷（ ）闭卷（ √ ）

题目	一	二	三	四	五	六	总分
得分							

一、名词解释（3+3+4=10 分）

1. Write out the definition of complete residue system modulo m, where m is a positive integer.

2. Write out the definition of $o_m(a)$, i.e. the order of a modulo m, where m is a positive integer, a is an integer coprime to m.

3. Write out the content of Euler's Theorem(proof is not required).

二、应用题（15 分），注意写清楚计算步骤。

4. Alice is using her computer to calculate

$$12345678909876543 21^{2017201820192020} (mod\, 31415926535897932626)$$

If the computer use 1 second to calculate the product of two 20-digits number, and 1 second to calculate the remainder of a 40-digits number divided by a 20-digits number, then is it possible to get the result in 5 minutes?
Give your reason and estimate a bound of time.

二、计算题（10+10+10+10=40 分），注意写清楚计算步骤。

5. a) Calculate the number of integers between 0 and 1000000 that are coprime to 1000000, i.e. the cardinality of the set

$$\{x \in \mathbb{Z} : 0 \le x \le 1000000, g.c.d.(x, 1000000) = 1\}.$$

b) Calculate the number of integers between 0 and 2022 that are coprime to 1000000, i.e. the cardinality of the set

$$\{x \in \mathbb{Z} : 0 \le x \le 2022, g.c.d.(x, 1000000) = 1\}.$$

6. Find the last four digits of the octal representation of 2021^{12306}.

7. a) Is there a primitive root modulo 250? Give your reason.
b) Is 3 a primitive root modulo 250? If your answer is yes, give your reason. If your answer is no, then calculate the order of 3 modulo 250.

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8. a) Calculate the order of 2021 modulo 108108.

b) Calculate $2021^{1800}(mod 108108)$.

三、证明题（8+20+7=35 分），注意写清楚证明细节。

9. Prove that the sum of squares of three consecutive integers(i.e. $n^2+(n+1)^2+(n+2)^2$) can't be a square.

10. a) Is 127 a prime or a composite? Give your reason. If 127 is a composite, factorize it into the product of prime powers.

b) Prove that: for any integer a coprime to 127, we have

$$a^{126} \equiv 1 (mod 127)$$

c) Is 1729 a prime or a composite? Give your reason. If 1729 is a composite, factorize it into the product of prime powers.

d) Prove that: for any integer a coprime to 1729, we have

$$a^{1728} \equiv 1 (mod 1729)$$

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11. Is $2^{251}-1$ a prime or a composite? Give your reason.