任课教师____________ 姓名____________ 学号____________ 班级____________ 专业____________ 学院____________

装 订 线

扬 州 大 学 试 题 纸

(2021 －2022 学年第 二 学期)

数学科学 学院 数学 21 级、信科 21 级 班(年)级课程

初等数论 自测题二

考试形式：开卷（ ）闭卷（ √ ）

题目	一	二	三	四	五	六	总分
得分							

一、名词解释（3+3+4=10 分）

1. Write out the definition of algebraic number.

2. Write out the definition of congruent(modulo m), where m is a positive integer.

3. State the content of Fundamental Theorem of Arithmetic(proof is not required).

二、应用题（20 分），注意写清楚计算步骤。

4. *A sunny afternoon, Sherlock Holmes and Doctor Watson are chatting after lunch.*

Holmes: "What are you doing, Watson?"

Watson: "I'm trying to find the last three digits of 504^{2022}. *You see, Sherlock, just note that the last digit of* $504, 504^2, 504^3, \cdots$ *are* $4, 6, 4, \cdots$, *it is quite easy to find the last digit of* 504^{2022}*."*

a) Find the last digit of the decimal representation of 504^{2022}.

Holmes smiles: "Yes."

Watson:"Simultaneously, I also calculate the last three digits of $504, 504^2, 504^3, \cdots$, but this time I don't see any cycles yet……*"*

Holmes:"Yes, it would be a great effort to find the cycles, but…… "

b) Find the last three digits of the decimal representation of 504^{2022}.

Watson: "Marvelous! Sherlock, Is this the power of deduction?"

Holmes laughs: "No, Watson. It is the power of number theory."

三、计算题（10+10+10=30 分），注意写清楚计算步骤。

5. a) Use Fermat Factorization to factorize 1891 into the product of prime powers.

b) Calculate the number of positive divisors of 1891.

c) Calculate the sum of all positive divisors of 1891.

6. Let n be a positive integer, find the number of nonnegative integral solutions of the Diophantine equation $x+y+2z=n$.

7. Find a fractional $\frac{p}{q}$ (where p, q are positive integers less than 100) satisfy the following conditions:

$$\left|\frac{p}{q}-0.123456789\right|<10^{-8}$$

Check your results.

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四、证明题（**6+10+14+10=40** 分），注意写清楚证明细节。

8. Prove: in any basis, the integer $(10101)_b$ (in base b-representation) is always a composite number.

9. Prove: Every integer greater than 2022 can be written as the product of primes(uniqueness is not required).
(Here you are not allowed to use the consequence of the Fundamental Theorem of Arithmetic directly.)

10. a)Let m be a positive integer, prove that m is a square if and only if for any prime p, $ord_p m$ is even.

b)Let m be a positive integer, m is not a square, prove that $\sqrt{m}$ is an irrational number.

c) Prove: $\sqrt{2^{251}-1}$ is an irrational number.

11. a)Prove that if $c > 31$, then the Diophantine equation $8x + 9y + 10z = c$ has an nonnegative integer solution.

b)Prove that if $c = 31$, then the Diophantine equation $8x + 9y + 10z = c$ has no nonnegative integer solution.