学院________ 专业________ 班级________ 学号________ 姓名________ 任课教师________

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扬 州 大 学 试 题 纸

(2021 －2022 学年第 二 学期)

数学科学 学院 数学 21 级、信科 21 级 班(年)级课程 初等数论 期中自测题

考试形式：开卷（ ）闭卷（ √ ）

题目	一	二	三	四	五	六	总分
得分							

一、名词解释（3+3+4=10 分）

1. Write out the definition of prime number.

2. Let $n = p_1^{a_1} p_2^{a_2} \cdots p_l^{a_l}$ be a positive integer, where $p_1, p_2, \cdots, p_l$ are distinct primes, $a_1, a_2, \cdots, a_l$ are positive integers. let $\sigma(n)$ be the sum of all positive divisors of n, write out the formula of $\sigma(n)$ (proof is not required).

3. Write out the content of Prime Number Theorem(proof is not required).

二、应用题（20 分），注意写清楚计算步骤。

4. A post office is left with stamps of only two values. The staffs found that there are exactly 24 postage amount that cannot be made up using these stamps, including 30fen. What are the values of stamps? In particular, If the postage amount is 48fen, Is it possible to combine the stamps to get the postage amount? If possible, How to combine the stamps to get the amount?(Note that the answer may not unique)

三、计算题（10+10+10+10=40 分），注意写清楚计算步骤。

5. a) Is 1 a prime or a composite?

b) Is 137 a prime or a composite? Give your reason. If 137 is a composite, factorize it into prime powers.

c) Is 100 a prime or a composite? Give your reason. If 100 is a composite, factorize it into prime powers.

6. Find all nonnegative integral solution of the Diophantine equation

$$666x + 468y = 38448$$

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7. a) Convert $(2227)_8$ to base-11 representation.

b) Convert $(5316)_8$ to hexadecimal representation.

8. Find all nonnegative integral solutions of the following linear system of Diophantine equations

$$\begin{cases} 2x + 3y + 5z = 29 \\ 25x + 30y + 35z = 245 \end{cases}$$

四、证明题（20+10=30 分），注意写清楚证明细节。

9. a) Let x, y be two real numbers, prove that：

$$[x]+[y]+[x+y]\le[2x]+[2y]$$

b) Prove that for any two positive integers m and n, $\dfrac{(2m)!(2n)!}{(m)!(n)!((m+n)!)}$ is an integer.

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10. Given coprime positive integers $a_1, a_2, \cdots, a_n$, the largest integer that cannot be expressed as an non-negative integral combination of these numbers is called the **Frobenius number** of $a_1, a_2, \cdots, a_n$, and denote by $g(a_1, a_2, \cdots, a_n)$.

We already know that the Frobenius number of two coprime positive integers x,y exists and $g(x, y) = xy - x - y$.

Now, let $a_1, a_2, \cdots, a_n$ be coprime positive integers, prove that the Frobenius number of $a_1, a_2, \cdots, a_n$ exists.