Elementary Number Theory
Spring 2022　Homework I

2022 年 2 月 24 日

Due: ? at 8am

Note: For all problems, you are allowed to use the textbook, class notes, and other book references. You can collaborate on problem solving for all but the "No collaboration" problems but ALL problems must be written on your own.

Problems

一、计算题

1. (No collaboration)
   Calculate the following, state your results in octal representation too.
   a)$(12345)_8 + (34567)_8$
   b)$(1235)_8 - (567)_8$

2. (No collaboration)
   a)Convert 2022 to hexadecimal and base-11 representation.
   b)Convert $(2022)_{16}$ to octal representation.

3. (No collaboration)
   a)Is 1 a prime or a composite?
   b)Is 139 a prime or a composite? Give your reason. If 139 is a composite, factorize it into prime powers.
   c)Is 2911 a prime or a composite? Give your reason. If 2911 is a composite, factorize it into prime powers.

4. a) Find the integer s such that $11^s||2022!$.
   b) Find the integer t such that $8^t||222!$.
   c) The duodecimal representation of $\binom{100}{50} = \frac{100!}{50!50!}$ ends in exactly m consecutive zeros, find the value of m.

5. a) Factorize 2022 into the product of prime powers.
   b)Find the number of positive divisors of 2022.
   c)Find the number of even positive divisors of 2022.
   d)Find the sum of all positive divisors of 2022.
   e)Find the sum of all even positive divisors of 2022.
   f)Find the sum of all positive divisors which are multiples of 4 of 10000.

6. a) Estimate the number of primes between 1000000000 and 10000000000(Take $\ln 10 \approx 2.302$).
   b) Estimate the size of the 10000000000th prime(Take $\ln 10 \approx 2.302$).

二、应用题

1. *A dozen equals 12, and a gross equals* 12^2*. Using duodecimal arithmetic, answer the following questions.*
   a) If 3 gross 7 dozen and 4 eggs are removed from a total of 11 gross and 3 dozen eggs, how many eggs are left?
   b) If 5 truckloads of 2 gross 3 dozen and 7 eggs each are delivered to the supermarket, how many eggs are delivered?
   c) If 11 gross 10 dozen and 6 eggs are divided in 3 groups of equal size, how many eggs are in each group?

三、证明题

1. Prove: in any basis, the integer $(10101)_b$(in base b-representation) is always a composite number.

2. Prove: Every integer greater than 2022 can be written as the product of primes(uniqueness is not required).
   (Here you are not allowed to use the consequence of the Fundamental Theorem of Arithmetic directly.)

3. Prove: $\sqrt{2^{251}-1}$ is an irrational number.

作业要求

1. 做题过程中，你可以使用教材、课堂笔记或者其它参考书。可以合作讨论，但是**所有的解答都应该用你自己的语言写出来。**

2. 最好用*A4*大小的纸书写，卷面整洁，减少涂改，保持字迹工整，字尽量写大一些，不要写的太密，留出批改的空间，各题之间留一些空白。作业有多页时标一下页码并且按顺序订一下以免纸张散掉。

3. 记得在作业封面写清楚自己的班级姓名学号。

4. 解方程要有过程，特别的，应用题设未知数要说明，还要有答。

5. 可以使用计算器，但不能使用数学软件。

6. 用蓝色的笔抄题（可缩写题干，但需要保留解题或证明题所需的所有信息），用黑色的笔写解答或证明。

7. 大作业用大写字母A到C评定等级（例如按要求把所有的题目都抄一遍但是所有题目都没做就会得到C的评分），**注意：这个等级评定是对作业的评价，和平时成绩中的作业分数无关（但超过*1* 次的*E* 评分和不交作业会影响平时分）**。
   平时成绩和总评成绩计算公式见习题课文件注意事项或第一次课的PPT。

8. 雷同或违反上面要求的作业会得到E的评分。

9. 除特殊原因（需要附假条）外，迟交作业会得到E的评分。

10. 第一次评分为E不扣平时成绩，从第二次评分为E开始，每次评分E扣10分平时分。

11. 每次不交作业扣10分平时分。