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14. 用牛顿迭代法和带参数的牛顿迭代法求方程 $f(x) = (\sin x - \frac{x}{2})^2 = 0$ 一个近似根精确到 10^{-5}，初值 $x_0 = \frac{\pi}{2}$

解：① $f'(x) = 2(\sin x - \frac{x}{2})(\cos x - \frac{1}{2})$

用牛顿迭代公式为：

$$x_{k+1} = x_k - \frac{(\sin x_k - \frac{x_k}{2})^2}{2(\sin x_k - \frac{x_k}{2})(\cos x_k - \frac{1}{2})}$$

$$= x_k - \frac{\sin x_k - \frac{x_k}{2}}{2(\cos x_k - \frac{1}{2})}$$

取 $x_0 = \frac{\pi}{2}$，　$x_1 = \frac{\pi}{2} - \frac{1 - \frac{\pi}{4}}{-1} = 1.785398$

$x_2 = 1.844562$

$\vdots$

$x_{19} = 1.89549$

$x_{20} = 1.895494$

$|x_{20} - x_{19}| = 0.000004 = 4\times10^{-6} < 10^{-5}$

因此近似根为 $x^* = x_{20} = 1.895494$

② 带参数的牛顿迭代法

由 $f(x) = (\sin x - \frac{x}{2})^2 = 0$ 可知：$f(x) = 0$ 时，

$x_1 = 0$ 或 $x = x^*$ 时成立，因此 x^* 为 $f(x)$ 的2重根.

用求根重根的迭代公式为：

$$x_{k+1} = x_k - m\frac{f(x)}{f'(x)}$$

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$$\Rightarrow x_{k+1} = x_k - 2\,\frac{\left(\sin x_k - \frac{x_k}{2}\right)^2}{2\left(\sin x_k - \frac{x_k}{2}\right)\left(\cos x_k - \frac{1}{2}\right)}$$

$$= x_k - \frac{\sin x_k - \frac{x_k}{2}}{\cos x_k - \frac{1}{2}}$$

取 $x_0 = \frac{\pi}{2}$，$x_1 = 2$ $\quad x_2 = 1.900995594$

$x_3 = 1.895511645$

$x_4 = 1.895494267$

$x_5 = 1.895494267$

因此 $x^* = x_4 = 1.895494267 \approx 1.895494$

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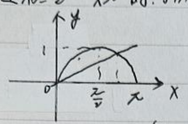
14.用牛顿迭代法和带参数的牛顿迭代法求方程的 $f(x)=(\sin x-\frac{x}{2})^2=0$ 一个近似根精确到 10^{-5}，初值 $x_0=\frac{\pi}{2}$ $x>0$ 时. 当为 $\sin x=\frac{x}{2}$. $y=\sin x$.

解：



$f'(x)=2(\sin x-\frac{x}{2})\cdot(\cos x-\frac{1}{2})$

单根牛顿迭代法. $x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}$ $k=0,1,2,\cdots$ $x_{k+1}=x_k-\frac{\sin x_k-\frac{x_k}{2}}{2(\cos x_k-\frac{1}{2})}$

初值 $x_0=\frac{\pi}{2}=1.57$ 和精度 10^{-5}.

$x_1=\frac{\pi}{2}-\frac{1-\frac{\pi}{4}}{2(0-\frac{1}{2})}=\frac{\pi}{2}+1-\frac{\pi}{4}=1+\frac{\pi}{4}\approx 1.78540$

$x_2=1.84456$ $x_7=1.89399$ $x_{12}=1.89545$

$x_3=1.87083$ $x_8=1.89474$ $x_{13}=1.89547$

$x_4=1.88334$ $x_9=1.89512$

$x_5=1.88946$ $x_{10}=1.89531$

$x_6=1.89249$ $x_{11}=1.89540$ $\therefore x^*=x_{13}=1.89547$

重根. 精确值 x^*. $\lim_{x\to x^{*+}} f(x)>0$ $\lim_{x\to x^{*-}} f(x)>0$. 是二重根.

使用带参数的牛顿迭代法. $m=2$

$x_{k+1}=x_k-m\cdot\frac{f(x_k)}{f'(x_k)}$ $\to$ $x_{k+1}=x_k-2\cdot\frac{\sin x_k-\frac{x_k}{2}}{2(\cos x_k-\frac{1}{2})}=x_k-\frac{\sin x_k-\frac{x_k}{2}}{\cos x_k-\frac{1}{2}}$. $k=0,1,2,\cdots$

初值 $x_0=\frac{\pi}{2}$. 精度 10^{-5}.

$x_1=2$

$x_2=1.90100$

$x_3=1.89551$ $|x_4-x_3|=0.00002$.

$x_4=1.89549$. $\therefore x^*=x_4=1.89549$.

陆怡静

① $f(x)$的导数 $f'(x)=2(\sin x-\frac{x}{2})(\cos x-\frac{1}{2})$

牛顿法迭代公式为 $x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}$ $k=0,1,2\cdots$

代入得 $$x_{k+1}=x_k-\frac{(\sin x_k-\frac{x_k}{2})^2}{2(\sin x_k-\frac{x_k}{2})(\cos x_k-\frac{1}{2})}$$

$$=x_k-\frac{\sin x_k-\frac{x_k}{2}}{2(\cos x_k-\frac{1}{2})}$$

初始值 $x_0=\frac{\pi}{2}$ ∴令 $k=0$ 代入

则 $x_1=1.785398$ $|x_{k+1}-x_k|<\varepsilon$

$x_2=1.844562$

$\vdots$

$x_{20}=1.895494$

$|x^*-1.895494|<10^{-5}$

② $f(x)$的根 x^*为2重根

$x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}$ $k=0,1,2\cdots$

$$=x_k-\frac{\sin x_k-\frac{x_k}{2}}{\cos x_k-\frac{1}{2}}$$

取 $x_0=\frac{\pi}{2}$.

则 $x_1=2.000000$ $x_3=1.895512$

$x_2=1.900996$ $x_4=1.895494$

$x_5=1.895494$ 四次迭代到法①上面 x_{20} 结果

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牛顿迭代法:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad n=0,1,2,\cdots$$

$$f(x_n) = \left(\sin x_n - \frac{x_n}{2}\right)^2 \qquad n=0,1,2,\cdots$$

$$f'(x_n) = 2\sin x_n \cos x_n - \sin x_n - x_n\cos x_n + \frac{x_n}{2} = \sin 2x_n - \sin x_n - x\cos x_n + \frac{x_n}{2} \qquad n=0,1,2\cdots$$

通过excel迭代运算. 取初值 $x_0=\frac{\pi}{2}$.

迭代至 $x_{15}=1.895488419$ 时. $|x_{15}-x_{14}| = 5.848\times10^{-6} < 10^{-5}$.

取近似根为 $x_{15}=1.895488419$.

带参数的牛顿迭代法

$$x_{n+1} = x_n - m\frac{f(x_n)}{f'(x_n)}, \quad n=0,1,2,\cdots$$

此时m不易知

则令 $g(x)=\frac{f(x)}{f'(x)}=\frac{(x-x^*)p(x)}{mp(x)+(x-x^*)p'(x)}$

则 $g'(x^*)=\frac{1}{m}\neq0$

则有 $x_{n+1}=x_n-\frac{g(x_n)}{g'(x_n)}=x_n-\frac{f(x_n)f'(x_n)}{[f'(x_n)]^2-f(x_n)f''(x_n)}, \quad n=0,1,2,\cdots$

取初值 $x_0=\frac{\pi}{2}$, 通过excel迭代运算.

迭代至 $x_4=1.8954942668$, $x_5=1.8954942670$

$|x_5-x_4| = 2.335\times10^{-10} < 10^{-5}$

取 $x^*=x_5=1.8954942670$

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牛顿迭代法

$f(x)=(\sin x-\frac{x}{2})^2=0$ 的根为二重根

$f'(x)=2[\sin x-\frac{x}{2}](\cos x-\frac{1}{2})$

用牛顿迭代公式为

$$x_{k+1}=x_k-\frac{(\sin x_k-\frac{x_k}{2})^2}{2(\sin x_k-\frac{x_k}{2})(\cos x_k-\frac{1}{2})}$$

$$=x_k-\frac{\sin x_k-\frac{1}{2}x_k}{2\cos x_k-1},\quad k=0,1,2\cdots$$

令$x_0=\frac{\pi}{2}$ $x_1=\frac{\pi}{2}-\frac{1-\frac{\pi}{4}}{0-1}=1.785398$

同理$x_2=1.844562$，$x_3=1.870834$，$x_4=1.886389$

一直迭代到$x_{20}=1.895494$，$|x^*-1.895494|<10^{-5}$.

带参数的牛顿迭代法

$m=2$. $x_{k+1}=x_k-2\times\frac{(\sin x_k-\frac{x_k}{2})^2}{2(\sin x_k-\frac{x_k}{2})(\cos x_k-\frac{1}{2})}$

$$x_{k+1}=x_k-\frac{\sin x_k-\frac{1}{2}x_k}{\cos x_k-\frac{1}{2}},\quad k=0,1,2\cdots$$

取$x_0=\frac{\pi}{2}$，则$x_1=\frac{\pi}{2}-\frac{1-\frac{\pi}{4}}{0-\frac{1}{2}}=2.000000$

$x_2=1.900996$，$x_3=1.895512$，$x_4=1.895494$，$x_5=1.895494$

$x_4=1.895494$，$|x^*-1.895494|<10^{-5}$.

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解: $f(x)=(\sin x-\frac{x}{2})$ 的根 x^* 为2重根.

$f'(x)=2(\sin x-\frac{x}{2})(\cos x-\frac{1}{2})$

由牛顿迭代法 $x_{k+1}=x_k-\dfrac{(\sin x_k-\frac{1}{2}x_k)^2}{2(\sin x_k-\frac{1}{2}x_k)(\cos x_k-\frac{1}{2})}$

$=x_k-\dfrac{\sin x_k-\frac{1}{2}x_k}{2\cos x_k-1}$, $k=0,1,2\cdots$

令 $x_0=\frac{\pi}{2}$, 则 $x_1=1.785398$, $x_2=1.844562\cdots$

迭代到 $x_{20}=1.895494$, $|x^*-1.895494|<10^{-5}$

由求重根的迭代公式: $x_{k+1}=x_k-\dfrac{\sin x_k-\frac{1}{2}x_k}{\cos x_k-\frac{1}{2}}$　$k=0,1,2\cdots$

取 $x_0=\frac{\pi}{2}$, 则 $x_1=2.000000$, $x_2=1.900996$, $x_3=1.895512$,

$x_4=1.895494$, $x_5=1.895494$.

4次迭代可以得到上面 x_{20} 的结果.