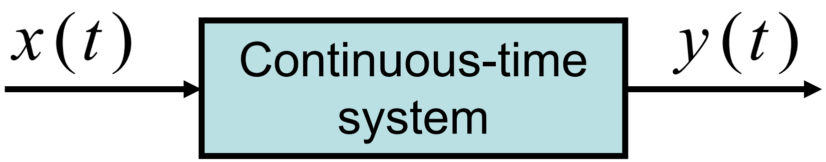
Systems

1.5 Continuous- and discrete-time systems

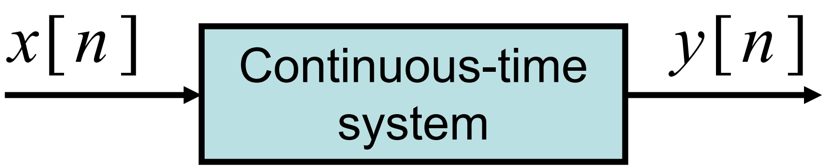
Systems are an interconnection of components or subsystems in the real world. In the signal processing, a system can be viewed as a process in which input signals are transformed by the system or cause the system to respond in some way, resulting in other signals as outputs.

Continuous-time system



$$x(t) \to y(t)$$

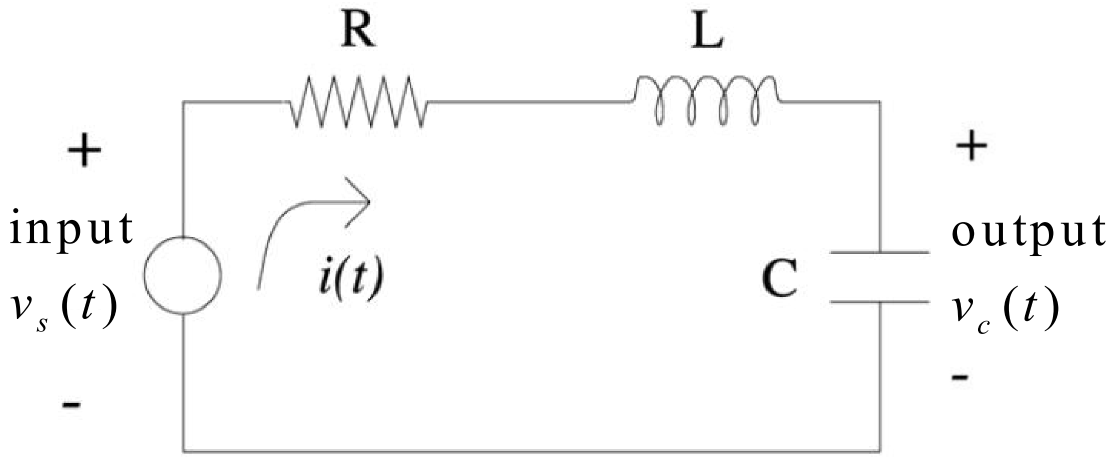
Discrete-time system



$$x[n] \to y[n]$$

1.5.1 Simple examples of systems

RLC circuit system



$$\begin{cases} \mathrm{R}\, i(t) + \mathrm{L}\dfrac{\mathrm{d}i(t)}{\mathrm{d}t} + v_c(t) = v_s(t) \\ i(t) = \mathrm{C}\dfrac{\mathrm{d}v_c(t)}{\mathrm{d}t} \end{cases}$$

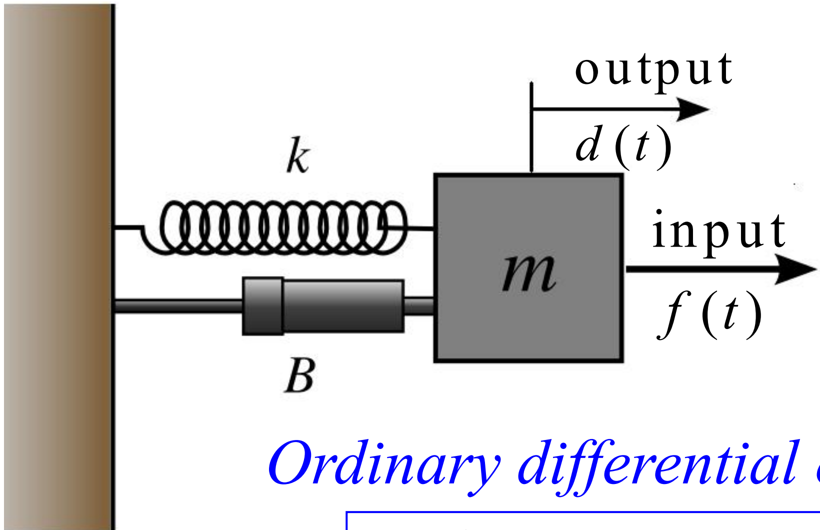
$$v_s(t) \triangleq x(t)$$
$$v_c(t) \triangleq y(t)$$

$$\boxed{\mathrm{LC}\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + \mathrm{RC}\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = x(t)}$$

Determining the output $y(t)$ with respect to $x(t)$ requires initial conditions.

1.5.1 Simple examples of systems

Mass-damper-spring dynamic system



$$m\ddot{d} = f(t) - kd - B\dot{d}$$

$$f(t) \triangleq x(t)$$

$$d(t) \triangleq y(t)$$

Ordinary differential equation (ODE)

$$\boxed{m\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + B\frac{\mathrm{d}y(t)}{\mathrm{d}t} + ky(t) = x(t)}$$

Comparing with $\boxed{\mathrm{LC}\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + \mathrm{RC}\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = x(t)}$

Very different physical systems may be modeled mathematically in very similar ways. So typical system properties are of interests.

1.5.1 Simple examples of systems

Month balance in a bank

Consider a simple model for the balance in a bank account from month to month. Specifically, let $y[n]$ denote the balance at the end of the nth month, and suppose that $y[n]$ evolves from month to month according to the *difference equation*

$$y[n] = 1.01\, y[n-1] + x[n]$$

where $x[n]$ represents the net deposit (i.e., deposits minus withdrawals) during the nth month and the term $1.01y[n\text{-}1]$ models the fact that we accrue 1% interest each month.

With some initial conditions: e.g. $y[0] = 1000$, we can calculate $y[n]$ according to the input $x[n]$.

1.5.1 Simple examples of systems

1) A very rich class of systems (but by no means all systems of interest to us) are described by differential and difference equations.

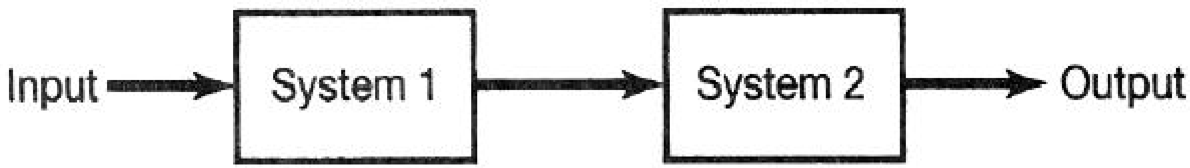
2) Such an equation, by itself, does not completely describe the input-output behavior of a system: we need auxiliary conditions (initial conditions, boundary conditions).

3) In some cases the system of interest has time as the natural independent variable and is causal. However, that is not always the case.

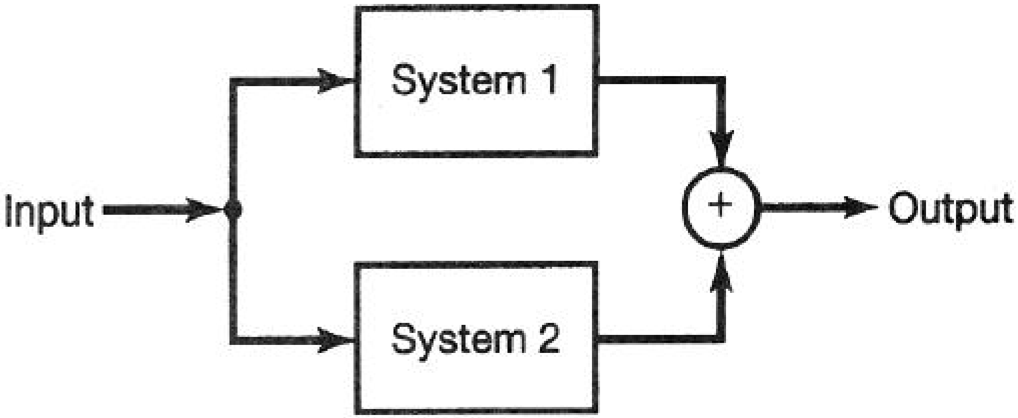
4) Very different physical systems may have very similar mathematical descriptions.

1.5.2 Interconnections of systems

Series (cascade) interconnection

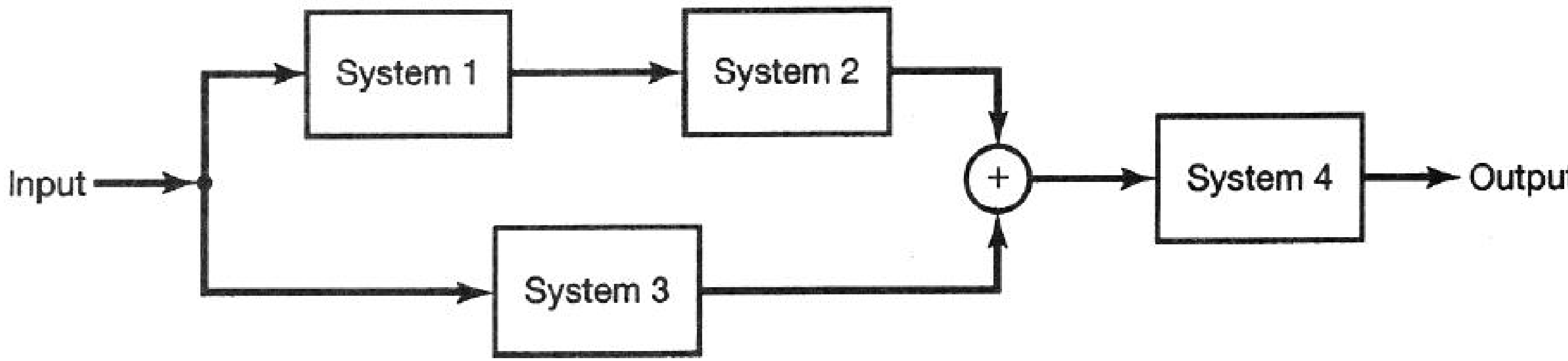


Parallel interconnection

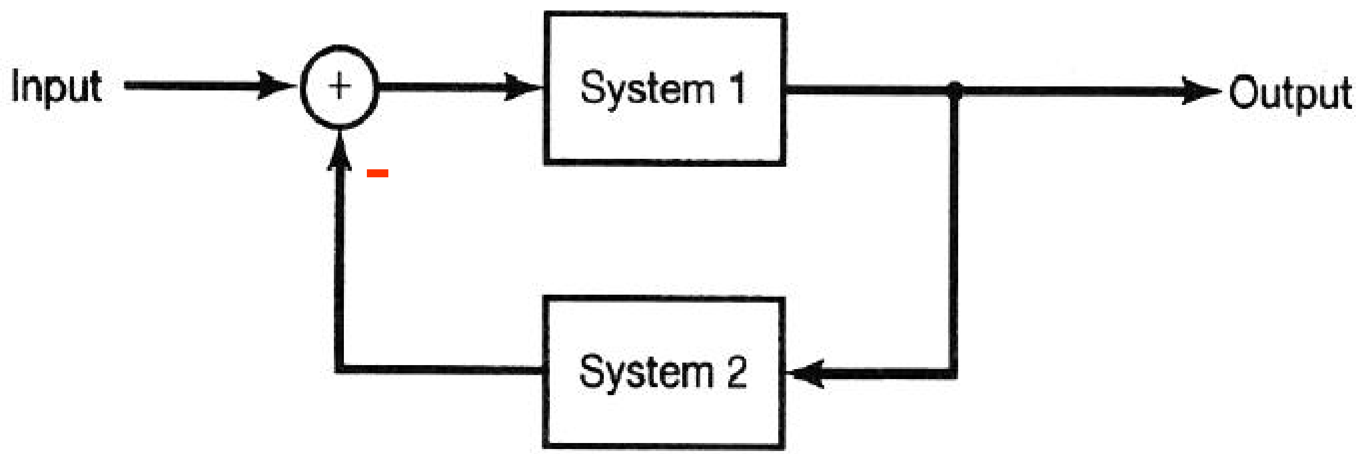


1.5.2 Interconnections of systems

Series-parallel interconnection



Feedback interconnection



1.6 Basic system properties

System properties have important physical interpretations and relatively simple mathematical descriptions. These properties are

- Memory
- Invertibility
- Causality
- Stability
- Time invariance
- Linearity

1.6.1 Systems with and without memory

A system is said to possess memory if its output signal depends on past or future values of the input and output signal.

A system is said to be memoryless if its output signal depends only on the present value of the input and output signal.

Memoryless systems

(1) $v(t) = Ri(t)$ (resistor)

(2) $y(t) = (2x(t) - x^2(t))^2$

(3) $y(t) = \frac{d}{dt}x(t)$ (differentiator)

Memory systems

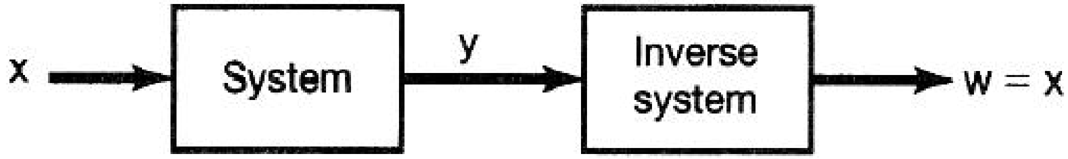
(4) $i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau$ (inductor)

(5) $y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$ (averager)

(6) $y[n] = \sum_{k=-\infty}^{n} x[k] = y[n-1] + x[n]$ (accumulator)

1.6.2 Invertibility and inverse systems

A system is said to be invertible if distinct inputs lead to distinct outputs, which means the inputs of the system can be recovered from the outputs. This type system is important in communication and encryption.



$$y(t) = H\{x(t)\} \quad x(t) = H^{inv}\{y(t)\} = H^{inv}\{H\{x(t)\}\}$$

$$\Rightarrow H^{inv} H = I$$

Example

$y(t) = 2x(t)$ whose inverse system is $w(t) = \frac{1}{2} y(t)$

Noninvertible systems: $y[n] = 0,\ y(t) = x^2(t)$

1.6.3 Causality

A system is *causal* if the output at **any** time depends only on values of the input at the present time and in the past. Such a system is often referred to as being nonanticipative, as the system output does not anticipate future values of the input. So all memoryless systems are causal.

Example $$y[n] = 2x[n] + x[n+1]$$

Causal or noncausal? $$y[n] = x[-n]$$

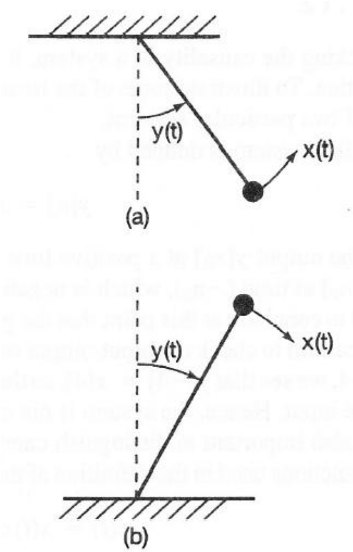
$$y(t) = x(t)\cos(t+1)$$

Causal systems are of great importance for real processing systems with the variable of time, but they are not essential constraint in applications in which the independent variable is not time, e.g. image processing and those recorded data.

1.6.4 Stability

A system is *stability* if the input to a stable system is bounded, then the output must also be bounded and therefore cannot diverge.

Bounded input and bounded output: BIBO principle



Example $$y[n] = \sum_{k=-\infty}^{n} u[k]$$

If we suspect that a system is unstable, then a useful way to verify this is to find a specific bounded input that leads to an unbounded output. However, if we want to verify a system stable, we must check for stability by using a way that does not utilize specific examples of input signals.

Example $$y(t) = e^{x(t)}$$

1.6.5 Time invariance

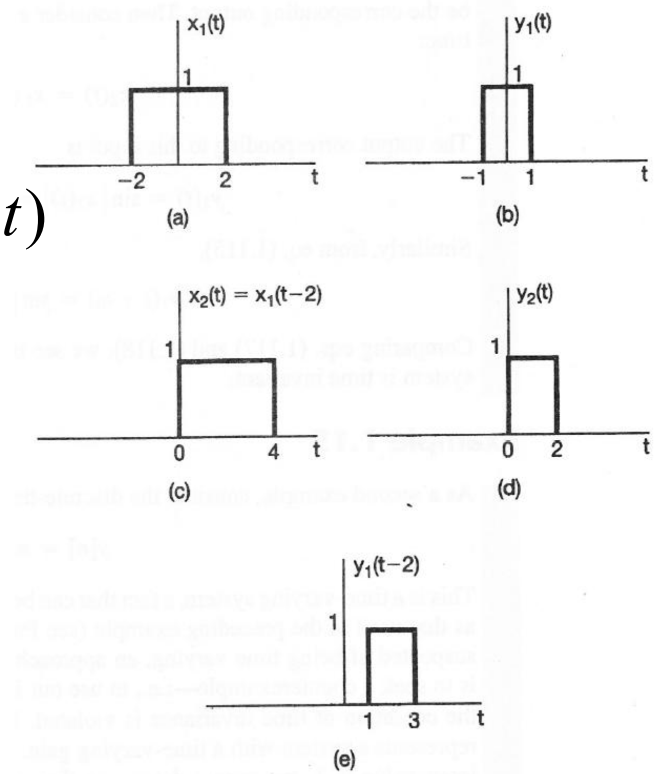
A system is *time invariant* if the behavior and characteristics of the system are fixed over time. In the signals and systems language, a system is time invariant is a time shift in the input signal results in an identical time shift in the output signal for any input and any time shift.

Examples

(1) $y(t) = \sin[x(t)]$

(2) $y[n] = nx[n]$

(3) $y(t) = x(2t)$



1.6.6 Linearity

A system is *linear* if it satisfies superposition (or additivity) and scaling (or homogeneity) properties.

Superposition: $x_1(t)+x_2(t) \to y_1(t)+y_2(t)$

Scaling: $ax_1(t) \to ay_1(t)$, where a is any complex constant

Combing two properties to give

$$\boxed{\text{Linearity: } ax_1(t)+bx_2(t) \to ay_1(t)+by_2(t)}$$

Examples

(1) $y(t)=tx(t)$

(2) $y(t)=2x(t)+3$

(3) $y(t)=x^2(t)$

Incrementally linear system
with zero-input response