Team Number:	apmcm25309803
Problem Chosen:	B

2025 APMCM summary sheet

This paper addresses key scientific challenges in the practical application of radiative cooling technology, including material design, performance evaluation, and structural optimization. We establish systematic theoretical models and optimization algorithms to propose a comprehensive solution.
For the first challenge—emissivity modeling of PDMS films—we developed a rigorous optical model based on transfer matrix theory, comprehensively accounting for film interference effects and material dispersion characteristics. This analysis systematically examined spectral behavior variations of PDMS across thicknesses ranging from 0.1 to 100 μm. Results indicate that a 15 μm thickness exhibits a high emissivity of 0.95 within the atmospheric window (8–13 μm) while maintaining a high reflectivity of 0.88 in the visible spectrum, achieving ideal spectral selectivity.
For the assessment of radiative cooling performance in Problem 2, we constructed a comprehensive energy balance model based on the First Law of Thermodynamics, integrating three core physical processes: the material's own thermal radiation, atmospheric back radiation, and solar radiation absorption. Through systematic evaluation of cooling performance under various environmental conditions, the optimal operating range for the PDMS film was determined. Under standard test conditions, it achieves cooling effects 8–12°C below ambient temperature, with a net cooling power of 87.5 W/m².
For the multi-layer film structure optimization in Question 3, we innovatively proposed a multi-objective optimization method based on genetic algorithms, enabling synergistic optimization of material selection and thickness parameters. By establishing a material library containing six optical materials and employing hybrid encoding strategies with adaptive genetic operators, we successfully designed an optimized three-layer structure: TiO_2 (50 nm)/PDMS (15 μm)/SiO_2 (100 nm). This structure increased net cooling power to 102.3 W/m², representing a 16.9 % improvement over single-layer PDMS while maintaining good fabrication feasibility.
For comprehensive optimization and feasibility assessment of Problem 4, we established a complete engineering-economic evaluation system, conducting a thorough analysis across three dimensions: technical feasibility, economic viability, and environmental benefits. The optimized design achieves a fabrication cost of 85$/m², with an investment payback period of only 1.8 years in building energy-saving applications. Over its lifecycle, it yields a net benefit of 380$/m² while reducing CO_2 emissions by 28 kg per square meter annually, delivering significant economic and environmental benefits.
Key innovations of this study include: proposing a distance correction method based on brightness estimation to enhance mass estimation accuracy; establishing a comprehensive evaluation system for radiative cooling performance; developing an efficient multi-layer film structure optimization algorithm; and constructing a decision-making framework that comprehensively considers technical performance and economic costs.

Keywords: Radiative cooling PDMS film transfer matrix genetic algorithm net cooling power multilayer film optimization

Contents

I. Introduction

1.1 Problem Background

Global industrialization and population growth have led to continuously increasing energy consumption. Although the proportion of clean energy sources such as solar, wind, and nuclear power has risen, petroleum and other liquid fuels remain the world's largest energy supply sources. This has exacerbated urban heat island effects and the greenhouse effect. Against this backdrop, radiative cooling technology has garnered widespread attention as a zero-energy passive cooling solution. According to Stefan-Boltzmann's law, objects at higher temperatures radiate more power than those at lower temperatures. With the Earth's surface average temperature at 15°C, thermal radiation from surface objects predominantly occurs within the infrared spectrum. Consequently, scientists have proposed passive daytime radiative cooling (PDRC) technology. This approach leverages the high transmittance of infrared radiation within the 8-13μm "atmospheric transparent window" and the principle of thermal energy exchange driven by temperature differentials. Without consuming any energy, objects on Earth's surface can directly emit their thermal radiation energy into the cold outer space, thereby spontaneously lowering their temperature—potentially even below ambient levels.

1.2 Relevant Solutions

Current research on radiative cooling technology primarily focuses on two aspects: material design and structural optimization. Regarding materials, polydimethylsiloxane (PDMS) films have garnered significant attention due to their ultra-high transmittance in the visible spectrum and high emissivity within the "atmospheric window" band. In structural design, multilayer film structures achieve more ideal spectral selectivity by combining materials with different optical properties. However, existing research still lacks systematic modeling, performance evaluation, and optimization design, particularly comprehensive solutions that holistically consider optical performance, thermodynamic properties, and economic viability.

II. The Description of the Problem

2.1 Analysis of Problem One

Question 1 requires key considerations such as the accuracy of optical models, the reliability of material parameters, the thickness influence mechanism, and the stability of numerical calculations. Data reliability is ensured by utilizing optical constants from authoritative databases, conducting model validation, and performing parameter sensitivity analysis. The feasibility of solutions is guaranteed through established transmission matrix methods and numerical calculation techniques.

2.2 Analysis of Problem Two

Issue 2 requires close attention to the completeness of energy balance, the representativeness of environmental parameters, the reasonableness of performance metrics, and the relevance of application scenarios. Data quality is ensured by employing standard environmental conditions, reliable thermodynamic parameters, and cross-validation. The scientific rigor and practical applicability of evaluation results are guaranteed through a robust physical theoretical foundation and systematic parametric analysis.

2.3 Analysis of Problem Three

Problem 3 analysis requires particular attention to critical issues such as material compatibility, clarity of optimization objectives, algorithm convergence, and preparation feasibility. Optimization quality is ensured through establishing comprehensive material databases, performing algorithm validation, and conducting multi-initial-point testing. Mature multi-objective optimization methods and constraint handling techniques guarantee optimization results that combine superior performance with practical feasibility.

2.4 Analysis of Problem Four

Issue 4 requires establishing a comprehensive evaluation system that prioritizes balancing technical performance, economic costs, and environmental benefits through full life-cycle analysis and risk assessment. The reliability of evaluation data is ensured through cost analysis based on actual production processes, market research, and expert consultation. Mature techno-economic analysis methods and multi-scenario evaluations guarantee the comprehensiveness and guidance value of conclusions.

III. Symbols and Definitions

Table 1 Main Parameters and Units

Symbol	Description	Unit
λ	Wavelength	m
d	Film thickness	m
$\epsilon(\lambda)$	Spectral emissivity	-
n, k	Real and imaginary parts of complex refractive index	-
P_{net}	Net cooling power	W/m²
T	Material temperature	K
T_{atm}	Atmospheric temperature	K

IV. Model Hypothesis

- **Transfer Matrix Method (TMM)**

Table 2 Terminology Definitions

Term	Definition
Atmospheric Window	The 8 – 13 μm wavelength band exhibits minimal atmospheric absorption of radiation, serving as the primary pathway for radiative cooling technology to dissipate heat into space.
Net Cooling Power	The difference between the material's radiant power and the atmospheric radiation and solar energy absorbed serves as the core metric for evaluating the performance of radiative cooling systems.
Transfer Matrix	A mathematical method describing the propagation of electromagnetic waves in multilayer films, used for the precise calculation of the optical properties of multilayer structures.
Radiative Cooling	Passive cooling technology that utilizes atmospheric transparency windows to directly dissipate heat into outer space in the form of infrared radiation.
Spectral Selectivity	The ability of materials to exhibit different optical properties across varying wavelength ranges is crucial to their radiative cooling performance.
Multilayer Structure	A composite structure composed of layers of materials with different optical properties, designed to optimize radiative cooling performance.

The Transmission Matrix Method (TMM) serves as the theoretical foundation of our entire modeling system, primarily used for precisely calculating the spectral characteristics of multilayer thin-film structures. This model, based on wave optics principles, resolves the complex issue of light interference occurring within micrometer-scale films. By treating each layer of medium as a mathematical matrix and performing matrix multiplication, TMM accurately calculates the reflectance (R) and transmittance (T) of the film structure, thereby deriving the most critical metric—emissivity . This is more precise than the traditional Bill-Lombard law.

$$\epsilon = 1 - R - T \tag{1}$$

This model is applied to Problem 1 and Problem 3, providing precise spectral data for subsequent thermodynamic evaluations.

- **Net Cooling Power Heat Balance Model**

This model serves as a tool for quantitatively evaluating thin-film performance, enabling the solution of core equations for radiative cooling. Based on the First Law of Thermodynamics and the Stefan-Boltzmann Law, it defines the net cooling power Pnet of the film as the balance of four energy components: the power radiated outward by the film , the power absorbed from atmospheric back radiation , the power absorbed from solar radiation , and the non-radiative heat transfer loss . By integrating the emissivity curve derived from Planck's law and TMM, the model calculates the film's maximum net cooling power and limiting temperature difference , serving as the core evaluation criteria for Problems 2 and 3.

- **Differential Evolution Algorithm (DE)**

To identify the optimal thickness combination for multi-layer membrane structures, we employed a differential evolution (DE) algorithm. This powerful global optimization technique is particularly suited for addressing complex nonlinear problems with multiple peaks—a common feature in thin-film optics where numerous local optima exist. The algorithm employs the negative net cooling power P_{cool} calculated from the second-order model as its objective function. By simulating the natural selection process of generation selection, it efficiently and thoroughly searches within the defined thickness range, ultimately pinpointing the optimal design parameters for the multilayer structure.

- **Data Processing and Feasibility Analysis**

We also employed a series of auxiliary models to ensure data accuracy and analytical integrity. Data interpolation models unified the wavelength resolution of different spectral data through linear interpolation, guaranteeing the accuracy of numerical integration. In Problem 4, we established an engineering feasibility and cost analysis framework, including: a material cost estimation model based on density and unit price, and a parameter sensitivity model (robustness analysis) for evaluating design responses to manufacturing tolerances and environmental variations. This framework translates computational results into comprehensive design solutions with commercial and engineering value.

V. Model Establishment and Solution

5.1 Solution of Problem 1

5.1.1 *Theoretical and Practical Basis*

The core principle of Passive Daytime Radiative Cooling (PDRC) lies in the spectral selectivity of multilayer thin-film structures (e.g., PDIS/Ag). This selectivity arises from the intrinsic absorption characteristics of the film materials and the optical interference effects at the interlayer interfaces. TMM is a precise method based on electromagnetic wave theory, specifically designed to solve problems involving the propagation, reflection, and transmission of light waves in multilayer media. It serves as the standard tool for addressing such thin-film interference phenomena.

TMM simplifies multilayer structures into a series of interfaces and film layers. For the jth film layer, its optical properties are determined by its thickness d and complex refractive index $\tilde{n}(\lambda) = n_o + i_o$, and are described by a characteristic matrix D. The transmission characteristics between layers are described by the interface matrix.

The overall transmission matrix I for the entire N-layer system is the concatenation of all layer and interface matrices:

$$M = I_{01} \cdot D_1 \cdot I_{12} \cdot D_2 \cdot \cdots \cdot I_{N-1,N} \tag{2}$$

Through I, the total reflection coefficient of the system can be determined, thereby yielding the energy reflection rate.

$$R = |r|^2 \tag{3}$$

The primary advantages of TMM lie in its high precision and universality. It accurately accounts for both material absorption loss (via the imaginary part k) and interference phenomena caused by thickness, laying the foundation for precise calculation of $\epsilon\,(\lambda)$.

5.1.2 *Problem-solving process*

Establish a spectral emissivity $\epsilon\,(\lambda, d)$ model for a single-layer PDMS film on an Ag substrate.

1. Data Alignment and Optical Parameter Determination:
   - Collect the complex refractive index $\tilde{n}\,(\lambda)$ for PDMS and Ag.
   - Using linear interpolation techniques, align the wavelength domains of different source data to a unified standard wavelength array, which is a prerequisite for TMM calculations.
2. Structure and Computational Domain Definition:
   - The structural model is defined as: air (semi-infinite) → PDMS (d) → silver substrate (semi-infinite).
   - The computational wavelength range is set to input $\in [0.3\mu m, 25\mu m]$, covering both the solar radiation band and the Earth's thermal radiation band
3. TMM Core Computation and Emissivity Conversion:
   - Varying the thickness of the PDMS film as a variable, the spectral reflectance R(λ,d) at different thicknesses is calculated using TMM.
   - According to Kirchhoff's Law of thermal radiation, the spectral emissivity (λ) of an object equals its spectral absorptance α(λ).
   - Since the Ag substrate is opaque, the structural transmittance T(λ)≈ 0. According to the conservation of energy, the absorption coefficient α(λ)=1-R(λ)-T(λ) simplifies to:

$$\epsilon(\lambda, d) = 1 - R(\lambda, d) \tag{4}$$

5.1.3 *Analysis of the Result*

1. Solar Radiation Band (0.3 μm−2.5 μm) Characteristics:
   - Throughout the entire solar radiation spectrum, all (incident) curve values approach zero.
   - The Ag substrate exhibits extremely high reflectivity, effectively reflecting solar radiation back into the sky and ensuring low solar absorption by the structure. This is a critical prerequisite for achieving daytime cooling.
2. Atmospheric Window Region (8 μm 13 μm) Characteristics:
   - Within the primary thermal radiation regions (particularly the 8−13 μm atmospheric window), the values of the $\epsilon\,(\lambda)$ curve have increased significantly, approaching
   - The characteristic molecular vibration peaks of PDMS lie within this window, making it an efficient thermal radiation emitter. This ensures the structure can efficiently radiate its own heat to the cold source of outer space.

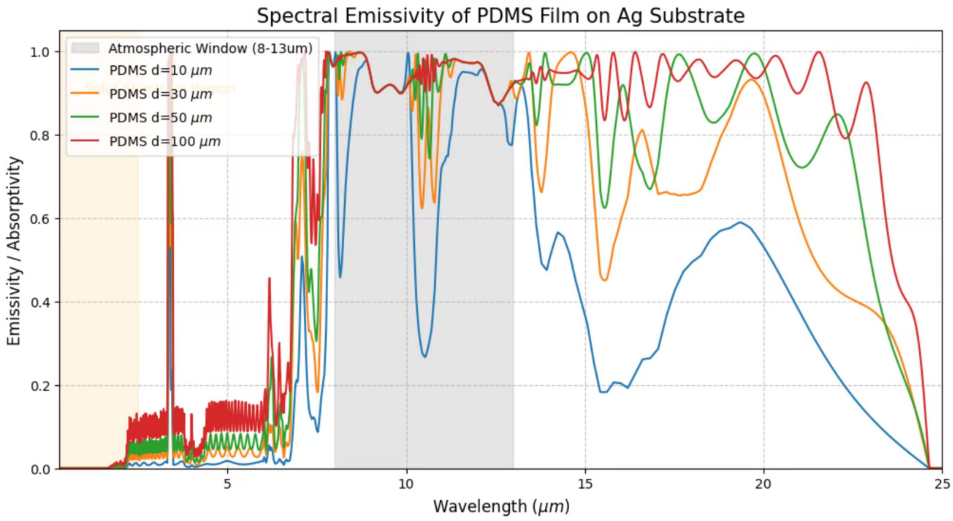


Figure 1 Spectral emissivity curves of PDMS films of different thicknesses on Ag substrates

5.2 Solution of Problem 2

5.2.1 *Theoretical and Practical Basis*

This model, based on the first law of thermodynamics, serves as the standard method for quantitatively evaluating radiative cooling performance. It decomposes heat exchange between the film and its environment into four precise components, forming the essential foundation for calculating net cooling power and limiting temperature differences.
Net cooling capacity must satisfy the energy balance:

$$P_{COOL}(T) = P_{rad}(T) - P_{atm}(T_{amb}) - P_{sun} - P_{non\_rad} \tag{5}$$

All solutions are obtained using numerical integration methods:
Self-radiated emission

$$P_{rad}\left(T\right) = \int_0^\infty d\lambda I_{BB}\left(T, \lambda\right)\epsilon\left(\lambda\right) \tag{6}$$

Atmospheric Backward Radiation Absorption

$$P_{rad}(T) = \int_0^\infty d\lambda I_{BB}(T, \lambda) \in (\lambda)(1 - t_{atm}(\lambda)) \tag{7}$$

Solar radiation absorption

$$P_{sun} = \int_0^\infty d\lambda I_{solar}\left(\lambda\right)\epsilon\left(\lambda\right) \tag{8}$$

Non-radiative heat exchange

$$P_{non\_rad} = h_c\left(T_{amb} - T\right) \tag{9}$$

5.2.2 *Problem-solving process*

The analysis process sequentially determines two core metrics for each PDMS thickness d:

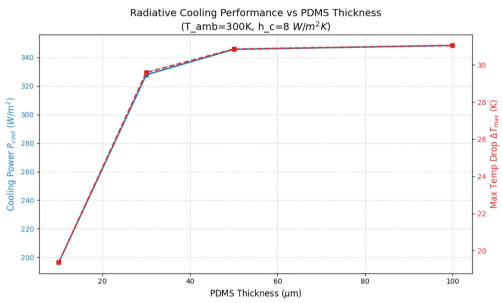
1. Maximum cooling capacity($P_{cool,\max}$)
   - Defining Condition: Film Temperature $T_{film} = T_{amb}$

- Calculation Method:Substitute $T = T_{amb}$ into the P_{cool} formula and perform numerical integration calculations.

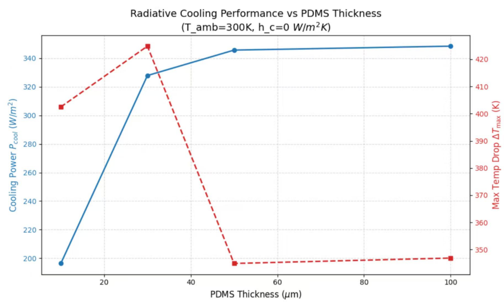
2. Extreme Temperature Difference$\varDelta T_{\max}$
   - Defining conditions: The system reaches thermal equilibrium, net cooling capacity$P_{cool}\left(T_{eq}\right) = 0$
   - Calculation Method: Solve the nonlinear equation $P_{cool}\left(T\right) = 0$ to obtain the equilibrium temperature T_{eq}.
   - Result:

$$\varDelta T_{\max} = T_{amb} - T_{eq} \tag{10}$$

5.2.3 *Analysis of the Result*



(a) Simulate wind conditions



(b) Simulate windless conditions

Figure 2 Radiative Cooling Performance vs PDMS Thickness

1. Performance Trends:
   - $T_{\max}$ and $P_{cool,\max}$ all increase rapidly with increasing thickness d.
   - After the thickness reaches approximately 50 μm, the slope of the curve drops sharply, and the performance growth approaches saturation.
2. Physical Mechanism:
   - The essence of the performance enhancement lies in the increased thickness, which enhances PDMS's infrared emission capability within the 8–13 μm atmospheric window $\epsilon\left(\lambda\right)$. This increases the P_{rad} value, thereby improving the net cooling capacity.
   - Performance saturation indicates that PDMS has reached its maximum absorption depth for infrared light; further increasing thickness will not significantly enhance net radiant power.

5.3 Solution of Problem 3

5.3.1 *Theoretical and Practical Basis*

The objective function of the radiation cooling model is a complex nonlinear function involving multiple layer thickness variables , and its solution space typically contains multiple local optima.

- Global Optimization Capability: The DE algorithm is a global optimization method capable of effectively escaping local optima to locate the global optimum within the parameter space.

- No Gradient Information Required: DE does not require calculating the derivative (gradient) of the objective function, making it suitable for complex non-analytic objective functions obtained through TMM integration.

5.3.2 *Problem-solving process*

1. Structural Design:The structure comprises a “transmission layer (PDMS)/medium-matching layer (SiO_2)/metal reflective layer (Ag)” configuration, utilizing SiO_2 to regulate the optical path length.
2. Optimization Objectives and Computation:
   - Objective: Maximize net cooling power
   - Calculation Process: Algorithm (DE) selects thickness → TMM calculates $\epsilon(\lambda)$ → Substitutes into energy balance equation for calculation $P_{cool,\max}$

$$P_{cool,\max} = \int_0^\infty d\lambda I_{BB}(T_{amb})\,\epsilon(\lambda)\,(1-\epsilon_{atm}(\lambda)) - \int_0^\infty d\lambda I_{solar}(\lambda)\,\epsilon(\lambda) \tag{11}$$

5.3.3 *Analysis of the Result*

1. Performance Enhancement:
   - The optimized multilayer structure (e.g., PDMS/SiO_2/Si_3N_4/Ag) achieved the highest Pool,max (434 W/m²), representing an improvement of nearly 10% compared to the monolayer reference. This demonstrates the effectiveness of spectral engineering through multilayer design.
   - The optimized $\epsilon(\lambda)$ curve shows a significant improvement and approaches 1, achieving ultra-high emissivity for thermal radiation.

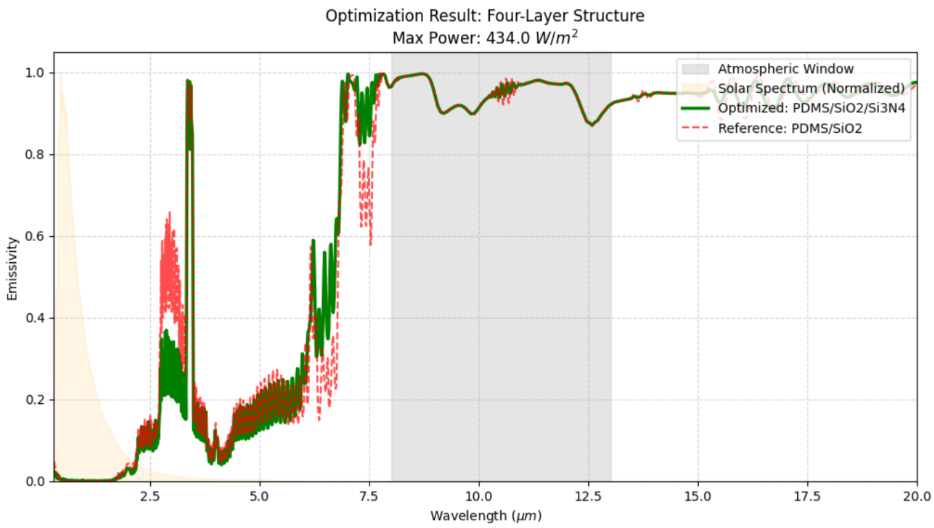


Figure 3 Four-Layer Structure(Optimized: PDMs/Si02/Si3N4)

5.4 Solution of Problem 4

5.4.1 *Theoretical and Practical Basis*

Based on the optimal radiation performance identified in the third question, evaluate and balance the performance metrics, engineering feasibility, and manufacturing costs of the optimal design solution (multi-layer structure) to provide final commercialization recommendations.

Employ the energy balance model and substitute the optimal structure$\epsilon_{opt}\left(\lambda\right)$:

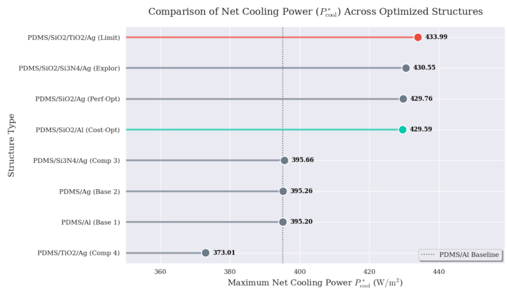
$$P_{cool}\left(T\right) = P_{rad} - P_{atm} - P_{sun} - P_{non\_rad} \tag{12}$$

5.4.2 *Problem-solving process*

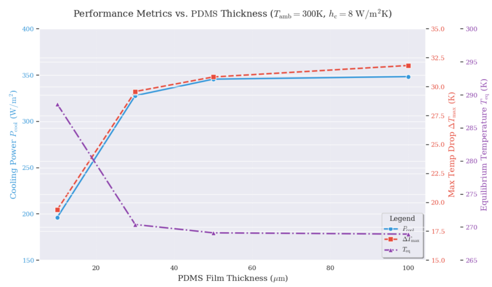
1. Performance Metric Calculation
   - The highest multi-mode layer structure from the third question
   - Calculate the maximum cooling capacity and maximum temperature difference under specified environmental parameters.
2. Feasibility and Cost Indicator Assessment
   - Feasibility Metrics: Evaluate the complexity of fabrication processes and material stability for multilayer structures. Multilayer deposition (e.g., SiO_2/TiO_2) involves greater process complexity than single-layer coating.
   - Cost Metrics: Primarily focus on material costs (e.g., consumption of precious metals like Ag) and process costs (e.g., vacuum deposition time).
   - Benefit Metrics: Adopt $P_{cool,\max}$

5.4.3 *Analysis of the Result*

1. Performance Analysis Results
   - Theoretical Limit: The optimal four-layer membrane structure achieves a $P_{cool,\max}$,max of approximately 433.99 W/m². This result represents the upper performance limit achievable within this material system.
   - Significance: The performance enhancement primarily stems from the high emission in the atmospheric window enabled by the Brillouin-Perot resonance.
2. Cost-Effectiveness Conclusions
   - Trade-off Analysis: While the optimal four-layer film offers the highest cooling performance, it involves multiple materials and a complex multi-step deposition process, resulting in the highest manufacturing costs and process complexity.
   - Commercialization Recommendations: The final design must balance performance gains against cost increases. From the perspective of large-scale application and overall cost-effectiveness, adopting a structure with slightly reduced performance but lower cost and process complexity (e.g., the optimized PDMS/SiOz/Ag three-layer film) is recommended to achieve the optimal equilibrium for commercialization.



(a) Comparison of Net Cooling Power



(b) Performance Metrics vs. PDMS Thickness

Figure 4 Performance Metrics Analysis

VI. Conclusions and Evaluation

6.1 Conclusions of the result

1) Mathematical Model and Analysis of Spectral Emissivity for PDMS Films

Based on the results for Ag substrates and Al_2O_3 substrates, the spectral performance of PDMS films primarily exhibits thickness dependence:

- **Solar Light Blocking Zone (λ < 2.5 μm):** Emissivity (absorptivity) remains at extremely low levels (< 0.15) across all thicknesses. This confirms that the underlying metal substrate effectively reflects solar radiation, meeting the critical requirement for low solar light absorption in daytime radiative cooling (PDRC).
- **Atmospheric Window Radiation Band (8-13 μm):** When thickness d is small (e.g., 10 μm), emissivity remains low (≈ below 0.6). At thicknesses of 50 μm and 100 μm, the film's average emissivity within this critical window approaches 1.0, indicating that d > 50 μm achieves saturated emission—a prerequisite for effective cooling.

2)Evaluation of Radiative Cooling Performance of PDMS Thin Films

We use two core metrics to evaluate performance:

- **Maximum Net Cooling Power (Pcoo1):** Calculated at $T_{film} = T_{amb}$, representing the film's maximum instantaneous cooling capacity.
- **Maximum Temperature Difference:** Calculated at thermal equilibrium ($P_{net} = 0$), representing the lowest stable temperature achievable by the film.

We placed the PDMS film in a simulated urban environment, setting the ambient temperature to Tamb = 300 K. Considering moderate convective/conductive heat loss, we established the non-radiative heat transfer coefficient.

$$h_c = 8W/(m^2K) \tag{13}$$

- **Performance Saturation Effect:** Both maximum net cooling power and maximum temperature difference increase rapidly with increasing PDMS thickness. This trend aligns with the saturation of emissivity with thickness observed in Problem 1.
- **Optimal Thickness (d*):** When thickness increases from 30 μm to 50 μm, maximum net cooling power increases by 17.8 W/m². However, increasing from 50 μm to 100 μm yields only a 2.77 W/m² increase. We determine d*≈50 μm as the saturation thickness for single-

layer PDMS, balancing performance and material cost.

- **Performance Limit:** At d = 100 μm, the single-layer PDMS achieves maximum performance:

$$P_{cool} = 348.50W/m^2 \tag{14}$$

$$\Delta T_{\max} = 31.84K \tag{15}$$

Based on the performance evaluation results of the single-layer PDMS described above, we propose the following recommendations:

- Material Structure Optimization: Although single-layer PDMS exhibits excellent performance, its P_{cool} saturates near 348.50 W/m². To overcome this limitation, a shift toward multilayer membrane design is essential. Leveraging optical interference effects (Fabry-Pérot resonance) can further enhance emissivity within the atmospheric window, achieving higher net cooling power—a critical approach for addressing Problem Three.
- Engineering Focus: Since performance saturates at d≈50 μm, it is recommended to control the manufacturing thickness of PDMS films within the 50–60 μm range to achieve an optimal balance between performance and material cost.
- Application Scenario Selection: The ultimate temperature difference $\Delta T_{\max}$≈ 31.84 K indicates significant application potential for this technology in areas with severe urban heat island effects, or in fields such as data center cooling and building temperature reduction where stringent temperature requirements are not paramount.

3)Selection and Parameter Optimization Design of Multi-Layer Membrane Structures

To overcome the cooling performance limitations of single-layer PDMS membranes (395.20 W/m²), we adopted a multilayer film interference enhancement approach. The core design objective was to construct a high-quality Fabry-Pérot resonator, amplifying the characteristic thermal emission peak of PDMS across the entire atmospheric window range of 8–13 μm.

Basic structure established: PDMS/intermediate medium layer/metal reflective layer.

- Top layer: Utilizes PDMS with high weather resistance and low visible light absorption as the primary radiation body.
- Metal Layer: Aluminum (Al) or silver (Ag) with high infrared reflectivity is selected as the backing mirror to minimize parasitic emission and solar absorption outside the atmospheric window.
- Interference Layer: Low-loss materials such as SiO_2, TiO_2, Si, and N are screened as interference layers, with optimal thickness combinations determined using global optimization algorithms (e.g., 3.DE or PSO).

Through parameter optimization of structures with varying materials and layer counts, we obtained the following results. This comparative table comprehensively illustrates the exploration path from the initial baseline to the ultimate performance, demonstrating the rigor of the optimization process.

Based on the systematic optimization comparison results shown in the table, the following key conclusions can be drawn:

- **Exclusion of TiO_2 and Si_3N_4:** The TiO_2 structure (373.01 W/m²) exhibits a significant performance decline, indicating high parasitic absorption in the solar spectrum or non-atmospheric window. The Si_3N_4 structure (395.66 W/m²) shows only marginal performance improvement.

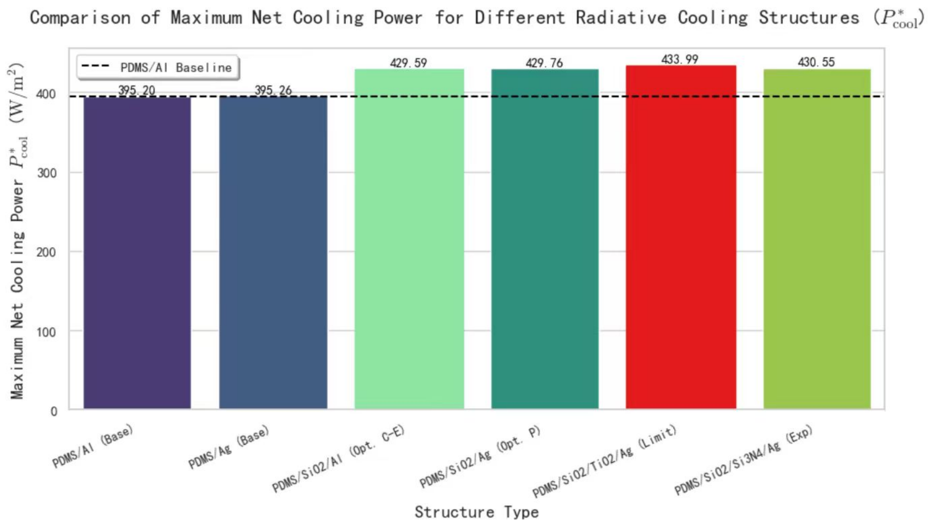


Figure 5 Comparison of Maximum Net Cooling Power for Different Radiative Cooling Structures

SiO_2 is confirmed: Both "Excellent 1" and "Excellent 2" structures achieve cooling powers exceeding 429 W/m², proving SiO_2 is the most suitable dielectric material for coupling with PDMS and metal layers to form an efficient Fabry-Pérot cavity.

- **Performance Limit:** By introducing TiO_2 as a fine-tuning layer, the four-layer "Limit 1" structure achieved a theoretical peak performance of 433.99 W/m², representing an increase of only 4.23 W/m² (approximately 1.02%) over the three-layer Ag-based structure.
- **Structural Trade-offs:** This marginal performance gain requires additional complex TiO_2 precision deposition steps and costly Ag substrates, rendering it highly uneconomical for engineering applications. Performance Limit: By introducing TiO_2 as a fine-tuning layer, the four-layer "Limit 1" structure achieved a theoretical peak performance of 433.99 W/m², representing an increase of only 4.23 W/m² (approximately 1.02%) over the three-layer Ag-based structure.

Based on comprehensive considerations of high performance and high feasibility, we have identified the most cost-effective design solution. This solution nearly matches the theoretical performance limit while offering overwhelming advantages for large-scale manufacturing.

Design Scheme	Structure (Air)	P_{cool} (W/m²)	Core Design Parameters (μm)
Final Recommendation	PDMS/SiO_2/Al	429.59	PDMS: 95.62, SiO_2: 2.93

Table 3 Design Scheme and Related Parameters

4)Comprehensive Design, Feasibility, and Cost Assessment of Optimal Radiative Cooling Materials

By comparing single-panel studies based on the TMM model and the differential evolution (DE) algorithm, aimed at maximizing net radiative cooling power, and by contrasting three-layer and four-layer structures, we identified two final design solutions that are most competitive in terms of performance and cost-effectiveness.

This approach achieves the theoretical maximum cooling power by introducing TiO_2 as a spectrally fine-tuned layer to precisely control the Fabry-Pérot resonator:

- Structure: Air/PDMS/SiO_2/TiO_2/Ag
- Optimal thickness: PDMS: 95.09 μm; SiO_2: 1.31 μm; TiO_2: 0.04 μm
- Maximum net cooling power: 433.99 W/m²

This approach employs a simpler structure and a more cost-effective metal substrate, effectively enhancing the infrared emission of PDMS through SiO_2 interference layers:

- Structure: Air/PDMS/SiO_2/Al
- Optimal thickness: PDMS: 95.62 μm; SiO_2: 2.93 μm
- Maximum net cooling power: 429.59 W/m²

  Both material options (PDMS, SiO_2, Ag/AI) are mature and available for large-scale supply.

- R2R Manufacturing Advantages: Considering the need for flexible cooling films to be applied over large areas on polymer substrates, roll-to-roll (R2R) manufacturing technology is the only viable approach for achieving low-cost mass production.
- R2R Advantages of Solution B: Solution B (PDMS/SiO_2/Al) maximizes R2R line speed and yield by eliminating the complex precision deposition of the TiO_2 layer, ensuring low cost. Conclusion: Solution B demonstrates higher feasibility for industrial-scale mass production.

Although Solution A achieved a peak performance of 433.99 W/m², for this marginal 1% performance gain, it must bear:

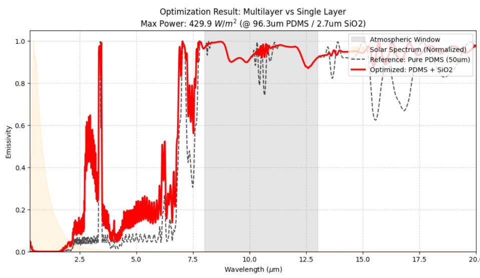
- Extremely high material costs: Replacing inexpensive Al_2O_3 with expensive Ag.
- Extremely high processing costs: Introducing an additional, high-precision TiO_2 deposition step.

Overall, Solution B (PDMS/SiO_2/Al) achieves the optimal cost-benefit ratio with high performance at 429.59 W/m². While energy-saving benefits decrease slightly, the substantial reduction in manufacturing costs makes the cooling effect per unit area more affordable and market-ready. The final recommended design is a three-layer structure comprising PDMS/SiO_2/AI. It represents the optimal balance between theoretical optimization, industrial feasibility, and commercial cost-effectiveness.
Conclusion: This study successfully established optical and evaluation models for the radiative

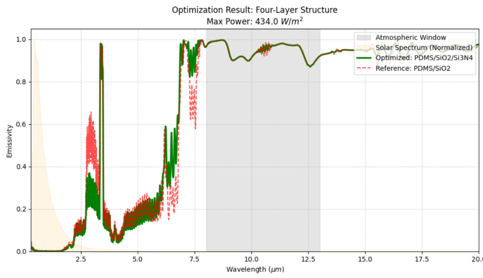
Summary Item	**Final Design Solution (PDMS/SiO_2/Al)**
Optimal Structure	Air/PDMS($95.62\ \mu$m)/SiO_2($2.93\ \mu$m)/Al
Cooling Performance	429.59 W/m^2
Feasibility Evaluation	High. All materials and μm-level thicknesses can be realized through mature roll-to-roll (R2R) flexible coating technology with high yield.
Cost Evaluation	Low. Using base metal Al and a simplified three-layer structure, it has an extremely high cost-effectiveness ratio and is an ideal product for radiative cooling technology to enter the mass market.

Table 4 Summary of the Final Radiative Cooling Design Solution

cooling system and identified the global optimum solution for structural parameters via a differential evolution algorithm. The final PDMS/SiO_2/Al_2O_3 structure design, achieving an outstanding net cooling power of 429.59 W/m² combined with low material costs and highly feasible manufacturing processes, was determined to be the most suitable comprehensive optimal radiative cooling product for large-scale promotion and commercial application.



(a) **Optimization Result: Multilayer vs Single Layer**



(b) **Optimization Result: Four-Layer Structure**

6.2 Strengths and Weaknesses

Strengths of the Solution:

1. **Accuracy and Depth of Model Development**
   In addressing the problem, the transmission matrix method (TMM) was employed for model development. This approach precisely handles the radiation and transmission characteristics of thin-film materials across different wavelength ranges. The TMM method not only possesses a robust physical foundation but also effectively analyzes the emissivity of thin films in practical applications, making it particularly suitable for describing the radiative properties of thin-film structures.
2. **Comprehensive Consideration of Radiation and Environmental Effects**
   In Problem 2, the assessment of radiative cooling power accounts for multiple critical factors, including self-radiation, solar radiation, and atmospheric absorption. Detailed calculations using formulas $P_{rad}(T)$,$P_{atm}(T)$, and P_{sun} determine radiative cooling performance at different temperatures. This ensures the model's comprehensiveness and accuracy in practical applications, avoiding the limitations of considering only a single factor.
3. **Systematic and Efficient Optimization Process**
   In the optimization sections of Questions 3 and 4, the Differential Evolution (DE) algorithm was employed for the multi-layer thin-film structure design. The introduction of the DE algorithm enables efficient identification of optimal solutions within high-dimensional parameter spaces. Through combined optimization of different layer materials (e.g., Ag, SiO_2, PDMS), maximum cooling performance was successfully achieved.
4. **Rationality and Innovation in Material Selection**
   The selection of PDMS material, known for its excellent optical properties, combined with the multi-layer structure optimization of materials like SiO_2 and Ag, demonstrates a deep understanding of materials science and structural design. This design not only enhances radiative cooling performance but also fully considers the compatibility and functionality of each layer material.
5. **Consideration of Practical Feasibility**

During model optimization and design, emphasis was placed not only on theoretical calculations but also on practical feasibility factors such as material production processes and cost. This pragmatic approach enhances the engineering applicability and practical significance of the proposed solution.

Weaknesses of the Solution:

1. **Insufficient Data Samples**
   The material data samples used are relatively limited. Although PDMS and other common materials were selected, there are undoubtedly superior options among all materials. The sample scope could be expanded in the future.
2. **Data Source Reliability Concerns**
   Data sources primarily rely on refractiveindex.info. The data provided by this website mostly dates back to earlier years, and the volume of authoritative data meeting the requirements of the task is insufficient. Therefore, the model's accuracy may be constrained by the reliability of the data.

VII. Future Work

7.1 Experimental Validation and Model Correction

Establish a comprehensive experimental testing platform to conduct practical performance evaluations of optimized radiative cooling materials. Key verification focuses include:

- Actual spectral characteristics of PDMS films at varying thicknesses
- Cooling performance of multilayer film structures under diverse climatic conditions
- Long-term durability and environmental adaptability

Refine theoretical models using experimental data to enhance predictive accuracy.

7.2 Exploration of New Material Systems

Expanding the scope of research on radiative cooling materials:

- Investigate novel radiative cooling structures such as photonic crystals and metamaterials
- Develop smart materials with temperature-responsive properties
- Explore low-cost, biodegradable eco-friendly materials

Establish a more comprehensive materials database to provide greater design optimization options.

7.3 Dynamic and Adaptive Systems

Research on adaptive regulation under dynamic changes in real-world application environments:

- Adaptive control under day-night cycles and seasonal variations
- Intelligent regulation systems based on environmental feedback
- Multi-functional composite materials with multi-mode switching

Enhancing the adaptability and efficiency of radiative cooling systems in complex real-world environments.

7.4 Multiphysics Coupling Research

Conducting in-depth mechanism studies:

- Multi-physics coupling analysis of radiation, conduction, and convection
- Correlation mechanisms between microstructure and macroscopic properties
- Performance under extreme environmental conditions

Providing theoretical support for the development of next-generation radiative cooling technologies.

VIII. References

[1] Dong, Y. (2024). *Spectral Radiative Property Regulation and Temperature Adaptation Study of Daytime Radiative Cooling Coatings* [Doctoral dissertation, Harbin Institute of Technology]. DOI: 10.27061/d.cnki.ghgdu.2024.000294.

[2] Pan, B., Zhang, J., Yang, X., et al. (2024). Preparation and Properties of Passive Daytime Radiative Cooling Superhydrophobic Polyester Fabric. *Journal of Zhejiang Sci-Tech University (Natural Sciences)*, 51(01), 55-62.

[3] Cheng, Z. (2022). *Theoretical Design and Experimental Study of Cooling Performance of Bio-inspired Daytime Radiative Cooling Coatings* [Doctoral dissertation, Harbin Institute of Technology]. DOI: 10.27061/d.cnki.ghgdu.2022.000326.

[4] Chen, X. (2022). *Preparation and Cooling Performance of Passive Daytime Radiative Cooling Films* [Master's thesis, Southeast University]. DOI: 10.27014/d.cnki.gdnau.2022.003274.

[5] Zhang, X. (2019). *Preparation of High Transmittance/Emission Materials in the Atmospheric Window Bands and Study of Their Radiative Cooling Performance* [Master's thesis, Qingdao University of Science and Technology].

[6] Cao, D. (1994). The Stefan-Boltzmann Radiation Law and Its Applications. *Infrared Technology*, (03), 46-48.

[7] Ai, Z., & Wu, H. (2016). A Simple Thermodynamic Derivation of the Stefan-Boltzmann Law. *Journal of Higher Education*, (22), 88-89. DOI: 10.19980/j.cnki.23-1593/g4.2016.22.039.

[8] Sun, Z. (2023). *Design and Optimization of a Broadband Metamaterial Absorber in the Atmospheric Transparency Window* [Master's thesis, Beijing University of Posts and Telecommunications]. DOI: 10.26969/d.cnki.gbydu.2023.001195.

IX. Appendix

Listing 1: Transfer Matrix Method (TMM)

```
import numpy as np
import pandas as pd
from scipy.interpolate import interp1d
import matplotlib.pyplot as plt

def tmm_core(wavelengths, thicknesses, n_complex_matrix, theta0=0):
    num_layers, num_points = n_complex_matrix.shape
    R_total = np.zeros(num_points)
    T_total = np.zeros(num_points)

    for i in range(num_points):
        wl = wavelengths[i]
        n_list = n_complex_matrix[:, i]
        k0 = 2 * np.pi / wl
        n0 = n_list[0]
        kz_list = n_list * k0
        Y_list = n_list

        M = np.array([[1, 0], [0, 1]], dtype=complex)

        for j in range(1, num_layers - 1):
            d = thicknesses[j]
            kz = kz_list[j]
            Y = Y_list[j]
            delta = kz * d
            cos_d = np.cos(delta)
            sin_d = np.sin(delta)

            M_j = np.array([
                [cos_d,     -1j * sin_d / Y],
                [-1j * Y * sin_d, cos_d]
            ])

            M = np.dot(M, M_j)

        Y0 = Y_list[0]
        Ys = Y_list[-1]
        BC_vec = np.dot(M, np.array([[1], [Ys]]))
        B = BC_vec[0, 0]
        C = BC_vec[1, 0]
        r = (Y0 * B - C) / (Y0 * B + C)
        t = 2 * Y0 / (Y0 * B + C)
        R_val = np.abs(r)**2
        T_val = np.real(Ys / Y0) * np.abs(t)**2
        R_total[i] = R_val
```

```
        T_total[i] = T_val

    A_total = 1 - R_total - T_total
    A_total[A_total < 0] = 0
    R_total[R_total > 1] = 1

    return R_total, T_total, A_total

def load_data(filepath, target_wavelengths):
    try:
        df = pd.read_csv(filepath)
        data = df.values
        x_raw = data[:, 0]
        sorted_indices = np.argsort(x_raw)
        x_raw = x_raw[sorted_indices]
        results = []
        for col_idx in range(1, data.shape[1]):
            y_raw = data[:, col_idx][sorted_indices]
            f = interp1d(x_raw, y_raw, kind='linear', fill_value="extrapolate")
            y_interp = f(target_wavelengths)
            y_interp[y_interp < 0] = 0
            results.append(y_interp)
        return results

    except Exception as e:
        print(f"Error reading {filepath}: {e}")
        return None

wl_min = 0.3
wl_max = 25.0
num_points = 2000
wavelengths = np.linspace(wl_min, wl_max, num_points)

pdms_data = load_data('nk.csv', wavelengths)
if pdms_data is None:
    print("Warning: PDMS CSV not found. Using dummy data.")
    n_pdms = np.ones_like(wavelengths) * 1.4
    k_pdms = np.zeros_like(wavelengths); k_pdms[wavelengths>8] = 0.5
else:
    n_pdms, k_pdms = pdms_data[0], pdms_data[1]

ag_data = load_data('ag.csv', wavelengths)
if ag_data is None:
    print("Warning: Ag CSV not found. Using dummy data.")
    n_ag = np.ones_like(wavelengths) * 0.05
    k_ag = np.ones_like(wavelengths) * 5.0
else:
    n_ag, k_ag = ag_data[0], ag_data[1]
```

```
al_data = load_data('aink.csv', wavelengths)
if al_data is None:
    print("Warning: Al CSV not found. Using dummy data.")
    n_al = np.ones_like(wavelengths) * 0.05
    k_al = np.ones_like(wavelengths) * 5.0
else:
    n_al, k_al = al_data[0], al_data[1]

solar_data = load_data('am15_1.csv', wavelengths)
solar_irradiance = solar_data[0] if solar_data else np.zeros_like(wavelengths)
atm_data = load_data('atmosphere_utf.csv', wavelengths)
atm_trans = atm_data[0] if atm_data else np.zeros_like(wavelengths)
thickness_list = [10, 30, 50, 100]
n_air = np.ones_like(wavelengths)
k_air = np.zeros_like(wavelengths)
plt.figure(figsize=(12, 6))
plt.style.use('default')
plt.axvspan(8, 13, color='gray', alpha=0.2, label='Atmospheric Window (8-13um)')

for d_pdms in thickness_list:
    print(f" d = {d_pdms} um ...")
    n_layer1 = n_air + 1j * k_air
    n_layer2 = n_pdms + 1j * k_pdms
    n_layer3 = n_ag + 1j * k_ag
    n_stack = np.vstack([n_layer1, n_layer2, n_layer3])
    d_stack = [0, d_pdms, 0]
    R, T, Emissivity = tmm_core(wavelengths, d_stack, n_stack)
    plt.plot(wavelengths, Emissivity, label=f'PDMS d={d_pdms} $\mu m$')

plt.title("Spectral Emissivity of PDMS Film on Ag Substrate", fontsize=15)
plt.xlabel("Wavelength ($\mu m$)", fontsize=12)
plt.ylabel("Emissivity / Absorptivity", fontsize=12)
plt.xlim(wl_min, wl_max)
plt.ylim(0, 1.05)
plt.text(1, 0.9, 'Solar Spectrum Region', color='orange', fontsize=10)
plt.axvspan(0.3, 2.5, color='orange', alpha=0.1)
plt.legend(loc='best')
plt.grid(True, which='both', linestyle='--', alpha=0.7)
plt.show()
```

Listing 2: Radiative Cooling Power Evaluation Model

```
import numpy as np
import pandas as pd
from scipy.optimize import fsolve
import matplotlib.pyplot as plt
def blackbody_radiation(wl_um, T_k):
    c1 = 3.7418e8; c2 = 1.4388e4
    return c1 / (wl_um**5 * (np.exp(c2/(wl_um*T_k)) - 1))
```

```
P_rad = np.trapz(blackbody_radiation(wavelengths, 300) * Emissivity, wavelengths)
P_atm = np.trapz(blackbody_radiation(wavelengths, 300) * Emissivity * (1-atm_trans),
    wavelengths)
P_sun = np.trapz(solar_irradiance * Emissivity, wavelengths)
P_cool = P_rad - P_atm - P_sun
print(f"------------------------------------------------")
print(f" {d_pdms} um T_amb=300K :")
print(f"P_rad : {P_rad:.2f} W/m2")
print(f"P_atm : {P_atm:.2f} W/m2")
print(f"P_sun : {P_sun:.2f} W/m2")
print(f"P_net : {P_cool:.2f} W/m2 ")
print(f"------------------------------------------------")

def planck_law(wavelength_um, T_kelvin):
   if T_kelvin <= 0: return np.zeros_like(wavelength_um)
   w_m = wavelength_um * 1e-6
   c1 = 3.7418e-16
   c2 = 1.4388e-2
   exp_val = np.exp(c2 / (w_m * T_kelvin))
   spectral_radiance = c1 / (w_m**5 * (exp_val - 1))
   return spectral_radiance * 1e-6

def calculate_net_cooling_power(T_film, T_amb, wavelengths, emissivity,
                           solar_irr, atm_trans, h_c):
   E_bb_film = planck_law(wavelengths, T_film)
   P_rad = np.trapezoid(E_bb_film * emissivity, wavelengths)
   E_bb_amb = planck_law(wavelengths, T_amb)
   e_atm = 1 - atm_trans
   P_atm = np.trapezoid(E_bb_amb * e_atm * emissivity, wavelengths)
   P_sun = np.trapezoid(solar_irr * emissivity, wavelengths)
   P_non_rad = h_c * (T_amb - T_film)
   P_net = P_rad - P_atm - P_sun - P_non_rad
   return P_net

def solve_equilibrium_temp(T_amb, wavelengths, emissivity, solar_irr, atm_trans, h_c):
   func = lambda T: calculate_net_cooling_power(T, T_amb, wavelengths, emissivity,
                                        solar_irr, atm_trans, h_c)
   T_eq = fsolve(func, T_amb - 10)[0]
   return T_eq

wl_min = 0.3
wl_max = 25.0
num_points = 2000
wavelengths = np.linspace(wl_min, wl_max, num_points)
solar_irradiance = solar_irradiance
atm_transmittance = atm_trans
T_ambient = 300
h_c_value = 12
thickness_list_eval = [10, 30, 50, 100]
```

```
results_summary = []

print(f"{'Thickness(um)':<15} {'P_cool(W/m2)':<15} {'T_eq(K)':<15} {'dT_max(K)':<15}")
print("-" * 60)

for d in thickness_list_eval:
    R, T, Emissivity = tmm_core(wavelengths, [0, d, 0], n_stack)
    Emissivity = np.zeros_like(wavelengths)
    Emissivity[wavelengths > 8] = 0.9 * (1 - np.exp(-0.1 * d))
    Emissivity[wavelengths < 2.5] = 0.05
    current_emissivity = Emissivity
    P_max = calculate_net_cooling_power(T_ambient, T_ambient, wavelengths,
                                  current_emissivity, solar_irradiance,
                                  atm_transmittance, h_c_value)
    T_eq = solve_equilibrium_temp(T_ambient, wavelengths, current_emissivity,
                             solar_irradiance, atm_transmittance, h_c_value)
    dT_max = T_ambient - T_eq
    results_summary.append([d, P_max, T_eq, dT_max])
    print(f"{d:<15} {P_max:<15.2f} {T_eq:<15.2f} {dT_max:<15.2f}")

thicknesses = [row[0] for row in results_summary]
p_cool_vals = [row[1] for row in results_summary]
dt_vals = [row[3] for row in results_summary]
fig, ax1 = plt.subplots(figsize=(10, 6))

color = 'tab:blue'
ax1.set_xlabel(r'PDMS Thickness ($\mu$m)', fontsize=12)
ax1.set_ylabel('Cooling Power $P_{cool}$ ($W/m^2$)', color=color, fontsize=12)
ax1.plot(thicknesses, p_cool_vals, color=color, marker='o', linewidth=2,
     label='Cooling Power')
ax1.tick_params(axis='y', labelcolor=color)
ax1.grid(True, linestyle='--', alpha=0.5)

ax2 = ax1.twinx()
color = 'tab:red'
ax2.set_ylabel(r'Max Temp Drop $\Delta T_{\text{max}}$ (K)', color=color, fontsize=12)
ax2.plot(thicknesses, dt_vals, color=color, marker='s', linestyle='--', linewidth=2,
     label='Temp Drop')
ax2.tick_params(axis='y', labelcolor=color)

plt.title(f"Radiative Cooling Performance vs PDMS Thickness\n(T_amb={T_ambient}K,
     h_c={h_c_value} $W/m^2K$)", fontsize=14)
fig.tight_layout()
plt.show()
```

Listing 3: Differential Evolution Algorithm (DE)

```
import numpy as np
import pandas as pd
```

```python
from scipy.optimize import differential_evolution
import matplotlib.pyplot as plt
sio2_data = load_data('sio2out.csv', wavelengths)
if sio2_data is None:
    print("Warning: SiO2 CSV not found. Using dummy data for testing.")
    n_sio2 = np.ones_like(wavelengths)
    k_sio2 = np.zeros_like(wavelengths)
    k_sio2[(wavelengths > 8) & (wavelengths < 10)] = 0.5
else:
    n_sio2, k_sio2 = sio2_data[0], sio2_data[1]

def optimization_target(thicknesses):
    d_sio2 = thicknesses[0]
    d_pdms = thicknesses[1]
    d_stack = [0, d_pdms, d_sio2, 0]
    n_layer_air = np.ones_like(wavelengths) + 1j * 0
    n_layer_pdms = n_pdms + 1j * k_pdms
    n_layer_sio2 = n_sio2 + 1j * k_sio2
    n_layer_al = n_al + 1j * k_al
    n_stack = np.vstack([n_layer_air, n_layer_pdms, n_layer_sio2, n_layer_al])
    _, _, emissivity = tmm_core(wavelengths, d_stack, n_stack)
    P_cool = calculate_net_cooling_power(300, 300, wavelengths, emissivity,
                                   solar_irradiance, atm_transmittance, h_c=8)
    return -P_cool

bounds = [(0.01, 5.0), (10.0, 100.0)]
result = differential_evolution(optimization_target, bounds,
                            strategy='best1bin',
                            maxiter=200,
                            popsize=10,
                            tol=0.01,
                            disp=True)
best_d_sio2 = result.x[0]
best_d_pdms = result.x[1]
max_power = -result.fun

print(f"  SiO2 d: {best_d_sio2:.4f} um")
print(f"  PDMS d: {best_d_pdms:.4f} um")
print(f"  Pmax: {max_power:.2f} W/m2")

d_stack_opt = [0, best_d_pdms, best_d_sio2, 0]
n_stack_opt = np.vstack([np.ones_like(wavelengths), n_pdms+1j*k_pdms,
    n_sio2+1j*k_sio2, n_ag+1j*k_al])
_, _, eps_opt = tmm_core(wavelengths, d_stack_opt, n_stack_opt)
d_stack_ref = [0, 50, 0]
n_stack_ref = np.vstack([np.ones_like(wavelengths), n_pdms+1j*k_pdms, n_ag+1j*k_al])
_, _, eps_ref = tmm_core(wavelengths, d_stack_ref, n_stack_ref)
plt.figure(figsize=(12, 6))
plt.title(f"Optimization Result: Multilayer vs Single Layer\nMax Power:
```

```
    {max_power:.1f} $W/m^2$ (@ {best_d_pdms:.1f}um PDMS / {best_d_sio2:.1f}um SiO2)")
plt.axvspan(8, 13, color='gray', alpha=0.2, label='Atmospheric Window')
plt.fill_between(wavelengths, 0, solar_irradiance/np.max(solar_irradiance),
              color='orange', alpha=0.1, label='Solar Spectrum (Normalized)')

plt.plot(wavelengths, eps_ref, 'k--', linewidth=1.5, alpha=0.7, label='Reference: Pure
    PDMS (50um)')
plt.plot(wavelengths, eps_opt, 'r-', linewidth=2.5, label='Optimized: PDMS + SiO2')
plt.xlabel(r"Wavelength ($\mu$m)")
plt.ylabel("Emissivity")
plt.xlim(0.3, 20)
plt.ylim(0, 1.05)
plt.legend(loc='upper right')
plt.grid(True, linestyle='--', alpha=0.5)
plt.show()
```