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Dynamic Allocation and Control of Coordinated UAVs to Engage Multiple Targets in a Time-Optimal Manner

Tomonari Furukawa[†], Frederic Bourgault[††], Hugh F. Durrant-Whyte[††] and Gamini Dissanayake[†††]
ARC Centre of Excellency in Autonomous Systems
[†]School of Mechanical and Manufacturing Engineering, The University of New South Wales, Sydney 2052 Australia
[††]Australian Centre for Field Robotics, The University of Sydney, Sydney 2006 Australia
[†††]Faculty of Engineering, University of Technology, Sydney, 2007 Australia
Email: {t.furukawa, f.bourgault, hugh, gdissa}@cas.edu.au

***Abstract*—This paper presents the real-time control of cooperative Unmanned Air Vehicles (UAVs) that dynamically engage multiple targets in a time-optimal manner. Techniques to dynamically allocate vehicles to targets and to subsequently find the time-optimal control actions are proposed. The decentralization of the proposed control strategy is further presented such that the vehicles can be controlled in real-time without significant time delay. The proposed strategy is then applied to various practical battlefield problems, and numerical results show the efficiency of the proposed strategy.**

I. Introduction

Multiple air vehicles are required to meet around a target as quickly as possible if the task that needs to be conducted for the target is not achievable by a single vehicle and the time-efficient arrival is indispensable. The necessity for this class of multiple vehicle cooperation can be found in a wide range of applications; forest fire fighting, where multiple air vehicles drop water around a fire to put it off, and battlefield bombing, where multiple air fighters drop munitions around a target facility for its complete destruction.

The actual field may have multiple targets; multiple fires to put off or multiple target facilities to destroy. The cooperation problem of multiple UAVs of this class can be thus stated in a general fashion as follows; given $m(\in N)$ vehicles and $q(\in N)$ targets, control the vehicles to engage every target as quickly as possible [1]. The form of engagement of a target is application-dependent. This paper will deal with a case where $m_j(< m)$ vehicles arrive *above* target $j(\in \{1, ..., q\})$ simultaneously [2] as the simplest and most common form.

In order to eliminate the natural complexity that is outside the scope of this paper, we assume that the vehicle model and the target positions are accurately known and that there is no disturbance in the environment. The only uncertain element is thus the fault of a vehicle, which can make the vehicle inactive at any moment.

Despite these simplifications, the problem is still of great complexity, leaving the following five issues to be explored; (1) In which order should the targets be assigned, (2) Which vehicles should be allocated to each target, (3) How can the cooperation be continued when a vehicle becomes inactive, (4) What control action should each vehicle that cooperates take, and (5) What control action should each vehicle that does not cooperate take. The former three issues are related to the dynamic vehicle allocation, while the latter two concerns the multi-vehicle control for a single target.

Although a significant number of techniques are available for the dynamic vehicle allocation [3]-[6] and multi-vehicle control [7]-[10], none of the techniques has handled multiple vehicles that may cause failure, multiple targets and the time-optimality simultaneously. Present dynamic vehicle allocation techniques are formulated to solve problems where the arrival time of each vehicle at each target is specified *a priori*, while many multi-vehicle control techniques are concerned with a single target or a formation rather than multiple targets. The authors have recently proposed a centralized off-line control approach that deals with multiple targets and time-optimality [11], [12].

With further improvements, this paper presents a decentralized on-line control strategy that enables cooperative UAVs to engage multiple targets time-optimally. The on-line time-optimal control can be achieved by the following two sequential techniques proposed in this paper; (1) A technique to dynamically allocate UAVs to targets for which UAVs have not cooperated, and (2) A technique to find the time-optimal control actions of the UAVs subject to the physical and kinematic constraints of the vehicles. In adition, the decentralization of the proposed techniques enables coordinated UAVs to be controlled in real time.

The paper is organized as follows. The following section describes the vehicle and target models. The on-line control of coordinated vehicles based on the dynamic vehicle allocation is presented in Section III, whereas its decentralization is dealt with in Section IV. Section V presents the refers to the canonical formulation of the time-optimal control problem as well as Control Parameterization and Time Discretization (CPTD) method [13], which is a numerical technique to solve this problem. Section VI demonstrates the effectiveness of the proposed control strategy through numerical examples.

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II. Vehicle and Target Models

A. Vehicle model

The model of i^{th} UAV ($\forall i \in \{1, ..., m\}$) moving within a horizontal plane can be approximated as

$$\begin{aligned} \dot{x}_i &= v_i \cos\theta_i, \\ \dot{y}_i &= v_i \sin\theta_i, \\ \dot{\theta}_i &= \alpha_{\theta_i}(\bar{\theta}_i - \theta_i), \\ \dot{v}_i &= \alpha_{v_i}(\bar{v}_i - v_i), \end{aligned} \tag{1}$$

where $\mathbf{x}_i^T \equiv [x_i, y_i, \theta_i, v_i]$ is a state vector representing the position, orientation (head angle) and velocity whereas $\mathbf{u}_i^T \equiv [\bar{\theta}_i, \bar{v}_i]$ represents a control vector describing the controlling orientation and velocity. $\alpha = [\alpha_{\theta_i}, \alpha_{v_i}]$ is a set of physical parameters.

The vehicle motion is constrained due to the physical and/or kinematic limitations. One of the typical continuous constraints for the UAV is on the state velocity:

$$(v_{min})_i \leq v_i(t) \leq (v_{max})_i. \tag{2}$$

Another typical constraint is that the rate of change of state orientation is

$$-(\dot{\theta}_{max})_i \leq \dot{\theta}_i(t) \leq (\dot{\theta}_{max})_i. \tag{3}$$

Note that the time-optimal control solution for this vehicle model is known. The real-world vehicle is however governed by much more complicated dynamic equations where the solution cannot be analytically derived. Computational techniques presented in this paper are developed to find an approximate solution of the real-world vehicle.

B. Target model

Restricting interest in target location to the horizontal plane, the position of each target is given by

$$\tilde{\mathbf{z}}_j^T \equiv [(\tilde{x})_j, (\tilde{y})_j], \forall j \in \{1, ..., q\}. \tag{4}$$

Additionally, each target is specified with the number of vehicles that must engage the target, m_j.

III. On-Line Control of Multiple Vehicles

A. Vehicle and Target States

Table I lists the types of vehicle and target states introduced to enable the dynamic vehicle allocation. Vehicle i is said to be 'active' if the vehicle can be driven as desired whilst Target j is 'engaged' if m_j vehicles allocated to the target successfully arrive at the target simultaneously. Vehicle i is said to be 'allocated' when the vehicle is allocated to a target, whereas Target j is said to be 'assigned' if m_j vehicles are allocated to the target. Vehicle i is said to be 'cooperating' for Target j if a total of m_j vehicles are allocated to the target and the vehicle is one of them.

TABLE I

VEHICLE AND TARGET STATES

	Vehicle	Target
State	Active/Inactive	Engaged/Non-engaged
	Allocated/Unallocated	Assigned/Unassigned
	Cooperating/Non-cooperating	

B. Control strategy

The flow diagram of Figure 1 describes the proposed control strategy. The states of all the vehicles are initially set to 'non-cooperating' and 'unallocated' whilst those of all the targets are set to 'unassigned' and 'non-engaged'. The first step is to allocate vehicles to targets. When vehicles are allocated, their state changes from 'unallocated' to 'allocated'. When the necessary set of allocated vehicles is identified, their state is changed from 'non-cooperating' to 'cooperating' and the corresponding target state is set to 'assigned'. The time-optimal trajectory for the assigned subset of vehicles is then computed, executed and the target is engaged. Once engaged, the target state is set to 'engaged'. Those vehicles which are 'allocated' but not yet cooperating must loiter. At every time step k the existence of newly engaged targets is checked. The vehicles stay in motion but once a target is engaged the engaging vehicles state is reset to 'non-cooperating' and 'unallocated' and the target state is set to 'engaged'. Unless all the targets are engaged, the vehicles continue in motion continuously checking for the existence of any remaining 'unassigned' targets. A new vehicle allocation is conducted between the 'non-cooperating' vehicles and 'unassigned' targets. In the terminal condition when all targets have been engaged, all vehicles become 'unallocated' and 'non-cooperating', and made to move in a straight with minimum velocity ($\bar{\theta}_i = \theta_i, \bar{v}_i = (v_{\min})_i$).

C. Dynamic Vehicle Allocation

The determination of control actions for 'non-cooperating' vehicles is as follows.

1) Let the 'non-cooperating' vehicles and 'unassigned' targets be the vehicles and targets in the current list, respectively.
2) Find the minimal terminal time for each vehicle to reach each target (**Time-Optimal Control Problem 1: A single vehicle to reach a target**).
3) For each target j, sort the terminal times, and find the largest terminal time of the m_j vehicles that reach the target first.
4) Compare all the targets and find target j that can be engaged first and the m_j vehicles that engage this target. Set the target and the vehicles to 'assigned' and 'cooperating'/'allocated', respectively.
5) Delete the vehicles and the target from the current list, and find the number of remaining targets and vehicles. If there are remaining targets, then proceed to Step 6. Otherwise, exit.

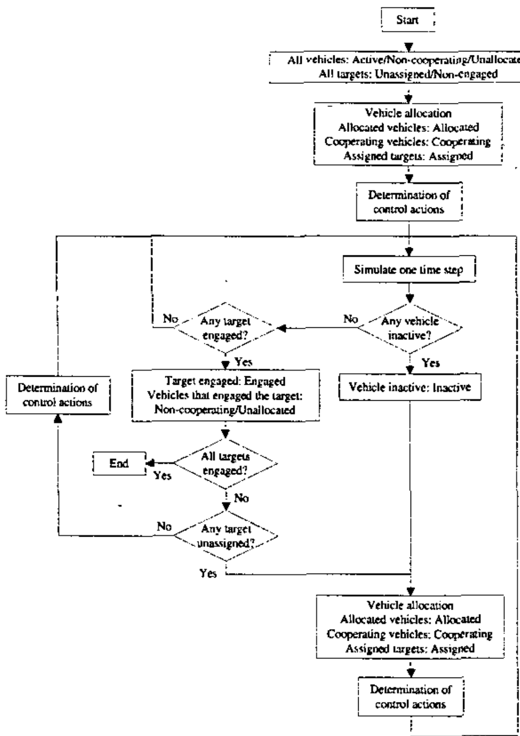

Fig. 1. Flowchart of the proposed control strategy

6) If the number of remaining vehicles is more than or equal to the smallest m_j of the remaining targets, then remove all the targets, that have m_j larger than the number of remaining vehicles, from the current list and return to Step 2. Otherwise, for each of the remaining vehicles, find, from the remaining targets, a target that allows the vehicle to reach first, by comparing the minimal terminal times computed in Step 2.

D. Determination of control actions

Table II describes the type of control action taken by each vehicle in each possible vehicle state. If m_j vehicles have been found to engage a target, control actions will be found such that the vehicles can reach a target in a time-optimal manner **(Time-Optimal Control Problem 2: Multiple vehicles to reach a target)**. There is a case that a vehicle could not find $(m_j - 1)$ vehicles to engage a target cooperatively. The control action proposed for such a vehicle is to loiter around a target such that the vehicle can participate in cooperation to engage this target.

The steps for allowing vehicle i to loiter around a target are as follows:

1) Find the control action of the vehicle such that the vehicle reaches the circumference of a circle (radius R_i) specified around the target tangentially in a time-optimal manner **(Time-Optimal Control Problem 3: A single vehicle to loiter around a target)**.

TABLE II

VEHICLE ACTIONS IN RELATION TO VEHICLE STATES

Allocation state	Cooperation state	Action
Allocated	Cooperating	Move cooperatively
Allocated	Non-cooperating	Loiter
Unallocated	Non-cooperating	Move straight

2) Make the vehicle follow the circumference in a feedback manner with the minimum velocity.

The control action of every vehicle can be determined provided that the three time-optimal control problems represented in bold type are solved. The problems will be formulated in the next section.

IV. DECENTRALIZED CONTROL

Figure 2 illustrates the comparison between the dynamic vehicle allocation and multi-vehicle cooperation by a central computer and their decentralization proposed by the authors for a case where seven vehicles require to engage four targets and three vehicles must reach a target at a time. In this case, Vehicles 1, 6 and 7 are allocated to Target 3, Vehicles 2, 4 and 5 to Target 1 and Vehicle 4 to Target 4.

The process consists of three main steps; (i) Find the minimum time for each vehicle to reach each target (Step 2 in III-C, (ii) Vehicle allocation (Steps 3-6 in III-C) and (iii) Determination of control actions. The figure shows that Steps (i) and (iii) are processed in a decentralized manner, thereby enabling fast computation.

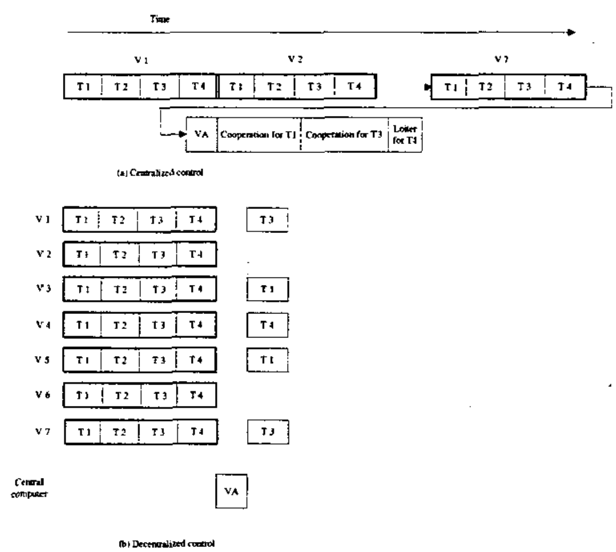

Fig. 2. Decentralized control

Step (iii), which is originally a centralized problem, can be decentralized as shown in the figure in sanction with the following theorem [12]:

The minimum time for a group of vehicles to reach a target simultaneously is that of the vehicle that takes longest.

In this case, Vehicles 6 and 2 take longest for Targets 3 and 1, respectively. As the terminal times for Vehicles 6 and 2 to reach Targets 3 and 1 are known through Step (i), the continuous control actions of Vehicles 1, 3, 5 and 7 can be derived independently by solving an optimal control problem for a target with a known terminal time. The payoff function proposed in this case is the energy function as the energy is the secondary important criterion for vehicles with fuel.

V. Optimal Control of a Single Vehicle to Engage a Target

A. Optimal Control Problem

Define the *continuous control action* of a vehicle as the complete control function from a time $t = 0$ to a terminal time T, $\mathcal{U} \boxminus \{\mathbf{u}(t) | \forall t \in [0, T)\}$. The general optimal control problem for a single vehicle to reach a single target can be formulated as finding the continuous control action $\mathcal{U}$ and the terminal time T that minimize a payoff function $g_0(\mathcal{U})$ [14], i.e.,

$$\min_{\mathcal{U},T} g_0(\mathcal{U}), \tag{5}$$

where the payoff function in canonical form [14] is given by

$$g_0(\mathcal{U}) \equiv \Phi_0(\mathbf{x}(T|\mathcal{U})) + \int_0^T \mathcal{L}_0(t, \mathbf{x}(t|\mathcal{U}), \mathbf{u}(t))dt. \tag{6}$$

The payoff function for the time-optimal control becomes $g_0(\mathcal{U}) \equiv \int_0^T dt = T$ whereas that for energy minimization is given by $g_0(\mathcal{U}) \equiv \int_0^T \mathcal{L}_0(t, \mathbf{x}(t|\mathcal{U}), \mathbf{u}(t))dt$ with energy function $\mathcal{L}_0$.

The problem is primarily subject to the terminal state constraint. For instance, if the vehicle is destined to a target $[\tilde{x}, \tilde{y}]$, the constraint is given by

$$g_1(\mathcal{U}) \equiv (x(T|\mathcal{U}) - \tilde{x})^2 + (y(T|\mathcal{U}) - \tilde{y})^2 = 0. \tag{7}$$

In addition to the terminal state constraint, the UAV is subject to the continuous state constraints described by the inequality (2):

$$g_2(\mathcal{U}) \equiv \int_0^T \min\{0, (s(t|\mathcal{U}) - s_{\min})\}^2 + \min\{0, (s_{\max} - s(t|\mathcal{U}))\}^2 dt = 0, \tag{8}$$

where $\min\{.,.\}$ takes the smaller value of its elements and $[s, s_{\max}, s_{\min}]$ are $[v, v_{\max}, v_{\min}]$ for the velocity constraint and $[\theta, \theta_{\max}, -\theta_{\max}]$ for the turning rate constraint.

B. CPTD Method

The CPTD method converts the dynamic time-optimal control problem to a static parameter optimization problem. First, the continuous control action $\mathcal{U}$ is approximated by a piecewise function with a set of static parameters $\Omega \equiv \{\sigma_j \in \Re^r \mid \forall j \in \{1, ..., n_p\}\}$:

$$\hat{\mathbf{u}}(t; n_p, \Omega) \equiv \sum_{j=1}^{n_p} \sigma_j \chi_j(t) \cong \mathbf{u}(t), \tag{9}$$

where $\chi_j(t)$ is given by

$$\chi_j(t) \equiv \begin{cases} 1 & (j-1)\Delta t_p \le t < j\Delta t_p \\ 0 & \text{Otherwise} \end{cases}. \tag{10}$$

and $\Delta t_p = T/n_p$. Note here that we use the expression $\cong$ to denote an approximation. The time-step interval used for iterative simulation, Δt, is then introduced as a function of T, $\Delta t = T/(n_p \cdot p)$. As a result, the payoff function and constraint functions are all described as a function of static parameters Ω and T. This static parameter optimization problem can be solved with a standard nonlinear programming method such as Sequential Quadratic Programming (SQP), though the global optimality of the solution depends upon the convexity of the payoff and the constraint functions.

VI. Numerical Examples

A. Common Parameters

Table III lists the properties of two types of vehicles that were used in the following numerical examples.

TABLE III
Properties of Vehicles

Parameter	Type 1	Type 2
$[\alpha_{\theta_i}, \alpha_{v_i}]$	[1.0, 0.21]	[1.2, 0.15]
$[(v_{\min})_i, (v_{\max})_i]$	[0.121, 0.148] km/s	[0.121, 0.148] km/s
$(\dot{\theta}_{\max})_i$	20 deg/s	20 deg/s

Table IV describes the parameters used for the CPTD method. A constrained nonlinear optimization program in IMSL mathematical library [15] was used for parametric optimization.

TABLE IV
Parameters used for CPTD method

Parameter	Value
n_p	5
p	10
Initial Δt	0.3
Initial $\sigma_j^{i\,T}$,	[0.148 km/s, 0 deg]

The computational cost of the proposed decentralized control strategy has to be examined. A PC cluster (16 processing elements, Pentium IV 2Hz, 100BaseT) was used to represent a maximum of 16 vehicles.

B. Example 1

1) Problem Setup: As a practical battlefield problem, five UAVs were assigned to engage ten targets ($m = 5$, $q = 10$) where two vehicles must cooperate for each target ($m_j = 2, \forall j \in \{1, ..., 10\}$). All the vehicles are of Type 1. Figure 3 shows the initial vehicle positions (o) and orientations (-) as well as the target positions (+). Each vehicle was made subject to a fault, every time the vehicle arrives above a target, with the probability $P(i) = 0.05$.

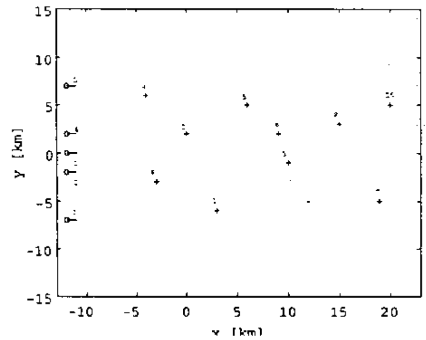

Fig. 3. Initial poses of vehicles and positions of targets in Example 1

2) Results: Figures 4 and 5 show the resulting vehicle motions until the first and seventh target is engaged, respectively. The initial states of all the vehicles and targets are 'active'/'unallocated'/'non-cooperating' and 'unassigned'/'unengaged', respectively. The dynamic vehicle allocation is first conducted, and Vehicles 2 and 3, Vehicles 4 and 5 and Vehicle 1 are respectively allocated to Target 4, Target 9 and Target 2. As a consequence, Targets 4 and 9 become 'assigned', Vehicles 2, 3, 4 and 5 'allocated'/'cooperating', and Vehicle 5 'allocated'/'non-cooperating'. The vehicle motions are based on these vehicle states; Vehicles 2 and 3 move cooperatively to Target 4, Vehicles 4 and 5 move cooperatively to Target 9, and Vehicle 1 loiters around Target 2. The vehicles proceed in a similar manner since no vehicle becomes inactive.

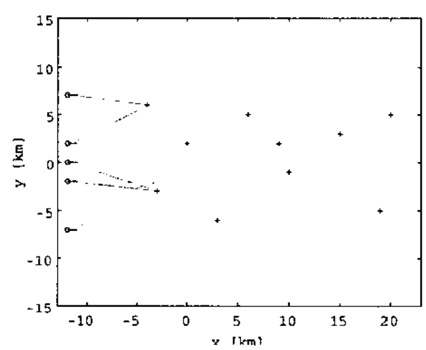

Fig. 4. Control actions of vehicles until the first target is engaged

Figure 6 shows the resulting vehicle motions until the last target (Target 7) is engaged. In the figure, it is seen that Vehicle 4 becomes inactive when Target 6 is engaged. The cooperation is however successfully continued thanks to the dynamic vehicle allocation strategy proposed in this paper. The largest time required to compute vehicle control actions was 0.18 sec whereas the total time required for engaging all the targets was 254.42 sec. This result indicates the real-time controllability of the proposed strategy.

The proposed control strategy has been implemented on a flight simulator, enabling investigation on the efficacy of the strategy in a virtual environment. The real-time control on the flight simulator is shown in Figure 7.

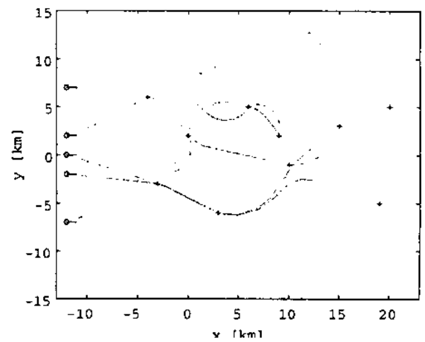

Fig. 5. Control actions of vehicles until the seventh target is engaged

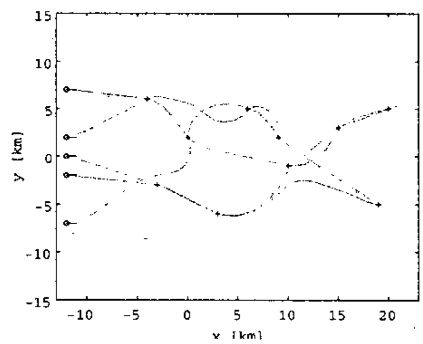

Fig. 6. Control actions of vehicles until all the targets are engaged

C. Example 2

The second example handles the same vehicle and target set-up as Example 1, but the differences are that

- Two types of vehicles cooperate (Type 1: Vehicles 1, 3 and 4, Type 2: Vehicles 2 and 3),
- All the vehicles are active.

Figure 8 shows the resulting vehicle motions until the last target is engaged. It is seen that the vehicles result in different cooperations.

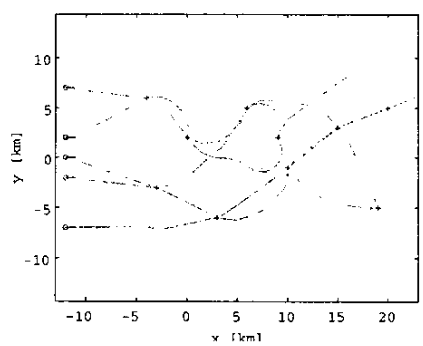

Fig. 8. Control actions of vehicles in Example 2

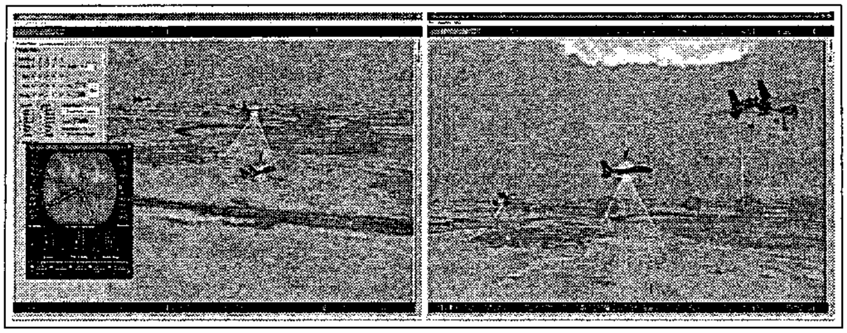

Fig. 7. Cooperation on flight simulator

D. Example 3

The third example adds ten vehicles and five targets to Example 2, thereby $m = 10$ and $q = 15$. Figure 9 shows the resulting vehicle motions. Despite of the large numbers of vehicles and targets, all the vehicles successfully cooperate to engage all the targets. The largest time required to compute vehicle control actions was 0.19 sec whereas the total time required for engaging all the targets was 290.11 sec.

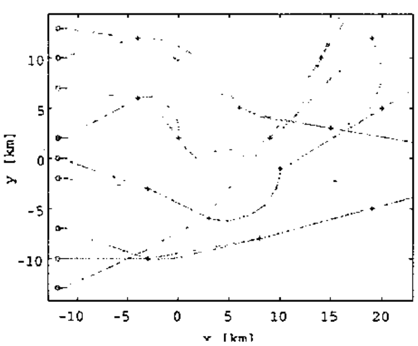

Fig. 9. Control actions of vehicles in Example 3

VII. Conclusions and Future Work

A decentralized control strategy that enables coordinated UAVs to engage multiple targets in a time-optimal manner has been presented. The proposed control strategy was successfully applied to engage multiple targets in various practical scenarios, including a case where a vehicle becomes inactive. In addition, the largest time required to compute control actions of ten vehicles engaging 15 targets was 0.19 sec, and the time did not increase significantly with respect to the increase of the numbers of vehicles and targets. This clearly indicate the effectiveness of the proposed control strategy of coordinated UAVs to engage multiple targets in real time.

VIII. Acknowledgment

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