An Experiment in Integrated Exploration

Alexei A. Makarenko, Stefan B. Williams, Frederic Bourgault, Hugh F. Durrant-Whyte

Australian Centre for Field Robotics
The University of Sydney, Australia
{*a.makarenko, stefanw, f.bourgault, hugh*}*@acfr.usyd.edu.au*

Abstract

Integrated exploration strategy advocated in this paper refers to a tight coupling between the tasks of localization, mapping, and motion control and the effect of this coupling on the overall effectiveness of an exploration strategy. Our approach to exploration calls for a balanced evaluation of alternative motion actions from the point of view of information gain, localization quality, and navigation cost. To provide a uniform basis of comparison of localization quality between different locations, a "localizability" metric is introduced. It is based on the estimate of the lowest vehicle pose covariance attainable from a given location.

1 Introduction

A mobile robot operating in the physical world must be aware of its environment. A large part of this awareness is knowing where in the world the robot is (the task of *localization*) and where it has been (the task of *mapping*). In the absence of external localization aids, the robot must be able to build a map and, at the same time, localize itself to the same partially built and imperfect map (*simultaneous localization and map-building* or SLAM [11]).

Often map creation is considered a goal in its own right (the task of *exploration*). An *exploration strategy* is then needed to answer the question of where to go next in order to build the map efficiently. Our approach to exploration calls for a balanced evaluation of alternative motion actions from the point of view of information gain, navigation cost, and localization quality. Existing solutions achieve varying degrees of integration between the tasks of localization, mapping and motion control. Figure 1 illustrates the field of robotic exploration with three partially overlapping regions. Region I graphically represents integration of localization and mapping implemented by several families of SLAM algorithms [11, 18, 5, 3]. Region II represents integration of motion control and mapping exemplified by virtually all exploration strategies [10, 20, 16, 17, 14]. Region III which integrates localization and motion control is the field of active navigation [8, 6, 15] and sensor management [12]. Full integration of all three components [5, 1] in Region IV is the goal of this work and will be referred to as *integrated exploration*.

This paper considers an exploration strategy which balances coverage, accuracy, and the speed of exploration – the basic driving factors in any exploratory mission. These drivers are often contradictory and in order to choose an action suitable metrics are required. In addition to the traditional metrics of information gain and time (or distance) traveled, a metric of *localizability* is introduced which allows to compare localization quality at different locations.

The paper is organized as follows. Section 2 gives a summary of work related to exploration. Section 3 presents our approach to integrated exploration. Section 4 derives the localizability metric in the context of feature-based SLAM. Section 5 presents results of simulation. Section 6 lists possible directions for future research.

2 Related Work

A large body of work exists on robotic exploration, due largely to the central role played by navigation and mapping in any mobile robotics application. Lee [10] gives a review of exploration strategies up to the mid-1990's. This section is organized according to the regions in Figure 1.

Classic exploration (II). Yamauchi [20] introduced the now popular frontier method of exploration. This work is extended in [16] by the use of an improved localization method and a more integrated map *representation* but the tasks of localization and exploration remain fully decoupled. Information driven exploration strategy is extended to multiple information sources in [14] but localization is not considered an integral part of the exploration strategy.

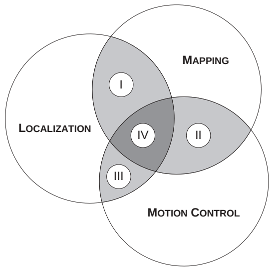


Figure 1: *The field of robotic exploration with highlighted regions of integration: (I) SLAM, (II) classic exploration, (III) active localization, (IV) integrated exploration*

SLAM and Exploration (I & II). In [17] multiple robots cooperatively explore an indoor environment each using a SLAM algorithm for localization and mapping. The exploration strategy is based on the frontier method but it does not address the issue of localization quality.

Active localization (III). A locally greedy algorithm is employed in [8] to select motion control actions which are likely to reduce the localization uncertainty. Coastal navigation, a path planning method aimed at increasing robustness of localization in large crowded spaces, is described in [15]. The distribution of the localizability metric is calculated off-line using an *a priori* occupancy grid map. The localizability metric introduced in this paper is intended for use on-line and in the context of SLAM.

SLAM and Active localization (I & III). Active stochastic localization is implemented in [6]. The algorithm selects from a list of possible motion actions by weighing the resulting reduction in localization uncertainty against the associated cost. The task of exploration is not considered.

Integrated exploration (IV). Feder et al. [5] is the most closely related work. The vehicle creates a map and localizes simultaneously and makes *local* decisions on where to move next in order to minimize the error in estimates of the vehicle pose and the landmark locations. This principle is applied to the problem of underwater exploration in [1]. The drive to minimize the vehicle pose and map errors is incorporated into a general behavior based architecture. In both cases the exploration strategy is spatially local and the localizability metric is based on a single-step look-ahead.

3 Exploration Strategy

Exploration is a high level activity. To be able to perform exploration, a robot must be able to localize itself relative to the environment, map the environment, make decisions on where to go next, and determine how to get there. Our approach utilizes proven algorithms for localization, mapping, and navigation which are tied together by the exploration strategy described in this section.

3.1 Approach

The task of exploration can have several possible objectives. The typical ones are map coverage, map accuracy, and the exploration time. Integrated exploration strategy advocated in this paper refers to a tight coupling between the tasks of localization, mapping, and motion control, the objectives of exploration, and the effect of this coupling on the overall effectiveness of an exploration strategy. Our approach to exploration calls for a balanced evaluation of alternative motion actions from the point of view of information gain, localization quality, and navigation cost. The exploration action sequence adopted in this work is as follows:

1. Generate potential destinations
2. Evaluate utility of each destination according to the multiple utility functions
3. Select a single destination using mission- or state-dependent weights
4. Plan a path to the selected destination using the current occupancy map
5. Navigate to the destination, if failed GOTO 4, if failed multiple times GOTO 1
6. Once at the destination, perform a 360° sensor sweep for maximum information gain, GOTO 1

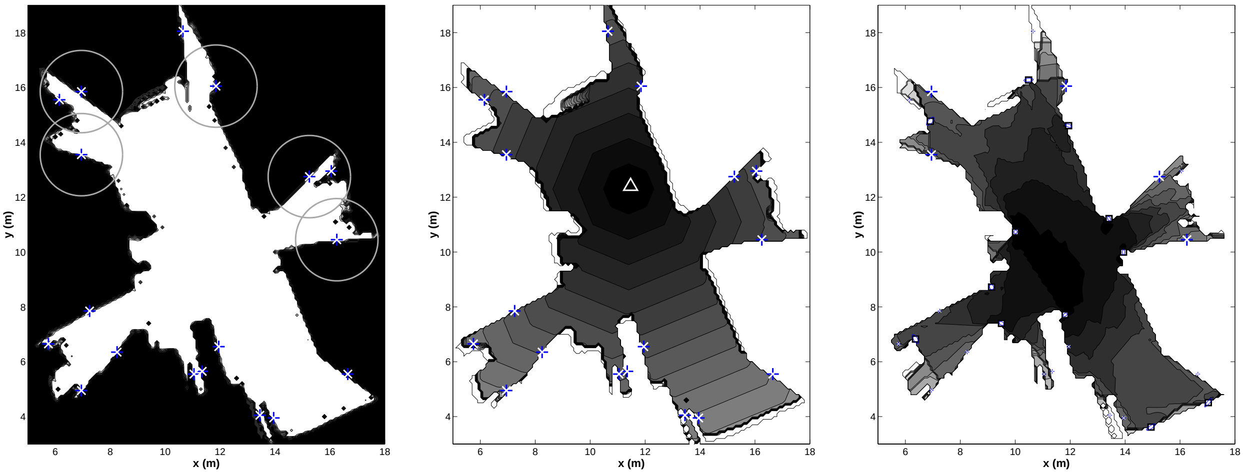
Note that this method breaks up the exploration mission into a series of discrete subtasks with purely terminal payoff utilities. While undoubtedly the optimal plan must take into account the expected integral payoffs (e.g. information gain along the path), the complexity of the problem effectively precludes this approach [14]. The following subsections detail the steps of the exploration sequence.

3.2 Proposing Destinations

The frontier method [20] is used to identify areas of interest which tend to lie on the edge of the explored and the unexplored regions. Currently, only the frontier module proposes destinations but other possibilities are under investigation. For example, if the vehicle pose covariance is getting dangerously large, the localization module may suggest locations where the chances of relocalizing are high (e.g. a cluster of several well known beacons.)

3.3 Evaluating and Selecting Destinations

In order to select a single destination from the list of potential candidates, the expected payoff and cost associated with moving to the proposed locations must



(a) (b) (c)

Figure 2: *Utility distributions for a section of the map, calculated off-line, (a) information gain, (b) navigation, and (c) localization ($k = 10$) utilities. All utilities favor dark colored regions. Also shown: current vehicle position ($\triangle$), landmarks ($\square$), potential destinations ($*$) and the expected sensed regions around them (–)*

be calculated. The utility measures from the point of view of information gain, navigation, and localization quality are described in detail below.

Utility of Information Gain. The main goal of an exploration strategy is to collect information about the world. The information utility is designed to favor destinations which offer higher information gain.

The information utility is obtained from the occupancy map. The occupancy grid map [4] is maintained in parallel with the SLAM feature map. The occupancy grid allows a simple and intuitive representation of distributed spatial information such as occupancy, traversability, etc. Similar to the SLAM algorithm, this framework allows incremental probabilistic update of each cell based on the sensor readings. Unlike in SLAM, however, it makes the critical cell independence assumption which precludes the occupancy grid framework from having the convergence property characteristic of the SLAM map [3]. To a certain extent the SLAM feature map and the occupancy grid map are complimentary, the former is superior for long term localization and the latter is more suitable for path planning and free space visualization.

Formally, the occupancy grid is a discrete-state binary random field. Each element encodes the probability of the corresponding grid cell being in a particular state. In the current implementation the map stores the estimated occupancy of the 2-D space. The element for each cell C can take on one of two values *occupied* and *empty*, written as $s(C) = \{OCC \,|\, EMP\}$. To measure the amount of information available for each cell, the entropy H of the binary distribution $s(C)$ is calculated as [2]

$$H(s(C)) = -\sum_{s(C)} P(s(C)) \log P(s(C)) \qquad (1)$$

The utility of making an observation from a destination point x_i is defined as the average entropy of a sensed region W_i around the destination point [4]

$$U_i^I = -\sum_{C \in W_i} H(s(C)) \qquad (2)$$

where the entropy H is calculated for each cell C in the region W_i. The higher the average entropy, the less information about the region is available, and the more attractive it is for the explorer.

Figure 2(a) plots the entropy distribution for a partially built occupancy map. Darker regions have higher entropy and, therefore, less information about them is available. The never-visited territory is completely unknown and has maximum entropy. A circular region around each frontier is shown for illustration. In practice the size and shape of the sensed region is determined in a manner similar to the one described in [17].

Utility of Navigation. Long travel time between destinations reduces the efficiency of exploration. The navigation utility is used to make shorter drives more appealing.

An instance of the navigation function family of algorithms [9] is used to calculate the cost of driving from the current position to the proposed destinations based on the information in the occupancy map. The navigation function encapsulates the cost of reaching any point on the map from the current position. The navigation utility at a destination point x_i is simply the negative of the navigation function V at that point.

$$U_i^N = -V(x_i) \tag{3}$$

Figure 2(b) shows the navigation utility calculated for a partially built occupancy map. Darker areas are closer and therefore are more appealing. Notice that the algorithm produces navigation utility for all destinations at once from the current starting point.

Utility of Localizability. Collecting data about the previously unmapped territory is of little use if this information cannot be integrated properly into the map due to the high vehicle localization error. The localization utility is used to distinguish between destinations with different localization quality. Conceptually, the quality of localization at any given point x_i is determined by the uncertainty in vehicle position achievable at that point. The uncertainty is described by the state covariance matrix $\mathbf{P}_{vv}(x_i, t)$ at the point x_i at time t.

To provide a uniform basis of comparison between different locations, the *localizability* metric is defined as the minimum vehicle covariance achievable by re-localizing a lost vehicle at a given location by observing only the features visible from that location. The vehicle remains stationary throughout the process. The number of observations is allowed to vary within $k \in [1\ \infty)$. The advantage of this definition is that the metric is clearly a function of both the map quality and the environment itself.

The details of calculating localizability depend on the specific localization algorithm. For the case of SLAM, the vehicle covariance $\mathbf{P}_{vv}^k$ achievable after k observations including the case of $k \to \infty$ is derived in Section 4. To convert the re-localized vehicle covariance matrix to a scalar utility, the Shannon information measure is used [2]. The entropy H for a Gaussian distribution in vehicle pose at a destination point x_i is

$$U_i^L = -H(\mathbf{P}_{vv}^k) = -\frac{1}{2}\log((2\pi e)^n|\mathbf{P}_{vv}^k|), \tag{4}$$

where n is the number of vehicle states and $\mathbf{P}_{vv}^k$ is the vehicle state covariance after k observations given by Equation 12. Figure 2(c) shows the localizability metric for a partially built occupancy map. The number of observations was set to $k = 10$.

Calculation of the localizability metric is an expensive operation. It is only performed during the planning stage and only for a limited number of proposed destinations. The costliest part of the current implementation is ray-tracing to determine visibility of beacons. The possibility of calculating the metric for the entire map or part of the map is currently under investigation. This information could then be incorporated into the path planning operation. Such a map would conceptually be very similar to the one described in [15] where it is computed *off-line* from an *a priori* map.

Total Utility. The total utility of a destination point is the weighted sum of individual utilities

$$U^{TOT} = w_I U^I + w_N U^N + w_L U^L \tag{5}$$

where the relative weights w_I, w_N, and w_L are mission dependent and may be adjusted either as an abrupt mode switch or varied continuously. In the experiments, the utilities were first normalized by an appropriate worst/best case scenario and the weights were set to unity. The combined effect of the three utilities can be observed in Figure 2. The information utility favors regions near the edges of the known map, the navigation utility prefers the vehicle to stay close to current location, and the localization utility leads the robot towards the well mapped regions. The location with the highest total utility is selected as the next destination

$$x^* = \underset{x}{\operatorname{argmax}}(U^{TOT}) \tag{6}$$

In this section three utility measures related to the information gain, navigation cost, and localization quality were described. Combined, they produce a balanced evaluation of the proposed destinations with respect to the overall goals of maximizing map coverage, accuracy, and the speed of exploration.

4 Localizability

The *localizability* metric was introduced in Section 3.3 to provide a basis of comparison between different locations from the standpoint of localization quality. The basic idea is to estimate the degree to which a vehicle can *relocalize* itself at a certain location given the current map. In this section the metric is formally derived after a brief introduction to the SLAM algorithm.

4.1 SLAM Algorithm

We use the Simultaneous Localization and Mapping (SLAM) algorithm [11, 3] to generate and maintain a map of environmental features and, at the same time, estimate the robot's position using relative observations of the map features. The method uses the Extended Kalman Filter (EKF) algorithm to optimally estimate the state vector $\mathbf{x}$, containing both the vehicle states $\mathbf{x}_v$ and the feature states $\mathbf{x}_m$, as well as the associated covariance matrix $\mathbf{P}$.

$$\mathbf{x} = \begin{bmatrix}\mathbf{x}_v \\ \mathbf{x}_m\end{bmatrix}, \mathbf{P} = \begin{bmatrix}\mathbf{P}_{vv} & \mathbf{P}_{vm} \\ \mathbf{P}_{vm}^T & \mathbf{P}_{mm}\end{bmatrix} \tag{7}$$

Details of the SLAM algorithm can be found in [3].

4.2 Localizability Metric

We now define the localizability metric for the case of SLAM feature-based localization. Assuming a partially known feature map and a partially known occupancy map, the following steps are taken:

1. Start with a completely unknown vehicle location (infinite covariance) and a partially known feature map $\mathbf{P}_{mm}$
2. Using the occupancy map, estimate which of the known features are observable from the location in question
3. Keep the vehicle stationary and estimate the vehicle covariance $\mathbf{P}^k_{vv}$ after k observations of visible features

In the derivation of $\mathbf{P}_{vv}(k)$ it is convenient to use the information filter form of the Kalman filter [13]. The initial information matrix for a "teleported" vehicle is

$$\mathbf{P}^{-1}_{ox} = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{P}^{-1}_{om} \end{bmatrix} \tag{8}$$

where $\mathbf{P}_{om}$ is the covariance matrix of the landmarks *observable* from the location in question. The unobservable features do not influence the vehicle covariance, greatly reducing the size of the problem. The zero sub-matrix in the upper-left corner reflects the lack of information (infinite variance) about the initial vehicle location. The off-diagonal terms are also set to zero. The update equation is

$$\mathbf{P}^{-1}(k \mid k) = \mathbf{P}^{-1}(k \mid k-1) + \begin{bmatrix} \mathbf{H}^T_v \\ \mathbf{H}^T_m \end{bmatrix} \mathbf{R}^{-1} \begin{bmatrix} \mathbf{H}^T_v & \mathbf{H}^T_m \end{bmatrix} \tag{9}$$

where $\mathbf{H}_v$ and $\mathbf{H}_m$ are the appropriately sized Jacobians of the generally non-linear observation function, and $\mathbf{R}$ is the variance of the sensor noise, assumed to be non-singular. Because the localizability metric is defined for a stationary vehicle, the linearization of the observation function has to be performed only once and the information state vector does not have to be updated. For the same reason, the process noise is zero ($\mathbf{Q}(k) = \mathbf{0}$) and the prediction stage is simply

$$\mathbf{P}^{-1}(k \mid k-1) = \mathbf{P}^{-1}(k-1 \mid k-1) \tag{10}$$

Following [3], the Equations 8 through 10 are combined to find the information matrix after k observations

$$\mathbf{P}^{-1}(k \mid k) = \mathbf{P}^{-1}_{ox} + k \begin{bmatrix} \mathbf{H}^T_v\mathbf{R}^{-1}\mathbf{H}_v & \mathbf{H}^T_v\mathbf{R}^{-1}\mathbf{H}_m \\ \mathbf{H}^T_m\mathbf{R}^{-1}\mathbf{H}_v & \mathbf{H}^T_m\mathbf{R}^{-1}\mathbf{H}_m \end{bmatrix} \tag{11}$$

Invoking the matrix inversion lemma, the vehicle covariance after k observations can be found. We are interested primarily in the sub-matrix corresponding to the vehicle states $\mathbf{P}_{vv}$

$$\mathbf{P}^k_{vv} = \left[\mathbf{H_m}^T\mathbf{R}^{-1}\left[\tfrac{1}{k}\mathbf{I} + \mathbf{H}_m\mathbf{P}_{om}\mathbf{H}^T_m\mathbf{R}^{-1}\right]^{-1}\mathbf{H}_v\right]^{-1} \tag{12}$$

In the limit for $k \to \infty$, the vehicle covariance reaches steady state at

$$\mathbf{P}^\infty_{vv} = \left[\mathbf{H}^T_v\left[\mathbf{H}_m\mathbf{P}_{om}\mathbf{H}^T_m\right]^{-1}\mathbf{H}_v\right]^{-1} \tag{13}$$

If the vehicle were to re-localize in practice at the given location, its true steady-state re-localized uncertainty would be not higher than the variance in Equation 13. The upper bound property of $\mathbf{P}^\infty_{vv}$ follows from the result that the map covariances in the SLAM algorithm cannot increase [3].

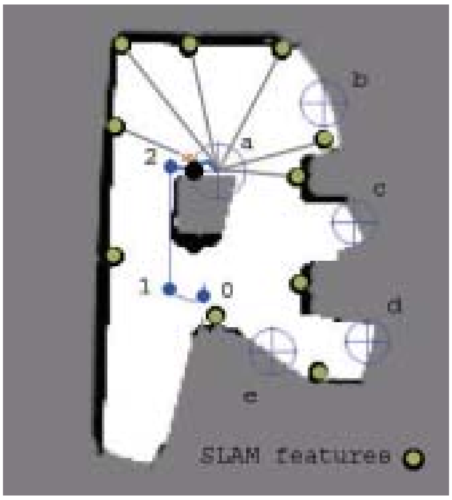


Figure 3: *Simulated path of an exploration mission*

5 Simulation

This section describes the simulations performed to validate the developments of this paper.

5.1 Implementation Details

Simulations were conducted in Stage [19]– a realistic robot simulator based on Player server [7]. The simulated environment included a Pioneer 2 indoor robot equipped with a laser range finder.

5.2 Results

Figure 3 shows an instance of selecting the next destination. Frontier centroids were found first.The top five frontiers according to the total utility measure were identified, marked with circles and designated (a) through (e). The best frontier (a) is selected and a path to it from the current location is found. Landmarks which are expected to be visible from the selected frontier are connected with gray lines. The path of the vehicle up to this moment is shown with increasing numbers starting at zero..

6 Conclusions

This paper advocates an integrated approach to mobile robotic exploration. The results of the experiments are encouraging. Among the areas of current and future research are

- experimental validation with Pioneer II robots;
- replacement of the occupancy grid representation with a functional one to improve scalability and allow tighter integration with the SLAM algorithm;
- direct use of the total utility in place of separate steps of proposing and evaluating potential destinations;
- extension of the current technics to a multi-robot system which offers cooperation opportunities in both localization and mapping.

References

[1] A. Bennett and J.J. Leonard. A behavior-based approach to adaptive feature mapping with autonomous underwater vehicles. *IEEE J. of Oceanic Engineering*, 25(2):213–226, 2000.

[2] T.M. Cover and J.A. Thomas. *Elements of Information Theory*. Wiley Series in Telecommunications. Wiley, New York, 1991.

[3] M.W.M.G. Dissanayake, P. Newman, S. Clark, H.F. Durrant-Whyte, and M. Csobra. A solution to the simultaneous localization and map building (SLAM) problem. *IEEE Trans. on Robotics and Automation*, 17(3):229–241, 2001.

[4] A. Elfes. Robot navigation: Integrating perception, environmental constraints and task execution within a probabilistic framework. In *Reasoning with Uncertainty in Robotics. International Workshop (RUR '95)*, pages 93–129, Amsterdam, Netherlands, 1995.

[5] H.J.S. Feder, J.J. Leonard, and C.M. Smith. Adaptive mobile robot navigation and mapping. *International J. of Robotics Research*, 18(7):650–668, 1999.

[6] D. Fox, W. Burgard, and S. Thrun. Active Markov localization for mobile robots. *Robotics and Autonomous Systems*, 1998.

[7] B.P. Gerkey, R.T. Vaughan, K. Sty, A. Howard, G.S. Sukhatme, and M.J. Mataric. Most valuable player: A robot device server for distributed control. In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS '01)*, pages 1226–1231, Maui, Hawaii, USA, 2001.

[8] L.P. Kaelbling, A.R. Cassandra, and J.A. Kurien. Acting under uncertainty: Discrete bayesian models for mobile robot navigation. In *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1996.

[9] J.-C. Latombe. *Robot Motion Planning*. The Kluwer International Series in Engineering and Computer Science. Kluwer Academic Publishers, Boston, 1991.

[10] D. Lee. *The Map-Building and Exploration Strategies of a Simple Sonar-Equipped Robot : An Experimental, Quantitative Evaluation*. Distinguished Disserations in Computer Science. Cambridge University Press, Cambridge ; New York, 1996.

[11] J.J. Leonard and H.F. Durrant-Whyte. Simultaneous map building and localisation for an autonomous robot. In *IEEE/RSJ International Workshop on Intelligent Robots and Systems (IROS '91)*, pages 1442–1447, Osaka, Japan, 1991.

[12] J. Manyika and H.F. Durrant-Whyte. *Data Fusion and Sensor Management: A Decentralized Information-Theoretic Approach*. Ellis Horwood Series in Electrical and Electronic Engineering. Ellis Horwood, 1994.

[13] P.S. Maybeck. *Stochastic Models, Estimation and Control*. Mathematics in Science and Engineering. Academic Press, New York, 1979.

[14] S. Moorehead, R. Simmons, and W.R.L. Whittaker. Autonomous exploration using multiple sources of information. In *IEEE International Conference on Robotics and Automation (ICRA '01)*, 2001.

[15] N. Roy, W. Burgard, D. Fox, and S. Thrun. Coastal navigation - mobile robot navigation with uncertainty in dynamic environments. In *the IEEE International Conference on Robotics and Automation (ICRA '99)*, volume 1, Detroit, MI, USA, 1999.

[16] A. Schultz, W. Adams, and B. Yamauchi. Integrating exploration, localization, navigation and planning with a common representation. *Autonomous Robots*, 6(3):293–308, 1999.

[17] R.G. Simmons, D. Apfelbaum, W. Burgard, D. Fox, M. Moors, S. Thrun, and H. Younes. Coordination for multi-robot exploration and mapping. In *AAAI National Conference on Artificial Intelligence*, pages 852–858, Austin, TX, USA, 2000.

[18] S. Thrun, W. Burgard, and D. Fox. A probabilistic approach to concurrent mapping and localization for mobile robots. *Machine Learning and Autonomous Robots*, 31(1-3):29–53, 1998.

[19] Richard T. Vaughan. Stage: A multiple robot simulator. Technical Report IRIS-00-394, Institute for Robotics and Intelligent Systems, University of Southern California, 2000.

[20] B. Yamauchi. A frontier based approach for autonomous exploration. In *IEEE International Symposium on Computational Intelligence in Robotics and Automation*, Monterey, CA, USA, 1997.