Proceedings of the 2006 IEEE International Conference on Robotics and Automation
Orlando, Florida - May 2006

Recursive Bayesian Search-and-Tracking Using Coordinated UAVs for Lost Targets

Tomonari Furukawa†, Frederic Bourgault††, Benjamin Lavis† and Hugh F. Durrant-Whyte††
ARC Centre of Excellence for Autonomous Systems
†School of Mechanical and Manufacturing Engineering, The University of New South Wales, Sydney 2052 Australia
††Australian Centre for Field Robotics, The University of Sydney, Sydney 2006 Australia
Email: {t.furukawa, f.bourgault, b.lavis, hugh}@cas.edu.au

***Abstract*— This paper presents a coordinated control technique that allows heterogeneous vehicles to autonomously search for and track multiple targets using recursive Bayesian filtering. A unified sensor model and a unified objective function are proposed to enable search-and-tracking (SAT) within the recursive Bayesian filter framework. The strength of the proposed technique is that a vehicle can switch its task mode between search and tracking while maintaining and using information collected during the operation. Numerical results first show the effectiveness of the proposed technique when a found target becomes lost and must be searched for again. The proposed technique was then applied to a practical marine search-and-rescue (SAR) scenario where heterogeneous vehicles coordinated to search for and track multiple targets. The result demonstrates the applicability of the technique to real search world scenarios.**

I. Introduction

Consider a possible marine disaster scenario where a ship is sinking in a storm and the crew and passengers need to be rescued before their survival expectancy vanishes. The only actions that the crew and passengers can take are to send a last mayday from the sinking ship, get on life rafts and wait, drifting due to strong winds and high waves, until a rescue team arrives. The goal of SAR in this scenario is to search for and find life rafts and rescue victims as efficiently and safely as possible.

A practically implementable robotics approach to this scenario is the use of a team of fast autonomous Unmanned Aerial Vehicles (UAVs) together with a team of autonomous helicopters with rescuers onboard. The cooperation of autonomous vehicles including fast UAVs can dramatically improve the efficiency of the search operation. In addition, if the UAVs can circle around the found life rafts to provide high beam lights and environmental information to the rescue helicopter while rescuers are retrieving victims, the efficiency and safety of the rescue operation will also improve significantly.

The autonomous control problem of this class is stated as follows: given initial estimates of multiple moving targets, how would heterogeneous autonomous aerial vehicles cooperatively search for the targets, track the found targets to achieve mission objectives, such as circling and following, and repetitively continue the SAT operation until the operation is complete on all the targets? The targets are moving under uncertainty, so that there is also a chance that a vehicle, having found a target, loses it and must resume searching while approaching the found target. The primary concern of the paper thus lies in the coordinated control which allows heterogeneous vehicles to autonomously search for and track multiple targets.

SAT studies have a long history. The primary issues regarding SAT were posed by B. O. Koopman and his colleagues in the Antisubmarine Warfare Operations Research Group (ASWORG) during World War II [1], but the two studies have evolved rather independently. The initial work on search was carried out by simplifying the search problem as an area coverage problem and applying area coverage techniques accordingly. The search study later has expanded with the introduction of the probability of detection as a probabilistic measure [2], [3], [4]. Bourgault, et al. [5], [6], [7] formulated a Bayesian approach for a target whose prior distribution and probabilistic motion model are known and generalized the approach for multi-vehicle search. Similarly, the tracking study commenced with a simple feedback motion tracking algorithm, and has evolved with the developments of a number of recursive filtering techniques. The filtering techniques include the extended Kalman Filter (EKF) [8] and Unscented Kalman Filter (UKF) [9], Gauss quadrature methods [10], grid-based methods, and Monte Carlo of particle filter methods [11]. Through the above individual studies, several unified SAT approaches have recently appeared, particularly as a result of rigorous discussion on multi-objective missions led by the UAV community [12], but the approaches are often either a collection of independent techniques or do not make full use of the knowledge available on the targets.

This paper presents a coordinated control technique that enables heterogeneous vehicles to autonomously search for and track multiple targets using recursive Bayesian filtering. Based on a filtering method, the proposed technique stochastically updates the Probability Density Functions (PDFs) of the target from their prior distributions and probabilistic motion models. In addition, the use of the recursive Bayesian filter allows nonlinear probabilistic target models to be handled in the presence of non-Gaussian noise as demonstrated in [5]. A unified sensor model and a unified objective function are proposed to enable both search and tracking within the recursive Bayesian framework. The further strength of the proposed technique is that a vehicle can switch its task mode between search and tracking without discarding information collected during the operation.

The paper is organized as follows. The following section reviews the recursive Bayesian filtering that predicts and updates the PDF of each target. In Sec. III, a coordinated control strategy to enable SAT in the recursive Bayesian filter framework is presented. Section IV demonstrates the efficacy of the proposed technique through numerical examples and conclusions are summarized in the final section.

II. Recursive Bayesian Filtering

This section describes the fundamentals of the recursive Bayesian filter when a single autonomous vehicle is concerned with a single target.

A. Target and Sensor Platform Models

Consider a target t to search for and track, the motion of which is discretely given by

$$\mathbf{x}_{k+1}^t = \mathbf{f}^t\left(\mathbf{x}_k^t, \mathbf{u}_k^t, \mathbf{w}_k^t\right) \tag{1}$$

where $\mathbf{x}_k^t \in \mathcal{X}^t$ is the state of the target at time step k, which in general describes its two-dimensional position but could also include other state variables such as attitude and velocity, $\mathbf{u}_k^t \in \mathcal{U}^t$ is the set of control inputs of the target, and $\mathbf{w}_k^t \in \mathcal{W}^t$ is the "system noise" of the target which includes external inputs such as wind and current in the case of a marine SAR scenario. The target is searched and tracked by an autonomous vehicle s, the state of which is assumed to be accurately known by the use of global sensors such as GPS, a compass and an IMU. The motion model is thus given by

$$\mathbf{x}_{k+1}^s = \mathbf{f}^s\left(\mathbf{x}_k^s, \mathbf{u}_k^s\right) \tag{2}$$

where $\mathbf{x}_k^s \in \mathcal{X}^s$ and $\mathbf{u}_k^s \in \mathcal{U}^s$ represent the state and control input of the vehicle, respectively. The vehicle carries a sensor to look for and observe a target. The observation from the sensor platform s, ${}^s\mathbf{z}_k \in \mathcal{X}^t$, is subject to noise $\mathbf{v}_k^s$ and given by

$${}^s\mathbf{z}_k = \mathbf{h}^s\left(\mathbf{x}_k^s, \mathbf{v}_k^s\right) \tag{3}$$

where $\mathbf{v}_k^s$ represents the observation noise. Note here that the terms "sensor platform" and "autonomous vehicle" are used interchangeably in this paper as the vehicle is assumed to carry only one sensor for observation.

B. Recursive Bayesian Filtering

Recursive Bayesian filtering forms a basis to the filtering of nonlinear non-Gaussian models. Computational approaches that enable Recursive Bayesian filtering include grid-based methods, Monte Carlo methods, Gauss quadrature methods and particle filter methods. Requiring computationally intensive numerical integration, they have become popular only recently, following the exponential increase in computer power.

Let a sequence of states of the sensor platform s and a sequence of observations by the sensor platform from time step 1 to time step k be $\tilde{\mathbf{x}}_{1:k}^s \equiv \{\tilde{\mathbf{x}}_i^s | \forall i \in \{1, ..., k\}\}$ and ${}^s\tilde{\mathbf{z}}_{1:k} \equiv \{{}^s\tilde{\mathbf{z}}_i | \forall i \in \{1, ..., k\}\}$, respectively. Note here that $\tilde{(\cdot)}$ represents an instance of variable $(\cdot)$. Given a prior distribution of the target $p\left(\tilde{\mathbf{x}}_0^t\right)$ and sequences of states $\tilde{\mathbf{x}}_{1:k}^s$ and observations ${}^s\tilde{\mathbf{z}}_{1:k}$, the PDF of the target at any time step k, $p\left(\mathbf{x}_k^t | {}^s\tilde{\mathbf{z}}_{1:k}, \tilde{\mathbf{x}}_{1:k}^s\right)$, can be estimated recursively through the two stage equations, update and prediction:

1) **Update**: The update equation computes the posterior density $p\left(\mathbf{x}_k^t | {}^s\tilde{\mathbf{z}}_{1:k}, \tilde{\mathbf{x}}_{1:k}^s\right)$ given the corresponding state estimated with the observations up to the previous time step $p\left(\mathbf{x}_k^t | {}^s\tilde{\mathbf{z}}_{1:k-1}, \tilde{\mathbf{x}}_{1:k}^s\right)$ and a new observation ${}^s\tilde{\mathbf{z}}_k$. The equation is derived by applying formulae for marginal distribution and conditional independence and given by

$$\begin{aligned} p&\left(\mathbf{x}_k^t | {}^s\tilde{\mathbf{z}}_{1:k}, \tilde{\mathbf{x}}_{1:k}^s\right) \\ &= \frac{p\left({}^s\tilde{\mathbf{z}}_k | \mathbf{x}_k^t, \tilde{\mathbf{x}}_k^s\right) p\left(\mathbf{x}_k^t | {}^s\tilde{\mathbf{z}}_{1:k-1}, \tilde{\mathbf{x}}_{1:k}^s\right)}{p\left({}^s\tilde{\mathbf{z}}_k | {}^s\tilde{\mathbf{z}}_{1:k-1}, \tilde{\mathbf{x}}_{1:k}^s\right)} \\ &= \frac{p\left({}^s\tilde{\mathbf{z}}_k | \mathbf{x}_k^t, \tilde{\mathbf{x}}_k^s\right) p\left(\mathbf{x}_k^t | {}^s\tilde{\mathbf{z}}_{1:k-1}, \tilde{\mathbf{x}}_{1:k}^s\right)}{\int p\left({}^s\tilde{\mathbf{z}}_k | \mathbf{x}_k^t, \tilde{\mathbf{x}}_{1:k}^s\right) p\left(\mathbf{x}_k^t | {}^s\tilde{\mathbf{z}}_{1:k-1}, \tilde{\mathbf{x}}_{1:k}^s\right) d\mathbf{x}_{k-1}^t} \end{aligned} \tag{4}$$

where $p\left({}^s\mathbf{z}_k | \mathbf{x}_k^t, \mathbf{x}_k^s\right)$ represents the observation likelihood given knowledge of the current state, or the sensor model, defined by Eq. (3). Notice that the update at $k = 1$ is carried out by letting $p\left(\mathbf{x}_k^t | {}^s\tilde{\mathbf{z}}_{1:k-1}, \tilde{\mathbf{x}}_{1:k}^s\right) = p\left(\tilde{\mathbf{x}}_0^t\right)$.

2) **Prediction**: The prediction step computes the PDF of the next state $p\left(\mathbf{x}_{k+1}^t | {}^s\tilde{\mathbf{z}}_{1:k}, \tilde{\mathbf{x}}_{1:k}^s\right)$ from the PDF in the current time step $p\left(\mathbf{x}_k^t | {}^s\tilde{\mathbf{z}}_{1:k}, \tilde{\mathbf{x}}_{1:k}^s\right)$. The prediction is carried out by Chapman-Kolmogorov equation, which is better known as Total Probability Theorem:

$$\begin{aligned} p&\left(\mathbf{x}_{k+1}^t | {}^s\tilde{\mathbf{z}}_{1:k}, \tilde{\mathbf{x}}_{1:k}^s\right) \\ &= \int p\left(\mathbf{x}_{k+1}^t | \mathbf{x}_k^t\right) p\left(\mathbf{x}_k^t | {}^s\tilde{\mathbf{z}}_{1:k}, \tilde{\mathbf{x}}_{1:k}^s\right) d\mathbf{x}_k^t \end{aligned} \tag{5}$$

where $p\left(\mathbf{x}_{k+1}^t | \mathbf{x}_k^t\right)$ is a probabilistic Markov motion model defined by Eq. (1) which maps the probability of transition from the current state $\mathbf{x}_k^t$ to the next state $\mathbf{x}_{k+1}^t$.

III. Bayesian Search and Tracking

This section extends recursive Bayesian filtering to multi-vehicle cases and formulates how multiple vehicles can search for and track multiple targets autonomously in a cooperative manner. The first subsection describes the recursive Bayesian filter framework whereas a unified sensor model and a unified objective function to enable SAT within the recursive Bayesian filter framework are proposed in the following subsections.

A. Recursive Bayesian Filter Framework

One form of coordination of multiple sensor platforms appears in the sensor data fusion. Let the states of n_s sensor platforms and their observations obtained at time k be $\tilde{\mathbf{x}}_k^s = \{\tilde{\mathbf{x}}_k^{s_i} | \forall i \in \{1, \ldots, n_s\}\}$ and ${}^s\tilde{\mathbf{z}}_k = \{{}^{s_i}\tilde{\mathbf{z}}_k | \forall i \in \{1, \ldots, n_s\}\}$, respectively. As the observation likelihood for sensor platform s_i given the state of target t_j is $p\left({}^{s_i}\mathbf{z}_k | \mathbf{x}_k^{t_j}, \mathbf{x}_k^{s_i}\right)$, the conditional independence of observation yields the multi-sensor observation likelihood as

$$p\left({}^s\tilde{\mathbf{z}}_k | \mathbf{x}_k^{t_j}, \tilde{\mathbf{x}}_k^s\right) = \prod_{i=1}^{n_s} p\left({}^{s_i}\tilde{\mathbf{z}}_k | \mathbf{x}_k^{t_j}, \tilde{\mathbf{x}}_k^{s_i}\right). \tag{6}$$

Assume that the sensors are fully connected with no communication failure and delay. The sensor platform i then can receive $p\left({}^{s_m}\tilde{\mathbf{z}}_k|\mathbf{x}_k^{t_j}, \tilde{\mathbf{x}}_k^{s_m}\right)$,$\forall m \neq i$, and decentrally construct Eq. (6). The substitution of Eq. (6) into Eq. (4) gives the multi-sensor update, and the prediction is performed accordingly by Eq. (5). This general decentralized data fusion approach proposed in [6] forms the recursive Bayesian filter framework and is used to introduce the unified sensor model and the unified objective function for SAT.

B. Unified Sensor Model

Let the probability of detecting a target by the sensor platform s_i be $p_d\left(\mathbf{x}_k^t|\mathbf{x}_k^{s_i}\right)$ where $0 \leq p_d\left(\mathbf{x}_k^t|\mathbf{x}_k^{s_i}\right) \leq 1$. The subspace of target in which a target is detectable by the sensor platform s_i is defined as:

$$
{}^{s_i}X_d^t\left(\mathbf{x}_k^{s_i}\right) = \left\{\mathbf{x}_k^t|\epsilon < p_d\left(\mathbf{x}_k^t|\mathbf{x}_k^{s_i}\right) \leq 1\right\} \tag{7}
$$

where ϵ is a positive threshold value. This means that if there exists a target $\mathbf{x}_k^{t_j} \in {}^{s_i}X_d^t$, then the sensor platform s_i observes the target as ${}^{s_i}\mathbf{z}_k = {}^{s_i}\tilde{\mathbf{z}}_k^{t_j}$. In this case, the accuracy of the target location depends on the intensity of return signals to the sensor, which is dependent not only on the state of the target relative to the sensor platform but also the properties of the target, $\pi^{t_j} = \tilde{\pi}^{t_j}$, such as reflectivity. As a result, a sensor model unified for SAT is stated as

$$
p\left({}^{s_i}\tilde{\mathbf{z}}_k|\mathbf{x}_k^{t_j}, \tilde{\mathbf{x}}_k^{s_i}\right) = \begin{cases} 1 - p_d\left(\mathbf{x}_k^t|\tilde{\mathbf{x}}_k^{s_i}\right) & \nexists \mathbf{x}_k^{t_j} \in {}^{s_i}X_d^t \\ p\left({}^{s_i}\tilde{\mathbf{z}}_k^{t_j}|\mathbf{x}_k^{t_j}, \tilde{\mathbf{x}}_k^{s_i}, \tilde{\pi}^{t_j}\right) & \exists \mathbf{x}_k^{t_j} \in {}^{s_i}X_d^t \end{cases} \tag{8}
$$

The upper formula is used when no target is detected whereas the lower formula is used when at least one target is detected. The next subsection will deal with how to relate the sensor model to the autonomous SAT.

C. Unified Objective Function

The objective of sensor platform s_i is to find its optimal control actions at n_k-step lookahead:

$$
J^{s_i}\left(\mathbf{u}_{k:k+n_k-1}^{s_i}|\mathbf{x}_k^{s_i}\right) \rightarrow \max_{\mathbf{u}_{k:k+n_k-1}^{s_i}} \tag{9}
$$

where the objective function in canonical form is given by

$$
\begin{aligned} & J^{s_i}\left(\mathbf{u}_{k:k+n_k-1}^{s_i}|\mathbf{x}_k^{s_i}\right) \\ & \quad = \Phi^{s_i}\left(\mathbf{x}_{k+n_k}^{s_i}\right) + \sum_{\kappa=0}^{n_k-1} \mathcal{L}^{s_i}\left(\mathbf{x}_{k+\kappa}^{s_i}, \mathbf{u}_{k+\kappa}^{s_i}\right)\Delta t. \end{aligned} \tag{10}
$$

In the function, $\Phi^{s_i}(\cdot)$ and $\mathcal{L}^{s_i}(\cdot)$ are the terminal objective and the integral objective, respectively. The control actions to take are concerned with the possible target locations or the target PDFs. The PDF of target t_j at time step $k+\kappa+1$ given the sequence of observations up to time step k, $p\left(\mathbf{x}_{k+\kappa+1}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right)$, can be recursively predicted by Chapman-Kolomogorov equation as

$$
\begin{aligned} & p\left(\mathbf{x}_{k+\kappa+1}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right) \\ & \quad = \int p\left(\mathbf{x}_{k+\kappa+1}^{t_j}|\mathbf{x}_{k+\kappa}^{t_j}\right) p\left(\mathbf{x}_{k+\kappa}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right) d\mathbf{x}_{k+\kappa}^{t_j}. \end{aligned} \tag{11}
$$

Let the PDFs in Eq. (11) be defined accurately for further convenience and associate them to the states of the sensor platforms:

$$
\begin{aligned} p\left(\mathbf{x}_{k+\kappa+1}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right) & \equiv p_{\mathbf{x}_{k+\kappa+1}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}}\left(g\left(\mathbf{x}_{k+\kappa+1}^{s_i}\right)|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right) \\ p\left(\mathbf{x}_{k+\kappa+1}^{t_j}|\mathbf{x}_{k+\kappa}^{t_j}\right) & \equiv p_{\mathbf{x}_{k+\kappa+1}^{t_j}|\mathbf{x}_{k+\kappa}^{t_j}}\left(g\left(\mathbf{x}_{k+\kappa}^{s_i}\right)|g\left(\mathbf{x}_{k+\kappa}^{s_i}\right)\right) \\ p\left(\mathbf{x}_{k+\kappa}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right) & \equiv p_{\mathbf{x}_{k+\kappa}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}}\left(g\left(\mathbf{x}_{k+\kappa}^{s_i}\right)|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right) \end{aligned}
$$

where $g : \mathcal{X}^s \rightarrow \mathcal{X}^t$ screens the state variables of sensor platform that are not used to represent the location of target. As the $p_{\mathbf{x}_{k+\kappa+1}^{t_j}|\mathbf{x}_{k+\kappa}^{t_j}}\left(g\left(\mathbf{x}_{k+\kappa+1}^{s_i}\right)|g\left(\mathbf{x}_{k+\kappa}^{s_i}\right)\right)$ is represented as a function of $\mathbf{u}_{k+\kappa}^{s_i}$:

$$
\begin{aligned} & p_{\mathbf{x}_{k+\kappa+1}^{t_j}|\mathbf{x}_{k+\kappa}^{t_j}}\left(g\left(\mathbf{x}_{k+\kappa+1}^{s_i}\right)|g\left(\mathbf{x}_{k+\kappa}^{s_i}\right)\right) \\ & \quad = p_{\mathbf{x}_{k+\kappa+1}^{t_j}|\mathbf{x}_{k+\kappa}^{t_j}}\left(g\left(\mathbf{f}^s\left(\mathbf{x}_{k+\kappa}^{s_i}, \mathbf{u}_{k+\kappa}^{s_i}\right)\right)|g\left(\mathbf{x}_{k+\kappa}^{s_i}\right)\right), \end{aligned} \tag{12}
$$

the recursive use of Eq. (11) indicates that $p_{\mathbf{x}_{k+\kappa+1}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}}\left(g\left(\mathbf{x}_{k+\kappa+1}^{s_i}\right)|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right)$ is governed by the sequence of control actions $\mathbf{u}_{k:k+\kappa}^{s_i}$.

As a consequence, the unified SAT objective function for sensor platform s_i can be formulated in the canonical form (10) as follows:

$$
\begin{aligned} & J^{s_i}\left(\mathbf{u}_{k:k+n_k-1}^{s_i}|\mathbf{x}_k^{s_i}\right) \\ & \quad = \sum_{\kappa=0}^{n_k-1}\sum_{j=1}^{n_t} w^{t_j} p_{\mathbf{x}_{k+\kappa+1}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}}\left(g\left(\mathbf{x}_{k+\kappa+1}^{s_i}\right)|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right)\Delta t \end{aligned} \tag{13}
$$

where w^{t_j} is a weighting factor for target t_j. This corresponds to $\Phi^{s_i}\left(\mathbf{x}_{k+n_k}^{s_i}\right) = 0$ and $\mathcal{L}\left(\mathbf{x}_{k+\kappa}^{s_i}, \mathbf{u}_{k+\kappa}^{s_i}\right) = \sum_{j=1}^{t_j} w^{t_j} p_{\mathbf{x}_{k+\kappa+1}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}}\left(g\left(\mathbf{x}_{k+\kappa+1}^{s_i}\right)|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right)$. Within the form of this unified objective function, SAT objective functions are described as follows.

1) Search Objective: Search is carried out when no target is detected. The search objective is thus formulated as the sum of all the target PDFs:

$$
\begin{aligned} & J^{s_i}\left(\mathbf{u}_{k:k+n_k-1}^{s_i}|\mathbf{x}_k^{s_i}\right) \\ & \quad = \sum_{\kappa=0}^{n_k-1}\sum_{j=1}^{n_t} p_{\mathbf{x}_{k+\kappa+1}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}}\left(g\left(\mathbf{x}_{k+\kappa+1}^{s_i}\right)|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right)\Delta t. \end{aligned} \tag{14}
$$

Note that w^{t_j} is dropped in search and the target PDFs are equally weighted. This is because it is meaningless to allocate targets to vehicles when targets are not found.

2) Tracking Objective: Tracking is carried out when at least one target has been detected. The tracking objective is thus formulated as

$$
\begin{aligned} & J^{s_i}\left(\mathbf{u}_{k:k+n_k-1}^{s_i}|\mathbf{x}_k^{s_i}\right) \\ & \quad = \sum_{\kappa=0}^{n_k-1}\sum_{\mathbf{x}^{t_j} \in X^t} w^{t_j} p_{\mathbf{x}_{k+\kappa+1}^{t_j}|{}^{s_i}\tilde{\mathbf{z}}_{1:k}}\left(g\left(\mathbf{x}_{k+\kappa+1}^{s_i}\right)|{}^{s_i}\tilde{\mathbf{z}}_{1:k}\right)\Delta t. \end{aligned} \tag{15}
$$

Weight w^{t_j} or which target that sensor platform s_i should track is determined by considering information available on all the targets. One simple method may be to assign the nearest target. Such target assignment or vehicle allocation is not further investigated in this paper as it is outside the scope of the paper.

IV. Numerical Examples

This section presents three scenarios in order to demonstrate the use of recursive Bayesian filtering for autonomous SAT. The first scenario, consisting of a single autonomous vehicle and a single target, acts as a proof of concept and to highlight the benefits of gathering the SAT processes under the recursive Bayesian filter framework. The second scenario subsequently deals with the cooperation of three vehicles in searching for and tracking a single lost target. Finally, a potential application of the proposed technique is presented where a heterogenous team autonomously searches for and tracks multiple lost targets.

The target models commonly used in the scenarios are those that move on a horizontal plane and given by

$$\begin{aligned} x_{k+1}^{t_j} &= x_k^{t_j} + \Delta t \cdot v_k^{t_j} \cos \gamma_k^{t_j} \\ y_{k+1}^{t_j} &= y_k^{t_j} + \Delta t \cdot v_k^{t_j} \sin \gamma_k^{t_j} \end{aligned} \quad (16)$$

where $v_k^{t_j}$ and $\gamma_k^{t_j}$ are the velocity and direction of the disturbance such as wind and current, each subject to a Gaussian noise, and Δt is a time increment. The prior distribution of each target is also given by a Gaussian noise. The sensor platforms are also assumed to move on a horizontal plane for simplicity and given by

$$\begin{aligned} x_{k+1}^{s_i} &= x_k^{s_i} + \Delta t \cdot v_k^{s_i} \cos \gamma_k^{s_i} \\ y_{k+1}^{s_i} &= y_k^{s_i} + \Delta t \cdot v_k^{s_i} \sin \gamma_k^{s_i} \\ \theta_{k+1}^{s_i} &= \theta_k^{s_i} + \Delta t \cdot \alpha \gamma_k^{s_i} \end{aligned} \quad (17)$$

where $v_k^{s_i}$ and $\gamma_k^{s_i}$ are the velocity and turn of the sensor platform. The probability of detection $p_d \left(\mathbf{x}_k^t | \tilde{\mathbf{x}}_k^{s_i} \right)$ is given by a Gaussian noise with zero mean and a constant covariance, whereas the observation likelihood $p\left({}^{s_i}\tilde{\mathbf{z}}_k^{t_j} | \mathbf{x}_k^{t_j}, \tilde{\mathbf{x}}_k^{s_i}, \tilde{\pi}^{t_j} \right)$ is given by a Gaussian noise with zero mean and a covariance proportional to the distance between the sensor platform and the target. Due to the limitation of space in the paper, the numerical examples show only results with one-step lookahead ($n_k = 1$).

A. Single-Vehicle Single-Target

In the single-vehicle single-target example, the vehicle starts moving from the position $\mathbf{x}_0^s = [-5, -5]^\top$ to search for a target whose distribution by prior belief is centered at $\mathbf{x}_{m0}^t = [-3.5, 0.5]^\top$ but the actual target is initially located at $\mathbf{x}_0^t = [-3.5, 0.0]^\top$. Importantly, the target is located outside the detection range of the sensor so that the vehicle starts moving in search mode.

Figure 1 shows the result of SAT in this example. In the figure, the simulated targets are marked with a yellow arrow and the target's path marked red. The search vehicles are marked with a cyan dot and their paths marked magenta. For time-steps $k = 1 : 5$ the vehicle is in search mode. At time step $k = 6$ the target comes within the sensor's detection range and is found by the vehicle. The update stage then produces

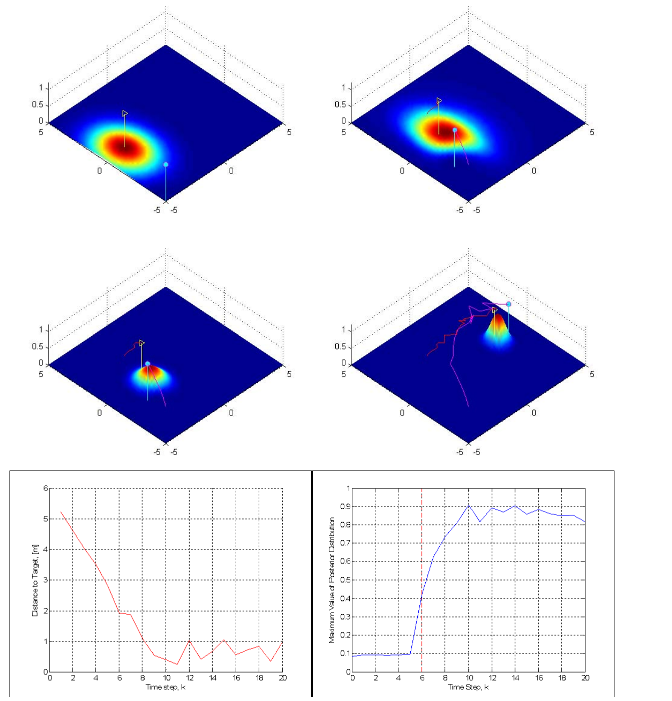


Fig. 1. The Single-Vehicle Single-Target system (a; upper-left) $k = 1$; (b; upper-right) $k = 5$; (c; middle-left) $k = 6$; (d; middle-right) $k = 20$; (e; lower-left) Distance between the vehicle and target at each time step; (f; lower-right) Peak values of the target's PDF.

a "peakier" and more constricted PDF as shown in Fig. 1(c) due to the use of the lower formula of the sensor model (8). In subsequent time steps the sensor successfully observes the target and the vehicle tracks the target by closing on it and following it.

The distance between the vehicle and the target at each time step is shown in Fig. 1(e). It can be seen that for $k = 10 : 20$, the vehicle is following the target, maintaining a distance from the target ranging between 0.2 and 1.1 and averaging 0.62. Figure 1(f) shows the change in the peak value of the target's PDF as a result of the detection and successful tracking of the target. When $k = 1 : 5$, the vehicle is in search mode, and the peak PDF value lies in the range [0.08, 0.10]. The peak value then starts increasing rapidly with the detection event at $k = 6$ and keep increasing until the vehicle approaches close enough to the target at $k = 10$. When $k = 10 : 20$, the vehicle is following the target, and here the peak value of the PDF plateaus, averaging 0.87 and remaining in the range [0.82, 0.91].

In this second example, the parameters used as in the initial example were repeated, with the exception that the randomly generated noise acting on the target's actual y-coordinate position was increased. Thus the motion of the target in the y direction was more erratic and consequently more difficult to predict than in the original example. Figure 2 shows the resulting motion where the sensor is again in search mode for time steps $k = 1 : 5$ and that an observation is made at time step $k = 6$. However at time step $k = 7$ the erratic motion of the target pushes it beyond the range of the sensor, such as may occur for a life raft afloat in a tumultuous sea and driven by swirling winds. Failing to detect the target at time step $k = 7$, the search vehicle again assumes search mode and the appropriate update is formed. Even though the target has been lost, given the prior observation, the update remains relatively peaky, maintaining information the target PDF (2(f)). This subsequently allows the search vehicle to reacquire the target and successfully tracks the target (Fig. 2(d-e)).

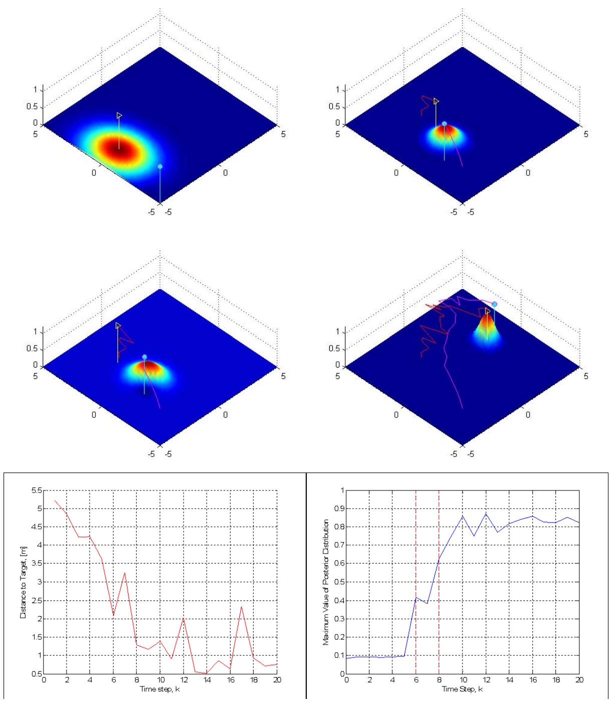


Fig. 2. The Single-Vehicle Single-Target system when target is subject to noisier inputs (a; upper-left) $k = 1$; (b; upper-right) $k = 6$; (c; middle-left) $k = 7$; (d; middle-right) $k = 20$; (e; lower-left) Distance between the vehicle and target at each time step; (f; lower-right) Peak values of the target's PDF.

B. Three-Vehicle Single-Target

In the three-vehicle single-target example, Vehicles 1, 2 and 3 start moving from positions $\mathbf{x}_0^{s_1} = [-5, -4]^\top$, $\mathbf{x}_0^{s_2} = [-5, -2]^\top$ and $\mathbf{x}_0^{s_3} = [-5, 0]^\top$. The target has a distribution by prior belief centered at $\mathbf{x}_{m_0}^t = [-1.0, 1.5]^\top$ but is actually located beyond the range of the sensors at $\mathbf{x}_0^t = [-1.5, 1.0]^\top$.

Figure 3 shows the result of SAT for this example. For time-steps $k = 1 : 6$, the vehicles are all in search mode. At time step $k = 7$, the target comes within the range of Vehicle 2's sensor and a detection event occurs. As a consequence, a narrower PDF, with a higher peak value, is generated at the update stage. In subsequent time steps, Vehicle 2 successfully observes and tracks the target, however the target remains beyond the range of Vehicles 1 and 3 and they continue to operate in search mode. For a single step planning horizon and the narrower PDF, Vehicle 1 is unable to determine an optimal control sequence, and simply continues to move straight ahead. Vehicle 1 continues doing this until $k = 11$, when the target drifts into a position that places its PDF within the scope of the vehicle's planning horizon. In contrast, Vehicle 3 is able to close on the target whilst operating in search mode. By time step $k = 12$, both Vehicles 1 and 3 have closed on the target sufficiently for detection and begin tracking.

The distance between the vehicles and the target at each time step is shown in Fig. 3(e). Fig. 3(f) again shows the familiar form of PDF's peak values over time. Note that the peak PDF value begins to plateau by $k = 10$ at a value of 0.81, as a result of being tracked by Vehicle 2 alone, but the peak value is further increased following the detection by Vehicles 1 and 3 at $k = 12$.

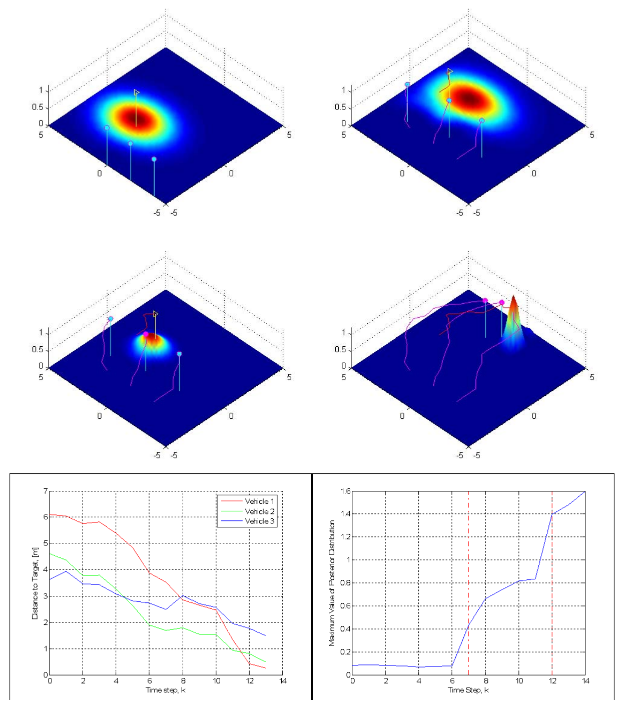


Fig. 3. The Three-Vehicle Single-Target system (a; upper-left) $k = 1$; (b; upper-right) $k = 6$; (c; middle-left) $k = 7$; (d; middle-right) $k = 13$; (e; lower-left) Distance between the vehicle and target at each time step; (f; lower-right) Peak values of the target's PDF.

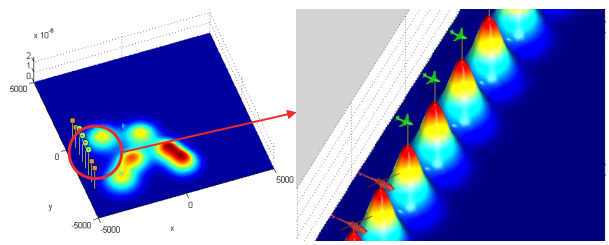
C. Marine Search and Rescue

The subsection deals with a marine SAR scenario introduced in Sec. I where a team of four autonomous UAVs and four autonomous manned helicopters cooperatively search for and rescue six life rafts. Table I compares the differences between the UAVs and the helicopters. The UAVs can move fast but at the expense of maneuverability whilst helicopters can even hover with high maneuverability but with low maximum speed. Both the vehicles carry out search when life rafts are not detected, but, from the physical characteristics, the UAVs circle around the found life rafts as a tracking action while rescue helicopters is following the life rafts to retrieve victims. Figure 4(a) shows the initial locations of the four UAVs, the four helicopters and the six actual life rafts. Notice that the

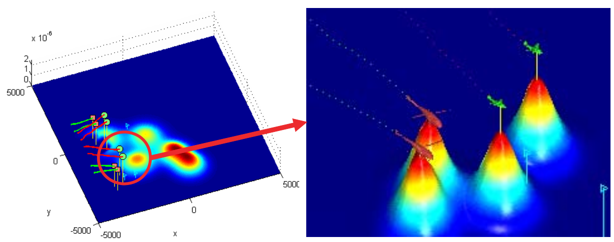
TABLE I

UAV AND HELICOPTER PARAMETERS

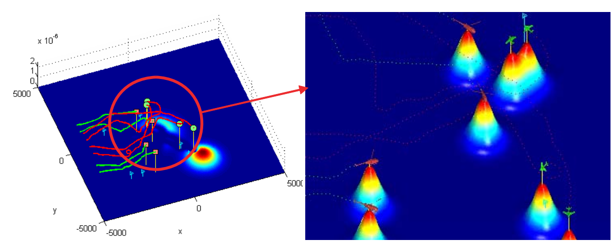
	UAV	Helicopter
Task	Search/Circle	Search/Follow
Maximum speed [m/s]	60	40
Minimum speed [m/s]	40	0
Maximum turn rate [deg/s]	30	90



(a) Initial states $k = 0$



(b) $k = 30$ when UAV 3 has detected Life Raft 2



(c) $k = 110$ when UAV 2 has found the last Life Raft 1

Fig. 4. Marine search and rescue

distribution shown in the left figure is the total prior belief on the six life rafts while the right figure shows the probability of detection of each sensor. Because no life raft is observable from vehicles, all the vehicles will start in search mode.

Figures 4(b)-(e) show the resulting SAR mission of the cooperative vehicles. Search by all the vehicles continues until the Life Raft 2 is detected at time step $k = 30$ by UAV 3 as shown in Fig. 4(b). Helicopter 2, which is nearest to the life raft, is then assigned to the life raft for rescue mission. Helicopter 2 switches its mode from search to following while UAV 3 switches from search to circling. The tracking by Helicopter 2 and UAV 3 continues until the helicopter reaches the life raft and hovers above the life raft for a specified period of time for the rescue mission. Upon the completion of the rescue mission, Helicopter 2 and UAV 3 resume searching.

The SAR tasks continue in a similar fashion until all the life rafts are found and rescued. Figure 4(c) shows the result at $k = 110$ when UAV 2 found the last Life Raft 1. By this time step, one can see the efficient movements of the vehicles where the vehicles move toward the life rafts without an attempt to cover the whole search area. The number of steps taken for the proposed technique to complete the entire SAR mission was 134 whereas the area coverage required 238 steps.

V. CONCLUSION AND FUTURE WORK

A coordinated control technique for heterogeneous vehicles to autonomously search for and track multiple targets using recursive Bayesian filtering has been presented. The unified sensor model and the unified objective function proposed within the recursive Bayesian filter framework allow a vehicle to switch its mode from search to tracking and vice versa by maintaining information accumulated previously. The proposed technique was first applied to the single-vehicle single-target problem, and the result shows that the vehicle can keep tracking even when the target motion is erratic. The result of the three-vehicle single-target SAT then indicates the improved tracking performance owing to the coordination of multiple vehicles. Finally, the application to marine SAR show the potential of the proposed technique to a variety of practical scenarios.

VI. ACKNOWLEDGMENT

This work is partly supported by the US Department of Air Force (AOARD-05-4026) and ARC Centre of Excellence programme, funded by the Australian Research Council (ARC) and the New South Wales State Government.

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