Managing Inventories

Logistics and Supply Chain Analytics

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Imperial College Business School

Imperial means Intelligent Business

Outline

- Fundamentals of inventory management
- Inventory management for deterministic demand
  - Stable demand: economic order quantity (EOQ)
  - Time varying demand
- Inventory management for stochastic demand
  - Safety inventory
  - Optimal product availability

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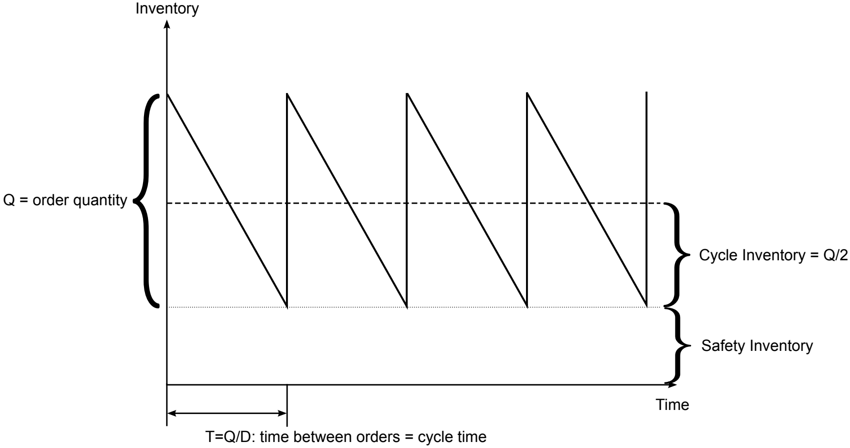
Role of Inventory in the Supply Chain

- It is a buffer in the supply chain
- Main tradeoff
  - Overstocking: amount available exceeds demand
    - Liquidation, obsolescence, holding cost
  - Understocking: demand exceeds amount available
    - Lost margin and future sales
- Goal: matching supply with demand

Inventory Profile with Safety Inventory



- Two questions of interest: When and How many?

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Cost of Inventory

- Purchase cost C (£ per unit)
  - Referred to as material cost
  - In many practical situations, material cost displays economies of scale
- Ordering cost S (£ per order)
  - All costs incurred each time an order is placed, regardless of the size of the order
  - May include transportation cost and receiving cost

Cost of Inventory

- Physical holding cost (out-of-pocket)
- Financial holding cost (opportunity cost)
- Obsolescence cost (markdowns, discounts)

⇨

Holding Cost H

- C: Unit product cost (£ per unit)
- h: Annual holding cost as % of unit product cost (% per year)
- $H = h \cdot C$: Annual unit holding cost (£ per unit per year)

Economic Order Quantity (EOQ) and Reorder Interval

- Inputs
  - D: Annual demand
  - C: Material cost
  - S: Setup cost
  - H: Holding cost per unit per year
- The total annual cost TC is given by

$$TC = CD + \left(\frac{D}{Q}\right) S + \left(\frac{Q}{2}\right) hC$$

  - Optimal lot size: $Q^* = \sqrt{\frac{2DS}{H}}$
  - Optimal reorder interval (or cycle time): $T^* = \sqrt{\frac{2S}{DH}}$

Economic Order Quantity (EOQ): An Example

Demand for the Deskpro computer at Argos is 1,000 units per month. Argos incurs a fixed transportation cost of £4,000 each time an order is placed. Each computer costs Argos £500 and the retailer has a holding cost of 20 percent.

What is the EOQ and optimal reorder interval?

Optimal Economies of Scale: Managerial Insights

$$EOQ = \sqrt{\frac{2SD}{H}} \qquad C_{EOQ} = \sqrt{2SDH}$$

- How to cut EOQ/cycle inventories (economically smart)?
- How to manage growth?
- Centralized inventory management?

Optimal Economies of Scale: Lessons

- In deciding the optimal order quantity, the trade off is between fixed order cost and holding cost
  - Aim for smaller order quantities for products that become obsolete quickly
  - Low cycle inventory and hence small order quantity are desirable, but economies of scale 'stand in the way' of this goal
- If demand increases by a factor of k, it is optimal to increase order quantity by a factor of $\sqrt{k}$ and produce (order) $\sqrt{k}$ as often
- EOQ is widely used despite its restrictiveness
  - A good starting point in most inventory systems

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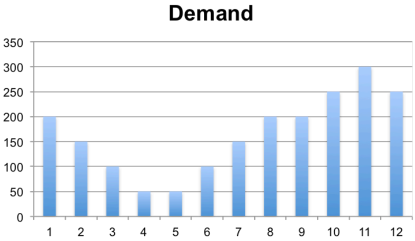
Overview of Different Approaches

- Simple heuristics
  - Fixed order quantity (FOQ)
  - Periodic order quantity (POQ)
- Optimal procedures
  - Mixed-integer linear programming (MILP)
- Specialty heuristics
  - Silver-Meal algorithm
  - Least unit cost (LUC)
  - Part-period balancing (PPB)

Managing Inventories with Time Varying Demand: An Example

Cost structure

- $D = 2000$ items per year
- $S = £500$ per order
- $C = £50$ per item
- $h = 24\%$ per item per year
- $H = £12$ per item per year



- Assumptions
  - Demand is required and consumed on first day of the period
  - Holding costs are not charged on items used in that period
  - Holding costs are charged for inventory ordered in advance of need

When should I order and for how much?

Approach: Fixed Order Quantity

Policy: Order Q^* (EOQ quantity) if $D(t) >$ inventory on hand

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	400	£200	£500	£700
2	150	0	£50	£0	£50
3	100	400	£350	£500	£850
4	50	0	£300	£0	£300
5	50	0	£250	£0	£250
6	100	0	£150	£0	£150
7	150	0	£0	£0	£0
8	200	400	£200	£500	£700
9	200	0	£0	£0	£0
10	250	400	£150	£500	£650
11	300	400	£250	£500	£750
12	250	0	£0	£0	£0
Totals	2,000	2,000	£1,900	£2,500	£4,400

Approach: Periodic Order Quantity

- Similar to EOQ
  - Find the optimal cycle time, T^*, for EOQ using annual demand
  - Set POQ $=$ round up of T^* to nearest integer
  - Every POQ time periods, order enough to satisfy demand for that POQ periods in the future
- In the example
  - $T^* = 0.204$ years $= 2.45$ months
  - POQ $= 3$ months

Approach: Periodic Order Quantity

Policy: Every POQ time periods, order an amount equal to the sum of demand for the next POQ periods

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	450	£250	£500	£750
2	150	0	£100	£0	£100
3	100	0	£0	£0	£0
4	50	200	£150	£500	£650
5	50	0	£100	£0	£100
6	100	0	£0	£0	£0
7	150	550	£400	£500	£900
8	200	0	£200	£0	£200
9	200	0	£0	£0	£0
10	250	800	£550	£500	£1,050
11	300	0	£250	£0	£250
12	250	0	£0	£0	£0
Totals	2,000	2,000	£2,000	£2,000	£4,000

Approach: Mixed-Integer Linear Programming

Decision variables

- Q_i = quantity purchased in period i
- Z_i = order variable; = 1 if $Q_i > 0$, and 0 o/w
- B_i = beginning inventory for period i
- E_i = ending inventory for period i

Data

- D_i = demand in period i
- S = ordering cost
- H = holding cost per period
- M = a very large number

MILP Model

- Objective function: minimize total relevant costs
- Subject to
  - Beginning and ending inventories must match
  - Conservation of inventory within each period
  - Nonnegativity for Q_i, B_i, E_i
  - Binary for Z_i

Approach: Mixed-Integer Linear Programming

$$
\begin{aligned}
\min \quad & S \cdot \sum_{i=1}^{n} Z_i + H \cdot \sum_{i=1}^{n} E_i \\
s.t. \quad & E_{i-1} = B_i \\
& E_i = B_i + Q_i - D_i \\
& M \cdot Z_i \geq Q_i \\
& B_i, E_i, Q_i \geq 0 \\
& Z_i \in \{0, 1\}
\end{aligned}
$$

Optimal order policy: order 550 in period 1, 450 in period 6, 450 in period 9, and 550 in period 11

Approach: Silver-Meal Algorithm

- Let $C(T) =$ (Ordering cost + Holding cost)/T
  - The average holding and ordering cost per period if the current order spans the next T periods
  - The objective is to minimize $C(T)$ for each replenishment
- Decision rule
  - Add next period's demand to the order if the average cost per period is reduced
- Algorithm
  1. Start at the first period
  2. Set $T = 1$
  3. If $C(T+1) > C(T)$ then
     - previous order goes for T periods with $Q = \sum_{i=1}^{T} D(T)$
     - go to 2 and begin the process again starting from period $T+1$
  4. Else, $T = T+1$ and go to 3

Approach: Silver-Meal Algorithm

Month	Demand	Order Quantity	Order Cost	Holding Cost	Total Costs	$C(T)$
1st	Order:					
1	200	200	£500	£0	£500	£500
2	150	350	£500	£150	£650	£325
3	100	450	£500	£150+£200	£850	£283
4	50	500	£500	£150+£200+£150	£1,000	£250
5	50	550	£500	£150+£200+£150 +£200	£1,200	£240
6	100	650	£500	£150+£200+£150 +£200+£500	£1,700	£283
2nd	Order:					
6	100	100	£500	£0	£500	£500
7	150	250	£500	£150	£650	£325
8	200	450	£500	£150+£400	£1,050	£350

Approach: Silver-Meal Algorithm

Month	Demand	Order Quantity	Order Cost	Holding Cost	Total Costs	$C(T)$
3rd	Order:					
8	200	200	£500	£0	£500	£500
9	200	400	£500	£200	£700	£350
10	250	650	£500	£200+£500	£1,200	£400
4th	Order:					
10	250	250	£500	£0	£500	£500
11	300	550	£500	£300	£800	£400
12	250	800	£500	£300+£500	£1,300	£433
5th	Order:					
12	250	250	£500	£0	£500	£500

Approach: Silver-Meal Algorithm

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	550	£350	£500	£850
2	150	0	£200	£0	£200
3	100	0	£100	£0	£100
4	50	0	£50	£0	£50
5	50	0	£0	£0	£0
6	100	250	£150	£500	£650
7	150	0	£0	£0	£0
8	200	400	£200	£500	£700
9	200	0	£0	£0	£0
10	250	550	£300	£500	£800
11	300	0	£0	£0	£0
12	250	250	£0	£500	£500
Totals	2,000	2,000	£1,350	£2,500	£3,850

Approach: Least Unit Cost (LUC)

- Let $CD(T) = (\text{Ordering cost} + \text{Holding cost})/\sum_{i=1}^{T} D(T)$
  - The average holding and ordering cost per item if the current order spans the next T periods
  - The objective is to minimize $CD(T)$ for each replenishment
- Decision rule
  - Add next period's demand to the order if the average cost per item is reduced
- Algorithm
  1. Start at first period
  2. Set $T = 1$
  3. If $CD(T+1) > CD(T)$ then
     - previous order goes for T periods with $Q = \sum_{i=1}^{T} D(T)$
     - go to 2 and begin the process again starting from period $T+1$
  4. Else, $T = T+1$ and go to 3

Approach: Least Unit Cost (LUC)

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	350	£150	£500	£650
2	150	0	£0	£0	£0
3	100	300	£200	£500	£700
4	50	0	£150	£0	£150
5	50	0	£100	£0	£100
6	100	0	£0	£0	£0
7	150	350	£200	£500	£700
8	200	0	£0	£0	£0
9	200	450	£250	£500	£750
10	250	0	£0	£0	£0
11	300	550	£250	£500	£750
12	250	0	£0	£0	£0
Totals	2,000	2,000	£1,300	£2,500	£3,800

Approach: Part Period Balancing (PPB)

- Let $H(T) =$ the total holding cost if the current order spans the next T periods
  - The objective is to minimize the (absolute) difference between $H(T)$ and S for each replenishment
- Algorithm
  1. Start at first period
  2. Set $T = 1$
  3. If $|H(T+1) - S| > |H(T) - S|$ then
     - previous order goes for T periods with $Q = \sum_{i=1}^{T} D(T)$
     - go to 2 and begin the process again starting from period $T + 1$
  4. Else, $T = T + 1$ and go to 3

Approach: Part Period Balancing

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	500	£300	£500	£800
2	150	0	£150	£0	£150
3	100	0	£50	£0	£50
4	50	0	£0	£0	£0
5	50	300	£250	£500	£750
6	100	0	£150	£0	£150
7	150	0	£0	£0	£0
8	200	650	£450	£500	£950
9	200	0	£250	£0	£250
10	250	0	£0	£0	£0
11	300	550	£250	£500	£750
12	250	0	£0	£0	£0
Totals	2,000	2,000	£1,850	£2,000	£3,850

Comparison of Approaches

Month	Demand	FOQ	POQ	Optimal	SM	LUC	PBB
1	200	400	450	550	550	350	500
2	150	0	0	0	0	0	0
3	100	400	0	0	0	300	0
4	50	0	200	0	0	0	0
5	50	0	0	0	0	0	300
6	100	0	0	450	250	0	0
7	150	0	550	0	0	350	0
8	200	400	0	0	400	0	650
9	200	0	0	450	0	450	0
10	250	400	800	0	550	0	0
11	300	400	0	550	0	550	550
12	250	0	0	0	250	0	0
	Holding Cost	£1,900	£2,000	£1,750	£1,350	£1,300	£1,850
	Order Cost	£2,500	£2,000	£2,000	£2,500	£2,500	£2,000
	Total Cost	£4,400	£4,000	£3,750	£3,850	£3,800	£3,850
	Pct $>$ Optimal	17%	7%	-	3%	1%	3%

Take Aways

- Many ways to solve the problem with implicit trade-offs
  - Heuristics: fast, simple, not always good
  - Optimal methods: most difficult to set up
    - An “optimal” solution might not be optimal in the real-world
  - Specialty heuristics: more focused, harder to set up, better “real-world” results
    - Silver-Meal algorithm, LUC and PPB are similar
    - Silver-Meal algorithm and LUC perform best if the costs change over time
    - PPB perform best if the costs do not change over time

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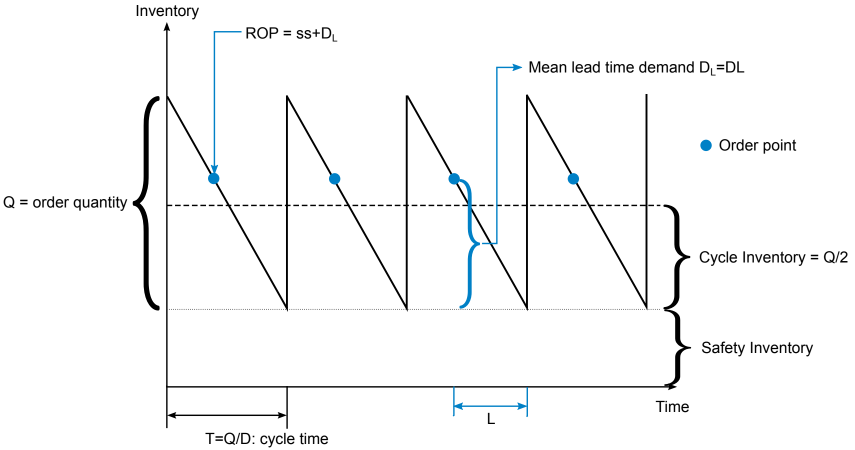
Replenishment Policy and Measure of Product Availability

- Why safety inventory? Hedge against uncertainty
  - Information uncertainty
  - Demand & supply uncertainty
- Replenishment policy: what form of inventory policy should I use?
  - Continuous review (Q, R): Order fixed quantity when total inventory drops below reorder point (ROP)
  - Periodic review (s, S): Order at fixed time intervals to raise total inventory to Order-up-to-Level (OUL)
- Measure of product availability: what service objectives should I set?
  - Cycle service level (CSL)
  - Product fill rate
  - Order fill rate

Inventory Profile with Safety Inventory



Relationship between ROP, Safety Inventory & Cycle Service Level

- Given ROP, what is the corresponding cycle service level?
  - Cycle service level = Probability(demand during lead time $\leq ROP$)
  - We can solve it using the Monte Carlo method
    - generate a sample $(X_1, \ldots, X_n)$ from the distribution of demand during lead time
    - approximate the probability with

$$\frac{1}{n}\sum_{j=1}^{n} \mathbf{1}_{\mathcal{D}_L \leq ROP}(x_j) \rightarrow E(\mathbf{1}_{\mathcal{D}_L \leq ROP}) = P(\mathcal{D}_L \leq ROP),$$

where $\mathbf{1}_{\mathcal{D}_L \leq ROP}(x_j)$ is an indicator function. It is equal to 1 if $x_j \leq ROP$, and 0 otherwise

Relationship between ROP, Safety Inventory & Cycle Service Level: An Example

Assume that weekly demand for phones at Carphone Warehouse is normally distributed, with a mean of $D = 2,500$ and a standard deviation of $\sigma_D = 500$. The manufacturer takes two weeks to fill an order placed by Carphone Warehouse manager, i.e., $L = 2$. The store manager currently orders $Q = 10,000$ phones when the inventory on hand drops to 6,000.

Questions

- What is the safety stock level?
- What is the cycle service level (CSL), i.e., the probability of NOT running out of stock in a given replenishment cycle?

Relationship between ROP, Safety Inventory & Cycle Service Level: An Example

- Demand per period
  - D: average demand per period
  - σ_D: standard deviation of demand per period
- Assumptions of demand
  - independent across periods
  - normally distributed
- Lead time demand $\mathcal{D}_L$
  - L: lead time for replenishment
  - Mean demand during lead time $D_L = D \cdot L$
  - Standard deviation of lead time demand $\sigma_L = \sqrt{L}\sigma_D$

Relationship between ROP, Safety Inventory & Cycle Service Level: An Example

- Demand distribution during lead time
  - Demand per week
    - Mean $D = 2,500$/week; standard deviation $\sigma_D = 500$
  - Demand during lead time
    - Mean $= D_L = D \cdot L =$
    - Standard deviation of lead time demand $\sigma_L = \sigma_D \sqrt{L} =$
- Cycle service level $= P(\mathcal{D}_L \leq ROP)$
- Safety Inventory $ss = ROP - DL =$
- Average Inventory $= Q/2 + ss =$

Relationship between ROP, Safety Inventory & Cycle Service Level

- Given desired cycle service level, what is the required safety inventory and ROP?
  - If CDF of the demand distribution (say F) is known, ROP is given by $F^{-1}(CSL)$
    - When demand is normally distributed, $ss = F_S^{-1}(CSL) \cdot \sigma_D \cdot \sqrt{L}$
  - In general, we can use the Monte Carlo Method in conjunction with optimization
    - For a given ROP, we can calculate the cycle service level CSL_{ROP} using the Monte Carlo method
    - Then find ROP^* that yields the desired cycle service level, which is given by

$$\min_{ROP} (CSL_{ROP} - CSL_{target})^2$$

  - Safety inventory $ss = ROP - D_L$

Given desired Cycle Service Level, what is the required Safety Inventory & ROP? An Example

Weekly demand for Legos at a Wal-Mart store is normally distributed, with a mean of 500 boxes and a standard deviation of 500. The replenishment lead time is four weeks. Evaluated the safety inventory that the store should carry to achieve a CSL of 90 percent.

Question: How much safety stock is needed for CSL of 90%?

- $ss =$
- $ROP = DL + ss =$

Factors Driving Safety Inventory

$$ss = f(CSL, \sigma_D, L)$$

- Factors driving safety inventory
  - Level of service
  - Demand uncertainty
  - Replenishment lead time
- How to reduce safety stock without hurting availability?
  - Reduce uncertainty in demand
  - Reduce replenishment lead time
  - Inventory Pooling

Impact of Inventory Pooling

Which of the two systems provides a higher level of service for a given level of safety stock?

System A (Decentralized)

(D_i, σ_i)

System B (Centralized)

(D^C, σ_D^C)

$$D^C = \sum_{i=1}^{k} D_i, \qquad \sigma_D^C = \sqrt{\sum_{i=1}^{k} \sigma_i^2 + 2\sum_{i>j} cov(i,j)}$$

$$cov(i,j) = \rho_{i,j}\sigma_i\sigma_j$$

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Value of Inventory Pooling

- Assumptions
  - k locations with identical demand
  - Normally distributed demand per location
- Amount of disaggregate safety inventory

$$ss_d = F_S^{-1}(CSL) \times \sqrt{L} \times \sigma_D \times k$$

- Amount of aggregate safety inventory

$$ss_a = F_S^{-1}(CSL) \times \sqrt{L} \times \sigma_D^C$$

- Safety inventory holding cost savings per unit

$$\frac{ss_d H - ss_a H}{kD} = F_S^{-1}(CSL) \times \sqrt{L} \times \left(\frac{\sigma_D}{D} - \frac{\sigma_D^C}{k \times D} \right) \times H$$

Factors Affecting Value of Inventory Pooling

- Coefficient of variation (cv) of demand (σ_D/D)
- Demand correlation
- Value of product
- Replenishment lead time
- Lead time uncertainty
- Desired level of product availability
- Number of facilities

Impact of Correlation on Aggregation

Example 12.8 in C&M:
A BMW dealership has four retail outlets serving the entire Chicago area (disaggregate option). Each outlet covers a separate geographic area, and the correlation of demand across any pair of areas is ρ.The dealership is considering the possibility of replacing the four outlets with a single large outlet (aggregate option).

ρ	Disaggregate Safety Inventory	Aggregate Safety Inventory
0	36.24	18.12
0.2	36.24	22.92
0.4	36.24	26.88
0.6	36.24	30.32
0.8	36.24	33.41
1.0	36.24	36.24

Impact of cv and Product Value on Aggregation

	Motors	Cleaner
Inventory is stocked in each store		
Mean weekly demand per store	20	1,000
Standard deviation	40	100
Coefficient of variation	2.0	0.1
Safety inventory per store	132	329
Total safety inventory	211,200	526,400
Value of safety inventory	$105,600,000	$15,792,000
Inventory is aggregated at the DC		
Mean weekly aggregate demand	32,000	1,600,000
Standard deviation of aggregate demand	1,600	4,000
Coefficient of variation	0.05	0.0025
Aggregate safety inventory	5,264	13,159
Value of safety inventory	$2,632,000	$394,770
Savings		
Total inventory saving on aggregation	$102,968,000	$15,397,230
Total holding cost saving on aggregation	$25,742,000	$3,849,308
Holding cost saving per unit sold	$15.47	$0.046
Savings as a percentage of product cost	3.09%	0.15%

Source: Example 12.10 in C&M

Tailored Aggregation

- Aggregation decreases inventories and facility costs, but increases transportation and hurts response time
- Tailored aggregation: aggregate items with
  - high cv
  - long replenishment lead time
  - high value
  - independent or negatively correlated demand (disaggregate items with positively correlated demand)
  - high desired level of availability

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End-of-Day Croissant Sale

How many croissants shall the store manager order each day?

Historical Sales Information

Demand	Probability	Probability of demand being this much or less	Probability of demand being greater than this much
4	.01	.01	.99
5	.02	.03	.97
6	.04	.07	.93
7	.08	.15	.85
8	.09	.24	.76
9	.11	.35	.65
10	.16	.51	.49
11	.20	.71	.29
12	.11	.82	.18
13	.10	.92	.08
14	.04	.96	.04
15	.02	.98	.02
16	.01	.99	.01
17	.01	1.00	.00

- On average, 10.26 croissants are sold each day.

Cost of Over- and Understocking

- Cost structure
  - Cost per croissant $= c = 24$
  - Regular price per croissant $= p = 79$
  - Sales price $= s = 19$
- Profit from selling a croissant

$$C_u = p - c = 79 - 24 = 55$$

- Cost of overstocking

$$C_o = c - s = 24 - 19 = 5$$

Profit from Ordering the Expected Demand

Order Quantity = 10 (Expected Demand = 10.26)

Probability	Demand	Sold	Overstocked	Understocked	Profit
.01	4	4	6	0	190
.02	5	5	5	0	250
.04	6	6	4	0	310
.08	7	7	3	0	370
.09	8	8	2	0	430
.11	9	9	1	0	490
.16	10	10	0	0	550
.20	11	10	0	1	550
.11	12	10	0	2	550
.10	13	10	0	3	550
.04	14	10	0	4	550
.02	15	10	0	5	550
.01	16	10	0	6	550
.01	17	10	0	7	550
Expected	10.26	9.15	0.85	1.11	499

Expected Marginal Contribution: Increasing Order Size by 1 unit

- If we order 10, the CSL=probability(demand ≤ 10) $= 0.51$
  - Additional 1 units sell with probability 1-CSL=0.49. We earn margin $C_u = p - c = 55$ per unit
  - Additional 1 units do not sell with probability CSL=0.51. We lost $C_o = c - s = 5$ per unit
- Expected marginal contribution of an additional 1 unit is given by

$$0.49 \times 1 \times 55 - 0.51 \times 1 \times 5 = 24.4$$

Expected Marginal Contribution as Availability Increases

Additional Unit	Expected Marginal Benefit	Expected Marginal Cost	Expected Marginal Contribution
11th	$55 \times .49 = 26.95$	$5 \times .51 = 2.55$	$26.95 - 2.55 = 24.40$
12th	$55 \times .29 = 15.95$	$5 \times .71 = 3.55$	$15.95 - 3.55 = 12.40$
13th	$55 \times .18 = 9.90$	$5 \times .82 = 4.10$	$9.90 - 4.10 = 5.80$
14th	$55 \times .08 = 4.40$	$5 \times .92 = 4.60$	$4.40 - 4.60 = -0.20$
15th	$55 \times .04 = 2.20$	$5 \times .96 = 4.80$	$2.20 - 4.80 = -2.60$
16th	$55 \times .02 = 1.10$	$5 \times .98 = 4.90$	$1.10 - 4.90 = -3.80$
17th	$55 \times .01 = 0.55$	$5 \times .99 = 4.95$	$0.55 - 4.95 = -4.40$

- Optimal order quantity = 13 croissants
- Service level = 92%

Optimal Level of Product Availability

- $p =$ regular price
- $s =$ salvage price
- $c =$ cost

$$\begin{aligned} C_u &= p - c \\ C_o &= c - s \end{aligned}$$

- Optimal cycle service level ("critical fractile")

$$CSL^* \geq \frac{C_u}{C_u + C_o}$$

- If demand is continuously distributed, $CSL^* = \frac{C_u}{C_u + C_o}$
  - The optimal order quantity $Q^* = F^{-1}(CSL^*)$, where F is the CDF of demand distribution

Optimal Order Quantity

- To determine optimal order quantity, we can use the Monte Carlo Method in conjunction with optimization
  - Generate a sample $(X_1, \ldots, X_n)$ from the distribution of demand
  - For a given Q, the expected profit is given by

$$\frac{1}{n}\sum_{j=1}^{n}\pi(x_j) = \frac{1}{n}\sum_{j=1}^{n}\left[p\min(Q, x_j) + s(Q - x_j)^+ - cQ\right] \to E(\pi_Q)$$

  - Then we find Q^* that yields the highest expected profit by solving the optimization problem

$$\max_Q E(\pi_Q)$$

Optimal Level of Product Availability: An Example

Consider a buyer at Bloomingdale's who is responsible for purchasing dinnerware with Christmas patterns. The dinnerware sells only during the Christmas season, and the buyer places an order for delivery in early November. Each dinnerware set cost $c = \$100$ and sells for a retail price $p = \$250$. Any set unsold by Christmas are heavily discounted in the post Christmas sales and are sold for a salvage value of $s = \$80$. The buyer has estimated that demand is normally distributed, with a mean of $\mu = 350$. Historically, forecast errors have had a standard deviation of $\sigma = 150$.

- How many units should be ordered?
- Assume ordering the optimal order quantity,
  - what is the expected profit?
  - what is the expected overstock?
  - what is the expected understock?

Levers to Improve Profitability

- Reduce the cost of mismatches
  - Increase salvage value (overstock outlets)
  - Decrease cost of under stocking (substitution)
- Reduce demand uncertainty
  - Improve forecast accuracy
  - Postponement of product differentiation
  - Tailored sourcing

Impact of Improving Forecasts: The Example Revisited

- The buyer has decided to conduct additional market research to get a better forecast
- Questions
  - How many units should be ordered as σ_D changes?
  - How will improving forecasts affect profits, overstock and understock?

Impact of Improving Forecasts: The Example Revisited

σ_D	Q^*	Expected Overstock	Expected Understock	Expected Profit
150	525	186.7	8.6	$47,469
120	491	149.3	6.9	$48,476
90	456	112.0	5.2	$49,482
60	420	74.7	3.5	$50,488
30	385	37.3	1.7	$51,494
0	350	0	0	$52,500

Profit grows, quantity ordered approaches expectation, and both over and understock shrink as forecasts become more accurate!

Value of Postponement: Benetton

- Two options
  - Option 1: traditionally, thread was dyed and then the garment was knitted
  - Option 2: Benetton developed a procedure whereby dyeing was postponed until after the garment was knitted
- The knitting/manufacturing process takes a total of 20 weeks
  - Option 1: Benetton makes the buying decision for each color 20 weeks before the sale period and holds separate inventories for each color
  - Option 2: The inventory held is based on the aggregate demand across all four colors. Benetton decides the quantity for individual colors after demand is known

Value of Postponement: Benetton

- Assume that Benetton sells garment in four colors
- Demand (uncorrelated)
  - Each color: mean = 1,000; SD = 500
  - Aggregate: mean = 4,000; SD = 1,000
- For each garment
  - Sale price = \$50
  - Salvage value = \$10
  - Production cost using option 1 (long lead time) = \$20
  - Production cost using option 2 (greige thread) = \$22
- What is the value of postponement?
  - Expected profit increases from \$94,576 to \$98,092

Value of Postponement with Dominant Product

- Consider the new demand as follows
  - Color with dominant demand: mean = 3,100, SD = 800
  - Other three colors: mean = 300, SD = 200
  - Aggregate: mean = 4,000, SD =872
- Expected profit without postponement = $102,205
- Expected profit with postponement = $99,872

Why is postponement not valuable with a dominant product?

How should we react?

Tailored Postponement: Benetton

- Under tailored postponement
  - Produce the amount that is likely to sell using the lower-cost production method without postponement
  - Produce the portion of demand that is uncertain using postponement
- It is complex to implement but can be valuable even when all products being postponed have similar demand
- Consider the scenario
  - Benetton is selling four colors
  - Each color is normally distributed with mean demand = 1,000, and SD=500
  - For each color, a quantity Q_1 to be manufactured using Option 1 and an aggregate quantity Q_A to be manufactured using Option 2

Tailored Postponement: Benetton

Manufacturing Policy		Average Profit	Average Overstock	Average Understock
Q_1	Q_A			
0	4,524	$97,847	510	210
1,337	0	$94,377	1,369	282
700	1,850	$102,730	308	168
800	1,550	$104,603	427	170
900	950	$101,326	607	266
900	1,050	$101,647	664	230
1,000	850	$100,312	815	195
1,000	950	$100,951	803	149
1,100	550	$99,180	1,026	211
1,100	650	$100,510	1,008	185

Cautions in Implementing Postponement

- Postponement often increases the manufacturing cost
- Cautions
  - Do a small set of products provide most of the sales?
  - Do products have low uncertainty?
  - Are individual product demands highly correlated?
- Tailored postponement
  - Higher manufacturing cost is justified only for uncertain portion of demand
  - Consider more efficient process for stable portion of demand

Optimal Product Availability: Lessons

- Optimal order quantities are obtained by trading off cost of understocking and cost of excess stock
  - This reasoning is very general!
  - The precise mechanics (equations) depend on the situation
- Levers for improving profitability
  - Reduce the cost of mismatches: increase salvage value/decrease stockout cost
  - Reduce the chance of mismatches
    - Improve forecast accuracy
    - Postponement of product differentiation
    - Tailored sourcing