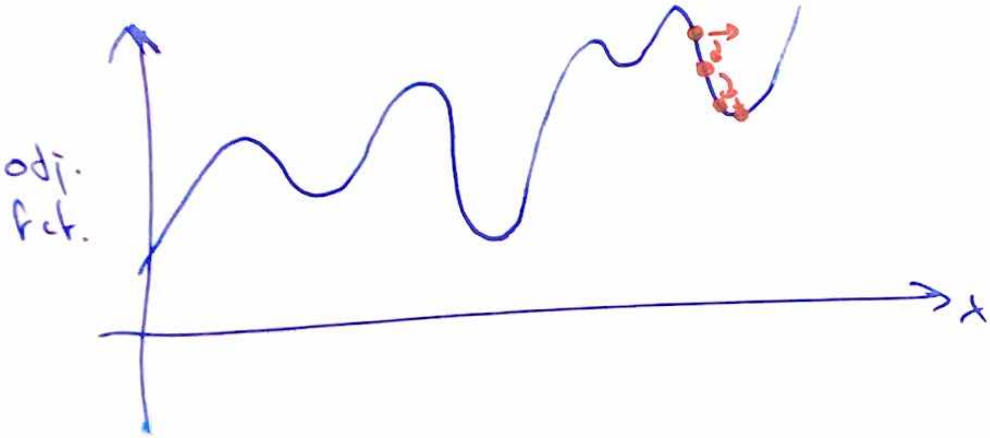
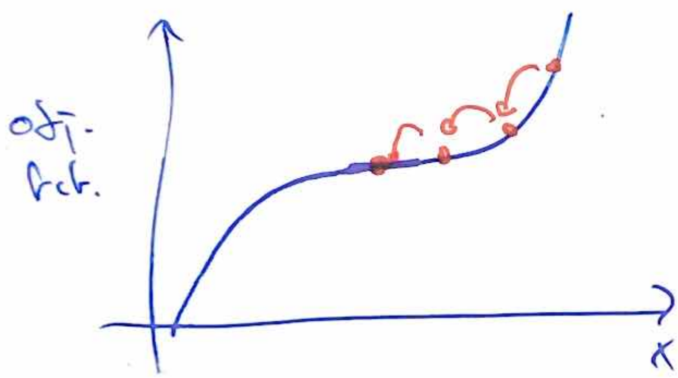
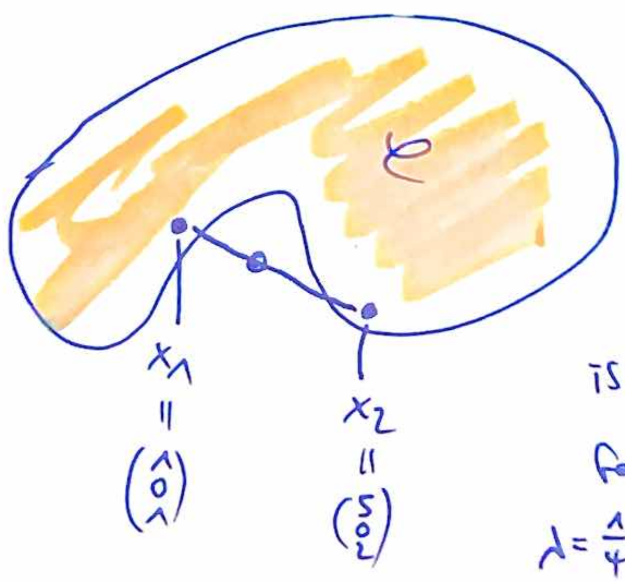
TWO "ENEMIES" WHEN WE MINIMISE:

① local minima



② saddle points



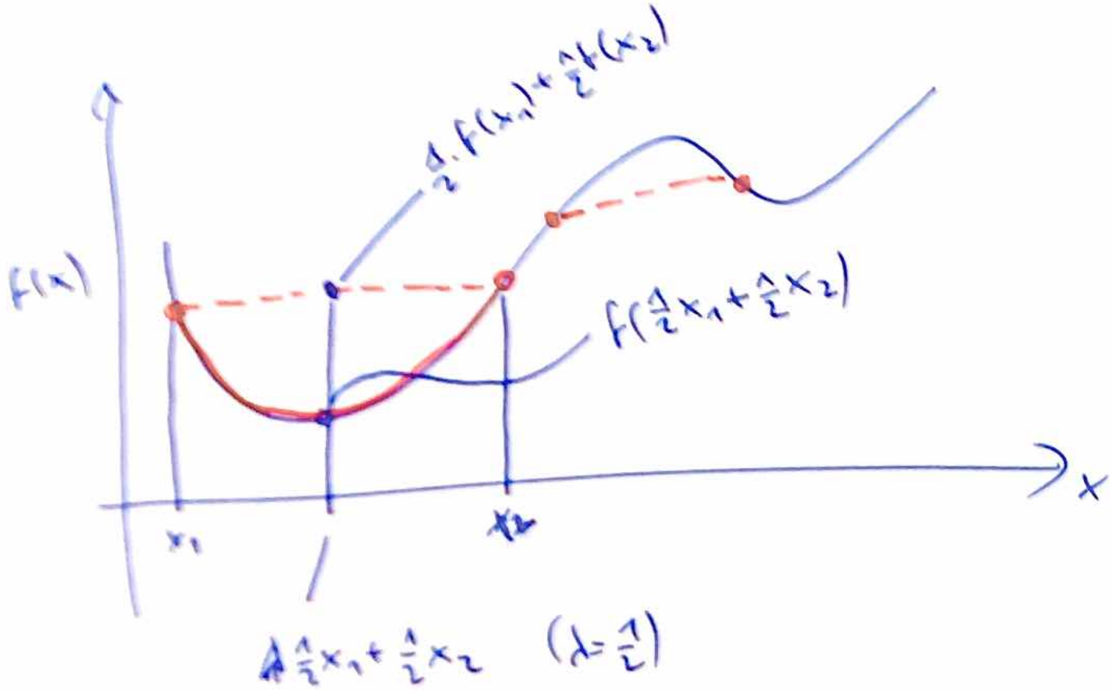


is $\lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + (1-\lambda) \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} \in \mathcal{C}$

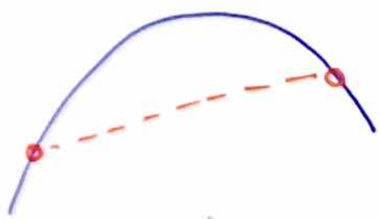
for all $\lambda \in [0,1]$?

$\lambda = \frac{1}{4}: \begin{pmatrix} 1/4 \\ 0 \\ 1/4 \end{pmatrix} + \begin{pmatrix} 5 \cdot 3/4 \\ 0 \\ 2 \cdot 3/4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 7/4 \end{pmatrix}$

A fct. is convex if:



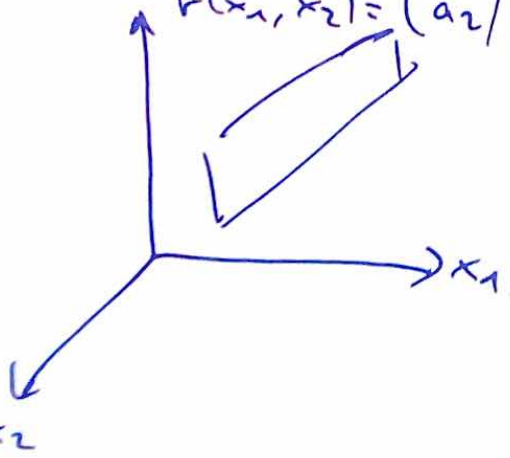
concave:



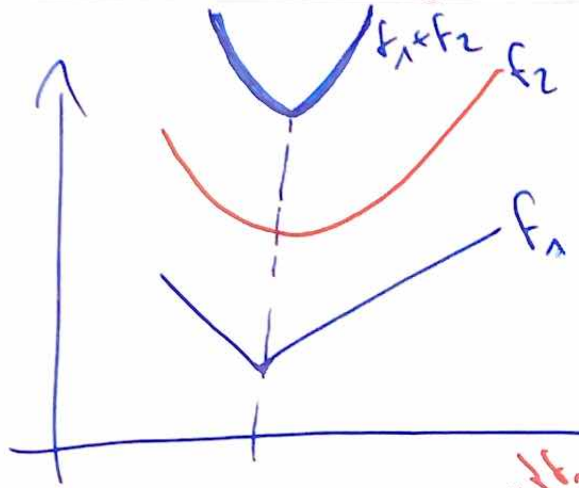
~~max~~

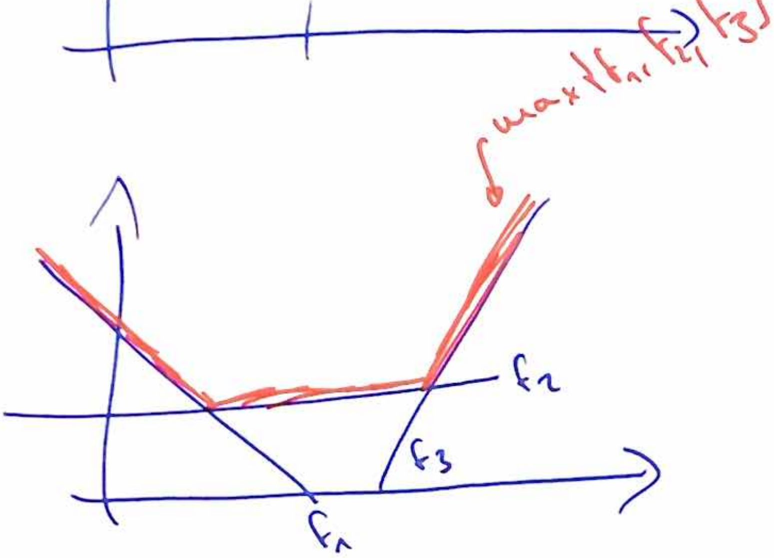
~~s.t.~~

$f(x_1, x_2) = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}^T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + b$



3/



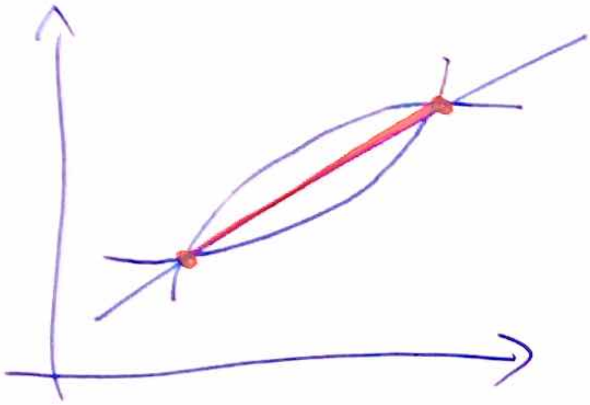


$h(x,y) = 2x - y$ affine

$f(x) = x^2$

$\Downarrow$

$(2x-y)^2$



An optimisation problem is $\boxed{\text{convex}}$ if

- the objective minimises a convex fct.
  or maximises a concave fct.

and • the feasible region is convex

Linear regression, on steroids:

minimise $\|y - X\beta\|_2$
~~st~~ subject to $\beta \in \mathbb{R}^k$

$\beta_1 \geq 0, \beta_2 \leq 0, \beta_3 \geq 2\beta_4$

$\Downarrow$

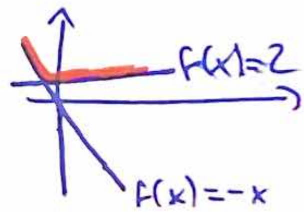
$-\beta_1 \leq 0$
$\beta_2 \leq 0$
$2\beta_4 - \beta_3 \leq 0$

Examples of Convex Functions

Show that the following functions are convex:

Pos. sum x^2+y^2 is convex as well

1. $f(x, y) = \max\{x^2 + y^2, |x|, 2x - y\}$
   - $f(x) = x^2$ as well as $f(y) = y^2$ are convex
   - $F(x) = |x|$ is convex
   - $2x - y$ is affine and hence both convex and concave
   - max of 3 convex fct's is convex
2. $f(x) = \max\{x, \max\{-x, 2\}\}$
   - $-x$ is affine
   - 2 is affine
   - $\max\{-x, 2\}$ is convex
   - x is affine
   - $\max\{x, \underbrace{\max\{-x, 2\}}_{\text{convex}}\}$ is convex

   → affine
3. $f(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|_2$

$\|\cdot\|_2$ is a norm and hence convex

→ $\|x\|_2 = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$

$f(x, y) = x - y$ is affine

$\|x - y\|_2$ is convex