confidence interval for average.

$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + u$$

educ = 12, exper = 4

$$\theta_0 = E[y \mid x_1 = c_1, x_2 = c_2, \cdots, x_k = c_k]$$

$$= \beta_0 + \beta_1 c_1 + \beta_2 c_2 + \cdots + \beta_k c_k$$

$$\Leftrightarrow \beta_0 = \theta_0 - \beta_1 c_1 - \beta_2 c_2 - \cdots - \beta_k c_k$$

$$y = (\theta_0 - \beta_1 c_1 - \beta_2 c_2 - \cdots - \beta_k c_k)$$

$$+ \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + u$$

$$= \theta_0 + \beta_1 (x_1 - c_1) + \beta_2 (x_2 - c_2) + \cdots + \beta_k (x_k - c_k) + u$$

CI vs PI

$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + u$$

If we want to predict average of y, when $x_1 = c_1$, $x_2 = c_2, \cdots$

we are predicting:

$$\theta_0 = E[y \mid x_1 = c_1, \cdots, x_k = c_k] = \beta_0 + \beta_1 c_1 + \cdots + \beta_k c_k$$

$$\left.\begin{array}{l} \hat{\theta}_0 = \hat{\beta}_0 + \hat{\beta}_1 c_1 + \cdots + \hat{\beta}_k c_k \\ se(\hat{\theta}_0) \end{array}\right\} \Rightarrow$$

95% CI

$$\hat{\theta}_0 \pm 1.96\, se(\hat{\theta}_0)$$

If we want to predict an individual i, with $x_1 = c_1, \cdots, x_k = c_k$, we are predicting:

$$y_i = \beta_0 + \beta_1 c_1 + \cdots + \beta_k c_k + u_i$$

$$\hat{\beta}_{k+1} c_{k+1}$$

$$\hat{y}_i = \hat{\theta}_0 = \hat{\beta}_0 + \hat{\beta}_1 c_1 + \cdots + \hat{\beta}_k c_k$$

95% prediction interval

$\Rightarrow \hat{\theta}_0 \pm 1.96 \cdot se(\hat{e})$

two sources of variations:

$$se(\hat{e}) = \left[se(\hat{\theta}_0)^2 + \hat{\sigma}^2 \right]^{1/2}$$

$se(\hat{y}_i)$

Prediction of Log model

$$\underline{\log(y) = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + u}$$

$$E[y \mid \vec{x}]$$

$$= E[\exp(\log(y)) \mid \vec{x}]$$

$$= E[\exp(\beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + u) \mid \vec{x}]$$

$$= \underline{\exp(\beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k)} \cdot \underline{E[\exp(u) \mid \vec{x}]}$$

$$\Uparrow \qquad\qquad\qquad \geq$$

$$\exp(\hat{\log y}) \qquad\qquad \exp(E[u \mid \vec{x}]) = 1$$

$$\hat{\log y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \cdots + \hat{\beta}_k x_k \qquad \frac{1}{n} \sum_{i=1}^{n} \exp(\hat{u}_i)$$