Imperial College London
BUSINESS SCHOOL

MSc Business Analytics 2020/21
Optimisation and Decision Models
Wolfram Wiesemann

Assignment 1 — In-Class Questions

Solutions to the in-class questions will be provided shortly after the tutorial. You do not need to submit your solutions to the in-class questions!

(1) **In-Class:** The Magnetron Company manufactures two types of microwave ovens: full-size and compact. Each full-size oven requires 2 hours of general assembly and 2 hours of electronic assembly, whereas each compact oven requires 1 hours of general assembly and 3 hours of electronic assembly. For the current production period, there are 500 hours of general assembly labour and 800 hours of electronic assembly labour available. The company estimates that it can sell up to 220 full-size and 180 compact ovens with an earnings contributions of £120 per full-size oven and £130 per compact oven. Magnetron wants to find a production plan that maximising earnings.

   (a) Formulate the above problem as a linear program.

   (b) Solve the linear program graphically. What is the optimal solution, and what is its objective value? Which constraints are binding at the optimal solution?

   (c) Solve the linear program via Excel and verify your solution from part (b).

   (d) Using overtime, Magnetron is considering to increase the number of general assembly labour hours by 10. What hourly rate should Magnetron be willing to pay at most?

(2) **In-Class:** Nature's Best Frozen Foods company produces 4 mixes of frozen ready-to-eat vegetables. The mixes consists of five different vegetables: carrots, mushrooms, green peppers, broccoli and corn. The mixes are "Stir Fry", "Barbecue", "Hearty Mushrooms" and "Veggie Crunch", and each mix is sold in 250g bags. Each bag of "Stir Fry", "Barbecue", "Hearty Mushrooms" and "Veggie Crunch" contributes £0.22, £0.20, £0.18 and £0.18 to the company's earnings, respectively. The compositions of the mixes and the monthly supplies of the ingredients are given in the following table:

Ingredients	Mixes				Monthly supply of ingredients
	Stir Fry	Barbecue	Hearty Mushrooms	Veggie Crunch	
Carrots	62.5g	50g	0g	62.5g	3,750kg
Mushrooms	75g	0g	100g	0g	2,000kg
Green peppers	62.5g	50g	75g	62.5g	3,375kg
Broccoli	50g	75g	75g	62.5g	3,500kg
Corn	0g	72g	0g	62.5g	3,750kg

(a) Assuming that the company can sell all the mixes that they produce, formulate a linear program that determines a production plan which maximises the total earnings.

(b) Solve the linear program using AMPL. What is the optimal solution, and what is its objective value? Which constraints are binding at the optimal solution?

(c) What is the value of an extra 100kg of green peppers?

Assignment 1 — Homework Questions

Your answers to both homework questions need to be submitted by the announced deadline. Proper explanations are as important as correct formulae! Please note that these are individual homework questions, and hence you need to address them by yourself.

(3) **Homework:** Upon review of its past projects, the ImpactNow consulting company has decided to focus on three different types of consulting projects: restructuring projects, growth projects and IT risk projects. Each project has different requirements on its workforce, as detailed below (all numbers are in employees required for the duration of 1 month; it is assumed that each project takes exactly one month to complete):

Project type	Workforce required per project	
	Strategy consultants	**IT experts**
Restructuring	2	1
Growth	3	1
IT risk	1	3

Assume that the company has 100 strategy consultants and 75 IT experts. On average, each restructuring project results in revenues of £25,000, each growth project results in revenues of £30,000, and each IT risk project results in revenues of £20,000.

The company wants to determine a project mix that maximises its revenues, subject to the available workforce. Assume that there is no shortage of projects. **[30 marks]**

(a) Formulate a linear program for this problem. Make sure you explain your model! **[10 marks]**

(b) Solve the model in AMPL. Make sure you submit the AMPL model! What is the optimal solution, and what is its objective value? Which constraints are binding at the optimal solution? Based on your model and solution, what advice do you give to ImpactNow? **[20 marks]**

(4) **Homework:** We now consider a variant of Gemstone Tool Company's problem where the company optimises its productions for each of the four quarters of the financial year, and where the company can keep its excess production on inventory. To this end, we make the following assumptions: **[70 marks]**

	Wrenches	Pliers	Availability per quarter
Required steel (lbs.)	1.5	1.0	15,000 lbs.
Required molding machine time (h)	1.0	1.0	12,000 h
Required assembly machine time (h)	0.3	0.5	8,000 h
Contributions to earnings ($/unit)	US$0.13	US$0.10	

Moreover, we assume that the demand for wrenches and pliers is going to be as follows:

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Demand for wrenches	5,000	9,000	10,000	15,000
Demand for pliers	6,000	11,000	12,000	15,000

(a) Formulate a linear program that decides how much to produce in each of the four quarters so as to maximise the overall earnings across the four quarters. Note that the company can decide to keep some of the production in each quarter for sales in a later quarter. Make sure you explain your model! **[20 marks]**

Hint: *Introduce the decision variables PW_1, …, PW_4 and PP_1, …, PP_4 for the production quantities of wrenches and pliers in each of the four quarters; the decision variables SW_1, …, SW_4 and SP_1, …, SP_4 for the sales quantities of wrenches and pliers in each of the four quarters; the decision variables IW_1, …, IW_4 and IP_1, …, IP_4 for the wrenches and pliers on inventory at the beginning of each quarter (before anything is produced or sold). Make sure you enforce that the inventories in the first quarter are all zero! (Depending on your formulation, you may even find it useful to introduce auxiliairy variables IW_5 and IP_5.)*

(b) Solve the problem in AMPL. Make sure you submit your AMPL file! Explain the optimal solution: What should we produce, sell and keep on inventory in every quarter, and what are the optimal earnings? **[25 marks]**

(c) In the past, Gemstone Tool Company has managed its inventories by producing as many wrenches as possible (due to their higher profit contribution) and subsequently using the remaining resources for the production of pliers. Critically appraise the advantages and shortcomings of your optimisation model relative to that strategy. **[25 marks]**