Imperial College London
BUSINESS SCHOOL

Exercises 4

(1) **In-Class:** The coach of the US national men's basketball team has to select a "dream team" of 12 players from the following list of 20 candidate players:

#	Avg. rebounds/game	Avg. assists/game	Height	Avg. points/game	Defensive ability
1	1	7	5'11"	10	10
2	2	14	6'0"	14	9
3	3	12	6'4"	19	8
4	4	4	6'0"	18	6
5	5	9	6'3"	20	8
6	7	6	6'5"	21	10
7	7	8	6'8"	23	10
8	4	2	6'5"	13	5
9	8	2	6'10"	17	8
10	5	5	6'4"	25	8
11	10	6	6'10"	20	9
12	8	8	6'9"	30	10
13	10	2	7'3"	24	9
14	9	5	6'10"	15	7
15	6	3	6'10"	17	6
16	16	2	6'9"	3	6
17	11	1	7'4"	27	9
18	12	5	7'2"	26	10
19	11	1	7'3"	21	9
20	9	1	7'0"	14	8

Players 1-5 are playmakers, players 4-11 are shooting guards, players 9-16 are forwards, and players 16-20 are centres. The coach would like to select the team with the highest average number of points/game, subject to the following restrictions:

1. The team should consist of at least 3 playmakers, 4 shooting guards, 4 forwards and 3 centres.

2. Players #4, #8, #15 and #20 come from the National Collegiate Athletic Association (NCAA), and at least two of them should be selected.

3. The average number of rebounds/game among the selected players should at least be 7, the average number of assists/game at least 6, the average number of points/game at least 18, and the average defensive ability at least 8.5.

4. The average height among the selected players should at least be 6'7" (note: 1 foot = 12 inches).

5. Player 5 would turn down the invitation if player 9 is selected.

6. Players 2 and 19 would only accept if both of them are selected.

7. At most three of the players #1, #7, #12, #16 should be selected as they all belong to the same franchise team.

(a) Construct a binary optimisation model that finds the optimal assignment.

(b) Use Excel to determine the optimal solution.

(2) **In-Class:** The office manager of a large City-based accounting firm needs to replace the ageing and out-of-style office furniture. The firm has decided to purchase new desk/chair/credenza furniture sets for all 2,000 offices in London. The company has received bids from four different furniture companies who are willing to supply the furniture sets, as follows:

   3. Caroline Woodworks has bid to deliver up to 1,000 furniture sets at a cost of £2,500 per set and with a one-time delivery charge of £10,000.

   4. Nashawtuc Millworks has bid to supply up to 1,200 furniture sets at a cost of £2,450 per set and with a one-time delivery charge of £20,000.

   5. Adirondack Furnishing Designs has bid to supply up to 800 furniture sets at a cost of £2,510 per set but with no delivery charge.

   6. Lancaster Artisan Company has bid to supply up to 1,100 furniture sets at a cost of £2,470 per set and with a one-time delivery charge of £13,000.

   (a) Construct a mixed-integer linear program (on paper) that purchases 2,000 furniture sets at minimum overall cost.

   (b) Suppose that a fifth company, Delaware Mills, has submitted a bid to supply up to 1,000 furniture sets at a cost of £2,530 per set and a one-time delivery charge of £9,000. However, if between 1,000 and 1,500 furniture sets are purchased from Delaware Mills, then Delaware Mills charges only £2,430/set for the additional furniture sets beyond the 1,000, with an additional delivery charge of only £7,000. Incorporate this new bid into your model.

(3) **Homework:** In class we have discussed the Markowitz portfolio optimisation problem. In this exercise, we want to explore its use for private investors. Private investors differ from investment banks in that they have limited means to diversify: If you have £50,000 to invest, for example, it is not usually meaningful to spread the money between 100 different assets (stocks, bonds, commodities) as otherwise you face large transaction costs (since every single buy/sell order is typically accommodated by a brokerage fee). In order to still achieve diversification, private investors often consider exchange traded funds (ETFs), which combine investments into many assets but incur lower fees than managed funds. **[100 marks]**

  (a) **Data collection:** Collect two years of weekly historical prices for 5 ETFs. The data can be collected, for example, from Yahoo Finance. The choice of ETFs is yours; you can get inspiration by searching "most popular etfs" or something similar online. Explain how you collected the data! **[20 marks]**

  (b) **Statistics:** Calculate the mean weekly return for each ETF, as well as the covariances of the weekly returns between all pairs of ETFs. Present your results, either in an Excel table or in text. Again, please explain how you accomplished the task! **[10 marks]**

  (c) **Optimisation:** By solving a sequence of Markowitz problems, produce a graph that shows the efficient frontier. Present the optimal portfolio composition for three selected portfolios (low return/low risk, medium return/medium risk and high return/high risk). Also plot each of the 5 individual ETFs that you selected in the graph. Provide the Markowitz problem that you used for computing the portfolios! (It can be implemented in Excel, AMPL or Python — please explain your implementation, however!)

  Which portfolio would you choose? Justify your answer! **[40 marks]**

  (d) **Additional constraints:** How would you incorporate additional constraints to ensure that you invest at least 10% and at most 35% of your wealth into any single ETF? You do not have to resolve your problem from part (c) — a description of the constraint is sufficient. **[5 marks]**

  (e) **Backtesting:** Prior to implementing a portfolio in practice, one would first verify the performance of the portfolio through a procedure called *backtesting*. Describe how you would implement a backtesting procedure for your portfolio problem! A verbal description is sufficient; you do not need to implement your backtesting procedure. (We assume that you are not familiar with the concept of backtesting; this requires some online research on your behalf. **[25 marks]**

<u>Hint:</u> *There are multiple ways in which the different steps can be performed (e.g., which ETFs to choose, how to measure the time frame of two years, which formula to use for calculating the covariances etc.). You can also use different software to accomplish the various steps (Excel, AMPL, Python (in the latter case, possibly with the help of some packages — but please formulate the optimisation model using a solver, not a pre-implemented Markowitz package). Please choose what you think is reasonable and justify your choice in the answer! Any reasonable choice will be accepted.*