Imperial College London
BUSINESS SCHOOL

MSc Business Analytics 2020/21
Optimisation and Decision Models
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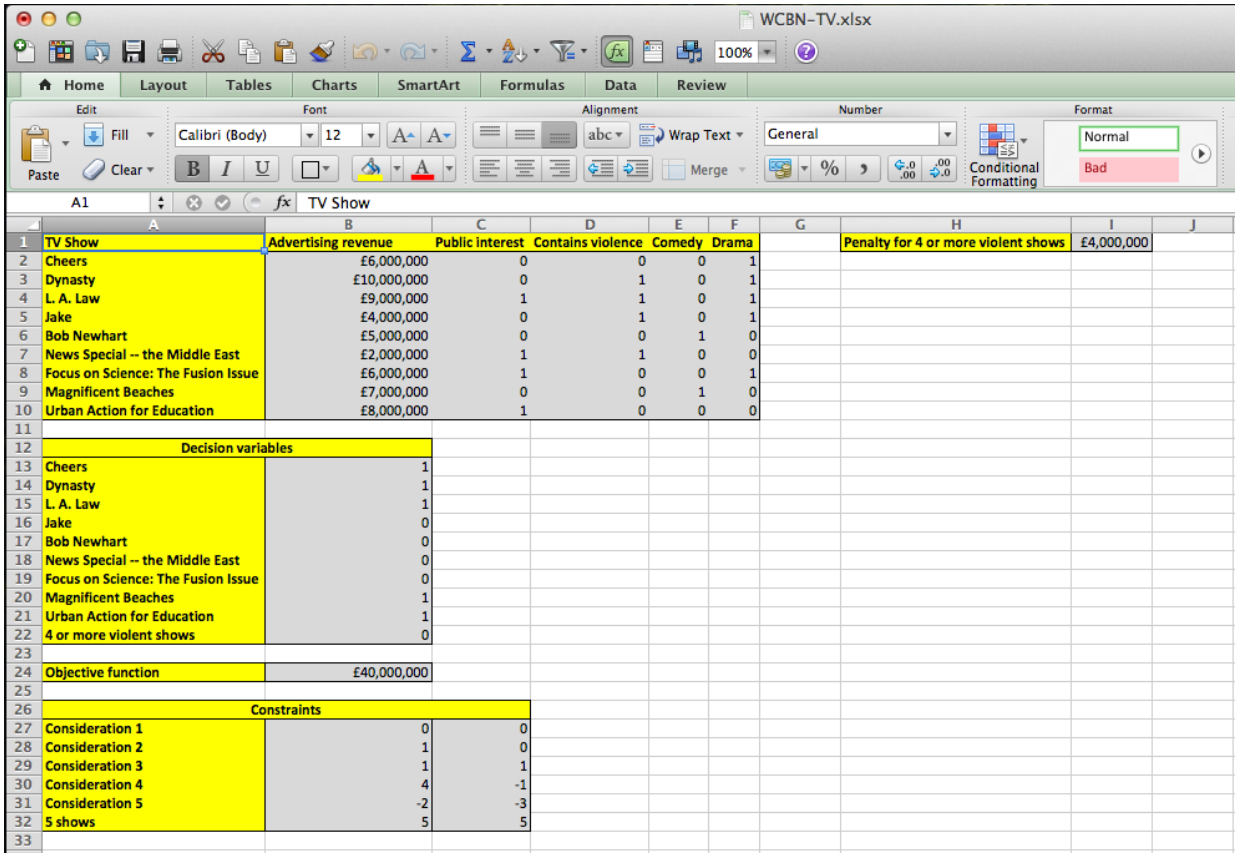
Solutions 3

Solution to (1) (a): We introduce the binary decision variables C, D, L, J, B, N, M and U for the different TV shows, with the interpretation that $C = 1$ if "Cheers" is scheduled and $C = 0$ otherwise (analogously for the other variables). We also introduce a variable V that attains the value 1 if 4 or more shows in the "contains violence" category are scheduled. WCBN-TV's scheduling problem can then be formulated as the following binary optimisation problem:

$$
\begin{array}{ll}
\text{maximise} & 6C + 10D + 9L + 4J + 5B + 2N + 6F + 7M + 8U - 4V \\
\text{subject to} & L + N + F + U \geq D + L + J + N \\
& F \leq J + L \\
& F + U \leq 1 \\
& C + B + M - 1 \leq 2 * (C + D + L + J + F) \\
& D + L + J + N - 3 \leq V \\
& C + D + L + J + B + N + F + M + U = 5 \\
& C, D, L, J, B, N, F, M, U, V \in \{0, 1\}
\end{array}
$$

Here, the first five constraints correspond to the five scheduling considerations from the question. In view of the fourth constraint, note that the left-hand side is positive if and only if at least two of the three variables C, B and M attain the value 1. In that case, the constraint requires at least one of the variables C, D, L, J or F on the right-hand side to attain the value 1.

Solution to (1) (b): The Excel model could look as follows (also provided separately):



	A	B	C	D	E	F	G	H	I	J
1	TV Show	Advertising revenue	Public interest	Contains violence	Comedy	Drama		Penalty for 4 or more violent shows	£4,000,000	
2	Cheers	£6,000,000	0	0	0	1				
3	Dynasty	£10,000,000	0	1	0	1				
4	L. A. Law	£9,000,000	1	1	0	1				
5	Jake	£4,000,000	0	1	0	1				
6	Bob Newhart	£5,000,000	0	0	1	0				
7	News Special -- the Middle East	£2,000,000	1	1	0	0				
8	Focus on Science: The Fusion Issue	£6,000,000	1	0	0	1				
9	Magnificent Beaches	£7,000,000	0	0	1	0				
10	Urban Action for Education	£8,000,000	1	0	0	0				
11										
12	Decision variables									
13	Cheers	1								
14	Dynasty	1								
15	L. A. Law	1								
16	Jake	0								
17	Bob Newhart	0								
18	News Special -- the Middle East	0								
19	Focus on Science: The Fusion Issue	0								
20	Magnificent Beaches	1								
21	Urban Action for Education	1								
22	4 or more violent shows	0								
23										
24	Objective function	£40,000,000								
25										
26	Constraints									
27	Consideration 1	0	0							
28	Consideration 2	1	0							
29	Consideration 3	1	1							
30	Consideration 4	4	-1							
31	Consideration 5	-2	-3							
32	5 shows	5	5							
33										

Thus, the optimal solution is to schedule Cheers, Dynasty, L. A. Law, Magnificent Beaches and Urban Action for Education, leading to revenues of £40m.

Solution to (2) (a): We introduce the binary variables A, B, C, L, S and W to indicate whether (value 1) or not (value 0) we place an ATM machine in Arlington, Belmont, Cambridge, Lexington, Somerville and Winchester, respectively. The optimisation problem can then be cast as follows:

$$
\begin{array}{ll}
\text{minimise} & A + B + C + L + S + W \\
\text{subject to} & A + B + C \geq 1 \\
& A + B + C + L \geq 1 \\
& A + B + C + W \geq 1 \\
& B + L + S \geq 1 \\
& L + S \geq 1 \\
& C + W \geq 1 \\
& A, B, C, L, S, W \in \{0, 1\}
\end{array}
$$

Solution to (2) (b): The AMPL model could look like this:

```
var A binary;
var B binary;
var C binary;
var L binary;
var S binary;
var W binary;

minimize objective: A + B + C + L + S + W;

subject to Arlington: A + B + C >= 1;
subject to Belmont: A + B + C + L >= 1;
subject to Cambridge: A + B + C + W >= 1;
subject to Lexington: B + L + S >= 1;
subject to Somerville: L + S >= 1;
subject to Winchester: C + W >= 1;
```

The optimal solution installs ATMs in Cambridge and Somerville. Another optimal solution would be to install ATMs in Cambridge and Lexington, resulting in the same optimal value.