

min

$$3x_{su} + 2x_{sv} + 0.7x_{uv} + x_{ut} + 7x_{vt}$$

flow conservation.

$$x_{su} + x_{sv} = 15 \leftarrow S$$

$$x_{su} = x_{uv} + x_{ut} \leftarrow u, \text{ flow-in} = \text{flow-out}$$

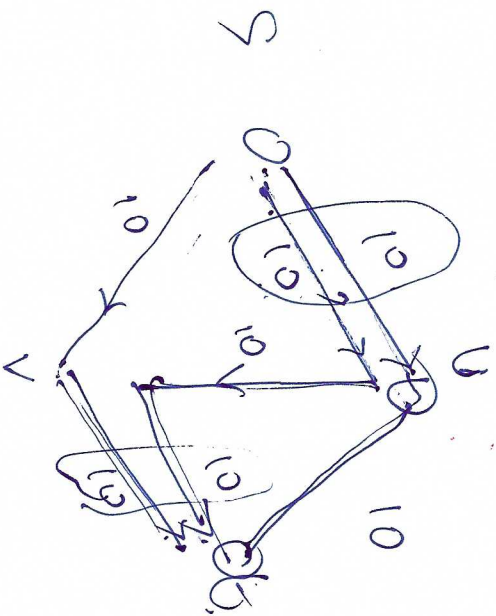
$$x_{sv} + x_{uv} = x_{vt} \leftarrow v$$

$$x_{ut} + x_{vt} = 15$$

$$0 \leq x_{su} \leq 20 \leftarrow \text{Capacity}$$

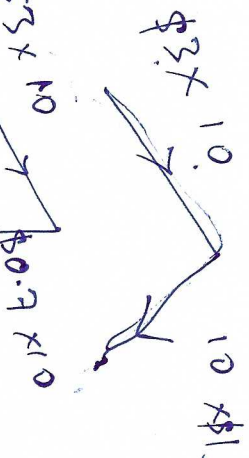
$$10 + 10 + 10$$

30

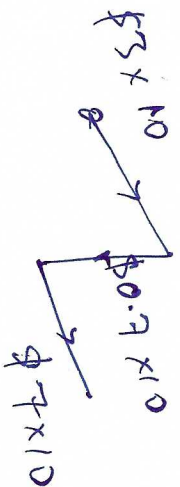


Cost of the flow

Min Cost

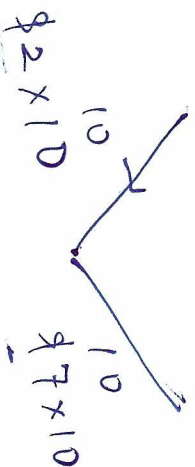


$$= \$40$$



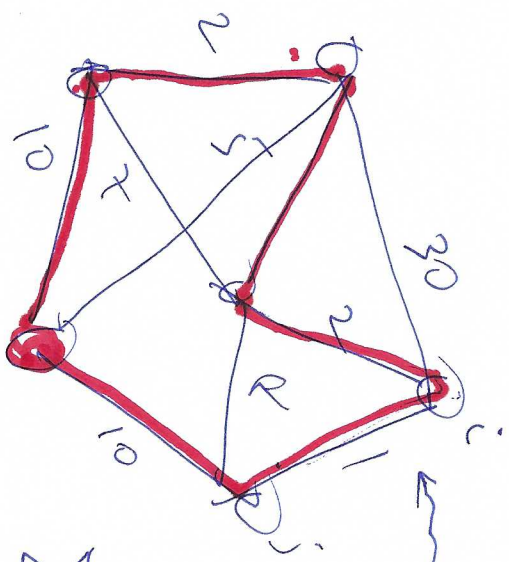
$$30 + 7 + 70 = \$107$$

Send 10 : \$40  
Send 10 : \$40  
Send 10 : \$40  
Send 10 : \$40



$$= 20 + 70 = \$90$$

$$\underline{\$237}$$



$$x_{e_{ij}} = \begin{cases} 1 & \text{if } (i,j) \text{ is in the tour} \\ 0 & \text{otherwise} \end{cases}$$

$$\min \sum_{(i,j)} c_{ij} \cdot x_{ij}$$

$$\sum_i x_{ij} = 2 \quad \forall j \text{ node}$$

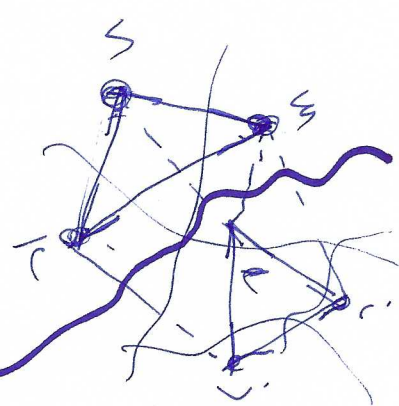
Sub-tours

$$x_{mi} + x_{me} + x_{nl} + x_{ic} + x_{ij} \geq 2$$

Sub-tour elimination constraint

All cuts

cut



$$N = \# \text{ of nodes}$$

$$\text{Min } 2x_{ab} + 3x_{ad} + 7x_{ae} + \dots$$

Degree constraints

$$a: x_{ab} + x_{ae} + x_{ad} = 2$$

$$d: x_{ad} + x_{ed} + x_{cd} = 2$$

$$b: \dots = 2$$

$$c: \dots = 2$$

$$e: \dots = 2$$

degree

Sub-tour elimination: (enumerate cuts)

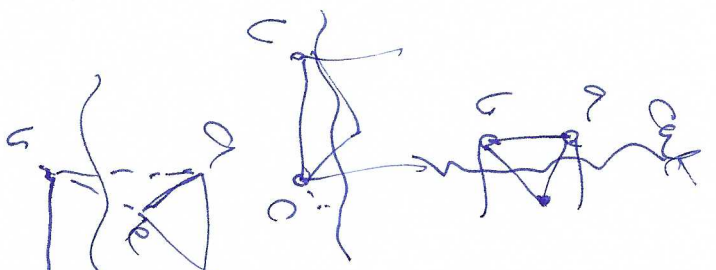
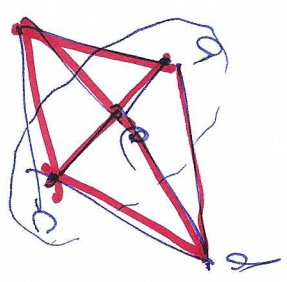
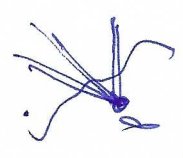
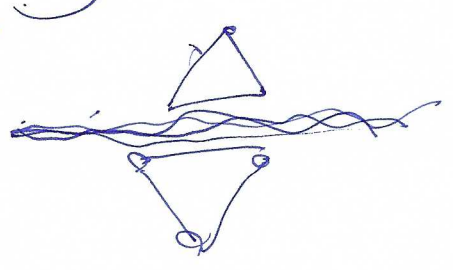
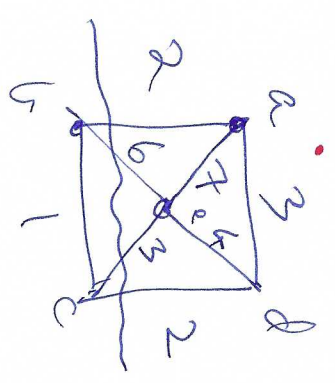
**{a,b}**

$$x_{ad} + x_{ae} + x_{be} + x_{bc} \geq 2$$

$$\{b,c\} \rightarrow x_{ba} + x_{be} + \dots + x_{ce} + x_{cd} \geq 2$$

{a,d,e}

$$\{a,e,c\} \rightarrow x_{ad} + x_{ed} + x_{cd} + x_{ab} + x_{be} + x_{bc} \geq 2$$



Ordering itself

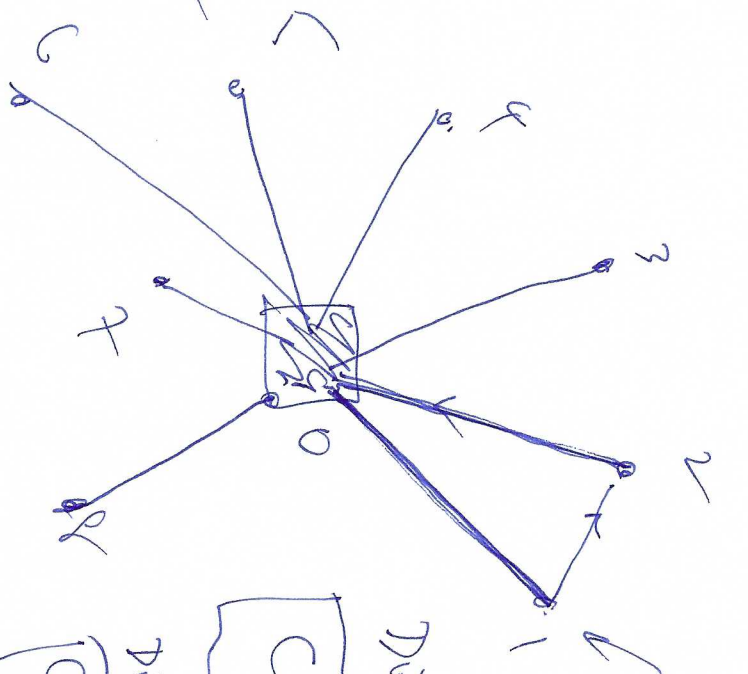


Heuristic



Savings: 12  
 Savings: 23

Greedy



customers

Distance (stupid)

$$C_{01} + C_{10} + C_{02} + C_{20}$$

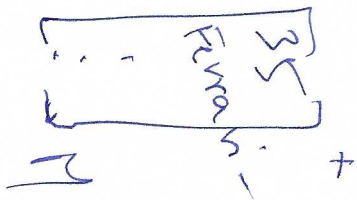
Distance with merge

$$C_{01} + C_{12} + C_{20}$$

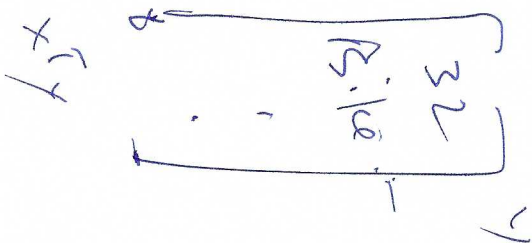
Subtract

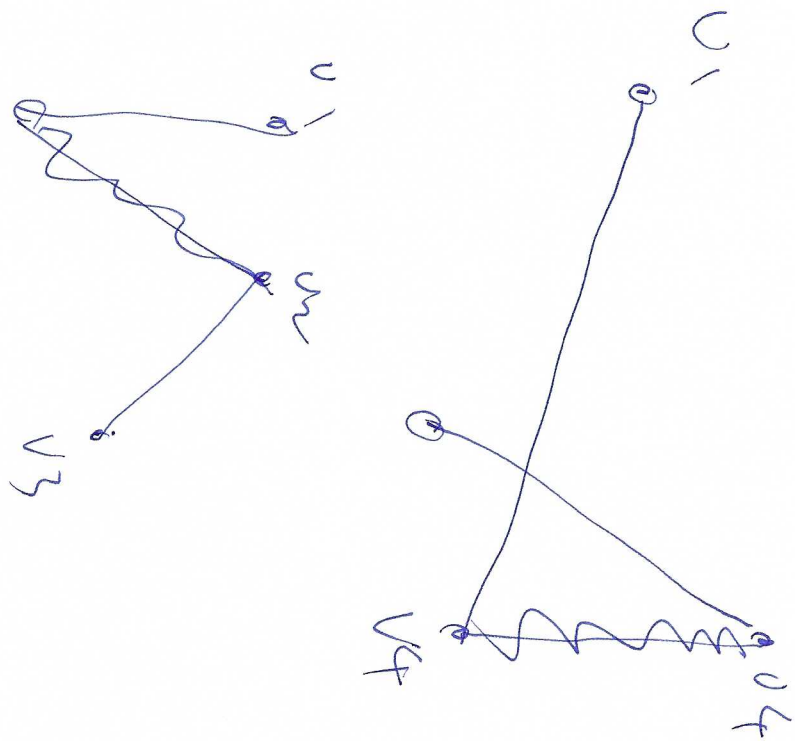
Savings:  $C_{10} + C_{02} - C_{12}$

Find the pair with maximum savings and then Merge

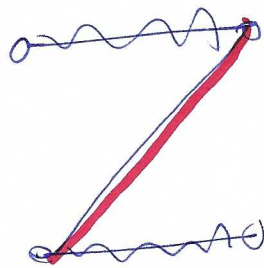
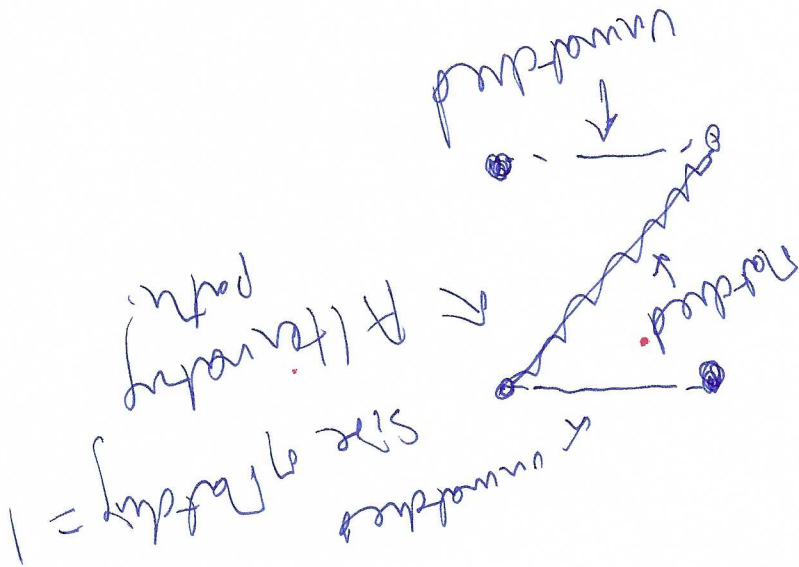
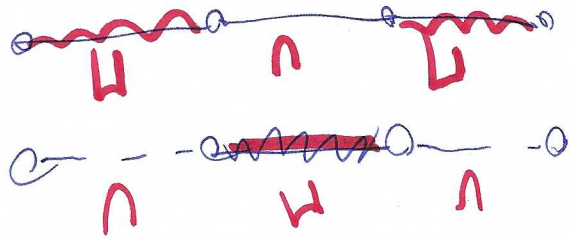


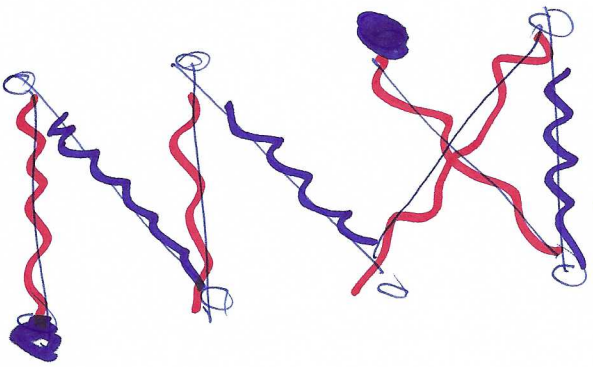
F











the markers

not  
a perfect  
match

match