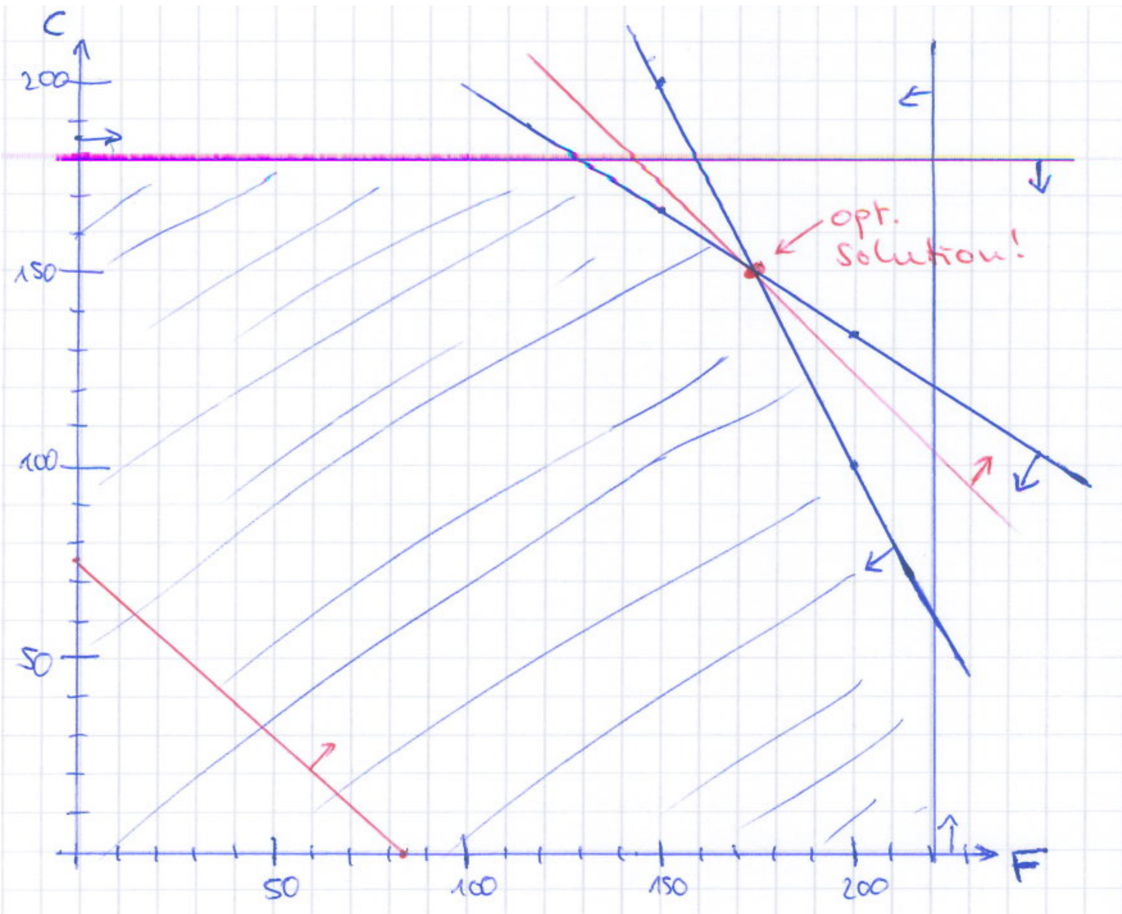
Solution to Assignment 1

Solution to (1) (a): Using the decision variables F and C for full-size and compact microwave ovens, respectively, the linear program reads as follows:

$$
\begin{array}{ll}
\text{maximise} & 120F + 130C \\
\text{subject to} & 2F + 1C \leq 500 \\
& 2F + 3C \leq 800 \\
& F \leq 220,\ C \leq 180 \\
& F,\ C \geq 0
\end{array}
$$

Here, the objective function maximises earnings, whereas the constraints ensure (in order) the satisfaction of the general assembly labour budget, the electronic assembly labour budget, the demand bounds and non-negativity of the production schedule.

Solution to (1) (b): The graphical solution looks as follows:



The optimal solution seems to be something like $F \approx 172$ and $C \approx 150$ with an objective value of

$$120 * 172 + 130 * 150 = 40{,}140.$$

The two labour constraints are binding at the optimal solution.

Solution to (1) (c): Please refer to the Excel sheet for the model. The optimal solution produces 175 full-size ovens and 150 compact ovens with a total earnings of £40,500.

Solution to (1) (c): Resolving the problem with 510 general assembly hours instead of 500 yields the higher earnings of £40,750, that is, an increase of £250. If we divide £250 by 10 to obtain the increase per hour, we obtain £25. In other words, Magnetron should pay at most £25 for each additional hour of general assembly time (within a certain range, as we will discuss later).

Solution to (2) (a): Using the decision variables S, B, H and V for the production amounts of stir fry, barbecue, hearty mushrooms and veggie crunch, respectively, the linear program reads as follows:

$$
\begin{array}{lrrrrl}
\text{maximise} & 0.22S + & 0.20B + & 0.18H + & 0.18V & \\
\text{subject to} & 62.5S + & 50B + & & 62.5V & \leq 3{,}750{,}000 \\
& 75S + & & 100H & & \leq 2{,}000{,}000 \\
& 62.5S + & 50B + & 75H + & 62.5V & \leq 3{,}375{,}000 \\
& 50S + & 75B + & 75H + & 62.5V & \leq 3{,}500{,}000 \\
& & 72B + & & 62.5V & \leq 3{,}750{,}000 \\
& \multicolumn{5}{l}{S,\ B,\ H,\ V \geq 0}
\end{array}
$$

Here, the objective function maximises earnings, whereas the constraints ensure (in order) the satisfaction of the carrots, mushrooms, green peppers, broccoli and corn constraints, as well as the non-negativity of the production schedule.

Solution to (2) (b): The AMPL model could look like this (see also the attached model):

```
# decision variables
var S;
var B;
var H;
var V;

# objective function
maximize earnings: 0.22 * S + 0.20 * B +  0.18 * H + 0.18 * V;

# constraints
subject to carrots: 62.5 * S + 50 * B + 62.5 * V <= 3750000;
subject to mushrooms: 75 * S + 100 * H <= 2000000;
subject to green_peppers: 62.5 * S + 50 * B + 75 * H + 62.5 * V <=
3375000;
subject to broccoli: 50 * S + 75 * B + 75 * H + 62.5 * V <= 3500000;
subject to corn: 72 * B + 62.5 * V <= 3750000;
subject to nn_S: S >= 0;
subject to nn_B: B >= 0;
subject to nn_H: H >= 0;
subject to nn_V: V >= 0;
```

The optimal solution is to produce 26,666.67 bags of Stir Fry, 18,333.33 bags of Barbecue and 12,666.67 bags of Veggie Crunch, with overall earnings of £11,813.33:

```
ampl: solve;
MINOS 5.51: optimal solution found.
3 iterations, objective 11813.33333

ampl: display S, B, H, V;
S = 26666.7
B = 18333.3
H = 0
```

```
V = 12666.7
```

We can verify which constraints are binding as follows:

```
ampl: display carrots.slack, mushrooms.slack, green_peppers.slack,
broccoli.slack, corn.slack;
carrots.slack = 375000
mushrooms.slack = 0
green_peppers.slack = 0
broccoli.slack = 0
corn.slack = 1638330
```

Thus, the mushrooms, the green peppers and the broccoli are fully used, whereas there are carrots and corn left.

Solution to (2) (c): There are two ways to answer this question. The first one is to resolve the problem with the updated quantity of green peppers:

```
# decision variables
var S;
var B;
var H;
var V;

# objective function
maximize earnings: 0.22 * S + 0.20 * B +  0.18 * H + 0.18 * V;

# constraints
subject to carrots: 62.5 * S + 50 * B + 62.5 * V <= 3750000;
subject to mushrooms: 75 * S + 100 * H <= 2000000;
subject to green_peppers: 62.5 * S + 50 * B + 75 * H + 62.5 * V <=
3475000;
subject to broccoli: 50 * S + 75 * B + 75 * H + 62.5 * V <= 3500000;
subject to corn: 72 * B + 62.5 * V <= 3750000;
subject to nn_S: S >= 0;
subject to nn_B: B >= 0;
subject to nn_H: H >= 0;
subject to nn_V: V >= 0;
```

We see that the overall earnings increase to £11,877.33, that is, the value of 100kg of additional green peppers is £11,877.33 - £11,813.33 = £64:

```
ampl: solve;
MINOS 5.51: optimal solution found.
3 iterations, objective 11877.33333
ampl: display S, B, H, V;
S = 26666.7
B = 14333.3
H = 0
```

```
V = 17466.7
```

Alternatively, we will soon see in class that we can use the sensitivity information to answer the question:

```
ampl: option solver cplex;
ampl: option cplex_options 'sensitivity';
ampl: solve;
CPLEX 12.6.3.0: sensitivity
CPLEX 12.6.3.0: optimal solution; objective 11877.33333
3 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;

ampl: display green_peppers.down, green_peppers, green_peppers.up;
green_peppers.down = 3111110
green_peppers = 0.00064
green_peppers.up = 3750000
```

The report shows that the Green peppers constraint has a shadow price of £0.00064/g. Since an increase by 100kg is within the allowable increase, the answer is 100,000g * £0.00064/g = £64.
(Please ignore this part of the answer until we have covered the topic of sensitivity analysis!)

Solution to (3) (a): Using the decision variables R, G and I for restructuring, growth and IT risk projects, respectively, the linear program reads as follows:

$$\begin{array}{ll} \text{maximise} & 25\,R + 30\,G + 20\,I \\ \text{subject to} & 2R + 3G + I \leq 100 \\ & R + G + 3I \leq 75 \\ & R,\ G,\ I \geq 0 \end{array}$$

Here, the objective function maximises the revenues from the three types of projects, and the constraints ensure (in order) the satisfaction of the strategy consultant and IT expert workforce availability, respectively. And we have, of course, the non-negativity constraints.

Solution to (3) (b): Please refer to the AMPL model. The optimal solution delivers 45 restructuring, no growth and 10 IT risk projects, earning revenues of £1,325,000.

Here is some of the advice that we could give to ImpactNow:

- At the moment, growth projects are not as profitable as restructuring and IT risk projects. ImpactNow therefore should reconsider their project portfolio; they could either focus on two types of projects only, consider a different mix of projects altogether, or check whether the price of growth projects can be increased to make them as profitable as the other ones.
- If we increase the availability of strategy consultants from 100 to 101, the revenues increase from £1,325,000 to £1,336,000, that is, by £11,000. If we increase the availability of IT risk experts from 75 to 76, on the other hand, the revenues increase from £1,325,000 to £1,328,000, that is, by only £3,000. This may have consequences on the salaries that can and should be paid to their workforce — different employees contribute differently to the overall revenues. (This may of course have legal and ethical consequences that need to be accounted for!)

It is also important to keep in mind the shortcomings of the current model:

- We assume that there is unlimited demand for projects.
- It is assumed that each project takes exactly one month.
- We don't account for staff holidays, sick leave etc.
- All data is considered to be deterministic, known and stationary (that is, not changing over time).

All of these shortcomings can be addressed in more elaborate variants of the problem. It is important, however, to keep models as simple as possible, as long as it provides useful advice. For strategic problems like this one, a simple model whose output can be interpreted may be preferable to a highly sophisticated model that is a "black box". (This is different in operational models that guide e.g. the weekly production plan — here, complicated models may well be suitable to "squeeze out" the last bit of efficiency.)

Solution to (4) (a): We can formulate the problem as follows:

maximise $0.13 * (SW_1 + SW_2 + SW_3 + SW_4) +$
$0.10 * (SP_1 + SP_2 + SP_3 + SP_4)$

subject to

$$1.5 * PW_t + 1.0\ PP_t <= 15{,}000 \quad \text{for } t = 1, \dots, 4$$
$$1.0 * PW_t + 1.0\ PP_t <= 12{,}000 \quad \text{for } t = 1, \dots, 4$$
$$0.3 * PW_t + 0.5\ PP_t <= 8{,}000 \quad \text{for } t = 1, \dots, 4$$

$$SW_1 <= 5{,}000,\ SW_2 <= 9{,}000,\ SW_3 <= 10{,}000,\ SW_4 <= 15{,}000$$
$$SP_1 <= 6{,}000,\ SP_2 <= 11{,}000,\ SP_3 <= 12{,}000,\ SP_4 <= 15{,}000$$

$$IW_1 = IP_1 = 0$$
$$IW_{t+1} = IW_t + PW_t - SW_t \quad \text{for } t = 1, \dots, 4$$
$$IP_{t+1} = IP_t + PP_t - SP_t \quad \text{for } t = 1, \dots, 4$$

$$SW_t,\ SP_t,\ PW_t,\ PP_t >= 0 \quad \text{for } t = 1, \dots, 4$$
$$IW_t,\ IP_t >= 0 \quad \text{for } t = 1, \dots, 5$$

Here, the objective function maximises the sales across all four quarters. The first constraint set is reminiscent of those presented in class, just repeated for all four quarters. The same goes for the second constraint set, which deals with the demand limits. The third constraint set manages the inventory: It ensures that the initial inventories are zero, and that the subsequent inventories equal the previous inventories plus the production minus the sales. Note that we have an inventory for an auxiliary fifth quarter, to ensure that we only sell what we have actually produced.

Solution to (4) (b): Please refer to the AMPL model. The optimal production plan looks as follows:

	Quarter 1	**Quarter 2**	**Quarter 3**	**Quarter 4**
Prod. wrenches	6,000	6,000	6,000	6,000
Prod. pliers	6,000	6,000	6,000	6,000
Sales wrenches	0	0	9,000	15,000
Sales pliers	6,000	6,000	6,000	6,000
Inventory wrenches	0	6,000	12,000	9,000
Inventory pliers	0	0	0	0

The overall earnings are US$ 5,520. The solution also unveils an unrealistic “feature” of our model: Since we do not impose costs on inventories (so-called inventory holding costs), we do not sell any wrenches in the first two quarters even if we could. (This actually depends on the solver that you use: the optimal solution to this problem is not unique, and hence different solvers could provide different optimal solutions.)

Solution to (4) (c): Under their old strategy, the company has produced as many wrenches as possible. Let us compute this quantity:

15,000 lbs. of steel allow us to produce 15,000 / 1.5 = 10,000 wrenches.
12,000 h of molding machine allow us to produce 12,000 / 1.0 = 12,000 wrenches.
8,000 h of assembly machine allow us to produce 8,000 / 0.3 = 26,666 wrenches.

In other words, the company can produce up to 10,000 wrenches per quarter. Since no more steel is left in that case, the company only sells wrenches. The overall sales generate revenues of:

5,000 + 9,000 + 10,000 + 15,000 = 39,000 wrenches, that is, 0.13 * 39,000 = $5,070.

Thus we see that our optimisation-based production plan leads to profits that are about 8.88% higher. In practice, a profit difference that large can make the difference between a thriving and a bankrupt company!

Let us now summarise some of the potential advantages and shortcomings of moving from a heuristic approach to an optimisation-based inventory management approach:

Advantages:

- As we have seen before, the optimisation-based solution leads to higher revenues.
- We can easily incorporate a range of constraints in the optimisation model that are difficult to handle with a heuristic approach (just remember the constraints that we included in the New Bedford Steel problem!).
- The optimisation-based solution can be readily obtained even for much larger problems (e.g., an annual production planning problem for thousands of products with a weekly granularity).
- By relying on trusted and tested third-party optimisation software, we are introducing a layer of abstraction that simplifies the problem for Gemstone Tool Company: We only need to formulate the optimisation problem, and we can (hopefully) rely on the optimisation software to provide us with the optimal and correct solution. There is no need for manual calculations or the design of tailored algorithms.

Disadvantages:

- The solution of the optimisation problem may be less intuitive for decision makers: It is not that easy to see why the optimisation-based solution does what it does.
- Because of the previous point, there may be less buy-in for the optimisation-based solution in the company.
- Going over to an optimisation-based solution may require an initial investment (possibly in optimisation software, although there is good open source software available, but also in optimisation experts).