Convex

in 1 variable	in n variables
$f(x) = ax+b$	$f(x) = a^T x + b$
$f(x) = x^2+bx+c$	$f(x) = x^T M x + c^T x$ where M is PSD
$f(x) = \lvert x \rvert$	$f(x) = \lVert x \rVert$ where $\lVert \cdot \rVert$ in any norm
$f(x) = -\ln(x)$ for $x>0$	$\sum_{i=1}^{m} -\ln(b_i - a_i^T x)$ for x satisfying $Ax<b$
$f(x) = \frac{1}{x}$ for $x>0$	
$f(x) = e^x$	

判断凸优化的方法 (which of the following optimisation problem is convex)

①. objective

minimize convex $\boxed{\text{or}}$ maximize concave

②. feasible region

convex $\leq c$

concave $\geq c$

Addition: if $f_1(x)$ and $f_2(x)$ are convex, $a,b>0$

$f(x) := a \cdot f_1(x) + b \cdot f_2(x)$ is also convex

Maximum: if $f_i(x)$ are convex, $i=1,2$

$f(x) = \max_i f_i(x)$ is also convex

Composition: if $f(x)$ is convex, then

$g(y) := f(Ay+b)$ is also convex

Nonlinear Programming Exercises

Objective:

(1). minimise $\sum_{i=1}^{m}(y_i - b_0 + b_1 x_{i1} + \cdots + b_n x_{in})^2$

- y_i, x_{i1}, x_{i2} are all affine
- $y_i - b_0 - x_{i1}b_1 - \cdots - x_{in}b_n$ is affine
- $(y_i - b_0 - x_{i1}b_1 - \cdots x_{in}b_n)^2$ is affine

so it is convex

Constraints:

(1) -①. $b_0, b_1, \cdots b_n \in R$

- entire space R^n
- so convex

(1) -②. $b_j \geq -10$, $b_j \leq 10$

- linear constraints
- so convex

(1) -③. $b_1 \geq 2b_2$

- $b_1 - 2b_2 \leq 0$
- linear constraint
- so convex

(1). -④. $b_3 = b_4$

- $b_3 - b_4 = 0$
- linear constraint
- so convex

(1). -⑤. $|b_j| \leq 10$

- Adding auxiliary variables y_j
- $y_j \geq b_j$, $y_j \geq -b_j$
- $y_1 + y_2 + \cdots y_{10} \leq 10$
- so convex

(1). -⑥. At most 5 of the slopes should be non-zero

- not convex

(1)-⑦. $b_5 \le 1$ or $b_5 \ge 2$

- not convex

(2). minimise $\frac{1}{x_1} + \frac{2}{x_2} + |x_3|$

- $\frac{1}{x_1}$, $\frac{2}{x_2}$, $|x_3|$ are all convex
- so it is convex

(2)-①. $\max\{x_1+x_2, x_1-x_3\} >= c$

- $\max\{x_1+x_2, x_1-x_3\}$ is convex
- but the form is convex $\le c$
- So not convex

This problem is not convex

(3). maximise $x_1 - x_2^2$

- x_1 is linear, hence convex and concave
- x_2^2 is convex, hence $-x_2^2$ is concave
- $x_1 - x_2^2$ is concave
- maximise concave

(3)-①. $(2x_1 - x_2)^2 \le x_1$

- $(2x_1 - x_2)^2 - x_1 \le 0$
- $2x_1 - x_2$ is linear, hence convex
- $(2x_1 - x_2)^2$ is convex
- $(2x_1 - x_2)^2 - x_1$ is convex
- convex $\le C$

This problem is convex

(4). maximise $3x_1 - 2x_2 + 5^2$

- $3x_1$ linear
- $2x_2$ linear
- $3x_1 - 2x_2 + 5^2$ is linear
- maximise concave

(4)-①. $x_1 + x_2 \le 2$

- linear
- concave $\ge c$

(4)-②. $x_2 >= x_1$

- $x_2 - x_1 \ge 0$
- linear
- concave $\ge c$

(4) -③. $x_1, x_2 \geq 0$

- concave $\geq c$

This problem is convex

(5). minimise x_1

- x_1 is linear
- maximise concave

(5) -①. $x_1 \cdot x_2 >= 2$ ($x_2 \geq 4$)

- since we know x_2 is positive
- $x_1 \geq 2/x_2$
- $x_1 - 2/x_2 \geq 0$
- x_1 linear
- $\frac{2}{x_2}$ convex, hence $-\frac{2}{x_2}$ concave
- $x_1 - \frac{2}{x_2}$ concave
- concave $\geq c$

This problem is convex.

(6). minimise $x^T \Sigma x$

- x^T linear
- Σx linear
- $x^T \Sigma x$ convex
- minimise convex

(6) -①. $e^T x = 1$

- $e^T x = 1$: linear
- $\mu^T x \geq \gamma$: linear

This problem is convex

(7).