

Interest Rates and Deterministic Cash-Flows

In these lecture notes we review concepts related to interest rates such as present and future values of deterministic cash-flows as well as internal rates of return. We also discuss various types of fixed-income securities including bonds, zero-coupon bonds, annuities and perpetuities. We introduce the term-structure of interest rates and show how one can move back and forth between spot rates / discount factors and forward rates discount factors. We end with a brief review of mortgage mathematics and how it leads very easily to the concept of securitization.

1 Basic Theory of Interest

Cash-flow Notation: We use $(c_0, c_1, \ldots, c_i, \ldots, c_n)$ to denote a series of cash-flows where c_i is received at time $t = i$. The length of a period, i.e. the interval of time between $t = i$ and $t = i + 1$, will usually be understood from the context. Negative cash-flows refer to cash payments. The initial cash-flow c_0 will often be negative while the remaining cash-flows are positive. This situation models the cash-flows of many securities such as stocks and bonds. In such circumstances, $c_0 < 0$ denotes the cost of the security, while later cash-flows refer to dividends, coupons or sale receipts all of which are positive.

Definition 1 *If A is invested in an account for n periods with a* **simple interest** *rate of r per period, then after n periods the account will be worth $A(1 + rn)$.*

Definition 2 *If A is invested in an account for n periods with a* **compound interest** *rate of r* per period, *then after n periods the account will be worth $A(1 + r)^n$.*

Interest rates are usually quoted on an annualized basis, even if the compounding period is less than a year. For example, the phrase "*10% interest, compounded quarterly*" implies that an investment of A will be worth $A(1 + .1/4)^4$ one year later. Similarly, the phrase "*10% interest, compounded semi-annually*" implies that an investment of A will be worth $A(1 + .1/2)^2$ after one year. In general, if there are n compounding periods per year and the interest rate is $r\%$, then an investment of A will be worth $V = A(1 + r/n)^{mn}$ after m years.

Definition 3 **Continuous compounding** *refers to the situation where we let the length of the compounding period go to 0. That is, after m years we see that an investment of A will be worth*

$$\lim_{n\to\infty} A(1 + r/n)^{mn} = Ae^{rm}.$$

Definition 4 *The* effective interest rate *is that rate which would produce the same result if compounding were done per year rather than per period.*

So if the length of a compounding period is one year, the effective interest rate is the same as the quoted or *nominal* rate. For example, A invested for 1 year at 10% interest, compounded quarterly, will be worth $1.1038A$ and so the effective interest rate is 10.38%.

1.1 Present and Future Values of Cash Flow Streams

The value of $\$1$ today is clearly not the same as the value of $\$1$ next year. However, the simple concepts of present value and future value allow us to fairly compare cash flows that occur at different dates. For example $\$1$ invested today at $t = 0$ would be worth $\$(1 + r)$ next year at $t = 1$ assuming[1] annual compounding. So for $r > 0$, we can conclude that $\$1$ at $t = 0$ is worth more[2] than $\$1$ at $t = 1$.

We can then reverse the argument to say that $\$(1 + r)$ at $t = 1$ is worth $\$1$ at $t = 0$. That is, the **present value** of $\$(1 + r)$ at $t = 1$ is $\$1$. We say that we are **discounting** the cash flow at $t = 1$ back to $t = 0$. Likewise the *future value* at $t = 1$ of $\$1$ at $t = 0$ is $\$(1 + r)$. More generally, the present value of the cash-flow $(c_0, c_1, \ldots, c_n)$ is

$$PV \;=\; c_0 + \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2} + \;\ldots\; + \frac{c_n}{(1+r)^n}$$

where we have assumed compounding is done per period. Likewise, the future value (at $t = n$) of the cash-flow is

$$FV \;=\; PV(1+r)^n \;=\; c_0(1+r)^n + c_1(1+r)^{n-1} + c_2(1+r)^{n-2} + \;\ldots\; + c_n.$$

You must be careful to observe the compounding convention. For example, if cash-flows occur yearly, interest rates are quoted on an annual basis (as usual) but are compounded m times per year then

$$PV \;=\; c_0 + \frac{c_1}{(1+r/m)^m} + \frac{c_2}{(1+r/m)^{2m}} + \;\ldots\; + \frac{c_n}{(1+r/m)^{nm}}$$

and

$$FV \;=\; PV(1+r/m)^{nm}.$$

Note that the individual cash flows c_i may be positive or negative.

1.2 Internal Rate of Return (IRR)

Definition 5 *Given a cash flow stream $(c_0, c_1, \ldots, c_n)$, the* **internal rate of return** *(IRR) is a number, r, that satisfies*

$$0 \;=\; c_0 + \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2} + \;\ldots\; + \frac{c_n}{(1+r)^n}. \tag{1}$$

- In most contexts, c_0 will be negative and we can interpret it as the price paid today for the cash flow stream, $(c_1, \ldots, c_n)$. Then r is just the number that makes this a "fair deal".
- If we let $z = 1/(1 + r)$ we can see that (1) is a polynomial in degree n. In general, (1) could then have zero, one or as many as n real roots so it is not always clear what the appropriate value of r is. (But see the following exercise.)

Exercise 1 *Show that when $c_0 < 0$ and $c_k \geq 0$ for all $k \geq 1$, there exists a unique positive root z^* to the equation*

$$0 = c_0 + c_1 z + c_2 z^2 + \;\ldots\; + c_n z^n.$$

Furthermore, if $\sum_{k=0}^{n} c_k > 0$ then show the corresponding IRR $r = 1/z^* - 1$ is positive.

- Even when we know there exists a unique positive solution to (1), we usually have to solve for it numerically.

[1] The compounding convention should always be observed so that if we are using quarterly compounding, for example, then 1 invested at $t = 0$ would be worth $(1 + r/4)^4$ at $t = 1$.

[2] This statement assumes that there will be no inflation over the next year. If there is inflation then we should use *real* interest rates before concluding that $\$1$ today is worth more than $\$1$ next year; see Luenberger, Section 2.6. We will not consider inflation or related issues in this course.

1.3 Project Evaluation Criteria: NPV versus IRR

Many investment problems may be formulated as the problem of optimally allocating funds among a number of competing projects. The word *project* is significant as it usually implies that the projects cannot be scaled up or down. For example, a project may represent developing an oil field or introducing a new product to a market and the level of investment required for such projects is usually fixed[3]. These investment allocation problems are often called *capital budgeting* problems and they constitute an important topic in corporate finance.

When faced with a capital budgeting problem, we need to be able to evaluate the competing projects so that the appropriate decision can be made regarding which (if any) projects to invest in. There are a number of different criteria available for evaluating projects but the two most common are *Net Present Value* (NPV) and *Internal Rate of Return* (IRR). The NPV criterion amounts to selecting the projects with the highest NPVs while the IRR criterion amounts to selecting the projects with the highest IRRs. The two criteria often give conflicting recommendations but such conflicts can often be reconciled when common sense is applied.

A particular difficulty that arises when applying the NPV criterion is that of choosing the correct *discount factor*. This problem does not arise when cash flows are deterministic in which case the appropriate risk-free interest rates should be used for discounting. More generally, however, it is not always clear how to discount cash flows that are stochastic since the *riskiness* of the cash-flows also needs to be considered. This is an important problem in finance and we will return to it later in the course.

Example 1 (Sunk Costs - from Luenberger's *Investment Science*)
A young couple has made a non-refundable deposit of the first month's rent (equal to $\$1,000$) on a 6-month apartment lease. The next day they find a different apartment that they like just as well, but its monthly rent is only $\$900$. They plan to be in the apartment only 6 months. Assuming a (somewhat crazy!) monthly interest rate of 12% answer the following questions.

(a) Should they switch to the new apartment?

(b) What if they plan to stay 1 year?

<u>*Solution*</u>: Different interpretations are possible here. For example, the deposit may *never* be refundable and therefore represent a brokerage free that is paid up front. Alternatively, the deposit may be non-refundable only if the couple choose not to take the apartment. We will adopt the latter interpretation but note that regardless of the interpretation, the important aspect of this example is that sunk costs are exactly that: sunk!

(a) Compare the two alternatives.

1. Stay in the original apartment, for an NPV, C_1, given by

$$C_1 = -\sum_{i=0}^{5} \frac{1000}{1.12^i} + \frac{1000}{1.12^6} \approx -4,222.$$

2. Take the new apartment where we assume a security deposit is again required. The NPV then is

$$C_2 = -900 - \sum_{i=0}^{5} \frac{900}{1.12^i} + \frac{900}{1.12^6} \approx -4,700.$$

The couple should therefore stick with the $\$1,000$ apartment.

(b) This is left as an exercise. What result do you expect? ∎

[3] This contrasts with portfolio optimization problems where it is commonly understood that an investor may invest as little or as much as she chooses in a particular asset such as a stock or bond. In this context of portfolio optimization, the term *asset* is preferred to the term *project*.

Example 2 (An Appraisal - from Luenberger's *Investment Science*)
You are considering the purchase of a nice home. It is in every way perfect for you and in excellent condition, except for the roof. The roof has only 5 years of life remaining. A new roof would last 20 years, but would cost $\$20,000$. The house is expected to last forever. Assuming that costs will remain constant and that the interest rate is 5%, what value would you assign to the existing roof?

Solution: We know a new roof costs $20,000$ and that it lasts 20 years. We can therefore infer the value per year, A, of a roof by solving

$$20,000 \;=\; \sum_{i=0}^{19} \frac{A}{1.05^i}.$$

We find that $A = 1,528.4$. If V is the value of a roof that has 5 years of life remaining, we can obtain that $V = 6,948$. ∎

Exercise 2 *Find an alternative method for solving the problem in Example 2.*

Example 3 (Valuation of a Firm and the Gordon Growth Model)
A simple model that is sometimes used to determine the value of a corporation is the Gordon Growth Model. It assumes there is a constant interest rate r and that dividends are paid annually and grow at a rate of g. The value of the firm may then be expressed as

$$V_0 \;=\; \frac{D_1}{1+r} \;+\; \frac{D_1(1+g)}{(1+r)^2} \;+\; \ldots \;=\; D_1 \sum_{k=1}^{\infty} \frac{(1+g)^{k-1}}{(1+r)^k}$$

so that we obtain $V_0 = D_1/(r-g)$ for $g < r$.

- Clearly V_0 is very sensitive to changes in g and r when $g \approx r$. For this reason, the Gordon model has been used to provide some intuition for the volatility of growth stock prices; e.g. internet, biotech stocks.
- Obviously this model is very simple and is easy to generalize. For example, we could assume that dividends only grow for a certain fixed number of years, possibly beginning at some future date.
- More realistic models should of course assume that dividends are stochastic. Moreover, since dividend policy is set by directors, we might prefer to focus not on dividends but instead on the underlying cash flows of the corporation.

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2 Fixed Income Securities

Traditionally, fixed income securities refers to securities whose cash flows are fixed in advance and whose values therefore depend largely on the current level of interest rates. We discuss some common examples of fixed-income securities in this section.

2.1 Bonds, Zero-Coupon Bonds and Corporate Bonds

The classic example of a fixed income security is a **bond** which pays a fixed **coupon** every period, e.g. every 6 months or every year, until maturity when the final coupon and **principal / face-value** are paid. A **zero-coupon bond (ZCB)** is a special kind of bond where there are no coupon payments and the only payment is the principal or face-value at maturity. If there is no default risk associated with these bonds then the bonds are said to be **risk-free**. Examples of risk-free bonds are government bonds issued in the government's home currency. These bonds are risk-free because the government in question can always just print more currency to ensure it can make the required coupon and principal payments.

In contrast, bonds are **risky** if the issuer is not guaranteed to be able to make the promised payments. Common examples of such bonds are **corporate bonds** and government bonds issued in currencies that they don't control. The risk of non-payment is commonly referred to as **default risk** or **credit risk**. Note that regardless of whether the bonds are risky or risk-free, they still have **price** or **market risk** because their prices will change randomly as interest rates change.

A **stock**, on the other hand, is the classic example of a non-fixed income security as the cash payments associated with owning a stock, i.e. the (possibly future) dividend payments, are uncertain in both their timing and magnitude. This additional uncertainly means that stock prices are generally more volatile than bond prices. But of course this isn't a hard rule and will depend on the relative financial strengths of the issuers. Finally we note that some securities, e.g. convertible bonds, have fixed-income and non-fixed-income characteristics so the distinction between the two can be blurred.

2.2 Annuities, Perpetuities and Amortization

Other types of fixed income securities include annuities and perpetuities.

Definition 6 *An* **annuity** *is a contract that periodically pays a pre-determined amount of cash over some interval of time.*

Annuities are issued by financial institutions and typically by life insurance companies. Traditionally annuity payments were made on an annual basis (hence the term 'annuity') and the interval of time was fixed. However, there are many variations. Pensions, for example, sometimes periodically pay a pre-determined amount of cash until a *random* time T that is usually the time of death of the recipient or the recipient's spouse.

Definition 7 *A* **perpetual annuity** *or a* **perpetuity** *pays a fixed amount of cash periodically for ever.*

Perpetuities are rare but do exist in some countries. In the UK, for example, they are called **consols**. A perpetuity is easily priced. Suppose it pays a fixed amount A per period beginning at the end of the current period, and that the interest rate per period is r. Then the price P of the perpetuity satisfies

$$P \;=\; \sum_{k=1}^{\infty} \frac{A}{(1+r)^k} \;=\; \frac{A}{r}. \tag{2}$$

The price of an annuity that pays A per period beginning at the end of the current period for a total of n periods, satisfies

$$P \;=\; \sum_{k=1}^{n} \frac{A}{(1+r)^k} \;=\; \frac{A}{r}\left(1 - \frac{1}{(1+r)^n}\right). \tag{3}$$

As always, the formulae in (2) and (3) depend on the compounding convention and how interest rates are quoted. They may also be inverted to express A as a function of P. For example, we may also write (3) in the form

$$A = \frac{r(1+r)^n P}{(1+r)^n - 1}. \tag{4}$$

This form of the annuity pricing formula is useful for determining the periodic payments that correspond to a fixed value P. It is also useful for **amortization**, which is the process of substituting periodic cash payments for an obligation today.

2.3 Yield-to-Maturity

The yield-to-maturity (YTM) of a bond is the interest rate (always quoted on an annual basis) that makes the present value of all associated future payments equal to the current value of the bond. In particular, the YTM is exactly the IRR of the bond at the current price. The YTM, λ, therefore satisfies

$$P \;=\; \sum_{k=1}^{n} \frac{C/m}{[1+(\lambda/m)]^k} \;+\; \frac{F}{[1+(\lambda/m)]^n} \;=\; \frac{C}{\lambda}\left[1 \;-\; \frac{1}{[1+(\lambda/m)]^n}\right] \;+\; \frac{F}{[1+(\lambda/m)]^n} \tag{5}$$

where F is the face value of the bond, C is the annual coupon payment, m is the number of coupon payments per year, n is the exact total number of coupon payments remaining, and it is assumed that compounding is done m times per year.

A lot of information is present in equation (5). In particular, it may be seen that P is a decreasing function of λ. When $\lambda = 0$, the bond price is simply the sum of the future payments, while if $\lambda = C/F$, then we obtain $P = F$. Different bonds can have different yields but they generally track one another quite closely, particularly when the bonds have similar maturities. It should also be emphasized again that bond yields, and therefore bond prices, generally change randomly through time. Hence bonds are risky[4] securities, despite the fact that their payment streams are fixed.

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3 The Term Structure of Interest Rates

If a bank lends you money for one year and lends money to someone else for ten years, it is very likely that the rate of interest charged for the one-year loan will differ from that charged for the ten-year loan. Term-structure theory has as its basis the idea that loans of different maturities should incur different rates of interest. This basis is consistent with what we observe in practice and allows for a much richer and more realistic theory than that provided by the yield-to-maturity (YTM) framework.

We will often assume that there are m compounding periods per year, but it should be clear what changes need to be made for continuous-time models and different compounding conventions. For example, we will also consider models where we compound on a per-period basis. Time may be measured in periods or years, but it should be clear from the context what convention we are using.

3.1 Spot Rates

Spot rates are the basic interest rates that define the term structure. Usually defined on an annual basis, the spot rate s_t is the rate of interest charged for lending money from today $(t = 0)$ until time t. In particular, this implies that if you lend A dollars for t years[5] today, you will receive $A(1 + s_t/m)^{mt}$ dollars when the t years have elapsed. The **term structure of interest rates** may be defined to constitute the sequence of spot rates $\{s_{t_k} :\ k = 1, \ldots, n\}$ if we have a discrete-time model with n periods. Alternatively, in a continuous-time model the set $\{s_t :\ t \in [0, T]\}$ may be defined to constitute the term-structure. The **spot rate curve** is defined to be a graph of the spot rates plotted against time. In practice, it is usually upwards sloping in which case $s_{t_i} < s_{t_j}$ whenever $i < j$.

3.2 Discount Factors

As before, there are discount factors corresponding to interest rates, one for each time t. The discount factor d_t for a deterministic cash-flow occurring t years from now is given by

$$d_t \ := \ \frac{1}{(1 + s_t/m)^{mt}}.$$

Using these discount factors we can compute the present value P of any deterministic cash flow stream $(x_0, x_1, \ldots, x_n)$. It is given by

$$P \ = \ x_0 + d_1 x_1 + d_2 x_2 + \ \ldots \ + d_n x_n.$$

where it is understood that x_i is received at time t_i and $d_i := d_{t_i}$.

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[4]This may seem to contradict our earlier statement that some bonds are risk-free. Unfortunately this because of ambiguity in the use of the term "risk". Sometimes risk refers to credit or default risk in which case some bonds are risk-free. On the other hand, risk may refer to market or price risk in which case all bonds are risky. One can usually tell from the context what kind of "risk" is intended.

[5]We assume that t is a multiple of $1/m$ both here and in the definition of the discount factor d_t above.

Example 4 In practice it is quite easy to determine the spot rate by observing the price of default risk-free bonds. Default risk-free bonds should be used as the contracted payments are sure to take place. For example the price P of a 2-year (default risk-free) ZCB with face value $\$100$ satisfies $P = 100/(1+s_2)^2$ where we have assumed an annual compounding convention. ∎

3.3 Forward Rates

A **forward rate** f_{t_1,t_2} is a rate of interest[6] that is agreed upon today for lending money from dates t_1 to t_2 where t_1 and t_2 are future dates. It is easy to compute forward rates given the set of spot interest rates. For example, if we express time in periods, have m periods per year, compound per period and quote rates on an annual basis, then we have

$$(1+s_j/m)^j \;=\; (1+s_i/m)^i \;(1+f_{i,j}/m)^{j-i} \tag{6}$$

where $i < j$.

Exercise 3 *Show that (6) must hold by using an* **arbitrage** *argument. That is, show you can make money for nothing unless (6) holds.*

3.4 Forward Discount Factors

We can also discount a cash flow that occurs at time j back to time $i < j$. The correct discount factor is

$$d_{i,j} \;:=\; \frac{1}{(1+f_{i,j}/m)^{j-i}}$$

where again time is measured in periods and there are m periods per year. In particular, the present value at date i of a cash-flow x_j that occurs at date $j > i$ is given by $d_{i,j}x_j$. It is also easy to see that these discount factors satisfy $d_{i,k} = d_{i,j}d_{j,k}$ for $i < j < k$ and they are consistent with earlier definitions.

3.5 Short Forward Rates

The term structure of interest rates may equivalently be defined to be the set of forward rates. There is no inconsistency in this definition as the forward rates define the spot rates and the spot rates define the forward rates. We also remark that in an n-period model, there are n spot rates and $n(n+1)/2$ forward rates. The set of short forward rates $\{r_k^f : \; k = 1,\ldots,n\}$ is a particular subset of the forward rates that also defines the term structure. The short forward rates are defined by $r_k^f := f_{k,k+1}$ and may easily be shown to satisfy

$$(1+s_k)^k \;=\; (1+r_0^f)(1+r_1^f)\ldots(1+r_{k-1}^f)$$

if time is measured in years and we assume $m = 1$.

Example 5 (Constructing a Zero-Coupon Bond)
Two bonds, A and B both mature in ten years time. Bond A has a 7% coupon and currently sells for $\$97$, while bond B has a 9% coupon and currently sells for $\$103$. The face value of both bonds is $\$100$. We want to compute the price of a ten-year zero-coupon bond that has a face value of $\$100$.

Towards this end, consider a portfolio that sells seven units of bond B and buys nine of bond A. The coupon payments in this portfolio cancel and the terminal value at $t = 10$ is $\$200$. The initial cost is $-7 \times 103 \;+ 9 \times 97 = 152$. The cost of a zero with face value equal to $\$100$ is therefore $\$76$. (The 10-year spot rate, s_{10} is then equal to 2.78%. ∎

Remark 1 *Note that once we have the zero-coupon bond price we can easily determine the corresponding spot rate. This was the point of Example 4.*

[6] It is usually quoted on an annual basis unless it is otherwise implied.

We now demonstrate that even simple and apparently reasonable term-structure models can contain arbitrage opportunities.

Example 6 (Arbitrage in a 1-Period Model)
Suppose at $t = 0$ the 1-year, 2-year and 3-year spot rates are given by 10%, 11% and 12%, respectively. One year from now the 1-year, 2-year and 3-year spot rates rates will either have increased to $[11\%,\ 12\%,\ 13\%]$ or decreased to $[9\%,\ 10\%,\ 11\%]$. Note that this example assumes that only parallel movements in the spot rates can occur.

If we assume continuous compounding, then we can see that the forward rate $f_{1,2}$ at $t = 0$ is given by

$$f_{1,2} = \frac{2(.11) - 1(.1)}{1} = 12\%.$$

This forward rate, however, is higher than either of the possible 1-year spot rates prevailing at $t = 1$ and so there is an arbitrage opportunity. The arbitrage opportunity is to today (at $t = 0$) lend money forward between years 1 and 2 at $f_{1,2} = 12\%$. Then in one year's time that lending can be funded by borrowing at the 1-year spot rate which will be either 11% or 9%. One could make an infinite amount of money doing this! Since that's not possible in practice, however, we should only ever consider models that don't have arbitrage opportunities. ∎

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4 Mortgage Mathematics

We briefly provide another example of how securitization can work by considering a standard level-payment mortgage. We assume the initial mortgage principal is $M_0 := M$ and that equal periodic payments of size B dollars are made. After n such payments the mortgage principal and interest will have all been paid in full. Each payment, B, therefore[7] pays both interest and some of the principal. If we assume the coupon rate is c per period then we can solve for B. In particular, let M_k denote the mortgage principal remaining after the k^{th} period. Then

$$M_k = (1+c)M_{k-1} - B \quad \text{for} \quad k = 1, 2, \ldots, n \tag{7}$$

with $M_n = 0$. We can iterate (7) to obtain

$$\begin{aligned} M_k &= (1+c)^k M_0 - B \sum_{p=0}^{k-1} (1+c)^p \\ &= (1+c)^k M_0 - B \left[\frac{(1+c)^k - 1}{c} \right]. \end{aligned} \tag{8}$$

But $M_n = 0$ and so we obtain

$$B = \frac{c(1+c)^n M_0}{(1+c)^n - 1} \tag{9}$$

which you may recognize as being identical to our formula (4) for an annuity payment. We can now substitute (9) back into (8) and obtain

$$M_k = M_0 \frac{(1+c)^n - (1+c)^k}{(1+c)^n - 1}.$$

Suppose now that we wish to compute the present value of the mortgage assuming a deterministic world with no possibility of defaults or prepayments. Then assuming a risk-free interest rate of r per period, we obtain that

[7] Such a mortgage is said to be fully amortizing.

the fair mortgage value as

$$
\begin{aligned}
F_0 &= \sum_{k=1}^{n} \frac{B}{(1+r)^k} \\
&= \frac{c(1+c)^n M_0}{(1+c)^n - 1} \times \frac{(1+r)^n - 1}{r(1+r)^n}.
\end{aligned} \tag{10}
$$

Note that if $r = c$ then (10) immediately implies that $F_0 = M_0$, as expected. In general, however, we will have $r < c$, to account for the possibility of default, prepayment, servicing fees, bank profits etc.

4.1 Scheduled Principal and Interest Payments

Since we know M_{k-1} we can compute the interest

$$I_k \;:=\; cM_{k-1} \tag{11}$$

on M_{k-1} that would be due in the next period, i.e. period k. This also means we can interpret the k^{th} payment as paying

$$P_k \;:=\; B - cM_{k-1} \tag{12}$$

of the remaining principal, M_{k-1}. In any time period, k, we can therefore easily break down the payment B into a scheduled principal payment, P_k, and a scheduled interest payment, I_k. Indeed we can take a large pool of these mortgages and assign all the interest payments (given by (11)) to an *interest-only* (IO) mortgage-backed security (MBS), and all the principal payments (given by (12)) to a *principal-only* (PO) MBS. There are many other classes of MBS including for example collateralized mortgage obligations (CMO's).