Tutorial for Network Analytics

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About myself

- Hanzhao(Hans) Wang
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- First year PhD student in Operations Management
- Office Hour:9:00 AM every Fri, but Wed (27 Jan) for the third week.

About this class

- Around 5 questions in each week, questions are from *A first course in Network Science*
- For each question, you will be given 10 mins to solve it, then I will share and explain the solutions
- Feel free to ask questions

Graph links

Exercise 1.12: Consider an undirected network with N nodes. What is the maximum number of links this network can have? (no self-loops)

Solution

The first node can link all $N-1$ nodes. Given this is undirected graph, the second node can only link node 3, 4,...,N, thus $N-2$ links... So the total links is $(N-1)+(N-2)+...+1+0=\frac{N(N-1)}{2}$

Extensions of adjacency matrix

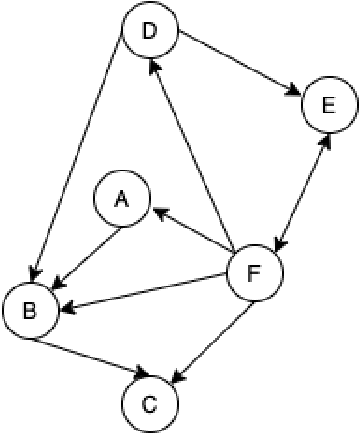
Exercise 1.16:Consider the adjacency matrix:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 2 & 1 & 3 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

An entry in the ith row and jth column indicates the weight of the link from node i to node j. For instance, the entry in the second row and third column is 2, meaning the weight of the link from B to C is 2.

- Is this network directed/undirected? weighted/unweighted?
- How many nodes and links? Are there any self-loops?
- Are there any nodes with outgoing links to every other node? If so, which nodes? Are there any nodes with in-links from every other node? If so, which nodes?

Solution

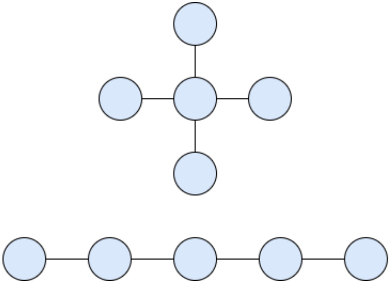


Nodes degree

Exercise 1.23: Imagine two different undirected networks, each with the same number of nodes and links. Must both networks have the same maximum and minimum degree? Explain why or why not. Must they have the same mean degree? Explain why or why not?

Solution

They do not necessarily have the same minimum and maximum degree. Consider the networks below a counter-example of networks with the same number of nodes ($N = 5$) and links ($L = 4$) but different k_{max}. Rewire an edge from either network to leave a singleton, so that k_{min} will be different as well. The average degree $<k> = \frac{2L}{N}$ depends on only N and L, so it must be the same.



Clique and hub

Exercise 3.3: Consider a network formed by 250 students in a dormitory. The links in this network represent room-mate relationships: two nodes are connected if they are currently room-mates. In this form, the rooms are mostly double occupancy with a few triples and quads.

- Is this graph connected?
- What is the mode (most frequent value) of the node degree distribution.
- How many nodes are in the largest clique?
- Would you expect this graph to have any hubs?

Solution

1. The graph is not connected. 2. The mode is one. 3. The largest cliques have four nodes. 4. This network does not have hubs.

- Clique: Maximal complete subgraphs of G; every node in the clique has an edge to every other node in the clique (maximum density possible, so to speak).
- Hub: A small number of nodes with an exceptional number of connections.

Erdos-Renyi Model

Exercise 5.3 5.4:

- Suppose we want to construct a random graph with 1000 nodes and about 3000 links. Give a value of the link probability p that could lead to this outcome.
- Suppose you are making a random network with 50 nodes, and you want the average node degree $< k >$ to be 10. What approximate value for p would you use?
- Erdos-Renyi Model:There are N nodes. For every pair of nodes: toss a coin with probability p of success to put an edge
- Hint: Binomial distribution with parameters M and p is the discrete probability distribution of the number of successes in a sequence of M independent experiments with success probability p. The mean is Mp given above parameters.

Solution

- The expected number of links in Erdos-Renyi random graph model is $\langle L \rangle = pN(N\text{-}1)/2$. If we want $\langle L \rangle = 3000$ and $N = 1000$ we need to have $p = 2\langle L \rangle/[N(N\text{-}1)] = 0.006$.
- The average node degree $\langle k \rangle = p(N\text{-}1)$, so $p = \langle k \rangle/(N\text{-}1)$. For $N = 50$ and $\langle k \rangle = 10$, $p = 10/49$.