Imperial College London
BUSINESS SCHOOL

MSc Business Analytics 2020/21
Optimisation and Decision Models
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Nonlinear Programming Exercises

(1) Consider the following multivariate linear regression problem: We are given m observations $(x_{11}, \ldots, x_{1n}; y_1), \ldots, (x_{m1}, \ldots, x_{mn}; y_m)$ where $x_{i1}, \ldots, x_{in}$ are the values of the n explanatory variables and y_i is the value of the response variable of the i-th observation, respectively, where $i = 1, \ldots, m$. We wish to find the slopes $b_1, \ldots, b_n$ as well as the intercept b_0 that minimise the sum of squared errors between the responses y_i and the regression line $b_0 + b_1 x_{i1} + \ldots + b_n x_{in}$, where the sum is taken over $i = 1, \ldots, m$.

(a) Write down the problem as a nonlinear optimisation problem.

(b) Is your problem from part (1) convex? Justify your response!

(c) How can you incorporate the following additional constraints:

- Each of the slopes $b_1, \ldots, b_n$ should be between -10 and 10.
- The slope b_1 needs to be at least twice as big as the slope b_2.
- The slopes b_3 and b_4 must be the same.
- The slope b_5 must either be ≤ 1 or ≥ 2.
- The sum of all absolute values of the slopes $|b_1|, \ldots, |b_n|$ must be 10 or less.
- At most 5 of the slopes $b_1, \ldots, b_n$ should be nonzero.

For each constraint, argue whether your optimisation problem remains convex!

(2) Which of the following optimisation problems is convex?

(a) minimise $1/x_1 + 1/x_2 + |x_3|$
subject to $\max\{x_1 + x_2, x_1 - x_3\} >= 2$
$x_1, x_2 >= 0$, x_3 unrestricted

(b) maximise $x_1 - x_2^2$
subject to $(2x_1 - x_2)^2 <= x_1$
$|x_1| <= 2$

(c) maximise $3x_1 - 2x_2 + 5^2$
subject to $x_1 + x_2 <= 2$
$x_2 >= x_1$
$x_1, x_2 >= 0$

(d) minimise x_1 **(Brainteaser!)**
subject to $x_1 x_2 >= 2$
$x_2 >= 4$