# MOCK Final Exam

**Exam Duration: 2 Hours**
**Total Marks Available: 100**

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## Instructions

1. This is a closed-book exam.
2. Keep your answers succinct and to the point. Long rambling answers with irrelevant details will work against you.
3. You may use a calculator or `Excel` for any calculations that you need to do.
4. Please read the entire exam carefully before starting on your answers!

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## Question 1. (25 marks)

(a) Let $C_0 = 3$ denote the time $t = 0$ price of a European call option on a non-dividend-paying stock with strike $K = 110$ and maturity $T = 1$ year. The current value of the stock price is $S_0 = 100$ and the discount factor is $d(0, T) = .99$. Can you compute the price $P_0$ of the European put option with the same strike and maturity? If so, what is the price? If not, why can't you compute it? (6 marks)

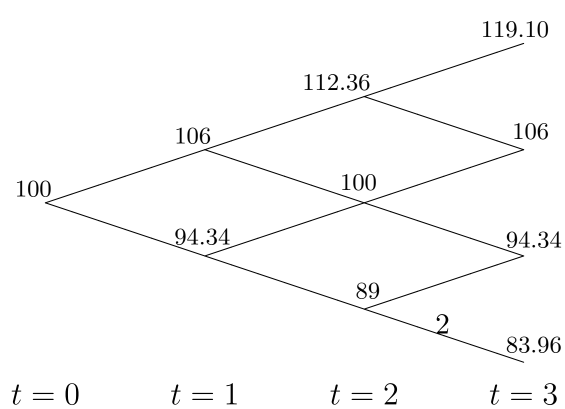
(b) Consider the pivot-table in Figure 1 at the end of this exam. It displays the P&L of an options and futures portfolio for various combinations of stresses to the underlying security and implied volatilities. You are told that the total dollar delta, i.e. ESP, and total dollar gamma for this portfolio are $-17,485$ and $-458,145$, respectively. What does a delta-gamma approximation suggest the P&L will be if the underlying increases by 10%? Is this consistent with the pivot-table? Explain your answer. (7 marks)

(c) Suppose the CAPM holds and the expected return on a particular risky asset is significantly less than the risk-free rate of interest. True or False: such an asset should rarely be held in an agents optimal portfolio? Give a reason for your answer. (6 marks)

(d) You're a market-maker and have just sold a put option on some underlying stock. You decide to delta-hedge your risk and so you put on a short position in the stock immediately after selling the option. True or false: if the underlying stock rises then will need to buy some stock. Explain your answer. (6 marks)

## Question 2. (25 marks)

Consider the binomial lattice below with $S_0 = 100$, $u = 1.06$ and $d = 1/u$. It describes the evolution of a non-dividend paying stock in a 3-period world. You may also assume there is a cash account which pays a total return of R = 1.01 per period and that borrowing or lending at that rate is possible. In addition short-sales are allowed.



(a) Compute the price of an American put option on the stock with strike = 98 and expiration date $t = 3$. (10 marks)

(b) Is it ever optimal to early exercise the put option from part (a). If so, when and does this violate put-call parity? (5 marks)

(c) A **chooser option** gives the owner the right to choose at time $t = 1$ either a European call option or a European put option. The call and put options in question both have the same strike $K$ and expiration at $T = 2$. Can you find an expression for the time $t = 0$ value of the chooser option for general values of $K$ that can be written as the sum of the original call option and a put option with a different expiration and a different strike. (Put-call parity helps!) (10 marks)

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**Question 3. (10 marks)**

A forward contract on a security is a contract agreed upon at date $t = 0$ to purchase or sell the security at date $T$ for a price $F$ that is specified at $t = 0$. The forward price $F$ is set in such a way that the initial *value* $f_0$ of the forward contract satisfies $f_0 = 0$. At the maturity date $T$ the value of the contract is then given by $f_T = \pm(S_T - F)$ where $S_T$ is the time $T$ value of the underlying security. (The time $T$ payoff is $+(S_T - F)$ if you went long the forward at $t = 0$ and $-(S_T - F)$ if you went short the forward.)

Assuming an arbitrage-free binomial model for the dynamics of $S_t$, compute an expression for $F$. (You can assume that $T$ corresponds to exactly $n$ periods in the binomial model.)

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**Question 4. (20 marks)**

In the Black-Scholes model the time $T$ payoff of an Asian call option is given by

$$h(\mathbf{X}) := \max\left(0,\ \frac{\sum_{i=1}^{m} S_{iT/m}}{m} - K\right)$$

where $\mathbf{X} = (S_{T/m}, S_{2T/m}, \ldots, S_T)$ and where $S_t$ denotes the time $t$ price of the underlying security. In words, the Asian call option is a European call option on the *average* price of the stock with the average taken over the stock price at the $m$ pre-specified times $iT/m$ for $i = 1, \ldots, m$. The risk-neutral stock-price is assumed to satisfy $S_t \sim \mathrm{GBM}(r, \sigma)$ and the arbitrage-free value of the option is then given by $\theta := \mathrm{E}[e^{-rT} h(\mathbf{X})]$.

(a) Provide pseudo-code for a Monte-Carlo algorithm that estimates $C_0$ using $n$ Monte-Carlo samples. Your pseudo-code should also calculate an approximate 95% confidence interval. (15 marks)

(b) What if anything can you say about the delta, gamma and vega of the Asian option? Will they be positive or negative? You don't have to prove anything here but you should give a 1 or 2 line answer to provide some intuition for your answers. (5 marks)

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**Question 5. (20 marks)**

(a) A common compromise on long-only portfolios in the mean-variance setting are the so-called (100+L)/L portfolios. Such a portfolio allows the long positions to be worth up to (100+L)% of the portfolio value and the short positions to be worth at most L% of the portfolio. Note that the full investment constraint $\mathbf{w}^\top \mathbf{1} = 1$ is always imposed. Suppose then that we wish to allow such portfolios. This results in the "short" constraint

$$\sum_{j=1}^{n} \min(w_j, 0) \geq -L \;\Leftrightarrow\; \sum_{j=1}^{n} \max(-w_j, 0) \leq L. \tag{1}$$

Note that there is no need to include the "long" constraint since (1) together with the constraint $\mathbf{w}^\top \mathbf{1} = 1$ will ensure it is satisfied. The problem with (1) is that it is not linear and so if we add it to our mean-variance problem formulation we will no longer have a convex quadratic program.

Show how we can *linearize* (1), i.e. replace it with linear constraints and justify your answer. (10 marks)

(b) The mean-variance optimization problem can be formulated as

$$\begin{aligned} \min_{w_1,\ldots,w_n} \quad & \frac{1}{2}\sum_{i,j=1}^{n} \sigma_{ij} w_i w_j \\ \text{subject to} \quad & \sum_{i=1}^{n} w_i \bar{r}_i = \bar{r} && (2) \\ & \sum_{i=1}^{n} w_i = 1 && (3) \end{aligned}$$

where $\sigma_{ij} = \mathrm{Cov}(r_i, r_j)$ and $\bar{r}$ is the target mean return of the portfolio. It can be shown that a solution to this problem is given by the solution $\{\mathbf{w} = (w_1, \ldots, w_n), \lambda, \mu\}$, to the $n$ linear equations

$$\sum_{j=1}^{n} \sigma_{ij} w_j - \lambda \bar{r}_i - \mu = 0, \quad \text{for } i = 1, \ldots, n \tag{4}$$

together with (2) and (3). Suppose we have two known solutions $\{\mathbf{w}^1, \lambda^1, \mu^1\}$ and $\{\mathbf{w}^2, \lambda^2, \mu^2\}$ corresponding to different target returns $\bar{r}^1$ and $\bar{r}^2$, respectively. Is it possible to determine the solution for an arbitrary target return $\bar{r}^3$ using these solutions?

Please justify your response and briefly discuss any possible implications for investing in the real world (assuming the mean-variance framework is true). (10 marks)

| Underlying | SX5E Index | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **Underlying and Volatility Stress Table** | | | | | | | | |
| Sum of PnL | Volatility Stress | | | | | | | | |
| Underlying Stress | -10 | -5 | -2 | -1 | 0 | 1 | 2 | 5 | 10 |
| -20 | 7,234 | (3,488) | (10,770) | (13,305) | (15,885) | (18,508) | (21,168) | (29,351) | (43,504) |
| -10 | 29,406 | 13,520 | 3,663 | 341 | (2,995) | (6,345) | (9,706) | (19,845) | (36,862) |
| -5 | 35,537 | 17,725 | 6,928 | 3,319 | (293) | (3,908) | (7,525) | (18,380) | (36,458) |
| -2 | 37,552 | 18,874 | 7,647 | 3,905 | 165 | (3,574) | (7,310) | (18,505) | (37,095) |
| -1 | 37,948 | 19,032 | 7,684 | 3,905 | 129 | (3,645) | (7,416) | (18,709) | (37,450) |
| 0 | 38,207 | 19,079 | 7,623 | 3,810 | 0 | (3,806) | (7,609) | (18,992) | (37,873) |
| 1 | 38,330 | 19,017 | 7,463 | 3,619 | (220) | (4,056) | (7,888) | (19,354) | (38,365) |
| 2 | 38,318 | 18,846 | 7,207 | 3,335 | (531) | (4,394) | (8,251) | (19,794) | (38,925) |
| 5 | 37,495 | 17,698 | 5,868 | 1,934 | (1,995) | (5,920) | (9,839) | (21,566) | (41,000) |
| 10 | 33,611 | 13,757 | 1,824 | (2,153) | (6,128) | (10,102) | (14,073) | (25,969) | (45,722) |
| 20 | 17,546 | (923) | (12,382) | (16,244) | (20,124) | (24,018) | (27,925) | (39,709) | (59,483) |

Figure 1: P&L for an options portfolio on the Eurostoxx 50 index under stresses to the underlying and implied volatility