EOQ Exercise – Solutions

StoreChain has 5 stores. Consider the sale of a particular jacket with annual revenues of £1M per store. Jackets sell at a retail price of £325, which represents a mark-up of 30% above what *StoreChain* pays its manufacturer. Being a profit center each store makes its own inventory decisions and is supplied directly from the manufacturer by truck. A shipment up to a full truck load, which can carry 2,600 jackets, is charged a flat fee of £2,200. The unit holding cost per year is 20% of the product cost.

1. What is the economic order quantity of a single store? What is the resulting annual holding and ordering cost at a single store and for the entire chain?

   Solution:
   Annual store demand: $D = 3,077$ jackets.
   Annual unit holding cost: $C = £250, h = 0.2, H = £50$.
   Fixed order per shipment: $S = £2,200$.
   Optimal order quantity at each store: $EOQ = \sqrt{\frac{2SD}{H}} = 520$ units.
   Annual holding + ordering cost:

   - **One store:** $C_{EOQ} = \sqrt{2SDH} = £26,018.07$**;**
   - $N = 5$ **stores:** $N \cdot \sqrt{2SDH} = £130,090.35$**.**

2. Suppose now that *StoreChain* replaces their five brick and mortar stores by an Internet store and places the required inventory in one central warehouse. Assuming the same total annual sales volume as for 5 stores and the same shipment fee as before, what is now the economic order quantity of the central warehouse and the annual holding and ordering cost for *StoreChain*? How does the total cost compare to Q1?

   Solution:
   Since the centralized store serves $N = 5$ times the demand of each of local store:

   - EOQ increases by $\sqrt{5}$: $EOQ = \sqrt{N}\sqrt{\frac{2SD}{H}} = \sqrt{N} \cdot 520 = 1,164$ units
   - So does the **total cost:** $C_{EOQ} = \sqrt{N}\sqrt{2SDH} = £58,178.17$
   - **Bottom line: total cost relative to multiple stores: reduction by** $\sqrt{5}$

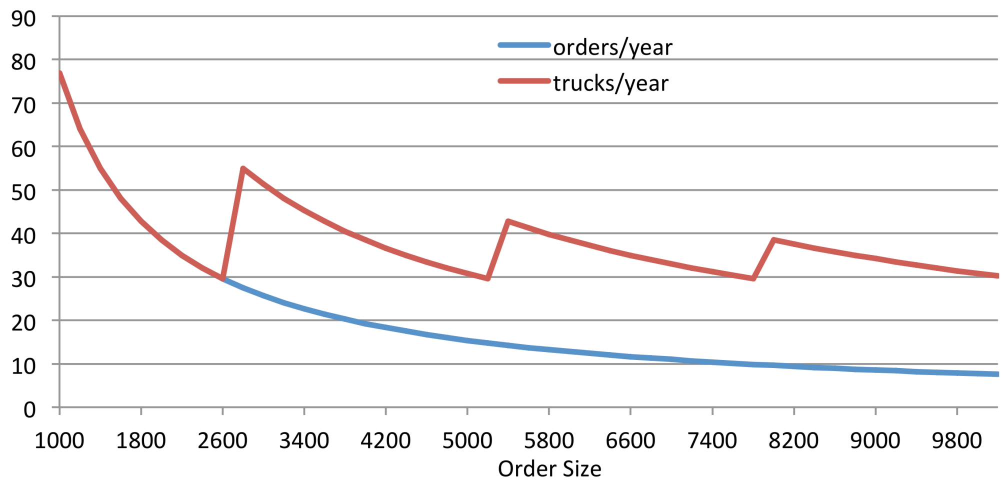
3. Repeat your analysis of Q1 and Q2, but now assume that the annual jacket revenue per store territory is £25M. How do your answers change?

   Solution:
   Answer to Q1

   - Assuming fixed unit price, this gives 25x unit volume at each store: $D = 25 \cdot 3,077$.
   - Relative to Q1, EOQ therefore increases by $5 = \sqrt{25}$ and is now 2,600 units.
   - **Store level annual order + holding cost**: increases by a factor 5, so $5 \times 26,018.07 = £130,090.35$ (£65,045.18 each for transportation and holding)
   - **Store chain total cost**: 5 stores $\times 130,090.35 = £650,451.77$ (£325,255.89 each for transportation and holding)

   Answer to Q2

- Since each local store already maximizes transportation scale economies (ordering full trucks) it is NOT correct to calculate the EOQ of the centralized store as $\sqrt{5} \cdot 2,600 = 5,818$ units. This ignores the truck size constraint.
- Now it is optimal for the centralized warehouse to order only one full truck load at a time, like the local store. Annual transportation cost as a function of lot size Q for given demand D:
$$C_T(Q) = D/Q \cdot S \cdot \left\lceil \frac{Q}{2,600} \right\rceil,$$
where $\left\lceil \frac{Q}{2,600} \right\rceil$ is the number of trucks required for an order size Q (rounding up to the nearest integer). This cost function has multiple minima: ordering any number of full trucks at a time means $D/2,600 = 29.6$ truck trips per year on average, as shown in the figure below. Sending less than full trucks increases the # of trucks per year and hence the transportation



cost. Since cycle inventory increases in batch size, it will not make sense to order more than one full truck per batch.
- Since each local store finds it optimal to order a full (as opposed to partially full) truck, the central warehouse will also find it optimal to do so. (Its unconstrained EOQ is 5,818 units as above).
- **Total cost with centralized warehousing**:
  - Transportation cost (total): £325,225.89, same as for all 5 local stores (same batch size and # of trips)
  - Holding cost (total): £65,045.18, which is same as at a single local store (same batch size)
  - Total cost: **£390,271.06**

General insight from this exercise:

(a) If the decentralized system is fall from full ordering scale economies, then centralizing N identical inventory locations cuts the system cost for ordering and holding cycle inventory by $\sqrt{N}$.

(b) If the decentralized system achieves (close to) full ordering scale economies, then centralizing N identical inventory location cuts the system cost for ordering and holding cycle inventory by LESS THAN $\sqrt{N}$. (If as in this example the transportation cost does NOT drop, the total holding cost is cut by N.)