$$\underset{\beta}{\text{minimise}} \quad \|y - X\beta\|_1$$

$$\Leftrightarrow \text{ minimise } |y_1 - (\beta_0 + \beta_1 x_{11} + \cdots \beta_k x_{1k})| +$$
$$|y_2 - (\beta_0 + \beta_1 x_{21} + \cdots \beta_k x_{2k})| +$$
$$\cdots$$
$$|y_n - (\beta_0 + \beta_1 x_{n1} + \cdots \beta_k x_{nk})|$$

such that $\beta_0, \ldots \beta_k \in \mathbb{R}$

$$\Leftrightarrow \text{ minimise } \theta_1 + \cdots + \theta_n$$

such that $\theta_1 = |y_1 - (\beta_0 + \beta_1 x_{11} + \cdots \beta_k x_{1k})|$

$$\vdots$$

$$\theta_n = |y_n - (\beta_0 + \beta_1 x_{n1} + \cdots \beta_k x_{nk})|$$

$$\beta_0, \ldots \beta_k \in \mathbb{R}, \ \theta_1, \ldots \theta_n \geq n$$

$$\Leftrightarrow \text{ minimise } \theta_1 + \cdots \theta_n$$

such that $\theta_1 \geq |y_1 - (\beta_0 + \beta_1 x_{11} + \cdots \beta_k x_{1k})|$

$$\vdots$$

$$\theta_n \geq |y_n - (\beta_0 + \beta_1 x_{n1} + \cdots \beta_k x_{nk})|$$

$$\beta_0, \ldots \beta_k \in \mathbb{R}, \ \theta_1, \ldots \theta_n \geq n$$

take any $a, b \in \mathbb{R}$, $a \geq |b|$ if and only if $a \geq b$ and $a \geq -b$

$a \leq -|b|$ if and only if $a \leq b$ and $a \leq -b$

$$\Leftrightarrow \text{ minimise } \theta_1 + \cdots \theta_n$$

such that $\theta_1 \geq y_1 - (\beta_0 + \beta_1 x_{11} + \cdots \beta_k x_{1k})$

$$\theta_1 \geq -y_1 + (\beta_0 + \beta_1 x_{11} + \cdots \beta_k x_{1k})$$

$$\cdots$$

$$\theta_n \geq y_n - (\beta_0 + \beta_1 x_{11} + \cdots \beta_k x_{1k})$$

$$\theta_n \geq -y_n + (\beta_0 + \beta_1 x_{11} + \cdots \beta_k x_{1k})$$

$$\beta_0, \ldots \beta_k \in \mathbb{R}, \ \theta_1, \ldots \theta_n \geq 0$$

max $x + 2y - |z|$

max $x + 2y + \theta$

s.t. $\theta \leq -|z|$

max $x + 2y + \theta$

s.t. $\theta \leq -z$

$\theta \leq z$

minimise $\|y - X\beta\|_\infty$

$= \max \{ |y_1 - [\ldots]|,$

$\vdots$

$|y_n - [\ldots]|$