

$$\begin{aligned}
\text{Var}(\hat{f}(x_0)) &= E[(\hat{f}(x_0) - E[\hat{f}(x_0)])^2] \\
&= E\left[\left(\frac{1}{k} \sum_{i=1}^k y_{(i)} - \frac{1}{k} \sum_{i=1}^k f(x_{(i)})\right)^2\right] \\
&= E\left[\left(\frac{1}{k} \sum_{i=1}^k (y_{(i)} - f(x_{(i)}))\right)^2\right] \\
&= E\left[\left(\frac{1}{k} \sum_{i=1}^k \varepsilon_{(i)}\right)^2\right] \\
&= \frac{1}{k^2} \sum_{i=1}^k E[\varepsilon_{(i)}^2] \quad (\text{because } E[\varepsilon_{(i)}\varepsilon_{(j)}] = 0 \text{ } \therefore \text{indep. training pts}) \\
&= \frac{1}{k^2} \sum_{i=1}^k \sigma_\varepsilon^2 = \frac{\sigma_\varepsilon^2}{k}
\end{aligned}$$