Imperial College London

BUSINESS SCHOOL

MSc BA MOCK Final Exam 2020/21

THERE MAY BE DIFFERENT VERSIONS WITH THE NUMBERS CHANGED

Network Analytics

Instructions

- THE TEST HAS ONE PROGRAMMING EXERCISE USING PYTHON AND NETWORKX.

- **WRITE ALL YOUR ANSWERS IN THE ANSWER BOOK**

Question 1 (10 marks):

A node X in an undirected graph is *pivotal* for a pair of distinct nodes Y and Z if X lies on *every* shortest path between Y and Z (and X is not equal to either Y or Z).

Give an example of a graph in which every node is pivotal for at least one pair of nodes. Explain your answer.

Question 2(10 marks):

Consider the following set of preferences for 3 males (represented by m_1 m_2 m_3) over females (w_1 w_2 w_3). Interpret the order as the preference; for instance, m_1 prefers w_2 over w_3 and w_1 over w_2.

m_1: w_1 w_2 w_3

m_2: $w_1$$w_3$ w_2

m_3: $w_1$$w_2$ w_3

w_1: m_2 $m_3$$m_1$

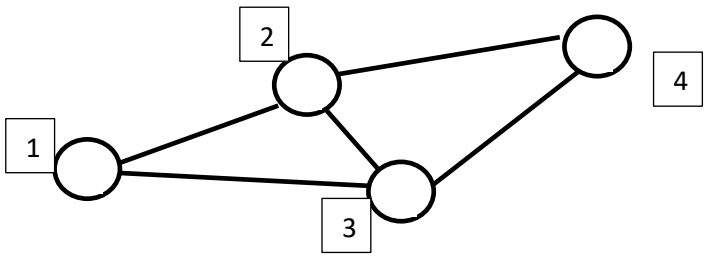
w_2: m_3 m_1 m_2

w_3: m_3 m_1 m_2

a) (5 points) Draw the above preferences as a bipartite graph
b) (5 points) Show the steps of an algorithm to find a stable matching in the above example.

Question 3 (15 marks):

Give a complete integer programming formulation of the traveling salesman problem on the following graph of four nodes (write down all the constraints, even if they are redundant or unnecessary for this particular graph). Assume the weight on each of the edges is 1.



Question 4 (30 points):
The following is a problem where departments in a company (eg: Marketing, Accounting etc.) want to have representation on a council. However no single category of employees (Category 1,2,3, 4 etc.) can have too many representative .

Specifically: There are *r* employees total, *q* departments and *p* categories. Each employee is a member of at least one department (some can belong to more than one) and exactly one category. Each department should nominate one of its members to represent on the council so that the number of employees of a particular category *k* is at most u_k.

a) Formulate this problem as a max-flow problem. A diagram with clear labeling on what corresponds to what in your model is sufficient (a math programming formulation is not necessary).
b) Explain why your formulation and its solution satisfies the conditions of the problem, and how to interpret the results in terms of the problem.

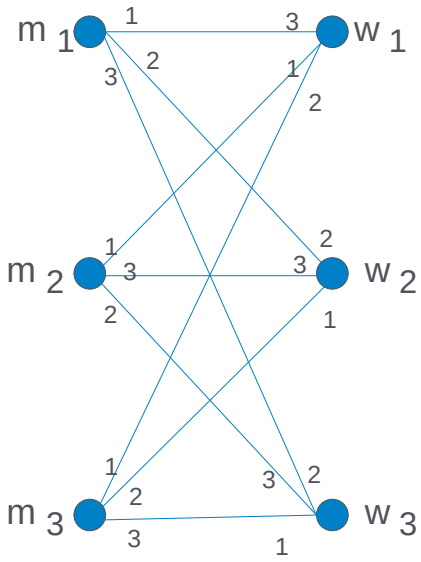
Question 5: (35 marks) In the dataset in your exam folder is the social network data of a video-game community (in edge_list format) *social_network.txt*. It is an undirected unweighted graph in edge list format. (note that it is a comma-separated file with comma as the delimiter). Submit your code and analysis

a. (15 marks) Analyze the dataset for important players using the concept of eigenvector centrality and the corresponding function in NetworkX. Explain in words what the eigenvector centrality concept is trying to capture.
b. (20 marks) A client has asked you to identify communities in this network for a possible targeting action. Plot the network and determine a community structure and give a plausible explanation to the client (using your plot as a reference) of why they appear to be communities. You are free to fold in your insights from part (a).

Solution (hints): These are NOT model answers nor a marking sheet. They are just hints for you to work through the mock exam. You are expected to give detailed explanation and clear answers like in any exam. No answer to the final programming question is provided.

1. *Try a Cycle with at least 4 nodes.*

2 a) The bipartite graph is shown below.



b) A stable matching would be a matching such that no pair consisting of an m node and a w node will *both* find it beneficial to match to each other than their current respective partners in the matching.

The proposal algorithm (males propose) will proceed as follows

Step1: m1, m2, m3 all three propose to w1 → w1 chooses m2

Step 2: m1 and m3 both propose to their next best choices w2 → w2 chooses m3

Step 3: m1 proposes to the only choice he hasn't proposed yet, w3, and w3 chooses m1

Algorithm stops.

3

Obj: Minimize x12+x24+x13+x23+x34

s.t
node 1: x12+x13=2
node 2: x12+x23+x24=2
node 3: x13+x23+x34=2
node 4: x24+x34=2

Subtour constraints:
Subset 12 : x13+x23+x24 >= 2
Subset 24: x12+x23+x34 >= 2
Subset 34: x13+x23+x24 >= 2
Subset 23: x12+x13+x24 +x34 >= 2
Subset 14: x12+x13+x24+x34 >= 2

Subset 123: x24+x34 >=2
Subset 234: x24+x13 >=2
Subset 124: x34+x13 +x23 >=2
Subset 134: x12+x23 +x24 >=2

$1 >= x_{ij} >= 0$, integer for all i, j

4.