Violation of homoskedasticity for LPM

$$
\begin{aligned}
&\mathrm{Var}(u \mid \vec{x}) \\
&= \mathrm{Var}(y \mid \vec{x}) \quad \text{(as conditional on } \vec{x}\text{, } \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k \text{ is a constant)} \\
&= E[y^2 \mid \vec{x}] - \left(E[y \mid \vec{x}]\right)^2 \\
&= 1^2 \cdot P(y=1 \mid \vec{x}) - \left(P(y=1 \mid \vec{x})\right)^2 \\
&= P(y=1 \mid \vec{x}) \cdot \left[1 - P(y=1 \mid \vec{x})\right] \\
&= (\beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k) \cdot (1 - \beta_0 - \beta_1 x_1 - \cdots - \beta_k x_k)
\end{aligned}
$$