

$$C_t = E_t^Q [e^{-r(r-t)} \max(0, S_T - K)]$$

①. Suppose $S_t \gg K \Rightarrow$ Very, very likely $S_T > K$

$$\begin{aligned} \text{Then } C_t &\approx E_t^Q [e^{-r(r-t)} (S_T - K)] \\ &= E_t^Q [e^{-r(r-t)} S_T] - e^{-r(r-t)} K \\ &= S_t - e^{-r(r-t)} K \end{aligned}$$

$$\Rightarrow \frac{\partial C}{\partial S_t} \approx 1 \quad \left(\text{and } \frac{\partial P}{\partial S_t} \approx 0, \frac{\partial P}{\partial \sigma} \approx 0 \right)$$

$$\frac{\partial^2 C}{\partial S_t^2} = 0$$

②. Suppose $S_t \ll K \Rightarrow$ very, very likely $S_T < K$

$$\Rightarrow C_t \approx 0 \Rightarrow \frac{\partial C}{\partial S_t} \approx 0 \quad \left(\text{and } \frac{\partial P}{\partial S_t} \approx -1, \frac{\partial P}{\partial \sigma} \approx 0 \right)$$

$$\frac{\partial^2 C}{\partial S_t^2} = 0$$