Imperial College London
BUSINESS SCHOOL

MSc Business Analytics 2020/21
Optimisation and Decision Models
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Solutions 4

Solution to (1) (a): We introduce the binary variables $x_1, \ldots, x_{20}$ with the interpretation that $x_i = 1$ if the i-th candidate player is chosen and $x_i = 0$ otherwise. The problem can then be formulated as the following binary optimisation problem:

$$
\begin{array}{lll}
\text{maximise} & (10x_1 + 14x_2 + \ldots + 14x_{20}) / 12 & \\
\text{subject to} & x_1 + x_2 + \ldots + x_{20} = 12 & \\
 & x_1 + x_2 + \ldots + x_5 \geq 3 & (1) \\
 & x_4 + x_5 + \ldots + x_{11} \geq 4 & (1) \\
 & x_9 + x_{10} + \ldots + x_{16} \geq 4 & (1) \\
 & x_{16} + x_{17} + \ldots + x_{20} \geq 3 & (1) \\
 & x_4 + x_8 + x_{15} + x_{20} \geq 2 & (2) \\
 & (1x_1 + 2x_2 + \ldots + 9x_{20}) / 12 \geq 7 & (3) \\
 & (7x_1 + 14x_2 + \ldots + 1x_{20}) / 12 \geq 6 & (3) \\
 & (10x_1 + 14x_2 + \ldots + 14x_{20}) / 12 \geq 18 & (3) \\
 & (10x_1 + 9x_2 + \ldots + 8x_{20}) / 12 \geq 8.5 & (3) \\
 & ((5*12+11)*x_1 + (6*12+0)*x_2 + \ldots + (7*12+0)*x_{20}) / 12 \geq 6*12+7 & (4) \\
 & x_5 + x_9 \leq 1 & (5) \\
 & x_2 = x_{19} & (6) \\
 & x_1 + x_7 + x_{12} + x_{16} \leq 3 & (7) \\
 & x_1, x_2, \ldots, x_{20} \in \{0, 1\} &
\end{array}
$$

Here, the numbers behind the constraints correspond to the restrictions from the question.

Solution to (1) (b): In Excel, the model could look like this:

#	Avg. rebounds/game	Avg. assists/game	Height (inches)	Avg. points/game	Defensive ability
1	1	7	71	10	10
2	2	14	72	14	9
3	3	12	76	19	8
4	4	4	72	18	6
5	5	9	75	20	8
6	7	6	77	21	10
7	7	8	80	23	10
8	4	2	77	13	5
9	8	2	82	17	8
10	5	5	76	25	8
11	10	6	82	20	9
12	8	8	81	30	10
13	10	2	87	24	9
14	9	5	82	15	7
15	6	3	82	17	6
16	16	2	81	3	6
17	11	1	88	27	9
18	12	5	86	26	10
19	11	1	87	21	9
20	9	1	84	14	8

Decision variables	
Choose player 1	0
Choose player 2	1
Choose player 3	1
Choose player 4	1
Choose player 5	1
Choose player 6	0
Choose player 7	1
Choose player 8	0
Choose player 9	0
Choose player 10	1
Choose player 11	0
Choose player 12	1
Choose player 13	1
Choose player 14	0
Choose player 15	1
Choose player 16	0
Choose player 17	1
Choose player 18	1
Choose player 19	1
Choose player 20	0

Objective function	22

Constraints		
12 players	12	12
Restriction 1 (a)	4	3
Restriction 1 (b)	4	4
Restriction 1 (c)	4	4
Restriction 1 (d)	3	3
Restriction 2	2	2
Restriction 3 (a)	7	7
Restriction 3 (b)	6	6
Restriction 3 (c)	22	18
Restriction 3 (d)	8.5	8.5
Restriction 4	80.16666667	79
Restriction 5	1	1
Restriction 6	0	0
Restriction 7	2	3

Thus, the optimal team consists of players 2-5, 7, 10, 12, resulting in an average number of points/game of 22.

Solution to (2) (a): We introduce the binary variables z_C, z_N and z_L to indicate whether (value 1) or not (value 0) we purchase from Caroline Woodworks, Nashawtuc Millworks or Lancaster Artisan Company, respectively. Similarly, we introduce the continuous variables x_C, x_N, x_A and x_L to represent the purchase amounts from Caroline Woodworks, Nashawtuc Millworks, Adirondack Furnishing Designs or Lancaster Artisan Company, respectively. (Since there are no delivery charges for Adirondack Furnishing Design, we do not introduce a binary variable for that company.) The optimisation problem can then be cast as follows:

$$
\begin{array}{ll}
\text{minimise} & 10{,}000z_C + 2{,}500x_C + \\
& 20{,}000z_N + 2{,}450x_N + \\
& \qquad\qquad 2{,}510x_A + \\
& 13{,}000z_L + 2{,}470x_L \\
\text{subject to} & x_C \leq 1{,}000z_C,\ x_N \leq 1{,}200z_N,\ x_A \leq 800,\ x_L \leq 1{,}100z_L \\
& x_C + x_N + x_A + x_L = 2{,}000 \\
& x_C,\ x_N,\ x_A,\ x_L \geq 0; \quad z_C,\ z_N,\ z_L \in \{0, 1\}
\end{array}
$$

The formulation is a (fairly) straightforward application of the "fixed charge" reformulation discussed in class.

Solution to (2) (b): Intuitively, we can treat Delaware Mills as two separate suppliers: a supplier "Delaware Mills small purchase" (with decision variables z_S and x_S) that delivers up to 1,000 furniture sets and a supplier "Delaware Mills big purchase" (with decision variables z_B and x_B) that delivers up to 500 additional furniture sets. We can only purchase from "Delaware Mills big purchase", however, if we purchase all 1,000 furniture sets from "Delaware Mills small purchase". To this end, we add the following terms to the objective function:

$$+\ 9{,}000z_S + 2{,}530x_S + 7{,}000z_B + 2{,}430x_B$$

Moreover, we add the following constraints to the optimisation model:

$$
\begin{array}{l}
x_S \leq 1{,}000z_S,\ x_B \leq 500z_B \\
z_B \leq x_S / 1{,}000 \\
x_S,\ x_B \geq 0; \quad z_S,\ z_B \in \{0, 1\}
\end{array}
$$

The second constraint can be understood as follows: as long as x_S is less than 1,000, z_B (and also x_B) has to be zero, that is, we cannot purchase anything from "Delaware Mills big purchase". Only if x_S is equal to 1,000 are we allowed to set z_B to 1 and hence purchase from "Delaware Mills big purchase".