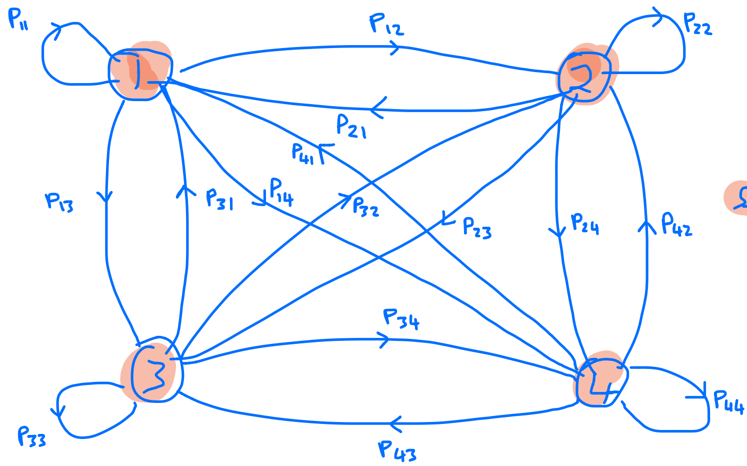
- $n=4$ possible states
- X_t = state of chain at time t (X_t is a r.var)
- X_t Markov $\iff P(X_t \mid X_0, X_1, \dots, X_{t-1}) = P(X_t \mid X_{t-1})$



$$\begin{array}{c|cccc} & & S_t & & \\ S_{t-1} & \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix} \end{array}$$

- A stationary distribution for the Markov chain is a probability distribution $\mu \in \mathbb{R}^n$ ($\Rightarrow \mu_i \geq 0, \sum_{i=1}^{n} \mu_i = 1$)

  so that $P(X_t \text{ is in state } i) = \mu_i$ for $t \to \infty$

- Assumption: t very large $\qquad P(X_t = i) = \sum_j P(X_t = i, X_{t-1} = j)$

  $\Rightarrow P(X_t = i) = \mu_i$

  $$P(X_t = i) = \sum_{j=1}^{n} \underbrace{P(X_t = i \mid X_{t-1} = j)}_{P_{ji}} P(X_{j-1} = j)$$

  $$= \sum_{j=1}^{n} P_{ji} \mu_j$$

  i.e. $\mu_i = \sum_{j=1}^{n} P_{ji} \mu_j$, $i = 1, \dots, n$

i.e. $\boxed{\mu = \mu P} \iff P^T \mu^T = \mu^T$