I.7.3

1. Lowest Order Contribution

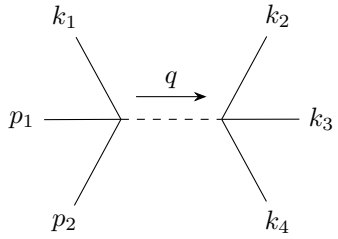
The interaction Lagrangian is $\mathcal{L}_{\text{int}} = -\frac{\lambda}{4!}\phi^4$. The scattering process is $\phi(p_1) + \phi(p_2) \to \phi(k_1) + \phi(k_2) + \phi(k_3) + \phi(k_4)$. The lowest order contribution is at $\mathcal{O}(\lambda^2)$, which involves two vertices ($V = 2$) connected by one internal propagator ($I = 1$). The total number of distinct diagrams is 10.

2. Diagram Classes

The 10 diagrams fall into two topological classes.

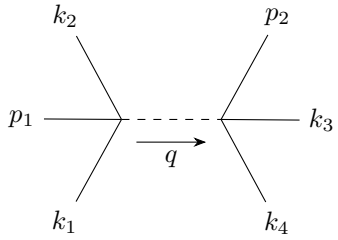
Class A (4 diagrams)

Both initial particles (p_1, p_2) attach to the same vertex.



Class B (6 diagrams)

The initial particles attach to different vertices.



3. Feynman Amplitudes

The total invariant amplitude $\mathcal{M}$ is the sum over all 10 diagrams.

$$\mathcal{M} = -\lambda^2 \left[\sum_{i=1}^{4} \frac{1}{(p_1 + p_2 - k_i)^2 - m^2} + \sum_{1 \le i < j \le 4} \frac{1}{(p_1 - k_i - k_j)^2 - m^2} \right]$$