I.7.2

We want to find the mathematical expression for the s-channel one-loop diagram in ϕ^4 theory.

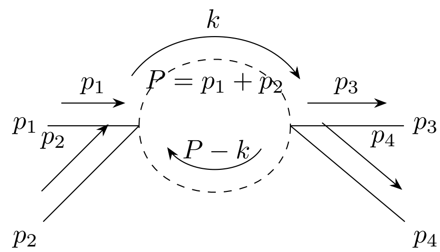

Figure 1: The s-channel one-loop Feynman diagram.

The final expression for the invariant amplitude, $i\mathcal{M}_s$, is:

$$i\mathcal{M}_s = \frac{1}{2}(-i\lambda)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(k_1 + k_2 - k)^2 - m^2 + i\epsilon}$$

The key is to derive the coefficient, especially the **symmetry factor** of 1/2.

Origin and Calculation

This diagram arises from the second-order term in the perturbative expansion of Formula B:

$$\frac{1}{2!}\left(-i \int d^4y \frac{\lambda}{4!}\phi^4(y)\right)^2$$

The coefficient is found by counting the number of ways to perform the Wick contractions that form the diagram's topology.

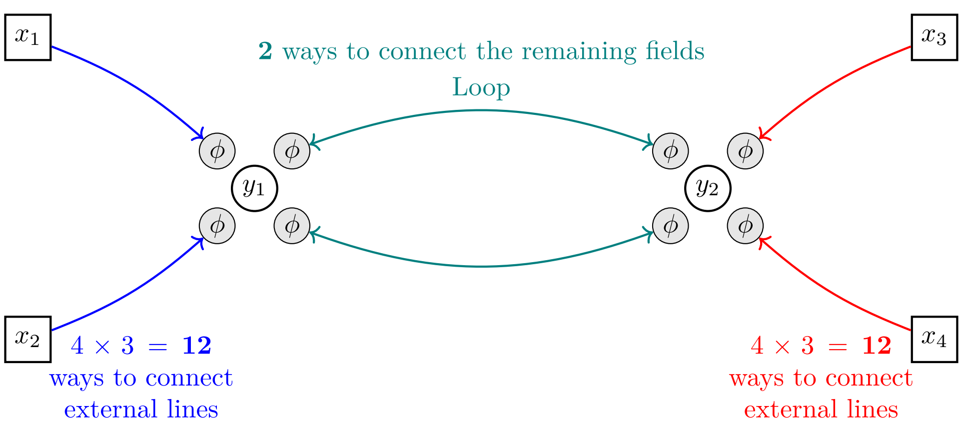

Figure 2: Visualizing the Wick contractions and counting the combinations.

Let's count the connections:

1. **Connect external lines to vertex** y_1: There are $4 \times 3 = \mathbf{12}$ ways.
2. **Connect external lines to vertex** y_2: There are $4 \times 3 = \mathbf{12}$ ways.
3. **Form the loop**: Two fields remain at each vertex. There are **2** ways to connect them.

Now, we combine this with the factors from the Lagrangian.

$$\begin{aligned}
\text{Coefficient} &= \underbrace{\left(\frac{-i\lambda}{4!}\right)^2}_{\text{Vertex Factors}} \times \underbrace{(12 \times 12 \times 2)}_{\text{Contractions}} \\
&= \frac{(-i\lambda)^2}{(24)^2} \times 288 \\
&= \frac{(-i\lambda)^2}{576} \times 288 \\
&= \frac{1}{2}(-i\lambda)^2
\end{aligned}$$

This calculation correctly produces the **symmetry factor of 1/2**.