Design of Experiments (DOE)

1.1 Executive Summary

In this project, the model of a generalized mechanical system is developed, the mechanical part used is the shock absorber. In this report, the methodology and steps by steps procedure for the development process. The first attempt is to develop the mathematical model of the shock absorber and stating the necessary assumption followed and also the limitation of the model. Some independent variables were combined to form a single-value objective function with the aim to attained a desired damper performance. A bending moment model was used to define the acceptable results, the error flag.

The mathematical model was represented with matlab code after defining some independent variables in excel sheet. The excel spreadsheet was interfaced with the matlab code. The condition for error flag was defined in the matlab code so as to determine the acceptable result. The result of the simulation is exported to an excel spreadsheet(summary.xlsx). This contains the combination of control factors used during the simulation, the measured error and defining a flag value that tells if the result is acceptable for design and not acceptable.

1.2 Development of generalized shock absorber Model

The basic function of the shock absorber in a machine is to absorb some shock by converting mechanical energy to heat energy and dissipating kinetic energy

The following assumptions is observed while developing the model

1. The damping coefficient is constant that is the rate of change in force with respect to velocity is constant.
2. The coefficient of drag of the valve is always 0.7

A generalized shock absorber model is similar to the model of a mechanical damper. The description of a hydraulic shock absorber is shown in figure 1, it is made up of cylinder with a sliding piston. A shock absorber model can be modelled by considering its linear relation, the compressive or extensive force of a shock absorber is directly proportional to the velocity of the shaft or piston. Mathematically, a shock absorber can be considered to have the following relationship:

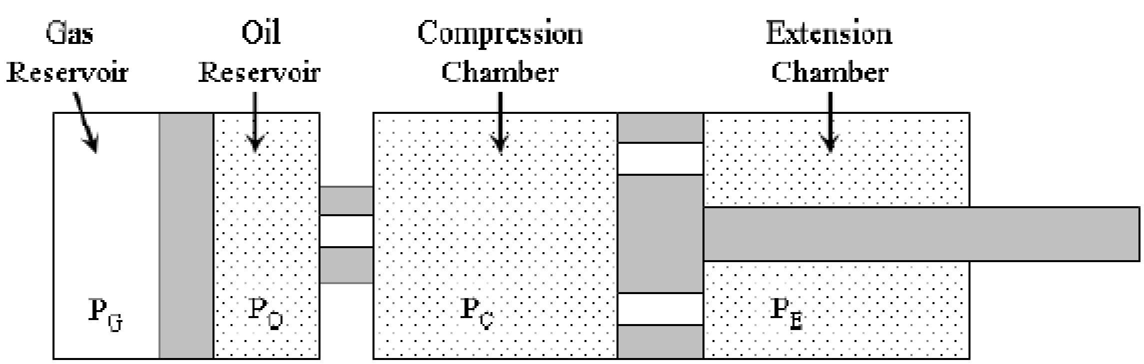
$$F = C\frac{dx}{dt} - - - - (1)$$

Where:

F = output force

$\frac{dx}{dt}$ = piston/shaft velocity

C = damping coefficient



The volumetric flow rate which indicate the rate of the volume of fluid displaced by the piston can be computed using the formula below

$$Q = A_{psiton} \times V - - - (2)$$

Where

$Q = Flow\ rate\ of\ oil\ displaced$

$A_{piston} = Area\ of\ the\ piston$

From the figure 1, the area of the piston is made up of area of annular and the area of the rod as described in the equation be low

$$A = A_{Rod} + A_{Annular}$$

$$Q = V \times (A_{Rod} + A_{Annular}) - - - - - \ (3)$$

The area of the piston is determined by adding the area of the rod and the annular. The area of the rod is found to be

Also the compressive force can be computed

$$F = P_c \times A_{piston} - - - (4)$$

The extensive force can be computed as

$$F = P_E \times A_{piston} \ - - - (5)$$

The output force is the resultant force between the compression chamber and the rebound chamber. Therefore, the resultant force is computed with the equation below:

$$F = P_c A_{piston} - P_E A_{Annular} - - - (6)$$

Shock absorber may use one or more valve; valve generally allows the flow of fluid in one direction. The valve design can either be seated valve or shim valve.

1.3 Valve Design

There are two valve design (the seated valve and the shim valve), in our design, we use a shim valve design. A shim valve is made of cantilever which could have either a circular or rectangular geometric. The main function of the valve is to control the flow of fluid getting into the shock absorber damper. The valve opens when the pressure caused by the flow rate exceeded a threshold. In this design, we implement the rectangular shim valve in our shock absorber model. Our design is aimed at computing the best control parameters that can accommodate the desired damping coefficient in the shock absorber model.

1.4 Basic valve model

The valve model is achieved by modelling the volumetric flow rate of oil in the orifice. The change in pressure as a result of the flow rate of oil is calculated as follows.

$$u = \sqrt{\frac{2\Delta P}{\rho}}$$

$$\Delta P = \frac{u^2 \rho}{2} - - - - - - - - (7)$$

The flow rate of the valve is given by the following equation:

$$Q = C_d \times A_{value} \times u = C_d \times A_{value} \times \sqrt{\frac{2\Delta P}{\rho}} - - - - - - (8)$$

Where,

ρ=density of the fluid

$A_{valve} = Area\ of\ valve$

$u = velocity\ of\ fluid$

The variable area caused by the change in pressure is computed with the formula below:

$$A_{valve} = b\frac{\Delta P A_{v-max} a^3}{3EI} - - - (9)$$

$where, b = width\ of\ opening\ of\ the\ shim$

$\Delta P = change\ in\ pressure$

$E = young\ Modulus\ of\ the\ shim\ materials$

$I = moment\ of\ inertia\ of\ the\ rectangular\ shim$

$A_{v-Max} = Maximum\ Area\ of\ the\ valve$

$$I = \frac{Lt^3}{12}$$

where,

L = length of the rectangular shim

$t = Thickness\ of\ the\ rectangular\ shim = \frac{D}{4}$

$D = Diameter\ of\ the\ shock\ absorber\ piston$

$$P = \Delta P(A_{fixed} + A_{valve}) - - - (10)$$

1.5 Selection of Control Factors

The following independent variables was used to developed objective functions

1. Diameter of the fixed valve.
2. Maximum diameter of the variable valve
3. Shim Thickness
4. The width of opening of the shim

1.6 Developing Predictive Model

The equation (10) was used to develop predictive model using the geometric of the aforementioned control parameters as the inputs. The input parameters are diameter of the rod, diameter of the piston and the aforementioned control factors. The output parameter is the system force.

1.7 Conditions for Error flag

In this section, we define the condition for error flag, this involves defining design result that satisfied the condition for acceptable results and defining the results that fails or exceed the constraints.

The regulation of the variable valve area is done by the rectangular shim, in this regards we assumed that the rectangular shim is a cantilever beam attached to the end of the valve. The pressure difference between the compression chamber and the extension chamber causes the deformation of the shim (Paulius Skačkauskas;Vidas Žuraulis;Vaidas Vadluga;Saulius Nagurnas, 2016). The maximum displacement of the cantilever beam is given by the stress equation shown below:

$$w_d(L) = \frac{PL^3}{6EI} --(11)$$

1.8 Developing error flag

When the maximum area of the valve (area of the piston) is lower than of the product of maximum displacement and the width of opening

$$b \times w_d \gg A_{v-max} --- (12)$$

The acceptable valve maximum area must be greater than the product of the width of the opening and the maximum deflation induced in the valve otherwise the design is flagged.

1.9 Determination of the optimal Performance of the shim valve

In order to determine the optimal performance of the shim valve, we define the desired force for a range of velocities using the formula in equation(1) . The figure 1 shows the graph of the desired force with respect to the piston speed. In our attempt to produce the desired force, we classified the damping coefficient into two which are the low velocity damping coefficient and the high velocity damping coefficient. The low velocity damping coefficient is defined for the low velocity to compute the force associated with it. Depending on our design, a velocity threshold is set for the defining the high velocity damping coefficient.

we compute the actual system force as a result of the change in pressure caused from the valve opening (10). A performance metrics given by equation (13) which measure the error between the desired force and the system force is used to obtained the best control factors by permutations.

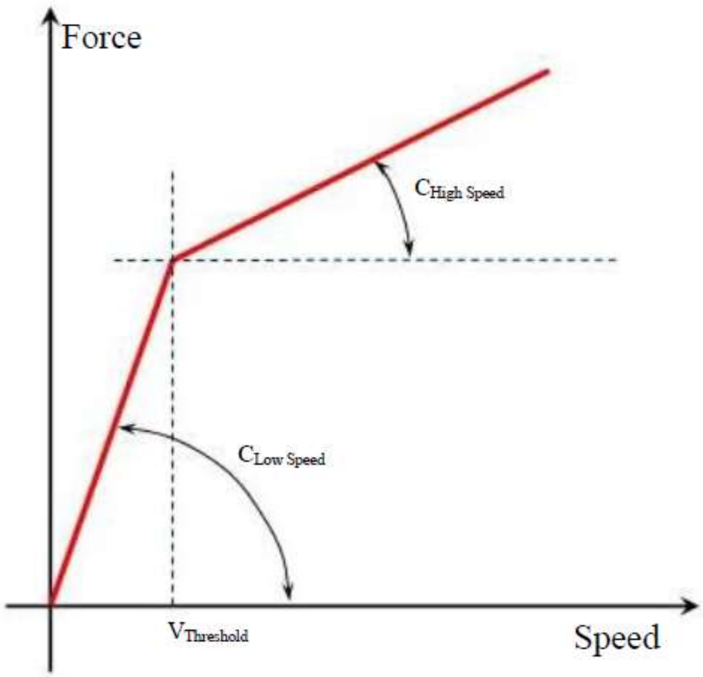


Figure 1: Desired force Vs piston speed

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(Y_i - X_i)^2 ----(13)$$

Where,

$X_i = predicted\ value$

$Y_i = intended\ value$

2.0 Summary of challenges overcome in integrating Excel mechanical system with matlab

The independent variables were defined with the excel spread sheet, the control factors, constant and other assigned parameters were arranged in rows and the columns. The control factors are defined for the fixed value, lower limit and upper limit. The first challenge encounter is reading the values from excel spreadsheet to the Matlab code. This challenge was overcome by arranging the values in the right ways understandable by Matlab code.

2.1 Improving the efficiency of the system

In order to increase the efficiency of the system, the control factors were combined to determine the optimal values. Every combination of control factor was subjected to the objective function; the system force was computed and compared with the desired force using the mean square error performance metric. The control factors that gave the minimum error is considered as the best for the valve system

2.2 Response of the model for a specific design

The shock absorber system was created using the design ID in the table below

	parameters	Unit	Geometric
1	Effective Diameter of piston	mm	49.80
2	Diameter of the rod	mm	6.0
3	Damping coefficient for low speed	N/m/s	350
4	Damping coefficient for high speed	N/m/s	200
5	Velocity Threshold	m/s	0.04

The best shim valve parameters found by permutation to accommodate the shock absorber model is described by the table below:

	Parameter	Unit	Value
1	Diameter of the constantly opened valve	mm	3
2	Maximum diameter of the hole with valve	mm	10
3	Shim Thickness	mm	0.5
4	Width of Opening	mm	6
5	Error		260.272

In order to optimize the initial design, the change in pressure was computed for a range of piston velocities (0.001-0.15). The figure below shows the ideal change in pressure for the range of velocities.

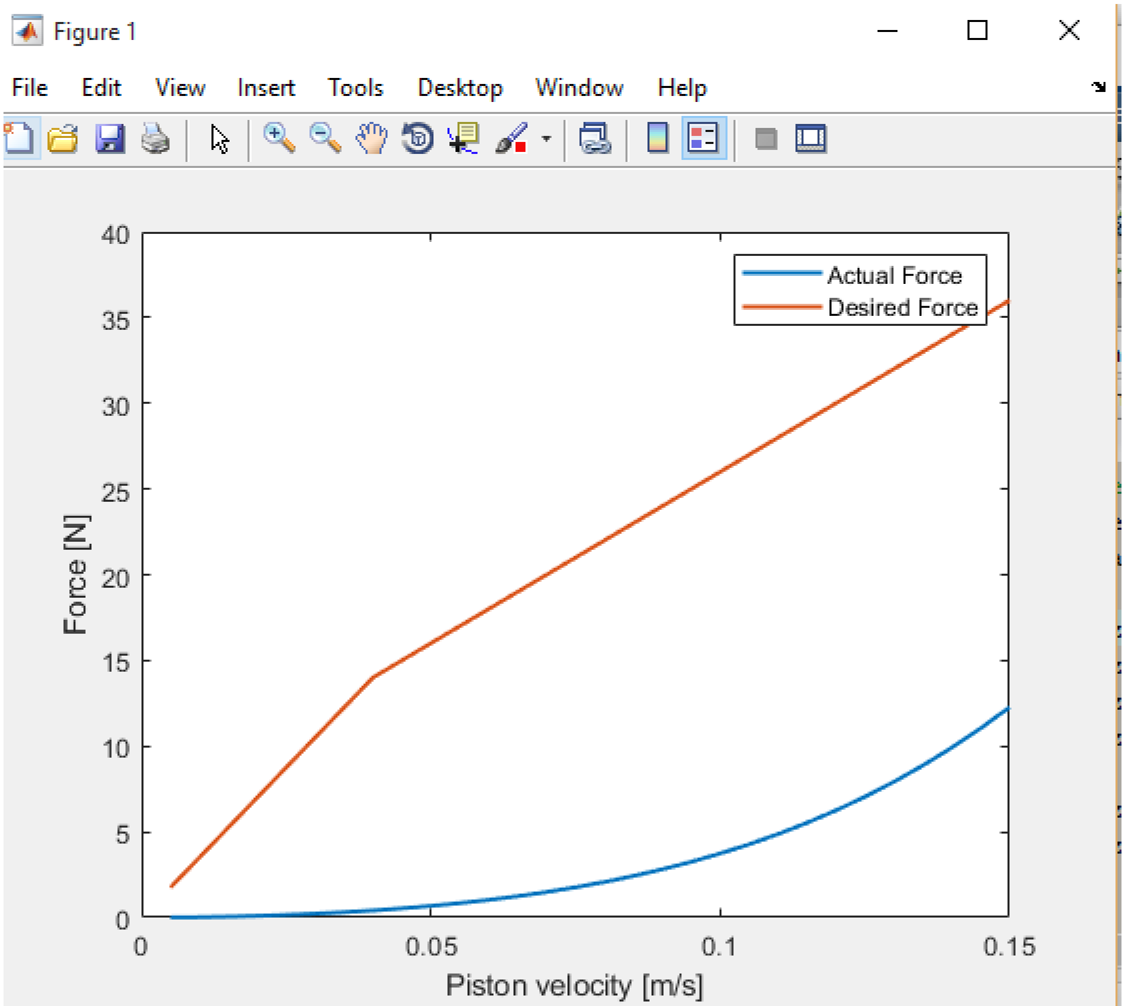


Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
Actual Force
Desired Force
Force [N]
Piston velocity [m/s]
0
5
10
15
20
25
30
35
40
0.05
0.1
0.15

Matlab code

```
clc
clear all
close all

%import data
[~,~,data]=xlsread('mechanical spreadsheet.xlsx');

global p Cd         %global
drod=data{13,3}*1e-3;     %diameter of the rod
dpiston=data{12,3}*1e-3;  %Effective diameter of the piston
p=data{26,3};       %density of lubricant
Cd= data{27,3};     %coeficient of drag
E=data{28,3};       %young Modulus



%dependent variables
Arod=pi*drod^2/4;        %Rod area
Apiston=pi*dpiston^2/4;     %Piston area
Aannular=Apiston-Arod; %Annular area
L=dpiston/4;            %shim length

V=cell2mat(data(3:32,8));         %velocities
Force=cell2mat(data(3:32,9));    %Actual force


dfvalveU=data{3,5};  %the upper limit of diameter of the fixed valve
dVvalveU=data{4,5};  %the upper limit of diameter of the variable valve
tshimU=data{5,5};  %the upper limit of  shim thickness
preloadU=data{6,5};  %the upper limit of diameter of the shim thickness
bshimU=data{7,5};     %the upper limit of the width of shim



dfvalveL=data{3,4};  %the lower limit of diameter of the fixed valve
dVvalveL=data{4,4};  %the lower limit of diameter of the variable valve
tshimL=data{5,4};    %the  lower limit of the shim thickness
preloadL=data{6,4};  %the upper limit of diameter of the shim thickness
bshimL=data{7,4};     %the  lower limit of the width of shim

dfvalves=linspace(dfvalveL,dfvalveU,data{3,6});    %valve diameter
dVvalves=linspace(dVvalveL,dVvalveU,data{4,6});    %valve diameter
tshim=linspace(tshimL,tshimU,data{5,6});          %shim thickness
preloads=linspace(preloadU,preloadL,data{6,6});    %valve diameter
bshim=linspace(bshimU,bshimL,data{7,6});    %width of shim


results=[]; %result
```

```
for i=1:length(dfvalves)
    for j=1:length(dVvalves)
        for k=1:length(bshim)
            for n=1:length(tshim)

                dfvalve=dfvalves(i)*1e-3;    %diameter of the constantly
opened valve
                dVvalve=dVvalves(j)*1e-3;    %maximum diameter of the hole
with valve

                t=tshim(n)*1e-3;        %shim thickness
                b=bshim(k)*1e-3;        %shim width


                Afvalve=pi*dfvalve^2/4; %area of the constantly opened valve
                AVvalve=pi*dVvalve^2/4; %maximum area of the hold with valve

                %fluid velocities
                Vf=V*Apiston/Afvalve;

                %compute the change in pressure
                dP=p*Vf.^2/2;

                %compute the moment of inertia
                I=L*t^3/12;
                a=abs(L-b);

                %compute the variable area
                Avalve=(b*dP*AVvalve*L^3)/(3*E*I);

                %total area of the valve
                Atotal=Afvalve + Avalve;

                %compute the flow rate when valve is fully open
                Qopen= Cd*Atotal.*V;

                 P=dP.*Atotal;
                 error=mse(P,Force);            %error

                 wd=max((P*L^3)/(3*E*I));       %Maximum  displacement

                flag=b*wd >max(Avalve);        %flag

                if flag
                else
                    d=[dfvalve,dVvalve,t,b,error];
                    results=vertcat(results,d);
                end

            end
        end
    end
  end
```

```
Fixed_valve_diameter=results(:,1)*1000;  %constant diameter of the valve
Maximum_valve_diameter=results(:,2)*1000;  %maximum diameter of the hold with
valve
shim_thickness=results(:,3)*1000;  %shim thickness
shim_width=results(:,4)*1000;      %shim width
error=results(:,5);                %error

[~,idx]=min(error); %find the minimum error


%create Table
T=table(Fixed_valve_diameter,Maximum_valve_diameter,shim_thickness,shim_width
,error)
writetable(T,'summary1.xlsx');

optimized_fixed_diameter=Fixed_valve_diameter(idx)*1e-3;
optimized_variable_diameter=Maximum_valve_diameter(idx)*1e-3;
optimized_shim_thickness=shim_thickness(idx)*1e-3;
optimized_shim_width=shim_width(idx)*1e-3;

optimized_fixed_Area=pi*optimized_fixed_diameter^2/4;
optimized_variable_Area=pi*optimized_variable_diameter^2/4;

%fluid velocities
Vf=V*Apiston/optimized_fixed_Area;

%compute the change in pressure
dP=p*Vf.^2/2;

%compute the moment of inertia
I=L*optimized_shim_thickness^3/12;

%compute the variable area
Avalve=(optimized_shim_width*dP*optimized_variable_Area*L^3)/(3*E*I);

%total area of the valve
Atotal=optimized_fixed_Area + Avalve;

%actual force
Factual=dP.*Atotal;


figure(1)
plot(V,Factual,'LineWidth',1.5)
hold on
plot(V,Force,'LineWidth',1.5)

xlabel('Piston velocity [m/s]')
ylabel('Force [N]')
legend('Actual Force','Desired Force','lcn','northeast')
```

```
Velocity_piston=V; %velocity of piston
Velocity_fluid=Vf; %velocity of fluid
change_pressure=dP; %change in pressure
variable_area=Avalve; %Area of variable valve
Total_area=optimized_fixed_Area + Avalve; %Total area of the valve
System_Force=Factual; %actual Force
Desired_Force=Force;  %Desired Force
Error=(System_Force-Desired_Force).^2; %error

T2=table(Velocity_piston,Velocity_fluid,change_pressure,variable_area,Total_a
rea,...
    System_Force,Desired_Force,Error)
writetable(T2,'summary2.xlsx');
```