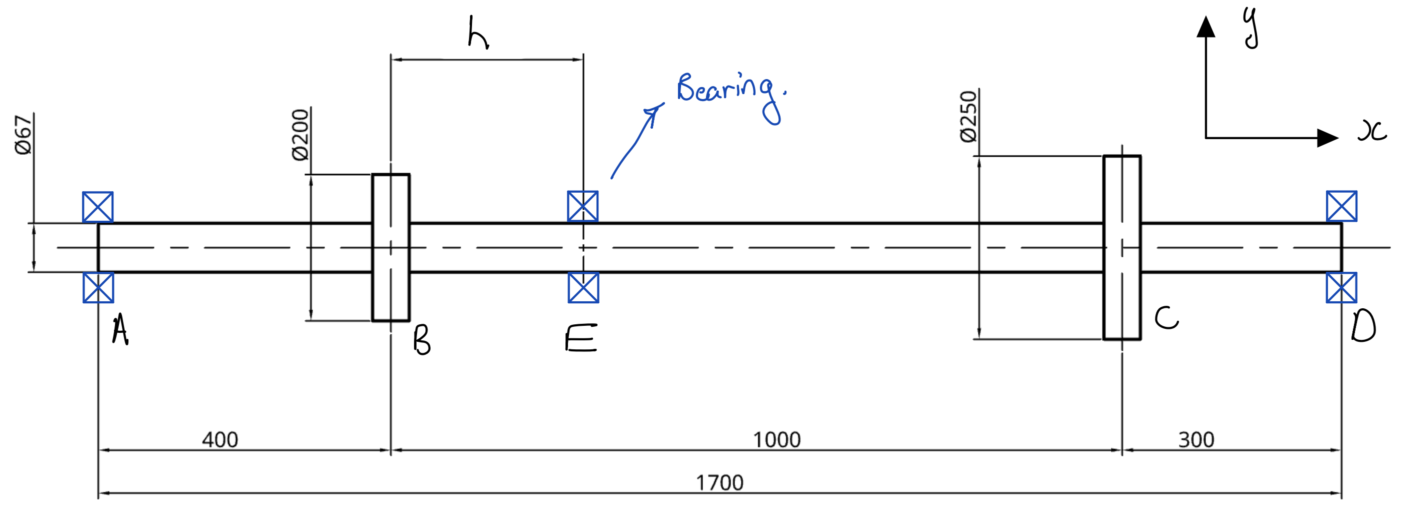
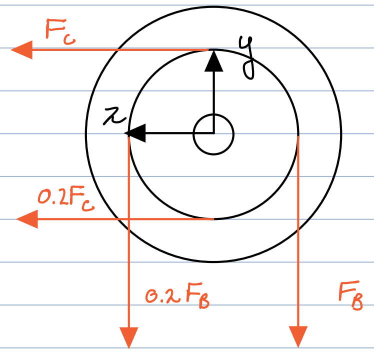
Solution A:

CONCEPTUALIZE



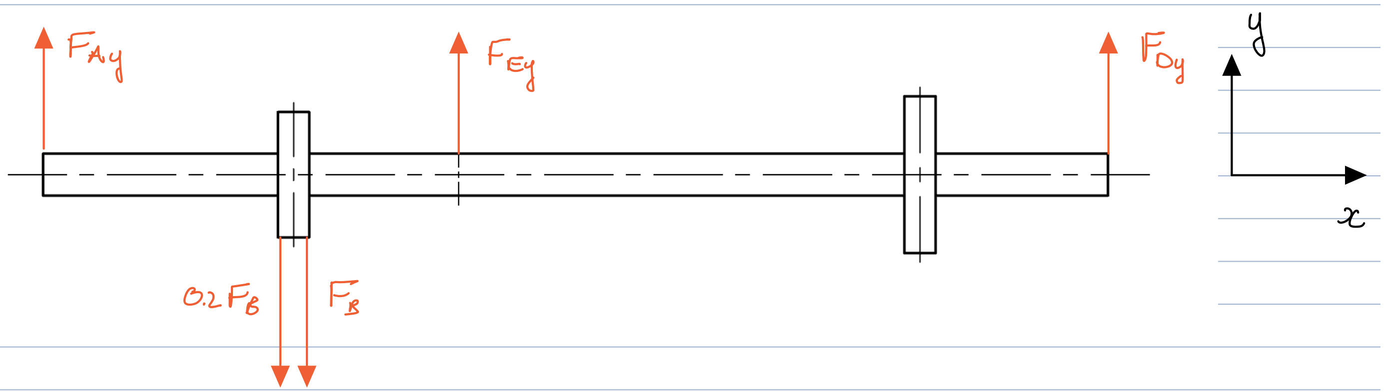
FBD:

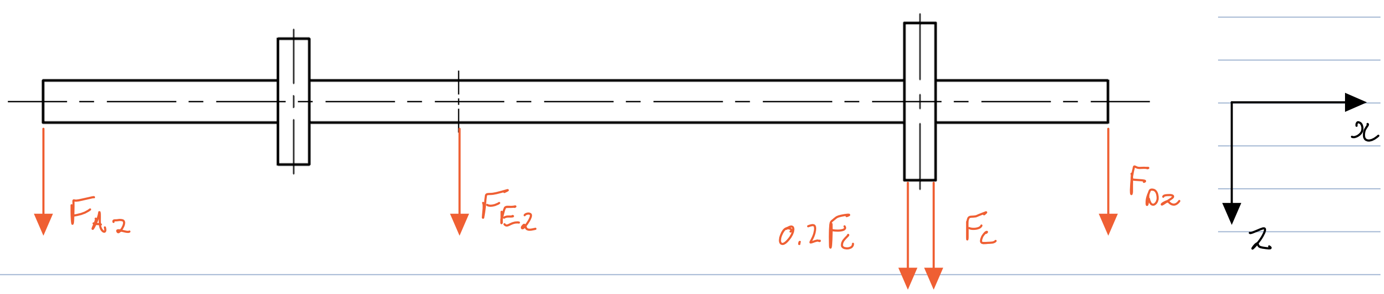


Shaft material: EN-8

$\therefore \quad E = 200 \text{ GPa}$

$\nu = 0.3$





Boundary conditions:

A:
$M_A = 0$
$\delta_A = 0$

D:
$M_D = 0$
$\delta_D = 0$

E:
$\delta_E = 0$

$$\Sigma F_y = 0 = F_{Ay} - 1.2 F_B + F_{Ey} + F_{Dy} \quad (1)$$

$$\Sigma F_z = 0 = F_{Az} + F_{Ez} + 1.2 F_C + F_{Dz} \quad (2)$$

Notes

- Self Alligning Bearings used → No moments is transfered.
- Neglect weight of shaft.

Limetations:

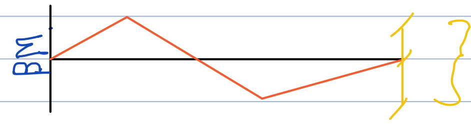
Normal stress $< \sigma_{Allow} = 80$ MPa
Shear stress $< \tau_{Allow} = 40$ MPa
Max deflection @ pulleys $< \delta_{allow} = 2$ mm
Max angle @ Bearings $< \theta_{allow} = 3$ degrees.

CATEGORIZE:

- Staticaly indeterminate: 2 displacement BC's and 2 Force BC's and +1 displacement design condition
- + One DC's @ E $\delta_E = 0$
- Calculate known forces from power:

  $P_{new} = 1.5P$ $\quad T = \frac{30 P}{N}$

- Use $EI v''''(x) = -q(x)$ and Integrate 4 times to obtain V, BMs, θ, and δ.
- Additionally use Macaulay's Method to represent F_A, F_B, F_C, F_D, F_E
- Solve for integration constants. By Applying BC's and ΣF, ΣM.
- Repeate for XZ Plane
- Combine Forces, Moments, θ, δ by pythagoras. to Find critical conditions
- Itterate for h to produce balanced Beam:

example:  Same distance

- Check $\sigma = \frac{My}{I}$, $\tau = \frac{Tr}{J}$ and $\tau_{max} = \frac{4V}{3A}$

with $\sigma_{max} = 80$ MPa $\tau_{max} = 40$ MPa

ANALYZE:

Calc known forces.

$P_{new} = 1.5 \times P$

$P_{new} = 1.5 \times 80$ kW
$= 120$ kW

$T = P/\omega$ $\qquad \omega \rightarrow$ rad/s

$\omega = \frac{N \cdot 2\pi}{60} = \frac{\pi N}{30}$

$\therefore T = \frac{30 P}{\pi N} = 573$ N·m

$\sum M_{Bx} = 573 = F_B (D_B/2) - 0.2 F_B (D_B/2)$

$5730 = 5 D_B F_B - F_B D_B$

$4 F_B D_B = 5730$

$F_B = 5730 / 4D_B = 7162$ N Checked

$\sum M_{Cx} = 573 = F_C (D_C/2) - 0.2 F_C (D_C/2)$

$F_C = \frac{5730}{4 D_C} = 5730$ N checked.

Goal $\rightarrow$ Select h so that All limitations are met.

$\therefore$ $F_B = 7162$ N $\longrightarrow$ $\qquad F_C = 5730$ N $\longrightarrow$

Differential eq's for x-y Plane.

(y.1) $EI\,v''''(x) = 0$

$M_B + C$ required by inspection,

(y.2) $V = EI\,v'''(x) = F_{Ay} - 1.2F_B\langle x-a\rangle^0 + F_{Ey}\langle x-(a+h)\rangle^0 + F_{Dy}\langle x-(a+b+c)\rangle^0$

(y.3) $M = EI\,v''(x) = F_{Ay}(x) - 1.2F_B\langle x-a\rangle^1 + F_{Ey}\langle x-(a+h)\rangle^1 + F_{Dy}\langle x-(a+b+c)\rangle^1 + C_1$

(y.4) $$EI\theta = EI\,v'(x) = \frac{F_{Ay}x^2}{2} - 0.6F_B\langle x-a\rangle^2 + \frac{F_{Ey}\langle x-(a+h)\rangle^2}{2} + \frac{F_{Dy}\langle x-(a+b+c)\rangle^2}{2} + C_1x + C_2$$

(y.5) $$EI\delta = EI\,v(x) = \frac{F_{Ay}x^3}{6} - 0.2F_B\langle x-a\rangle^3 + \frac{F_{Ey}\langle x-(a+h)\rangle^3}{6} + \frac{F_{Dy}\langle x-(a+b+c)\rangle^3}{6} + \frac{C_1x^2}{2} + C_2x + C_3$$

Sum Of Forces:

(1) $\Sigma F_y = 0 = F_{Ay} - 1.2F_B + F_{Ey} + F_{Dy}$

6 unknowns: F_{Ay}, F_{Dy}, F_{Ey}, C_1, C_2, C_3

6 - equations (With BC's):
- (y.3) - ($M_A = 0$)
- (y.3) - ($M_D = 0$)
- (y.5) - ($\delta_A = 0$)
- (y.5) - ($\delta_D = 0$)
- (y.5) - ($\delta_E = 0$)
- (1) - (Sum of Forces y-dir)

Program eq and BC's in python solver (sympy.optimize.minimize)

for $h = 0.32$ m the solution is.

$C_1 = 0$ $C_2 = -204.4$ $C_3 = 0$

$F_{Ay} = 3121$ N $F_{Dy} = -514$ N $F_{Ey} = 5987$ N

Differential eq's for x-z Plane.

(2.1) $q = EI v''''(x) = 0$

From inspection no + k required

(2.2) $V = EI v'''(x) = F_{Az} + F_{Ez}\langle x-(a+h)\rangle^0 + 1.2F_c\langle x-(a+b)\rangle^0 + F_{Dz}\langle x-(a+b+c)\rangle^0$

(2.3) $M = EI v''(x) = F_{Az}(x) + F_{Ez}\langle x-(a+h)\rangle^1 + 1.2F_c\langle x-(a+b)\rangle^1 + F_{Dz}\langle x-(a+b+c)\rangle^1 + k_1$

(2.4) $$EI\theta = EI v'(x) = \frac{F_{Az}x^2}{2} + \frac{F_{Ez}\langle x-(a+h)\rangle^2}{2} + 0.6F_c\langle x-(a+b)\rangle^2 + \frac{F_{Dz}\langle x-(a+b+c)\rangle^2}{2} + k_1 x + k_2$$

(2.5) $$EI\delta = EI v(x) = \frac{F_{Az}x^3}{6} + \frac{F_{Ez}\langle x-(a+h)\rangle^3}{6} + 0.2F_c\langle x-(a+b)\rangle^2 + \frac{F_{Dz}\langle x-(a+b+c)\rangle^2}{6} + \frac{k_1 x^2}{2} + k_2 x + k_3$$

Sum of forces.

(2) $\Sigma F_z = 0 = F_{Az} + F_{Ez} + 1.2F_c + F_{Dz}$

6-unknowns: F_{Az}, F_{Dz}, F_{Ez}, k_1, k_2, k_3

6 equations (When BC's applied): (2.3) – ($M_{Ay} = 0$)
(2.3) – ($M_{Dy} = 0$)
(2.5) – ($\delta_A = 0$)
(2.5) – ($\delta_D = 0$)
(2.5) – ($\delta_E = 0$)
(2) – (Sum forces in z-dir)

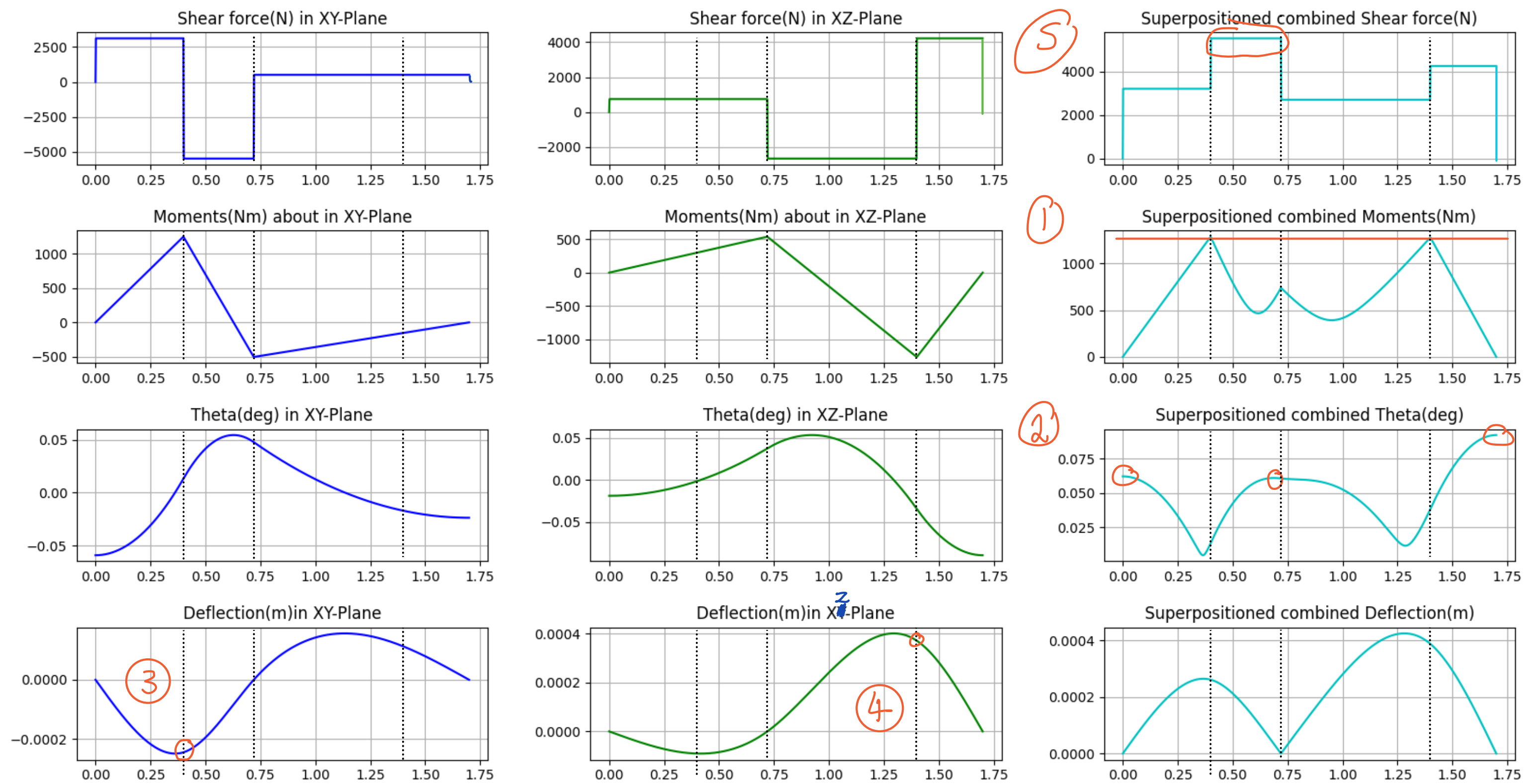
Program eq and BC's in python solver (sympy.optimize.minimize)

Solution for $h = 0.32$ m: use a starting $h = 0.1$ m and itterate

$k_1 = 0$ $k_2 = -64.7$ $k_3 = 0$

$F_{Az} = 748.4$ N $F_{Dy} = -4221$ N $F_{Ey} = -3403$ N

XY: XZ: Combined.



Beam equations in xy, xz, and combined directions.
For h=0.32m

Design constraints are indicated in Orange:

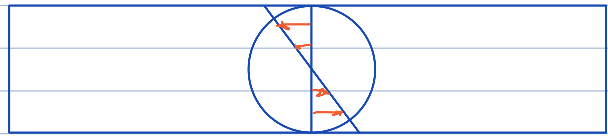
1 - The value of h was changed iteratively by trial an error until the 2 positions of highest BM were the same. This resulted in a Beam with ballenced stresses.

2 - The journal bearings had a design limetation of angular misalignment of 3 deg. And from the plot it is visable that al angles is < 1 deg.

3 - The pulley at C has a design restriction of a deflection of 2mm to keep the tension in the belts. A seen the deflection does not exceed 0.3 mm at $x = 0.4$ m

4 - Similar to 3 but the pulley works in a different direction so deflection at $x = 1.4$ in the XZ plane is < 0.4 mm.

Still need to check max stress state on surface.



Use: $\tau = \frac{Tr}{J}$ and $\sigma = \frac{My}{I}$

$M = 1288.5$ Nm
= from graph (combined.)

where $J = \frac{\pi D^4}{32}$ $\quad I = \frac{\pi D^4}{64}$

$T = 573$ Nm

$$\tau = \frac{32T\left(\frac{D}{2}\right)}{\pi D^4} = \frac{16T}{\pi D^3} = \frac{16(573)}{\pi(0.067)^3} = 9.70 \text{ MPa} \leq 40 \text{ MPa}$$ Checked.

$$\sigma = \frac{64 M D/2}{\pi D^4} = \frac{32M}{\pi D^4} = \frac{32(1288.5)}{\pi(0.067)^3} = 43.64 \text{ MPa} \leq 80 \text{ MPa}$$ Checked.

Max Shear stress in center of shaft.

$\tau_{max} = \frac{4V}{3A}$ max shear stress in circular section

$V_{max} = 5525$ N (Between C-E, see figure plots)

$A = \pi r^2 = \frac{\pi D^2}{4}$

$$\tau_{max} = \frac{4 V_{max} 4}{3\pi D^2} = \frac{16 V_{max}}{3\pi D^2} = 2.10 \text{ MPa}$$ Checked.

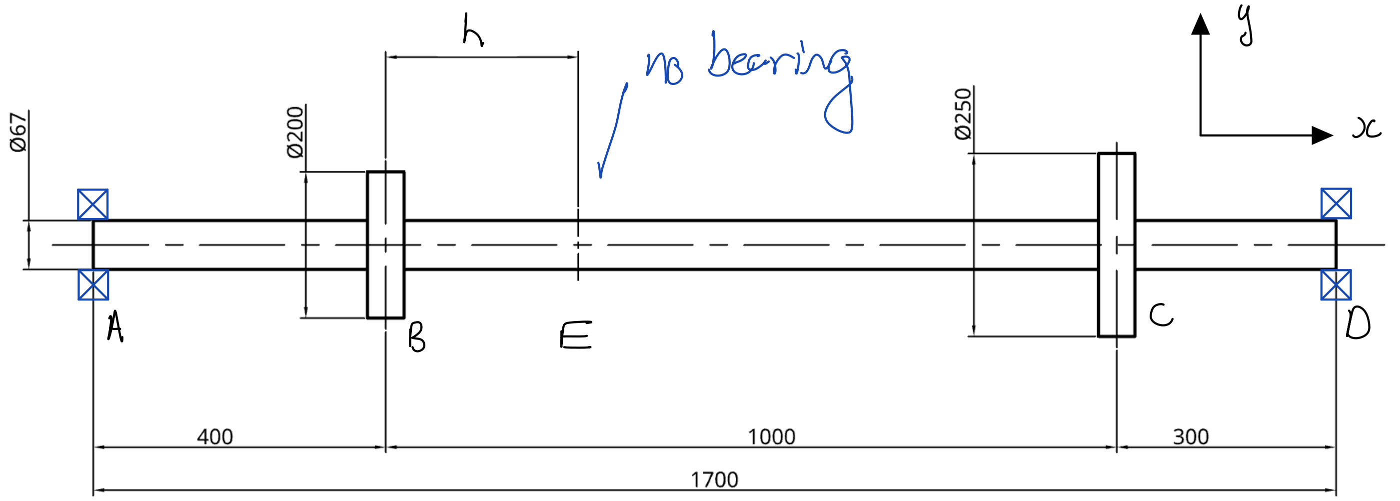
Reflect:

- All the Shear force and Bending Moments Plots start and end at zero. Makes sense and verifies $\Sigma F = \Sigma M = 0$.
- All the deflection Plots has is $\delta = 0$ at the Bearings which verify the displacement equations.
- $h = 0.32$ m has been choosen as the best position by minimizing the maximum positions on the combined Bending moment plot.
- Normal stress due to bending ≈ 44 MPa $<$ 80 MPa (Yeild criterion including SF)
- Shear stress due to torque ≈ 10 MPa $<$ 40 MPa (Shear Yield criteria includin SF)
- Shear stress due to Shear forces ≈ 2 MPa $\leq$ 40 MPa (Shear Yield criteria includin SF)
- Conclusion the shaft will carry the loads by a large margin if $h = 0.32$m and will also satisfy the angle and deflections constraints.

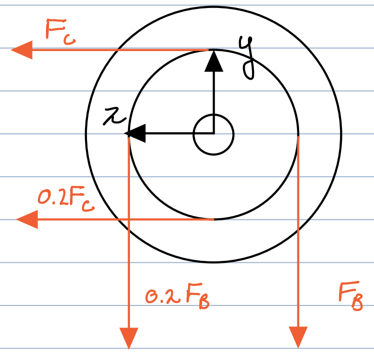
Note the shaft will be compaired with Problem B.

Solution B:

CONCEPTUALIZE



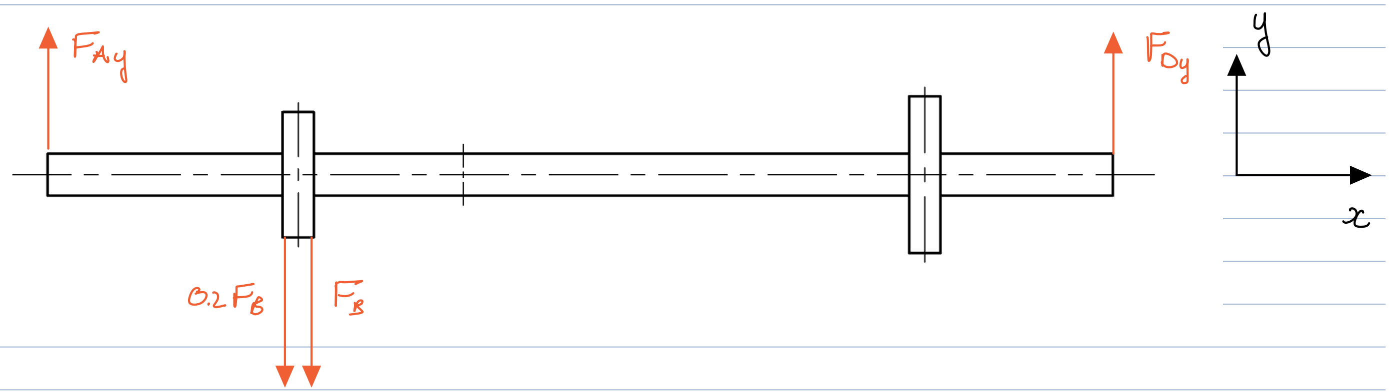
FBD:

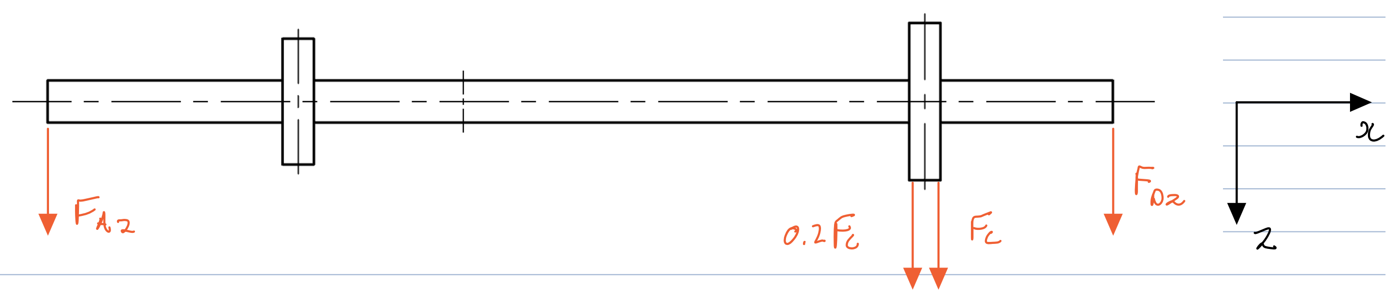


Shaft material: EN-8

$\therefore \quad E = 200 \text{ GPa}$

$\nu = 0.3$





Boundary conditions:

A:

$M_A = 0$

$\delta_A = 0$

D:

$M_D = 0$

$\delta_D = 0$

$\Sigma F_y = 0 = F_{Ay} - 1.2 F_B + F_{Dy}$

$\Sigma F_z = 0 = F_{Az} + 1.2 F_C + F_{Dz}$

Notes

- Self Alligning Bearings used → No moments is transfered.
- Neglect weight of shaft.

Limitations:

Normal stress $< \sigma_{Allow} = 80$ MPa

Shear stress $< \tau_{Allow} = 40$ MPa

Max deflection @ pulleys $< \delta_{allow} = 2$mm

Max angle @ Bearings $< \theta_{allow} = 3$ degrees.

CATEGORIZE:

- Statically determinate: 2 Force + 2 Displacement BC's
- Solve for forces in y and z
- Solve whole beam with $EI v''''(x) = -q(x)$ and integrate 4 times.
- Substitute all known forces ons solve for integration constants
- Determine the max stress element location.
- Use $\sigma = \frac{My}{I}$ $\tau = \frac{Tr}{J}$ $\tau_{max} = \frac{4V}{3A}$
- Use $\sigma_{max} = 80$ MPa and $\tau = 40$ MPa to solve for the required diameter.
- Also Check the required diameter for the Design constraints $\theta_{max} < 3$ deg, $\delta_{max} < 2$mm

ANALYSE:

$T = 573$ Nm $\quad F_B = 7162$ N $\quad F_C = 5730$ N

from Solution A:

Calculating known forces

$$\Sigma F_y = 0 = F_{Ay} - 1.2 F_B + F_{Dy}$$

$$F_{Ay} + F_{Dy} = 1.2 F_B$$

$$\Sigma M_z = 0 = -1.2(a)F_B + (a+b+c)F_{Dy}$$

$$F_{Dy} = \frac{1.2(a)F_B}{a+b+c} \quad (1)$$

$$\therefore F_{Ay} = 1.2F_B - \frac{1.2(a)F_B}{a+b+c}$$

$$F_{Ay} = F_B\left(1.2 - \frac{1.2a}{a+b+c}\right) \quad (2)$$

$$\Sigma F_z = 0 = F_{Az} + 1.2F_C + F_{Dz}$$

$$F_{Az} + F_{Dz} = -1.2F_C$$

$$\Sigma M_y = 0 = \overset{1.2}{(a+b)}F_C + (a+b+c)F_{Dz}$$

$$F_{Dz} = \frac{-(a+b)F_C 1.2}{(a+b+c)} \quad (3)$$

$$\therefore F_{Az} = \frac{(a+b)F_C 1.2}{(a+b+c)} - 1.2F_C$$

$$F_{Az} = F_C\left(\frac{(a+b)1.2}{a+b+c} - 1.2\right) \quad (4)$$

y.1 $\quad EI\,v''''(x) = 0$

y.2 $\quad V = EI\,v'''(x) = F_{Ay} - 1.2F_B\langle x-a\rangle^0 + F_{Dy}\langle x-(a+b+c)\rangle^0$

y.3 $\quad M = EI\,v''(x) = F_{Ay}x - 1.2F_B\langle x-a\rangle^1 + F_{Dy}\langle x-(a+b+c)\rangle^1 + C_1$

y.4 $\quad EI\theta = EI\,v'(x) = \frac{F_{Ay}x^2}{2} - 0.6F_B\langle x-a\rangle^2 + \frac{F_{Dy}\langle x-(a+b+c)\rangle^2}{2} + C_1x + C_2$

y.5 $\quad EI\delta = EI\,v(x) = \frac{F_{Ay}x^2}{6} - 0.2F_B\langle x-a\rangle^3 + \frac{F_{Dy}\langle x-(a+b+c)\rangle^3}{6} + \frac{C_1x^2}{2} + C_2x + C_3$

6unknowns: C_1 3equation (With BC's): $(y.3)-(M_A=0)$
C_2 $(y.5)-(\delta_A=0)$
C_3 $(y.5)-(\delta_D=0)$

Program eq and BC's in python solver (sympy.optimize.minimize)

<u>Differential eq's for x-z Plane.</u>

(2.1) $q=EI\,v''''(x)=0$

(2.2) $V=EI\,v'''(x)=F_{Az}+1.2F_c\langle x-(a+b)\rangle^0+F_{Dz}\langle x-(a+b+c)\rangle^0$

(2.3) $M=EI\,v''(x)=F_{Az}(x)+1.2F_c\langle x-(a+b)\rangle^1+F_{Dz}\langle x-(a+b+c)\rangle^1+k_1$

(2.4) $EI\theta=EI\,v'(x)=\frac{F_{Az}x^2}{2}+0.6F_c\langle x-(a+b)\rangle^2+\frac{F_{Dz}\langle x-(a+b+c)\rangle^2}{2}+k_1x+k_2$

(2.5) $EI\delta=EI\,v(x)=\frac{F_{Az}x^3}{6}+0.2F_c\langle x-(a+b)\rangle^3+\frac{F_{Dz}\langle x-(a+b+c)\rangle^3}{6}+\frac{k_1x^2}{2}+k_2x+k_3$

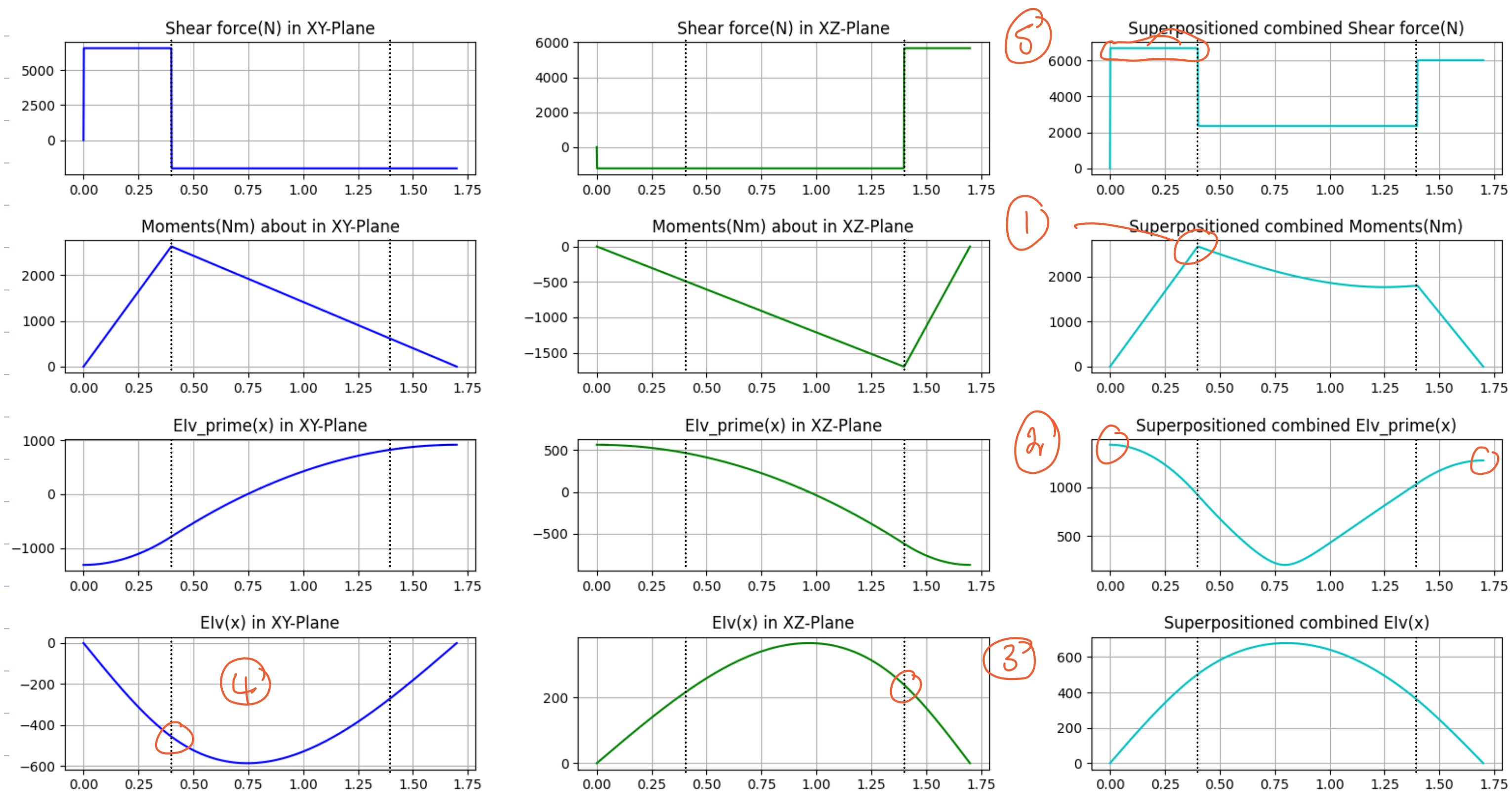
3-unknowns: k_1 3 equations (When BC's applied): $(2.3)-(M_{Ay}=0)$
k_2 $(2.5)-(\delta_A=0)$
k_3 $(2.5)-(\delta_D=0)$

Program eq and BC's in python solver (sympy.optimize.minimize)

$F_{Ay}=6572\ N$ $F_{Az}=-1213.3\ N$
$F_{Dy}=2022\ N$ $F_{Dz}=-5662\ N$

$C_1=0$ $k_1=0$
$C_2=-1314$ $k_2=566$
$C_3=0$ $k_3=0$

Solution Plots : Note EIv'(x) and EIv(x) is plotted.



Critical locations:

1 Max Bending moment.

2 $EIv'(x)$ at Bearings (Angle)

3 4 $EIv(x)$ at pulleys in the direction of tension. (displacement)

5 ~ Max location of shear force.

Next step: Calculate the minimum required diameter of shaft for each of these loadcases.

Max BM:

$$\sigma = \frac{My}{I} \qquad \sigma = 80\,MPa \qquad I = \frac{\pi D^4}{64} \qquad y = \frac{D}{2}$$

$M = 2673$ Nm (from Python plots)

$$80\times10^6 = \frac{2673 \cdot \frac{D}{2} \cdot 64}{\pi D^4} = \frac{85536}{\pi D^3}$$

$$D = \sqrt[3]{\frac{85536}{\pi \times 80\times10^6}} = 0.0698 \text{ m} \quad \therefore \text{ min integer} = 70 \text{ mm.}$$

Max Shear Force:

$$\tau_{max} = \frac{4V}{3A} \qquad V = 6683 \text{ N (from Python plots)}$$

$$A = \frac{\pi D^2}{4} \qquad \tau_{max} = 40\,MPa$$

$$A = \frac{4V}{3\tau_{max}}$$

$$\frac{\pi D^2}{4} = \frac{4V}{3\tau_{max}}$$

$$D = \sqrt{\frac{16V}{3\pi\tau_{max}}} = \sqrt{\frac{16(6683)}{3\pi\, 40\times10^6}} = 0.0168 \text{ m} \quad \therefore \text{ 17 mm smallest int}$$

Max Torque:

$$\tau = \frac{Tr}{J} \qquad T = 573 \text{ Nm} \qquad r = \frac{D}{2} \qquad J = \frac{\pi D^4}{32}$$

$$\tau_{max} = 40\,MPa$$

$$\tau = \frac{32T\,D}{2\pi D^4} = \frac{16T}{\pi D^3}$$

$$D = \sqrt[3]{\frac{16T}{\pi\, 40\times10^6}} = \sqrt[3]{\frac{16(573)}{\pi\, 40\times10^6}} = 0.0418 \text{ m} \quad \therefore \text{ 40 mm is min.}$$

Max θ at Bearings: From the Plots → Bearing @ A

$EI\,v'(0) = 1432$ $\qquad E = 200$ Gpa $\qquad I = \frac{\pi D^4}{64}$

Now $v'(x) =$ rads $\left[\frac{360 \text{ deg}}{2\pi}\right]$ → deg.

$\therefore\ \theta = \frac{v'(0)\,180}{\pi}$ $\qquad \theta_{max} = 3$ deg.

And $v'(0) = \frac{1432}{EI} = \frac{64(1432)}{E\pi D^4}$

$\therefore\ \theta = \frac{64(1432)\,180}{E\pi^2 D^4}$

$D = \sqrt[4]{\frac{64(1432)180}{\theta_{max} E \pi^2}} = \sqrt[4]{\frac{64(1432)180}{3(200\times10^9)\pi^2}} = 0.0409$ $\therefore$ 41 mm shaft.

Max deflection at pulleys: From plots → Pulley B has greatest deflection.

$EI\,v(0.4) = 456$

$E = 200$ Gpa $\qquad I = \frac{\pi D^4}{64}$

$v_{max} = 2\text{mm} \rightarrow 0.002$ m

$\frac{\cancel{\pi} E D^4}{\cancel{64}}(\cancel{0.002}) = \cancel{456}$

$D^4 = \frac{(64)\,456}{0.002\pi(200\times10^9)}$

$D = \sqrt[4]{\frac{64(456}{0.002\pi(200\times10^9)}} = 0.0694$ $\therefore$ 70 mm min int diameter.

The most critical design constraints requiers the largest diameter.

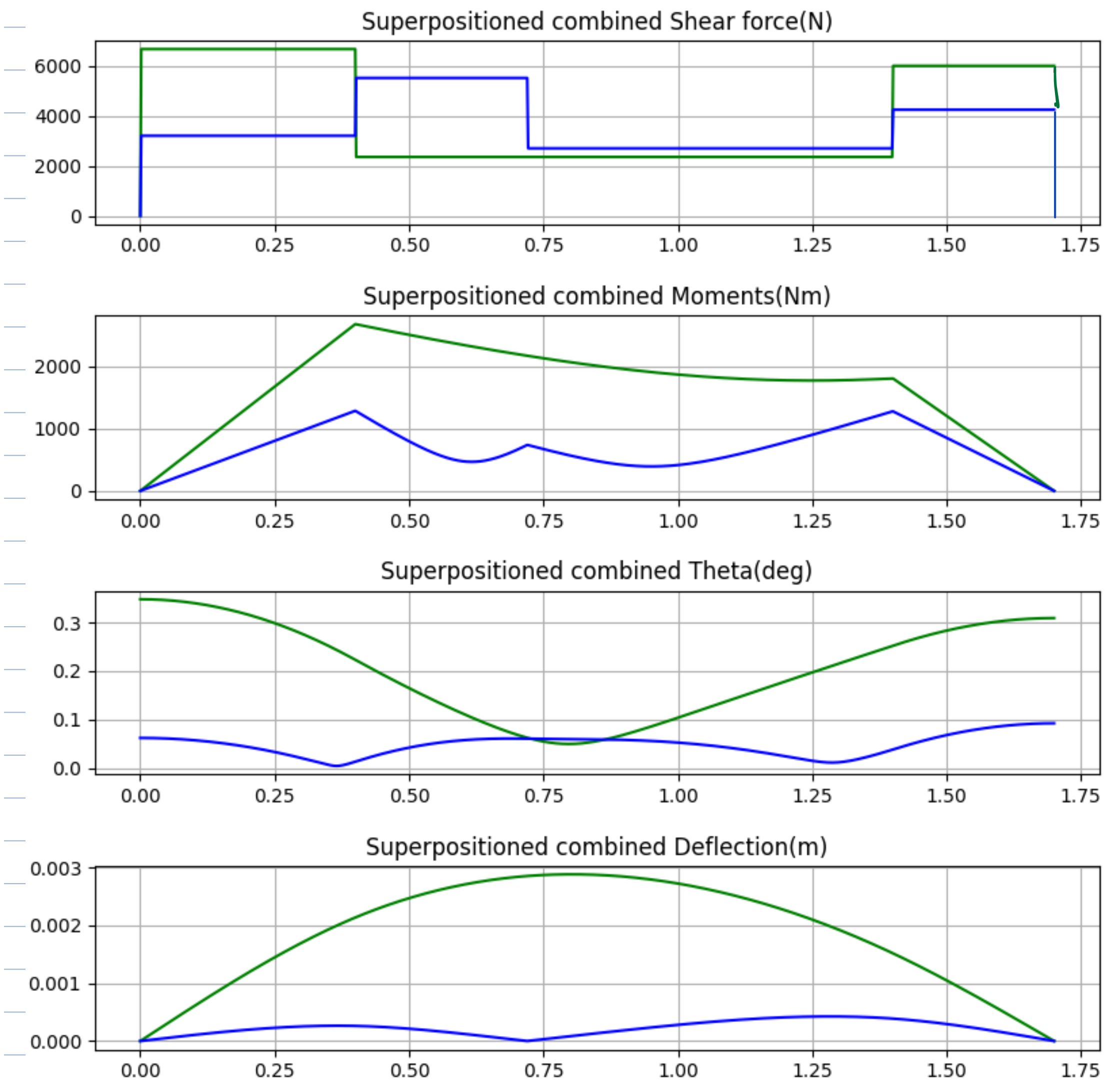
2 of them produce the same diameter.

· BM and pully deflection. @ 70 mm ∅

REFLECT:

- Min shaf diameter of 70 mm is required to satisfy the BM and displacement constant of 2mm.

- Comparison between solutions



Blue → Sol A
Green → Sol B

Notes:

1.) Solution A has almost double the stress benifit compaired to B

2.) Solution B is also much better in θ for the bearings but Solution A is still within bounds.

3.) Max Shear Force is marginaly better than Solution A

Other Considerations:

Cost: Solution B requires a whole new shaft to be made and at least major modifications to be made to the Pulleys.

Solution A will require only minor modifications (likely)

Installment: Consider how to install a journal bearing in solution A Between the 2 pulleys.

Make provisions for the removal of the pulleys and Check that the bearing can slide from either end of the shaft to its intended instalment location.

Choosen Solution → Solution A (But check Bearing installment procedure) !!!