

## Department of Mechanical and Aeronautical Engineering

# Simulation-based design MOW 323

Semester test 2: November 2021

Lecturer

Dr. S. Schmidt

### Test information

Full marks: 85 marks
Time: 3 hours
Questions: 1 - 4
Pages: 1 - 14

#### Instructions

- This is an open book test in which you are allowed to use the material given to you on clickUP, your class notes and jupyter notebooks, and the prescribed textbook (Shigley's). You are allowed to use the material (e.g. spreadsheets) given to you in previous machine design modules and dynamics.
- The semester test files are distributed as a .zip file with the question paper, answer books, supplementary jupyter notebooks and engmod python module included.
- Each question has a separate answer book and might have separate jupyter notebook files:
  - Question 1: ANSWER\_BOOK\_Q1.xlsx\*.
  - Question 2: ANSWER\_BOOK\_Q2.xlsx\*, notebook\_question2.ipynb
  - Question 3: ANSWER\_BOOK\_Q3.docx\*\*,
  - Question 4: ANSWER\_BOOK\_Q4.docx\*\*, notebook\_question4.ipynb
  - \*These excel files will be marked by a computer using a fixed set of rules. You must insert the answers in the required format in the correct excel cells. You are not allowed to insert new rows or new columns. You are not allowed to change the file name of the document nor change the names of the sheets in the file.
  - \*\*These answer books can also be handwritten notes that were scanned in (e.g. using camscanner) or jupyter notebook files. Do not upload photos as these files are very large. It is however critical that you have a single answer book for each question. The document must be called ANSWER\_BOOK\_QXXX, where XXX is the question number and will replace the template that was provided. All answers must be clearly labelled in this book and all the essential evidence should be presented in the answer book.
- You are only allowed to have one answer book per question in your submissions.
- There will be three submission links on the day:
  - 1. ST 2- Normal submission
  - 2. ST 2 Extra time submission 10min
  - 3. ST 2 Extra time submission 15min

You must submit your semester test under the correct link. You are only allowed to submit under  $ST\ 2$  -  $Extra\ time\ submission$  - Xmin if you have submitted an official letter from the dean's office to the lecturer that states that you are allowed to obtain the additional time. All incorrect submissions/submissions under the inappropriate links will be deleted.

- All submitted files must have your name and student number on the top of the first page.
- Ensure that you submit the correct files. We can only mark the files that were successfully submitted. A recommendation is that you work in a specific directory on your computer and that you check that the files in the directory are updating as you are completing the test and saving your work. If the files are not updating, you might be working on files in a different directory.
- Multiple submissions are allowed and encouraged before the deadline. You are encouraged to submit
  your latest drafts of your work throughout the test. These submissions will serve as backups of
  your work.
- Each submission must contain all of the answer books and jupyter notebooks for all of the questions. Attach all the files in a single submission. Do not submit them in separate submissions. Do not upload zip-files.
- The files in the last successful submission are marked.
- At the end of the test period (e.g., 3 hours for normal submissions), you must stop writing. You have 15 minutes to upload your work to clickUP. The link will disappear after the 15 minutes and we will mark and process the last files that were successfully uploaded.
- Unless you are only expected to provide an answer (e.g. multiple choice), you must show the necessary evidence of your understanding, knowledge and thought process. This means that you should explain the decisions and show the necessary sample calculations. Python is only used as a calculator in this module, which means that we will not be marking code.
- As there might be multiple documents that need to be completed during the test, it is recommended that only the documents of a specific question must be open on the computer at the same time. This is because if you have too many documents open the document processing software might not respond. Please ensure that you regularly save your work to avoid information loss.
- You are not allowed to contact/discuss/share anything with other people (including any fellow students) during the full test period. You are only allowed to contact the lecturer via his email or via the online session that will be communicated on clickUP.

#### **Declaration statement**

The University of Pretoria commits itself to produce academic work of integrity. By submitting this test on clickUP, I affirm that I am aware of and have read the Rules and Policies of the University, more specifically the Disciplinary Procedure and the Tests and Examinations Rules, which prohibit any unethical, dishonest or improper conduct during tests, assignments, examinations and/or any other forms of assessment. I am aware that no student or any other person may assist or attempt to assist another student, or obtain help, or attempt to obtain help from another student or any other person during tests, assessments, assignments, examinations and/or any other forms of assessment.

# Question 1 [20 marks]

All multiple choice answers must be inserted in the correct cells of the book ANSWER\_BOOK\_Q1.xlsx. Only the answers in the answer cells will be marked. Do not insert new rows or columns. Do not insert new sheets or change the sheet name.

- (a) The safety factor of a product is evaluated for three load cases with two failure modes considered, namely, yielding and buckling. The following safety factors are obtained for the product after performing the analyses: [4 marks]
  - Buckling (Load case 1): 8.85497
  - Yielding (Load case 1): 2.5651
  - Buckling (Load case 2): 9.12315
  - Yielding (Load case 2): 4.82794
  - Buckling (Load case 3): 0.94885
  - Yielding (Load case 3): 8.8498

What is the overall safety factor of the product according to the analyses that were performed?

- (A) 0.87443
- (B) 0.94885
- (C) 4.56158
- (D) 5.99497
- (E) 9.12315
- (F) None of the above.
- (b) The modified endurance limit of a specimen in bending (e.g. shaft, beam) is not adequate and it must be increased. The specimen will be in a reversed stress state. Which one of the following statements is correct? [4 marks]
  - (A) The modified endurance limit of this specimen decreases with an increase in cross-sectional dimension.
  - (B) The modified endurance limit of this specimen increases with an increase in cross-sectional dimension.
  - (C) The modified endurance limit of this specimen is not dependent on its cross-section, but only on its material properties.
  - (D) The modified endurance limit increases with cross-sectional dimension if the specimen rotates and decreases with cross-sectional dimension if the specimen does not rotate.
  - (E) The modified endurance limit increases with cross-sectional dimension if the specimen does not rotate and decreases with cross-sectional dimension if the specimen rotates.
  - (F) None of the above

(c) Calculate the stress amplitude in the bar shown in Figure 1.

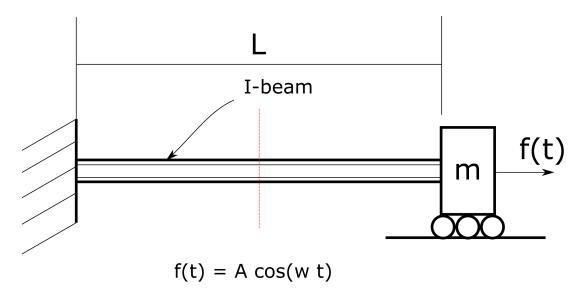


Figure 1

A mass of  $m=100 \mathrm{kg}$  is fixed to a bar with the following properties: Young's modulus E=70 GPa, Length of the bar  $L=2 \mathrm{m}$ , Area moment of inertia  $I=1290 \cdot 10^{-6}$  m<sup>4</sup>, Cross-sectional area  $A=0.0198 \mathrm{m}^2$ . Use an undamped response approximation and assume all transients are 0. The following forcing function is used:  $f(t)=10^5 \cdot \cos(1000 \cdot t)$  [N].

- (A) 1.217 MPa
- (B) 2.586 MPa
- (C) 5.050 MPa
- (D) 5.902 MPa
- (E) 150.35 MPa
- (F) None of the above
- (d) A concern is that the component in Figure 2 will buckle under the compressive loading. The component has the following dimensions: D = 20 mm and d = 10 mm. Which one of these estimates would be a reasonable **conservative** moment of inertia estimate for the critical buckling load calculation. [4 marks]
  - (A)  $490.9 \text{ mm}^4$
  - (B)  $685.2 \text{ mm}^4$
  - (C) 833.3 mm<sup>4</sup>
  - (D) 7854.0 mm<sup>4</sup>
  - (E) 13333.3 mm<sup>4</sup>
  - (F) 125663.7 mm<sup>4</sup>

Choose the answer that is closest to your estimate.

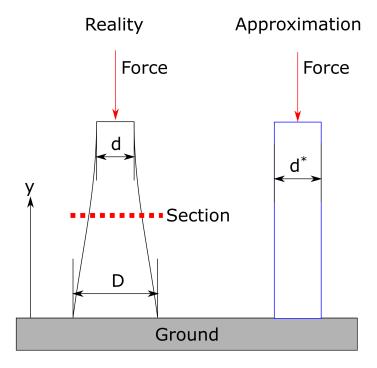


Figure 2: The cross-section of the column is circular for all y. The diameters at the ends are given by  $D=20 \mathrm{mm}$  and  $d=10 \mathrm{mm}$  respectively. Reality refers to the actual column and Approximation refers to a simplification of the column to get an order-of-magnitude estimate of the critical buckling load and buckling safety factor.  $d^*$  is used to estimate the moment of inertia.

- (e) A structural steel element is subjected to a fluctuating stress with a midrange stress of 200 MPa and an amplitude of stress of 150 MPa. The ultimate stress is 600 MPa and the modified endurance limit is 180 MPa. Which of the following options match the fatigue life the **closest**? Use the modified Goodman's fatigue theory. [4 marks]
  - (A) 0 cycles
  - (B) 12968 cycles
  - (C) 232670 cycles
  - (D) 502339 cycles
  - (E) 3291630 cycles
  - (F) Infinite cycles

# Question 2 [20 marks]

All answers should be inserted in the excel spreadsheet ANSWER\_BOOK\_Q2.xlsx. Only the answers in the answer cells will be marked. Do not insert new rows or columns. Do not insert new sheets or change the sheet name. You must however submit your jupyter notebook that was used to perform the calculations.

Consider the beam in Figure 3.

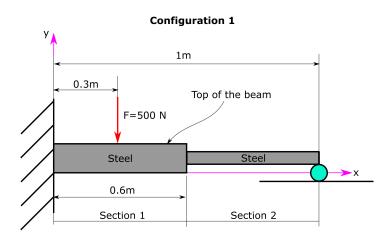


Figure 3

The beam has the following properties:

- Young's modulus: E = 200 GPa.
- Density:  $\rho = 7800 \text{ kg/m}^3$
- Cross-sectional area of section 1: 314.1592653589793 mm<sup>2</sup>
- Cross-sectional area of section 2: 1256.6370614359173 mm<sup>2</sup>
- Area moment of inertia of section 1: 7853.981633974483 mm<sup>4</sup>
- Area moment of inertia of section 2: 125663.70614359173 mm<sup>4</sup>

Please insert the values in the given spreadsheet with an accuracy of at least two decimals.

All loads are applied slowly to the beam.

### Configuration 1: (Figure 3)

- (a) What is the first natural frequency of the structure in Hz? [4 marks]
- (b) What is the vertical (y-direction) displacement of the beam at x = 0.6m in mm? Do not discard the sign. Please include a positive number if the deflection is in the positive y-direction and include a negative number if the deflection is the negative y-direction. [3 marks]

- (c) What is the value of the maximum absolute bending moment in the structure in N.m, i.e. what is the maximum value of the absolute bending moment |M(x)| for all x values? [3 marks]
- (d) At a specific x position, the circular beam has a diameter of 20mm and is exposed to a bending moment of M = -20 N.m. What is the normal stress xx on the top<sup>1</sup> of the beam at that x-location? Please insert the answer in MPa and indicate a tensile stress as a positive number and indicate a compressive stress as a negative number. [3 marks]

# Addition of a vertical roller support to the structure

Another vertical roller support needs to be added to one of the following nodes of the beam:

- Node 5
- Node 6
- Node 7
- Node 8
- Node 9

so that the second natural frequency is minimized.

- (e) At which node should the support be added to minimize the second natural frequency?
  - (A) Node 5
  - (B) Node 6
  - (C) Node 7
  - (D) Node 8
  - (E) Node 9

Only insert A, B, C, D or E in your answer book. [3 marks]

(f) What is the minimized natural frequency? Insert the value in Hz in your answer book. [3 marks]

#### Material change

(g) For the beam shown in load configuration 1 in Figure 3, what is the third natural frequency if the material is changed to a material with a density of  $3000 \text{kg/m}^3$  and a Young's modulus of E = 150 GPa? Return the answer in Hz correct up to at least 2 decimals. [3 marks]

<sup>&</sup>lt;sup>1</sup>Please refer to Figure 3 for a reference of "top of the beam".

# Configuration 2: (Figure 4)

Consider the second load configuration in Figure 4. The beam has the following properties as the first part of the question with only a moment added to the tip of the beam:

- Young's modulus: E = 200 GPa.
- Density:  $\rho = 7800 \text{ kg/m}^3$
- Cross-sectional area of section 1: 314.1592653589793 mm<sup>2</sup>
- $\bullet$  Cross-sectional area of section 2: 1256.6370614359173  $\mathrm{mm}^2$
- Area moment of inertia of section 1: 7853.981633974483 mm<sup>4</sup>
- Area moment of inertia of section 2: 125663.70614359173 mm<sup>4</sup>

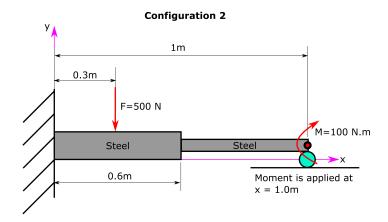


Figure 4: A moment is added to the tip of the beam

(h) What is the vertical reaction force of the beam at the fixed support, i.e. x = 0.0m? Include the answer in Newtons, with a positive value indicating that it is in the positive y-direction and a negative value indicating that it is in the negative y-direction. [3 marks]:

# Question 3 [20 marks]

You are supplied an ANSWER\_BOOK\_Q3. docx answer book. This allows you to insert figures, insert equations and explain your calculations. However, you are welcome to submit scanned documents in PDF format. It is however important that the generated file(s) are submitted as a single document and has the name ANSWER\_BOOK\_Q3.

A part needs to be designed that will be used under dynamic loading conditions. The CAD of the part is shown in Figure 5 with the coordinate system used in all subsequent discussions also shown.

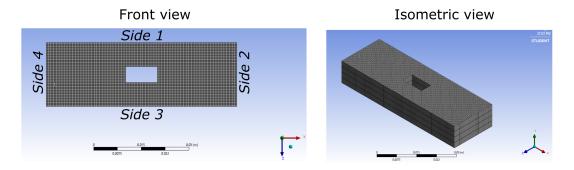


Figure 5: Two views of the part in ANSYS.

A modal analysis was performed. The results of the modal analysis is included in Figure 7 on page 11.

The results of one of the identified modes are shown in Figure 6. A supplementary video mode\_example.mp4 is included if you need to see a video of the response.

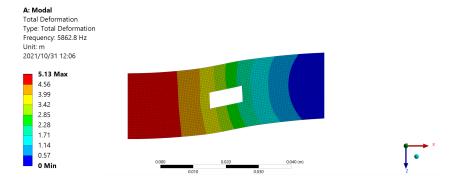


Figure 6: The modal analysis results of a mode.

## Questions

- (a) Which mode is shown in Figure 6? [3 marks]
- (b) Which mode participates the most in the y-direction and what is its participation factor? [3 marks]
- (c) The structure will be excited in the z-direction at a frequency of 4000 Hz. Is a quasi-static analysis appropriate? Briefly motivate. [3 marks]
- (d) Using the modal analysis results in Figure 6, what are the displacement boundary conditions that are applied to the four external sides of the part in the xz-plane? Include a drawing where you show the boundary conditions and show the prescribed displacements. If there are no displacements applied to a side, then please explicitly state so. [5 marks]
- (e) Another analysis is performed, with the results shown in Figure 8 on page 12. This is referred to as Case 2. The boundary conditions for Case 1 and 2 are different. What are the prescribed displacements that are used in the Case 2's model and briefly motivate? [3 marks]
- (f) Do you agree with the following statement: "The mode shape can be used to determine the displacement of the part when it is excited at resonance"? Briefly motivate your answer. [3 marks]

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	PARTIE	TIAITON TACTOR				C	
						CUMULATIVE	RATIO EFF.MASS
MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	MASS FRACTION	TO TOTAL MASS
1	5862.84	0.17057E-03	0.0000	0.000000	0.00000	0.00000	0.00000
2	12671.4	0.78918E-04	0.0000	0.000000	0.00000	0.00000	0.00000
3	20504.1	0.48771E-04	0.0000	0.000000	0.00000	0.00000	0.00000
4	25057.5	0.39908E-04	0.0000	0.000000	0.00000	0.00000	0.00000
5	29193.7	0.34254E-04	0.0000	0.000000	0.00000	0.00000	0.00000
6	37876.2	0.26402E-04	0.0000	0.000000	0.00000	0.00000	0.00000
7	42902.4	0.23309E-04	0.0000	0.000000	0.00000	0.00000	0.00000
8	43993.1	0.22731E-04	0.26746	1.000000	0.715323E-01	1.00000	0.792382
9	50181.9	0.19927E-04	0.0000	0.000000	0.00000	1.00000	0.00000
10	60837.8	0.16437E-04	0.0000	0.000000	0.00000	1.00000	0.00000
sum					0.715323E-01		0.792382
	****						
	***** PARTIC	.IPATION FACTOR	CALCULATION ***	*** Y DIRE	CTION	CUMULATIVE	RATIO EFF.MASS
MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	MASS FRACTION	TO TOTAL MASS
1	5862.84	0.17057E-03	0.0000	0.000000	0.00000	0.00000	0.00000
2	12671.4	0.78918E-04	0.24996	1.000000	0.624818E-01	0.813578	0.692127
3	20504.1	0.48771E-04	0.0000	0.000000	0.00000	0.813578	0.00000
4	25057.5	0.39908E-04					
5			0.0000	0.000000	0.00000	0.813578	0.00000
_	29193.7	0.34254E-04	0.37640E-03	0.001506	0.141678E-06	0.813580	0.156940E-05
6	37876.2	0.26402E-04	0.0000	0.000000	0.00000	0.813580	0.00000
7	42902.4	0.23309E-04	0.0000	0.000000	0.00000	0.813580	0.00000
8	43993.1	0.22731E-04	0.0000	0.000000	0.00000	0.813580	0.00000
9	50181.9	0.19927E-04		0.478682	0.143168E-01	1.00000	0.158591
10	60837.8	0.16437E-04	0.0000	0.000000	0.00000	1.00000	0.00000
sum					0.767988E-01		0.850720
	**** DARTI	:TD4TTON 545TOD	CALCULATION ***	*** 7 0705	CTTON.		
	***** PARTIC	IPATION FACTOR	CALCULATION ***	*** Z DIRE	CTION	CUMULATIVE	RATIO EFF.MASS
MODE		CIPATION FACTOR					
MODE 1	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	MASS FRACTION	TO TOTAL MASS
1	FREQUENCY 5862.84	PERIOD 0.17057E-03	PARTIC.FACTOR 0.24765	RATIO 1.000000	EFFECTIVE MASS 0.613308E-01	MASS FRACTION 0.772435	TO TOTAL MASS 0.679378
1 2	FREQUENCY 5862.84 12671.4	PERIOD 0.17057E-03 0.78918E-04	PARTIC.FACTOR 0.24765 0.0000	RATIO 1.000000 0.000000	EFFECTIVE MASS 0.613308E-01 0.00000	MASS FRACTION 0.772435 0.772435	TO TOTAL MASS 0.679378 0.00000
1 2 3	FREQUENCY 5862.84 12671.4 20504.1	PERIOD 0.17057E-03 0.78918E-04 0.48771E-04	PARTIC.FACTOR 0.24765 0.0000 0.0000	RATIO 1.000000 0.000000 0.000000	EFFECTIVE MASS 0.613308E-01 0.00000 0.00000	MASS FRACTION 0.772435 0.772435 0.772435	TO TOTAL MASS 0.679378 0.00000 0.00000
1 2 3 4	FREQUENCY 5862.84 12671.4 20504.1 25057.5	PERIOD 0.17057E-03 0.78918E-04 0.48771E-04 0.39908E-04	PARTIC.FACTOR 0.24765 0.0000 0.0000 0.12699	RATIO 1.000000 0.000000 0.000000 0.512783	EFFECTIVE MASS 0.613308E-01 0.00000 0.00000 0.161267E-01	MASS FRACTION 0.772435 0.772435 0.772435 0.975543	TO TOTAL MASS 0.679378 0.00000 0.00000 0.178640
1 2 3 4 5	FREQUENCY 5862.84 12671.4 20504.1 25057.5 29193.7	PERIOD 0.17057E-03 0.78918E-04 0.48771E-04 0.39908E-04 0.34254E-04	PARTIC.FACTOR 0.24765 0.0000 0.0000 0.12699 0.0000	RATIO 1.000000 0.000000 0.000000 0.512783 0.000000	EFFECTIVE MASS 0.613308E-01 0.00000 0.00000 0.161267E-01 0.00000	MASS FRACTION 0.772435 0.772435 0.772435 0.772435 0.975543 0.975543	TO TOTAL MASS 0.679378 0.00000 0.00000 0.178640 0.00000
1 2 3 4 5	FREQUENCY 5862.84 12671.4 20504.1 25057.5 29193.7 37876.2	PERIOD 0.17057E-03 0.78918E-04 0.48771E-04 0.39908E-04 0.34254E-04 0.26402E-04	PARTIC.FACTOR 0.24765 0.0000 0.0000 0.12699 0.0000 0.0000	RATIO 1.000000 0.000000 0.000000 0.512783 0.000000	EFFECTIVE MASS 0.613308E-01 0.00000 0.00000 0.161267E-01 0.00000 0.00000	MASS FRACTION 0.772435 0.772435 0.772435 0.975543 0.975543 0.975543	TO TOTAL MASS 0.679378 0.00000 0.00000 0.178640 0.00000 0.00000
1 2 3 4 5 6 7	FREQUENCY 5862.84 12671.4 20504.1 25057.5 29193.7 37876.2 42902.4	PERIOD 0.17057E-03 0.78918E-04 0.48771E-04 0.39908E-04 0.34254E-04 0.26402E-04 0.23309E-04	PARTIC.FACTOR 0.24765 0.0000 0.0000 0.12699 0.0000 0.0000	RATIO 1.000000 0.000000 0.000000 0.512783 0.000000 0.000000 0.177938	EFFECTIVE MASS 0.613308E-01 0.00000 0.00000 0.161267E-01 0.00000 0.00000 0.194184E-02	MASS FRACTION 0.772435 0.772435 0.772435 0.975543 0.975543 1.00000	TO TOTAL MASS 0.679378 0.00000 0.00000 0.178640 0.00000 0.00000 0.215103E-01
1 2 3 4 5 6 7 8	FREQUENCY 5862.84 12671.4 20504.1 25057.5 29193.7 37876.2 42902.4 43993.1	PERIOD 0.17057E-03 0.78918E-04 0.48771E-04 0.39908E-04 0.34254E-04 0.26402E-04 0.23309E-04 0.22731E-04	PARTIC.FACTOR 0.24765 0.0000 0.0000 0.12699 0.0000 0.0000 -0.44066E-01 0.0000	RATIO 1.000000 0.000000 0.000000 0.512783 0.000000 0.000000 0.177938 0.000000	EFFECTIVE MASS 0.613308E-01 0.00000 0.00000 0.161267E-01 0.00000 0.00000 0.194184E-02 0.00000	MASS FRACTION 0.772435 0.772435 0.772435 0.975543 0.975543 0.975543 1.00000 1.00000	TO TOTAL MASS 0.679378 0.00000 0.00000 0.178640 0.00000 0.80000 0.215103E-01 0.00000
1 2 3 4 5 6 7 8	FREQUENCY 5862.84 12671.4 20504.1 25057.5 29193.7 37876.2 42902.4 43993.1 50181.9	PERIOD 0.17057E-03 0.78918E-04 0.48771E-04 0.39908E-04 0.34254E-04 0.26402E-04 0.23309E-04 0.19927E-04	PARTIC.FACTOR 0.24765 0.0000 0.0000 0.12699 0.0000 0.0000 -0.44066E-01 0.0000 0.0000	RATIO 1.000000 0.000000 0.000000 0.512783 0.000000 0.177938 0.000000 0.000000	EFFECTIVE MASS 0.613308E-01 0.00000 0.00000 0.161267E-01 0.00000 0.00000 0.194184E-02 0.00000 0.00000	MASS FRACTION 0.772435 0.772435 0.772435 0.975543 0.975543 1.00000 1.00000	TO TOTAL MASS 0.679378 0.00000 0.00000 0.178640 0.00000 0.215103E-01 0.00000 0.00000
1 2 3 4 5 6 7 8	FREQUENCY 5862.84 12671.4 20504.1 25057.5 29193.7 37876.2 42902.4 43993.1	PERIOD 0.17057E-03 0.78918E-04 0.48771E-04 0.39908E-04 0.34254E-04 0.26402E-04 0.23309E-04 0.22731E-04	PARTIC.FACTOR 0.24765 0.0000 0.0000 0.12699 0.0000 0.0000 -0.44066E-01 0.0000	RATIO 1.000000 0.000000 0.000000 0.512783 0.000000 0.000000 0.177938 0.000000	EFFECTIVE MASS 0.613308E-01 0.00000 0.00000 0.161267E-01 0.00000 0.00000 0.194184E-02 0.00000	MASS FRACTION 0.772435 0.772435 0.772435 0.975543 0.975543 0.975543 1.00000 1.00000	TO TOTAL MASS 0.679378 0.00000 0.00000 0.178640 0.00000 0.80000 0.215103E-01 0.00000

\*\*\*\*\* PARTICIPATION FACTOR CALCULATION \*\*\*\*\* X DIRECTION

Figure 7: Modal analysis: Case 1

	***** PARTICI	PATION FACTOR	CALCULATION ***	*** X DIRE	ECTION		
						CUMULATIVE	RATIO EFF.MASS
MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	MASS FRACTION	TO TOTAL MASS
1	0.00000	0.0000	0.59673E-02	0.029593	0.356086E-04	0.394446E-03	0.394446E-03
2	0.00000	0.0000	-0.11842	0.587286	0.140245E-01	0.155747	0.155353
3	0.00000	0.0000	0.98384E-01	0.487899	0.967936E-02	0.262968	0.107221
4	0.00000	0.0000	-0.10077	0.499750	0.101553E-01	0.375461	0.112493
5	0.00000	0.0000	-0.12537	0.621742	0.157184E-01	0.549578	0.174117
6	0.365434E-01	27.365	0.20165	1.000000	0.406619E-01	1.00000	0.450422
7	12473.0	0.80173E-04	0.0000	0.000000	0.00000	1.00000	0.00000
8	16916.7	0.59113E-04		0.000000	0.00000	1.00000	0.00000
9	22517.0	0.44411E-04	0.0000	0.000000	0.00000	1.00000	0.00000
10	31293.8	0.31955E-04	0.0000	0.000000	0.00000	1.00000	0.00000
sum					0.902750E-01		1.00000
	***** PARTICI	PATION FACTOR	CALCULATION ***	*** Y DIRE	ECTION		
						CUMULATIVE	RATIO EFF.MASS
MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	MASS FRACTION	TO TOTAL MASS
1	0.00000	0.0000	0.15971	0.828545	0.255088E-01	0.282567	0.282567
2	0.00000	0.0000	0.90988E-01	0.472012	0.827873E-02	0.374273	0.917057E-01
3	0.00000	0.0000	-0.12710E-01	0.065933	0.161532E-03	0.376062	0.178933E-02
4	0.00000	0.0000	0.19277	1.000000	0.371584E-01	0.787676	0.411613
5	0.00000	0.0000	-0.11170	0.579465	0.124770E-01	0.925887	0.138211
6	0.365434E-01	27.365	0.81796E-01	0.424328	0.669052E-02	1.00000	0.741127E-01
7	12473.0	0.80173E-04	0.0000	0.000000	0.00000	1.00000	0.00000
8	16916.7	0.59113E-04	0.0000	0.000000	0.00000	1.00000	0.00000
9	22517.0	0.44411E-04	0.0000	0.000000	0.00000	1.00000	0.00000
10	31293.8	0.31955E-04	0.0000	0.000000	0.00000	1.00000	0.00000
sum					0.902750E-01		1.00000
	***** PARTICI	PATION FACTOR	CALCULATION ***	*** Z DIRE	ECTION	C A T T. / C	DATES 555 WASS
MODE	EDECLIENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
	FREQUENCY						
1 2	0.00000 0.00000	0.0000 0.0000	0.21929E-03	0.000924 0.002378	0.480884E-07	0.532688E-06 0.406300E-05	0.532688E-06
3			0.56453E-03		0.318699E-06		0.353031E-05
4	0.00000 0.00000	0.0000 0.0000	0.68040E-01 0.76658E-01	0.286626 0.322931	0.462946E-02 0.587650E-02	0.512858E-01 0.116381	0.512818E-01 0.650956E-01
5				1.000000			
	0.00000	0.0000	0.23738		0.563507E-01	0.740593	0.624211
6	0.365434E-01	27.365	0.15303	0.644652	0.234180E-01	1.00000	0.259407
7	12473.0	0.80173E-04	0.0000	0.000000	0.00000	1.00000	0.00000
8	16916.7	0.59113E-04	0.0000	0.000000	0.00000	1.00000	0.00000
9	22517.0	0.44411E-04	0.0000	0.000000	0.00000	1.00000	0.00000
10	31293.8	0.31955E-04	0.0000	0.000000	0.00000	1.00000	0.00000
sum					0.902750E-01		1.00000

Figure 8: Modal analysis: Case 2

# Question 4 [20 marks]

You are supplied an ANSWER\_BOOK\_Q4. docx answer book. This allows you to insert figures, insert equations and explain your calculations. However, you are welcome to submit scanned documents in PDF format. It is important that the generated file(s) are submitted as a single document and has the name ANSWER\_BOOK\_Q4. Please submit the jupyter notebook files that were used to perform the simulations.

Consider the following truss structure in Figure 9:

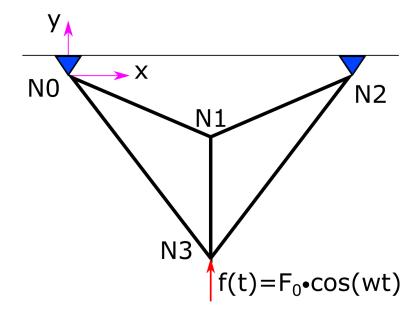


Figure 9: The structure under consideration for Question 4. The dimensions of the nodes are given in the notebook.

Each node is pin-jointed. Each truss element is made from steel with the following properties:

- Young's modulus: E = 200 GPa.
- Yield strength:  $S_y = 300$  MPa.
- Ultimate strength:  $S_{ut} = 500 \text{ MPa}.$
- Modified endurance limit:  $S_e = 165$  MPa.
- Density of the truss element:  $\rho = 7800 \text{ kg/m}^3$ .
- Cross-sectional area of each element is square with an area h = 0.010m.

The force that is applied to Node 3 has the following form:

$$f(t) = 1 \cdot \cos(\omega \cdot t) \text{ [kN]}$$

where  $\omega$  is the excitation frequency in rad/s.

## Questions

- (a) Perform a modal analysis and use this to determine which of the following excitation frequencies are expected to result in largest displacement amplification when applying the force in Figure 9: [4 marks]:
  - $\omega = 2 \cdot \pi \cdot 18.2 \text{ rad/s}.$
  - $\omega = 2 \cdot \pi \cdot 181.6 \text{ rad/s}.$
  - $\omega = 2 \cdot \pi \cdot 279.2 \text{ rad/s}.$

Briefly motivate.

- (b) If it was required to increase the first natural frequency of the design, show and comment on the effectiveness of the following changes [4 marks]:
  - Decrease the cross-sectional area?
  - Shift Node 3 upwards in the y-direction?
  - Shift Node 3 downwards in the y-direction?
- (c) Assume the particular solution of the equation of motion is an approximate for the steady state solution of the problem for the load case for  $\omega = 2 \cdot \pi \cdot 120 \text{ rad/s}$ . Calculate,
  - the vertical displacement of node  $N_3$ ; and
  - the stress in the element connecting nodes  $N_1$  and  $N_3$ .

at the time t = 0. Discuss/show the process. [5 marks]

- (d) What is a risk of only using the particular solution in the design of a structure? [2 marks]
- (e) What is the overall safety factor of the structure against fatigue failure for an excitation frequency of  $\omega = 2 \cdot \pi \cdot 120 \text{ rad/s}$ ? Show evidence of the calculation process in your script [5 marks]

### End of the paper.