Surnmame and Initials

Student Number

Semester Test 1 MKM321 Full marks [45]

Duration: 90 minutes (90 minutes of writing and 15 minutes to upload and complete the ClickUP test)

Allowable Material: Prescribed Textbook and material available on ClickUP.

Questions 1 - 3

For Questions 2-3, grades are only awarded for the rationale, reasoning, methodology or evidence that backup the answer.

All questions must also be answered on ClickUP which includes uploading this notebook and filling in the multiple choice questions.

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Question 1 [15] - Multiple choice to be answered on ClickUP

1.1 Given the stress matrix (defined for the unit cube x, y and z from -1 to 1 and units MPa),

$$\sigma(x,y,z) = \begin{bmatrix} 2x^2 & -4yx & 2 \\ -4yx & 2y^2 & 0 \\ 2 & 0 & 2z^2 \end{bmatrix}$$

give the traction vector on the x-y surface at $z = 1$ [3]

A) $$

\mathbf{T}(x,y,z)=

$$[\,2x^2 \quad -4yx \quad 2\,]$$

$$

B) $$ \mathbf{T}(x,y)=

$$[\,-4yx \quad 2y^2 \quad 0\,]$$

C)

\mathbf{T}(x,y,z)=

$$[\,2 \quad 0 \quad 2z^2\,]$$

$$

D) $$ \mathbf{T}(x,y)=

$$[\,2 \quad 0 \quad 2\,]$$

$$

E) None/All of the above

In [ ]:

1.2 Which of the following vectors is a unit vector that is normal to the vector $\mathbf{n} = [0.707, \quad 0.707]$ (given up to three significant digits) in two dimensions?: [3]

A) $\mathbf{n} = [0.000, \quad 0.000,]$

B) $\mathbf{n} = [1.000, \quad 0.000,]$

C) $\mathbf{n} = [0.707, \quad 0.707]$

D) $\mathbf{n} = [0.707 \quad -0.707]$

E) None/All of the above

In [ ]:

1.3 Given the displacement field

$$u(x, y, z) = x,$$
$$v(x, y, z) = -0.3y,$$
$$z(x, y, z) = -0.3z,$$

with x defined between 0 and 1, y between -0.5 and 0.5 and z between -0.5 and 0.5. Which description best matches the displacement field? [3]

A) Uniaxial tension

B) Bending

C) Rigid body translation

D) Rigid body rotation

In [ ]:

1.4 Consider the following three force vectors (in units N) acting through the xy, xz and yz surfaces of a cube with side lengths 1:

$$f_{xy} = [\,1, 0, 0\,],$$
$$f_{xz} = [\,0, 2, 0\,],$$
$$f_{yz} = [\,0, 1, 3\,].$$

Is the state of stress in equilibrium at this point? [3]

True / False

In [ ]:

1.5 Consider the following three force vectors (in units N) acting through the xy, xz and yz surfaces of a cube with side lengths 1:

$$f_{xy} = [\,1, 0, 0\,],$$
$$f_{xz} = [\,0, 2, 0\,],$$
$$f_{yz} = [\,0, 1, 3\,].$$

Give the associated stress tensor: [3]

A) $$

\mathbf{\sigma}(x,y,z)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

B)

\mathbf{\sigma}(x,y,z)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

C)

\mathbf{\sigma}(x,y,z)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

D)

\mathbf{\sigma}(x,y,z)

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & -3 \end{bmatrix}$$

$$

E) None of the above.

In [ ]:

Question 2 [Full Marks 15]

Consider the displacement field (x-displacement field u(x,y) and y-displacement field v(x,y)) given in Python code below of a structure in the xy-plane:

The material has a Young's modulus of E and Poisson's ratio of $\nu = 0$ (pronounced nu). The cross sectional shape (zy plane) of the structure is rectangular with width 1 (m) and height $2c$. The beam has distributed loads T (N/m^2) and moments M (Nm) applied.

Geometry is defined over a rectangular domain of x (-L to L) and y (-c to c) and z between (-0.5 to 0.5). Assume $2 << L$ and $2 << c$

2.1. Interpret the displacement field at $x = 0$ and $x = L$? [5]

In [2]:
```
import sympy as sp
import numpy as np
from IPython.display import display, Math

x,y,z,E,nu,c,L,I,T,M = sp.symbols('x,y,z,E,nu,c,L,I,T,M')

u = -3/2*x*(M*y - 2/3*T*c**3)/(E*c**3)
v = 3/4*M*(L**2 + x**2 + 2*y**2)/(E*c**3)

print('u:');display(u.factor())
print('v:');display(v.factor())

print('------------------------------------------------------------------------')
```

u:

$$-\frac{1.5x\left(1.0My - 0.666666666666667Tc^3\right)}{Ec^3}$$

v:

$$\frac{0.75M\left(L^2 + x^2 + 2y^2\right)}{Ec^3}$$

2.2 In the abscence of body forces is the stress in equilibrium? [5]

In [3]:
```
# Given are two constitutive matrices, select an appropriate one for the problem
C1 = E/(1-nu**2)*sp.Matrix([[1,nu,0],[nu,1,0],[0,0,(1-nu)/2]]);
print('C1:');display(C1)
C2 = E/((1+nu)*(1-2*nu))*sp.Matrix([[1-nu,nu,0],[nu,1-nu,0],[0,0,(1-2*nu)/2]]);
print('C2:');display(C2)

print('-----------------------------')
print('Start of your answer')
print('-----------------------------')
```

C1:

$$\begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E\left(\frac{1}{2}-\frac{\nu}{2}\right)}{1-\nu^2} \end{bmatrix}$$

C2:

$$\begin{bmatrix} \frac{E(1-\nu)}{(1-2\nu)(\nu+1)} & \frac{E\nu}{(1-2\nu)(\nu+1)} & 0 \\ \frac{E\nu}{(1-2\nu)(\nu+1)} & \frac{E(1-\nu)}{(1-2\nu)(\nu+1)} & 0 \\ 0 & 0 & \frac{E\left(\frac{1}{2}-\nu\right)}{(1-2\nu)(\nu+1)} \end{bmatrix}$$

```
-----------------------------
Start of your answer
-----------------------------
```

In [1]:
```
print('-----------------------------')
print('Start of your answer')
print('-----------------------------')
```

```
-----------------------------
Start of your answer
-----------------------------
```

2.3. What are the tractions on the left, right, top and bottom edges? [5].

In [2]:
```
print('-----------------------------')
print('Start of your answer')
print('-----------------------------')
```

```
-----------------------------
Start of your answer
-----------------------------
```

Question 3 [Full Marks 15]

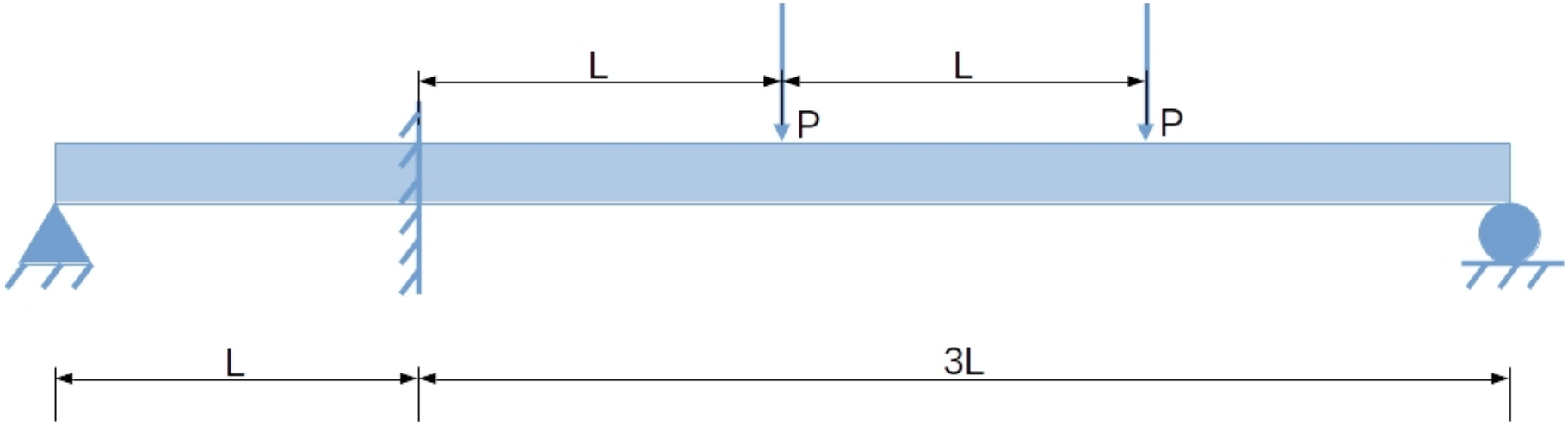
Given below are claimed solutions to the stated problem. In your motivations include qualitative and quantitative motivations.

In this section only state motivations that you are certain of. Wrong motivations will be graded negatively. No beam deflection equation tables are required when answering this question. All observations and quantifications must be clearly motivated.

Consider the problem depicted below. L = 0.5m. The beam is made of steel E = 210 GPa, with rectangular cross sectional area of height 200mm and width 150mm. $P = 2$kN.

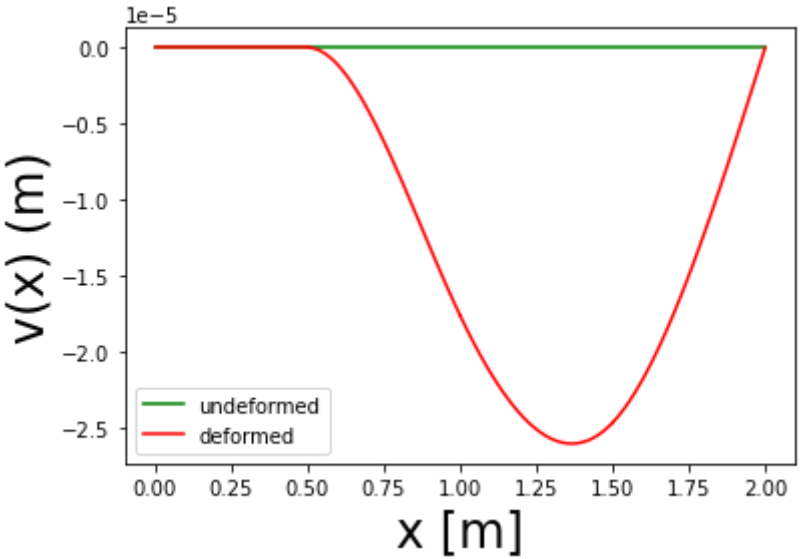
In [3]:
```
from IPython.display import Image
Image('question3_2021.jpg',embed=True)
```

Out[3]:



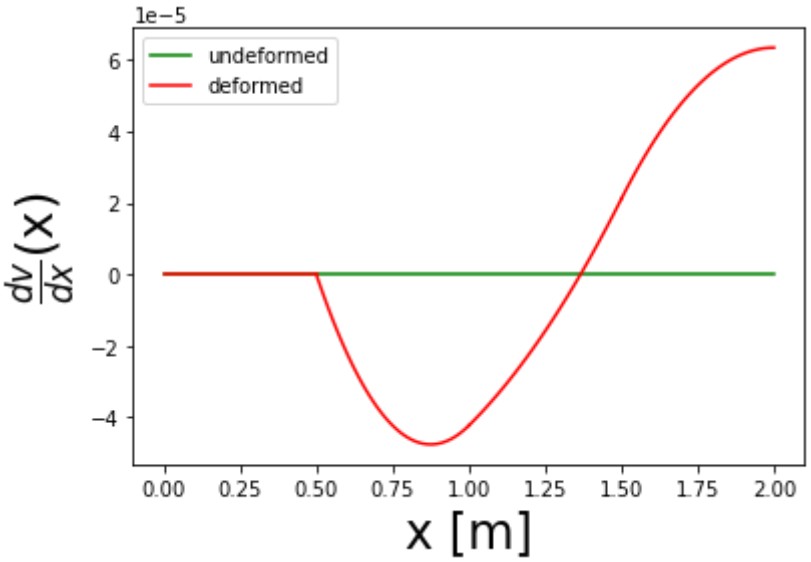
In [11]:
```
from IPython.display import Image
Image('q31.png',embed=True)
```

Out[11]:



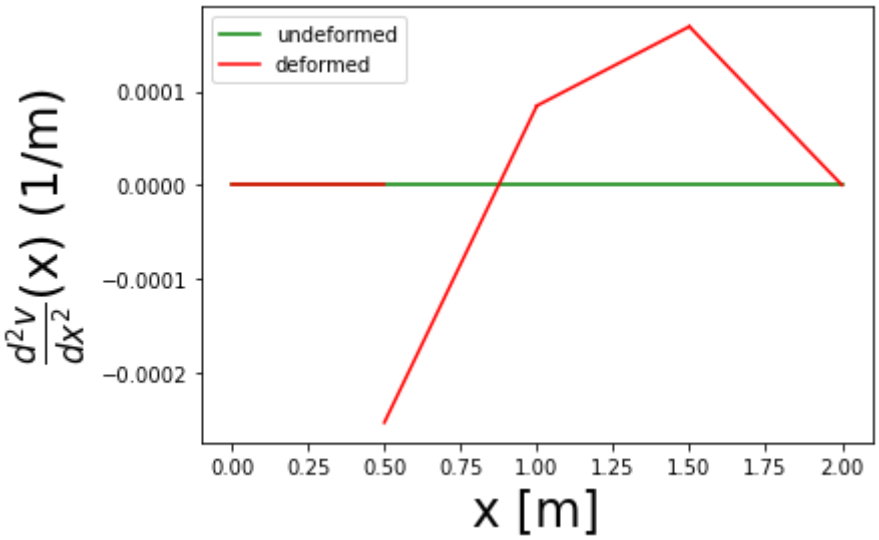
In [12]:
```
from IPython.display import Image
Image('q32.png',embed=True)
```

Out[12]:



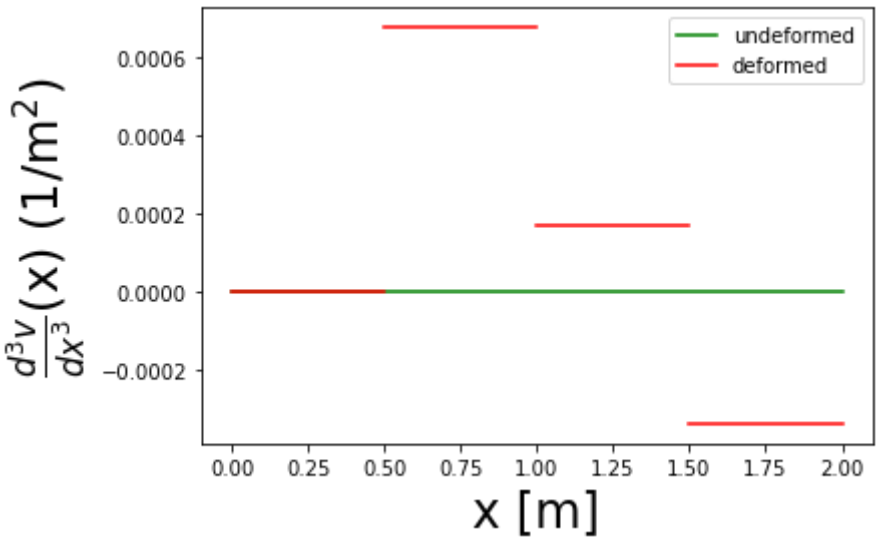
In [13]:
```
from IPython.display import Image
Image('q33.png',embed=True)
```

Out[13]:



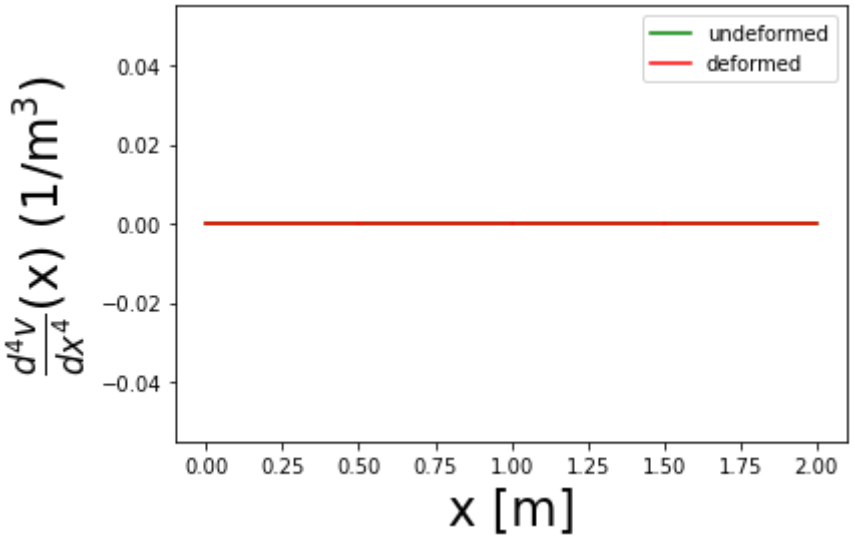
In [14]:
```
from IPython.display import Image
Image('q34.png',embed=True)
```

Out[14]:



In [15]:
```
from IPython.display import Image
Image('q35.png',embed=True)
```

Out[15]:



3.1. Critically evaluate the given vertical displacement field i.e. highlight all aspects that you deem consistent and all aspects you deem inconsistent w.r.t. the stated problem [3].

Example: Consistent: displacement is downwards as expected [1]

------ ANSWER HERE (Double Click This Cell) -------

In [ ]:

3.2 Critically evaluate the given first derivative field $\frac{dv}{dx}$ i.e. highlight all aspects that you deem consistent and all aspects you deem inconsistent w.r.t. the stated problem. [3]

------ ANSWER HERE (Double Click This Cell) -------

In [ ]:

3.3 Critically evaluate the given second derivative field $\frac{d^2v}{dx^2}$ i.e. highlight aspects that are consistent and inconsistent w.r.t. the stated problem. [3]

------ ANSWER HERE (Double Click This Cell) -------

In [ ]:

3.4. Critically evaluate the given third derivative field $\frac{d^3v}{dx^3}$ i.e. highlight aspects that are consistent and inconsistent w.r.t. the stated problem. Include a quantitative evaluation as well, i.e. are the numbers correct? [3]

------ ANSWER HERE (Double Click This Cell) -------

In [ ]:

3.5. Critically evaluate the given fourth derivative field $\frac{d^4v}{dx^4}$ i.e. highlight aspects that are consistent and inconsistent w.r.t. the stated problem. Critically interpret the implication of the provided result? [3]

------ ANSWER HERE (Double Click This Cell) -------

In [ ]: