Department of Mechanical and Aeronautical Engineering

Simulation-based design MOW 323

Exercises - Dynamic loading conditions

October 22, 2021

Shigley's:

- **Chapter 3** (Loads and stress)
- **Chapter 4** (Deflection and stiffness)
- **Chapter 5** (Failure modes for static loading)
- **Chapter 6** (Failure modes for dynamic loading)
- **Chapter 19** (Finite element analysis)

1 Static analysis

Consider the following beam:

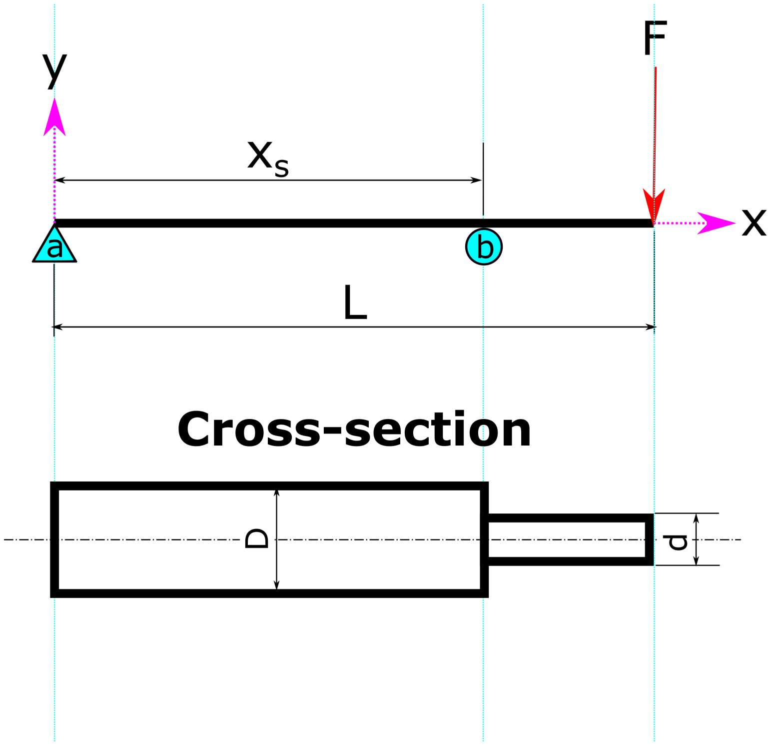

Figure 1: Problem 1

The beam has the following properties:

- Young's modulus: 200 GPa.
- Length of the beam: 2 m.
- Cross-sectional area: Circular.
- Diameters: $D = 10$ mm, $d = 5$ mm
- Density: 7800 kg/m^3
- $F = 50$N.
- $x_s = 0.75 \cdot L$

Answer the following questions:

(a) Calculate the area moment of inertia of the two sections of the beam.

(b) Prepare the following sanity checks:

  (i) Calculate the reaction forces.
  (ii) Calculate the bending moment diagram.
  (iii) Calculate the normal stress xx at the top of the beam as a function of x.
  (iv) Estimate the displacement at $x = L$ by assuming the beam has a uniform thickness equal to D. If you were concerned about large displacements, would this approximation be conservative or not?

(c) Implement the model in `engmod`:

  (i) Calculate the reaction forces.
  (ii) Calculate the bending moment diagram.
  (iii) Calculate the normal stress xx at the top of the beam as a function of x.
  (iv) Calculate the displacement at $x = L$.
  (v) Use the displacement plot to read the estimate of the displacement at $x = 0.3 \cdot L$. Hint: Use the `%matplotlib notebook` magic, and use the cursor.
  (vi) Verify the correctness of the results.

(d) The beam needs to be designed to adhere to the following requirements:

  (i) The safety factor against yielding is at least 2. The yield strength is 300 MPa.
  (ii) The magnitude of the vertical displacement at $x = L$ is less than 1mm.

Parametrise the design so that:

$$d = 0.5 \cdot D \tag{1}$$

This means you only have one unknown D (or d).

Specify a feasible D and d.

2 Dynamic analysis

Consider the following structure:

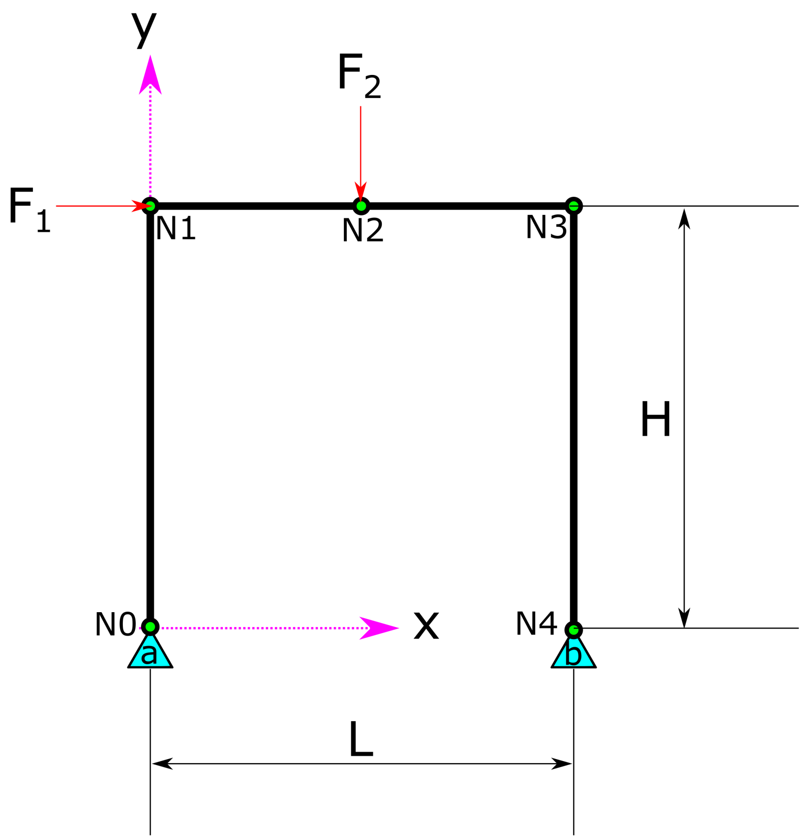

Figure 2: Geometry layout

- Material: Steel.
- Young's modulus: 200 GPa.
- Density: 7800 kg/m^3.
- Yield strength: 300 MPa.
- $H = 2$m.
- $L = 1$m.
- Cross-sectional area: Square with a side length of 100mm, i.e. the cross-sectional area is 100^2mm^2

(a) Perform a modal analysis and plot the first five mode shapes. Reflect on the symmetry of the mode shapes and the order of the modes.

  - Which excitation frequency do you expect is most dangerous for F_1?
  - Which excitation frequency do you expect is most dangerous for F_2?

(b) Apply a static load to node 2 of 1000N in the same direction as the figure. Assume $F_1 = 0$. Calculate:

- The vertical displacement at node 2.
- The stiffness for a vertical force at node 2 and a vertical displacement measured at node 2.
- Plot bending moment in the frame.
- Plot the stress in the frame due to bending only. Since the geometry is constant, you can easily parametrise the stress as:

$$\sigma(M) = M \cdot \gamma \tag{2}$$

  where the constant $\gamma = y/I$.
- Plot the normal stress in the frame in element 0.

(c) Perform a dynamic analysis by assuming the damping is zero and assuming that the particular solution is equal to the steady-state solution. Calculate the (i) largest absolute bending moment in the structure and (ii) the vertical displacement at node 2[1] for the following excitation forces:

  (a) $F_2 = 1000 \cdot \cos(\omega \cdot t)$ [N], $\omega = 1$ rad/s.

  (b) $F_2 = 1000 \cdot \cos(\omega \cdot t)$ [N], $\omega = 2 \cdot \pi \cdot 10$ rad/s.

  (c) $F_2 = 1000 \cdot \cos(\omega \cdot t)$ [N], $\omega = 2 \cdot \pi \cdot 50$ rad/s.

  (d) $F_2 = 1000 \cdot \cos(\omega \cdot t)$ [N], $\omega = 2 \cdot \pi \cdot 81$ rad/s.

  (e) $F_2 = 1000 \cdot \cos(\omega \cdot t)$ [N], $\omega = 2 \cdot \pi \cdot 120$ rad/s.

(d) Repeat the same process where the magnitude of F_1 is equal to 50 N and $F_2 = 0$.

(e) Repeat the step (c), but use proportional damping of the form $\boldsymbol{C}_{ff} = 0.1 \cdot \boldsymbol{K}_{ff}$.

[1] You can access this with $U[7]$ where U is the global displacement vector.

3 Compliance minimization

Consider the following beam:

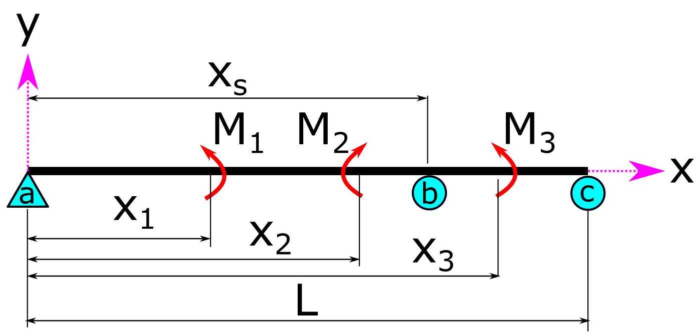

Figure 3: Problem 3

The beam has the following properties:

- Young's modulus: 70 GPa. $\rho = 2700$ kg/m^3 .
- Length of the beam: 1m.
- Cross-sectional area: Rectangular. Cross-sectional dimensions: $y \times z = 0.020 \times 0.010$m^2.
- $M_1 = 50$ N.m is applied to Node 4 with the same direction shown in the figure.
- $M_2 = -100$ N.m is applied to Node 10 with the same direction shown in the figure.
- $M_3 = 100$ N.m is applied to Node 15 with the same direction shown in the figure.

Twenty-one node positions are defined: Node 0 has an x-position of 0m and the last node, Node 20, has a x position of L.

(a) Calculate the compliance of the structure for $x_s = 0.5 \cdot L$.

(b) Calculate the first five natural frequencies of the structure for $x_s = 0.5 \cdot L$.

(c) Perform the following calculation in a for loop: Calculate the compliance, the maximum absolute vertical displacement and the natural frequencies of the structure for the different node positions.

- Plot the compliance vs. the support position.
- Plot the natural frequencies vs. the support position.
- Plot the maximum absolute vertical displacement of the beam vs. the support position.

(d) What should x_s be to minimize the compliance?

(e) What benefits would there be to refine the mesh in this calculation?

4 Design

Consider the structure in Figure 4. Each element is connected via truss joints (i.e. cannot transfer moments). Each element should be made from the same cross-sectional geometry (e.g. the cross-sectional properties are the same). The parts will be machined.

The steel has a yield strength of 580MPa and an ultimate strength of 690MPa. $E = 200$ GPa. $\rho = 7800$ kg/m^3.

The following force is applied to the system:

$$f(t) = A_f \cdot \cos(\omega \cdot t) \text{ [N]} \tag{3}$$

The structure is operated in two load cases:

- $A_f = 3000$, $\omega = 2 \cdot \pi$ rad/s.
- $A_f = 2000$, $\omega = 2 \cdot \pi \cdot 100$ rad/s.

(a) Perform a modal analysis using $h = 0.01$m. Are the modes sensitive to h?

(b) Use $h = 0.01$m, calculate the stresses in the structure for the two load cases.

(c) The system must be designed to adhere to the following specifications: The cross-section must be square, i.e. $A = h^2$. The structure must have a safety factor of five against buckling using Euler's theory, a safety factor of two against yielding and a safety factor of one against fatigue failure using the modified Goodman theory. All elements have a notch radius of 3mm and a stress concentration factor of 3.0 that needs to be incorporated. Assume that the temperature factor is $k_d = 1.0$ and use a reliability of 99%.

Does $h = 0.01$m satisfy the specifications? If it does not, choose a suitable h.

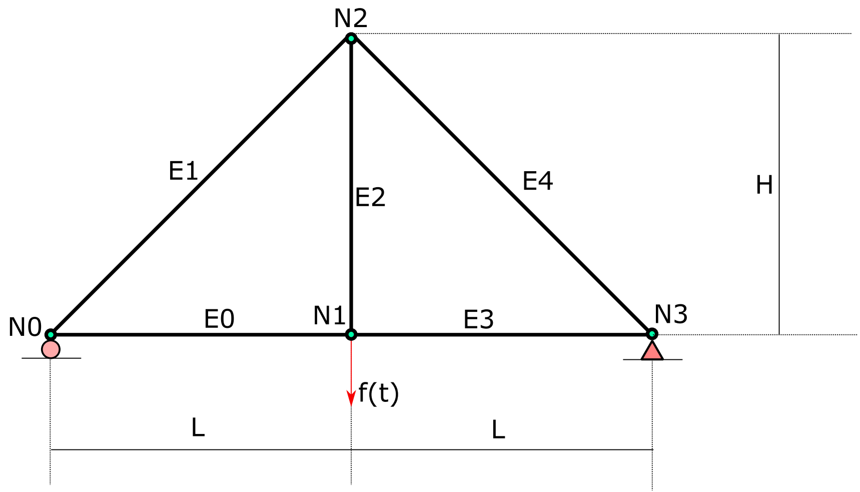

Figure 4: Truss system: $L = 2$m; $H = 2$m