Department of Mechanical and Aeronautical Engineering

Simulation-based design MOW 323

Exercises - Dynamic loading conditions

November 4, 2021

Shigley's:

- **Chapter 3** (Loads and stress)
- **Chapter 4** (Deflection and stiffness)
- **Chapter 5** (Failure modes for static loading)
- **Chapter 6** (Failure modes for dynamic loading)
- **Chapter 19** (Finite element analysis)

1 Static analysis

Consider the following beam:

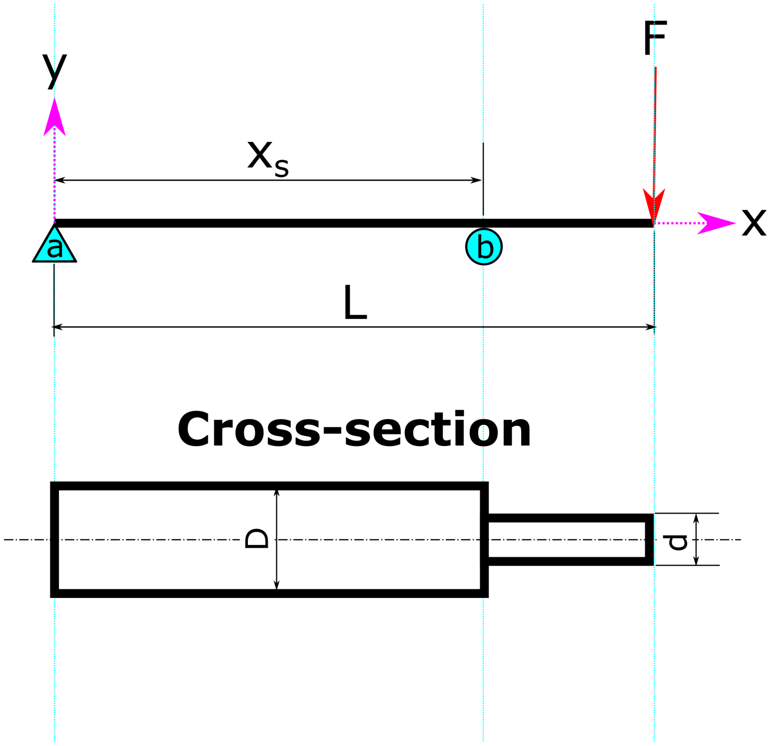

Figure 1: Problem 1

The beam has the following properties:

- Young's modulus: 200 GPa.
- Length of the beam: 2 m.
- Cross-sectional area: Circular.
- Diameters: $D = 10$ mm, $d = 5$ mm
- Density: 7800 kg/m^3
- $F = 50$N.
- $x_s = 0.75 \cdot L$

Answer the following questions:

(a) Calculate the area moment of inertia of the two sections of the beam.
$A0 = 78.54 \times 10^-6m^2, A1 = 19.63 \times 10^-6m^2, I0 = 490.87 \times 10^-12m^4, I1 = 30.68 \times 10^-12m^4$

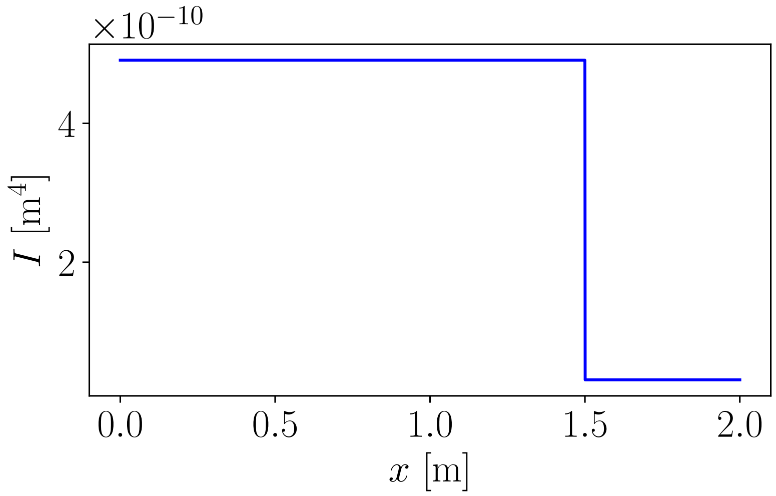

Figure 2: I

(b) Prepare the following sanity checks:

(i) Calculate the reaction forces.
$R_a = -16.667$ N, $R_b = 66.667$ N. (From equilibrium or the table in Shigley's)

(ii) Calculate the bending moment diagram. (From first principles or the table in Shigley's)
3

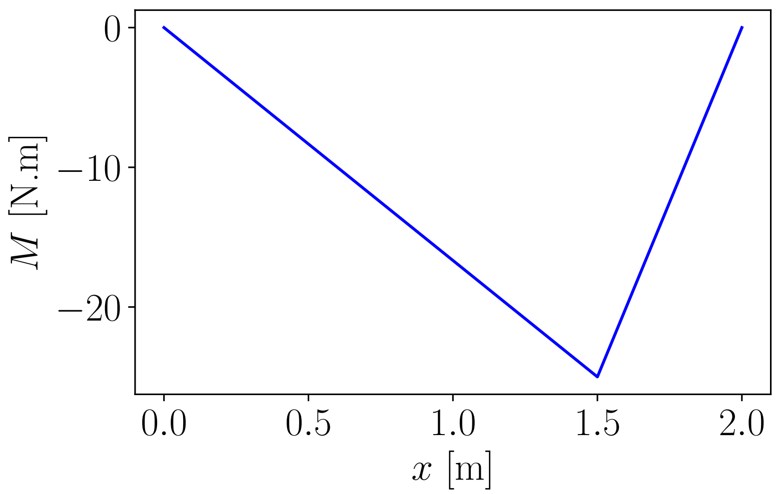

Figure 3: Bending moment

(iii) Calculate the normal stress xx at the top of the beam as a function of x.
Figure 4. Stress is too large. Linear elastic analysis is wrong and we cannot use the results.

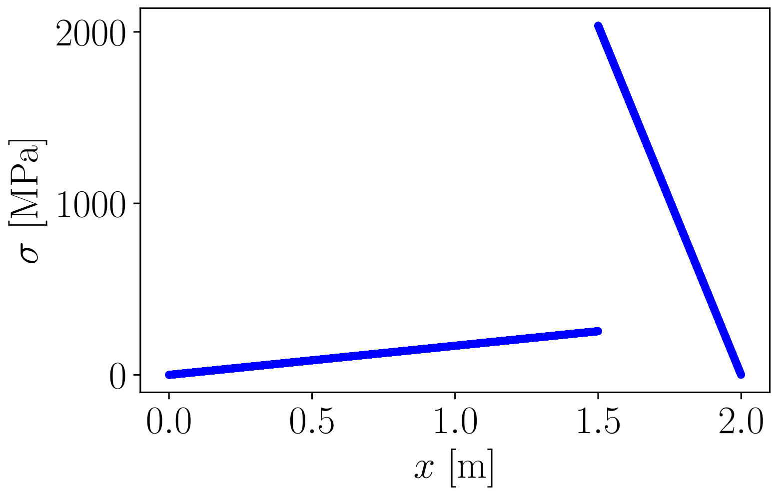

Figure 4: Stress xx in the top of the beam. Stress concentration factors are not incorporated in this calculation.

(iv) Estimate the displacement at $x = L$ by assuming the beam has a uniform thickness equal to D. If you were concerned about large displacements, would this approximation be conservative or not?
Deflection of 84.8826 mm. This is very large. Small displacement approximation is violated.

(c) Implement the model in `engmod`:

(i) Calculate the reaction forces.
$R_a = -16.667$ N, $R_b = 66.667$ N.

(ii) Calculate the bending moment diagram.
See Figure 5.
The exact values of the bending moment can be accessed with the following equation: `dict_moment = fmn.post_get_bending_moment(element_number)`
The bending moment can easily be visualised with `fmn.post_plot_bending_moment()`. However, this only provides the minimum and maximum values of the bending moment.

(iii) Calculate the normal stress xx at the top of the beam as a function of x.
See Figure 6.

(iv) Calculate the displacement at $x = L$.
Displacement is -403.192522mm. This is very large. Linear analysis not appropriate.

(v) Use the displacement plot to read the estimate of the displacement at $x = 0.3 \cdot L$. Hint: Use the `%matplotlib notebook` magic, and use the cursor.

(vi) Verify the correctness of the results.

- Bending moment diagram direction is correct.
- Bending moment values are the same as sanity check. Sanity check will be exact.
- Displacement approximation is far off. However, When calculating the displacement of a beam with uniform thickness of $r_0 = 5$mm, the deflection is $1.3m$. Hence, the predicted displacement is in the expected range.
- Stresses are exactly the same.
- **However, the stresses and displacements are too large. Linear elastic analysis is incorrect and inappropriate.**

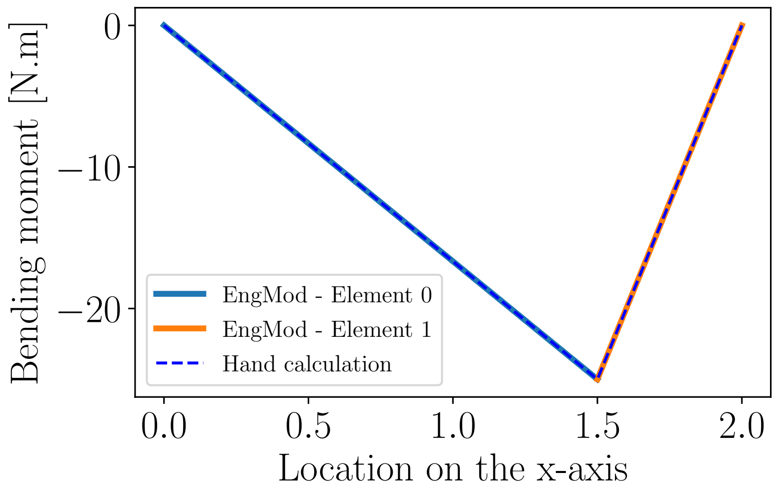

Figure 5: EngMod-Bending Moment

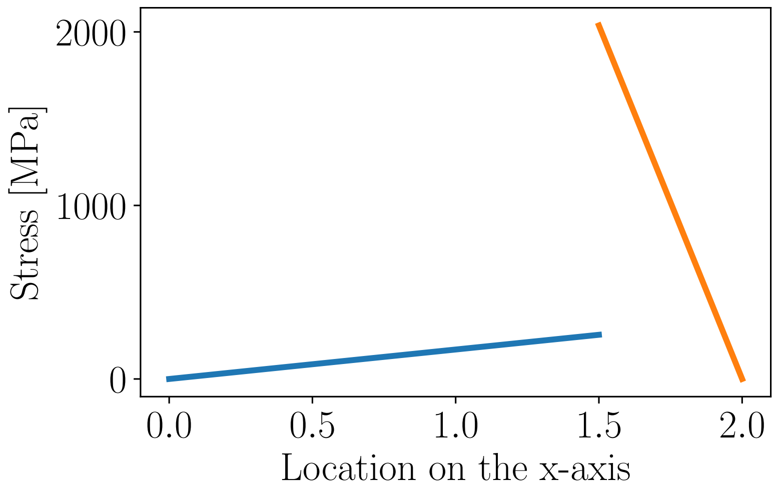

Figure 6: EngMod - Normal stress

(d) The beam needs to be designed to adhere to the following requirements:

(i) The safety factor against yielding is at least 2. The yield strength is 300 MPa.

(ii) The magnitude of the vertical displacement at $x = L$ is less than 1mm.

Parametrise the design so that:

$$d = 0.5 \cdot D \tag{1}$$

This means you only have one unknown D (or d).

Specify a feasible D and d.

There are two constraints for the design: Stress and displacement. One of the constraints will be active for the final design to ensure that the beam is not overdesigned. This equation has a closed form stress calculation. We can solve for r_0 in closed-form

$$\sigma_{xx}(x) = -\frac{M(x)r(x)}{I(x)} \tag{2}$$

If $x \leq 1.5 \cdot m$,

$$\sigma_{xx}(x) = -\frac{M(x)D/2}{\pi/64 \cdot D^4} \tag{3}$$

else if $x > 1.5$:

$$\sigma_{xx}(x) = -\frac{M(x)d/2}{\pi/64 \cdot d^4} = -\frac{M(x)D/4}{\pi/64 \cdot (D \times 0.5)^4} \tag{4}$$

by using $d = 0.5 \cdot D$.

Von-Mises stress (ignoring transverse shear)

$$\sigma_{vm} = |\sigma_{xx}(x)| \tag{5}$$

Therefore, the yield safety factor is given by

$$\text{SF} = \frac{S_y}{\max(\sigma_{vm})} \tag{6}$$

Solving for $SF = 2$:

$$\max(\sigma_{vm}) = \max(|\sigma_{xx}|) = \frac{S_y}{2} \tag{7}$$

Therefore, we need to solve for D so that $\max(|\sigma_{xx}|) = \frac{S_y}{2}$.

The maximum bending moment in both sections is 25 N.m

The stress would be larger in Section 2 (with the smaller diameter) and therefore this would be the limiting case:

$$\frac{S_y}{2} = \frac{25\text{N.m} \cdot D/4}{\pi/64 \cdot (D \times 0.5)^4} \tag{8}$$

Rearranging:

$$D = \sqrt[3]{\frac{2}{S_y} \cdot \frac{25 \cdot \frac{1}{4}}{\pi/64 \cdot \frac{1}{2^4}}} = 23.85867277172399mm \tag{9}$$

This results in: $D = 23.85628643\text{mm}$ and $d = 11.92814321\text{mm}$. You do not have to use iterative methods to perform this.

Displacement: Use EngMod. Iterative/Numerical Method[1].

Iterative method:

[1] It is possible to call engmod in a function and then to solve for r_0 that will result in the maximum displacement at the tip.

- D = 23.85mm, d = 11.925mm, σ_{xx} = 150.16369671429422 [MPa], displ. = -12.461173294774591mm (Not adequate)

  Iterate:
- D = 25.0mm, d = 12.5mm, σ_{xx} = 130.37972938088063 [MPa], displ. = -10.321728575986382mm (Not adequate)
- ...
- D = 45.0mm, d = 22.5mm, σ_{xx} = 22.35592067573401 [MPa], displ. = -0.9832465112012639mm (Good enough)

The displacement is the dominant constraint in the design.

Stress concentration factors? Read Section 5-2 Stress Concentrations in Shigley's. The stress in the beam is much lower than the yield strength (approximately 15 times).

2 Dynamic analysis

Consider the following structure:

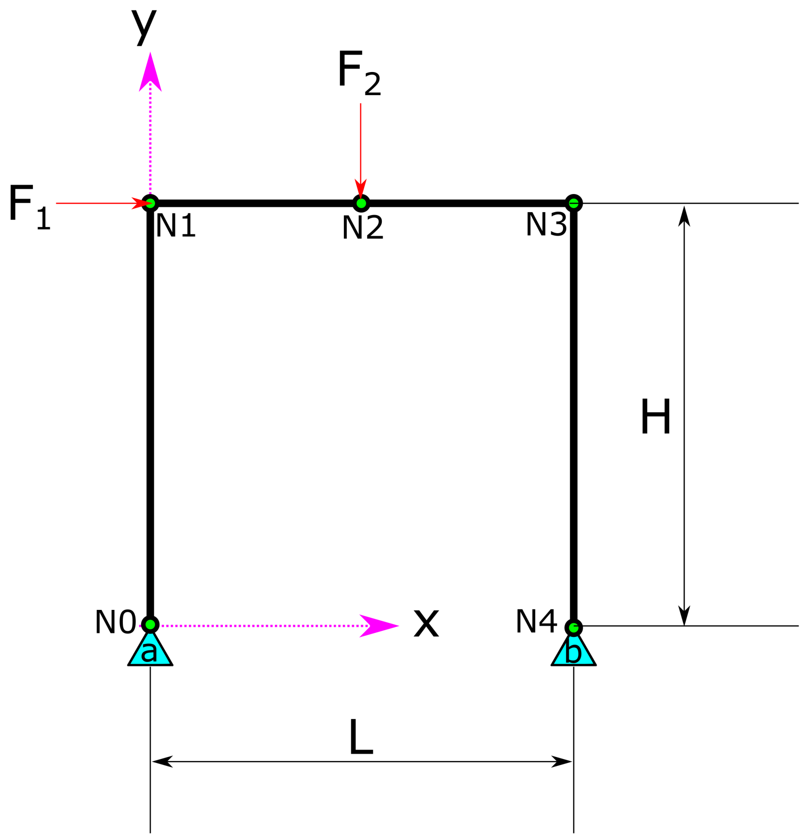

Figure 7: Geometry layout

- Material: Steel.
- Young's modulus: 200 GPa.
- Density: 7800 kg/m^3.
- Yield strength: 300 MPa.
- $H = 2$m.
- $L = 1$m.
- Cross-sectional area: Square with a side length of 100mm, e.g. the cross-sectional area is 100^{2}mm^2

A = 0.010 m^2, I = 8.333333333333334e-06 m^4

(a) Perform a modal analysis and plot the first five mode shapes.

First five natural frequencies: 10.69383737, 82.33087645, 122.86239663, 262.11234228, 472.88418177. Mode shapes not shown.

Reflect on the symmetry of the mode shapes and the order of the modes.

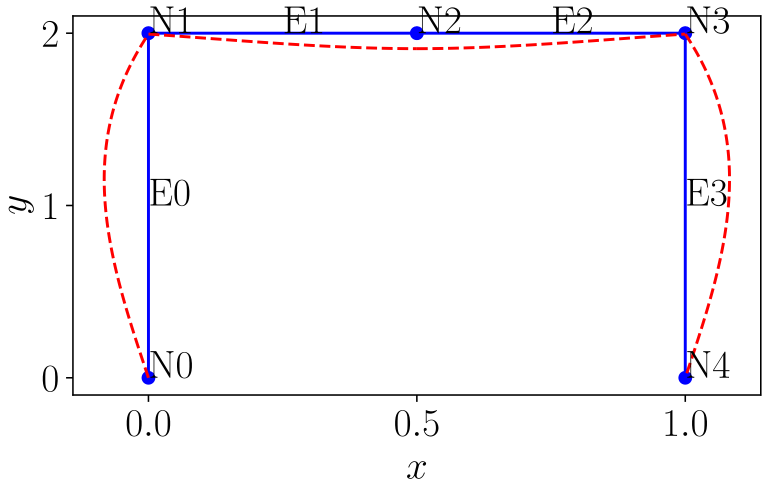

Figure 8: EngMod - Displacement

- Which excitation frequency do you expect is most dangerous for F_1? Expected - 10.693Hz
- Which excitation frequency do you expect is most dangerous for F_2? Expected - 82.33Hz or 262.112Hz. Need to check quantitative data (calculate the stresses in the structure).

(b) Apply a static load to node 2 of 1000N in the same direction as the figure. Assume $F_1 = 0$. Calculate:

- The vertical displacement at node 2.
  -8.982501562360502μm
- The stiffness for a vertical force at node 2 and a vertical displacement measured at node 2.

$$\frac{1000N}{8.982501562360502\mu m} \tag{10}$$

  or fmn.post_get_force(2,1)/fmn.post_get_displacement(2,1) /1E6
  k = 111.32756204466621 MN/m
- Plot bending moment in the frame.
  Use `fmn.post_plot_bending_moment()`.
  See Figure 9.
  - Minimum bending moment: -53.56664583519329 N.m
  - Maximum bending moment: 196.43335416480676 N.m
- Plot the stress in the frame due to bending only. Since the geometry is constant, you can easily parametrise the stress as:

$$\sigma(M) = M \cdot \gamma \tag{11}$$

  where the constant $\gamma = y/I$.

```
function = lambda M: M * h/2 / I /1E6
fmn.post_plot_bending_moment(function=function)
```

- Plot the normal stress in the frame in element 0, i.e. it connects node N_0 and N_1.

$$\sigma_{yy,0,axial} = P/A = 50 \text{ kPa} \tag{12}$$

$$\sigma_{yy,0,bending}(y) = -M \cdot c/I \tag{13}$$

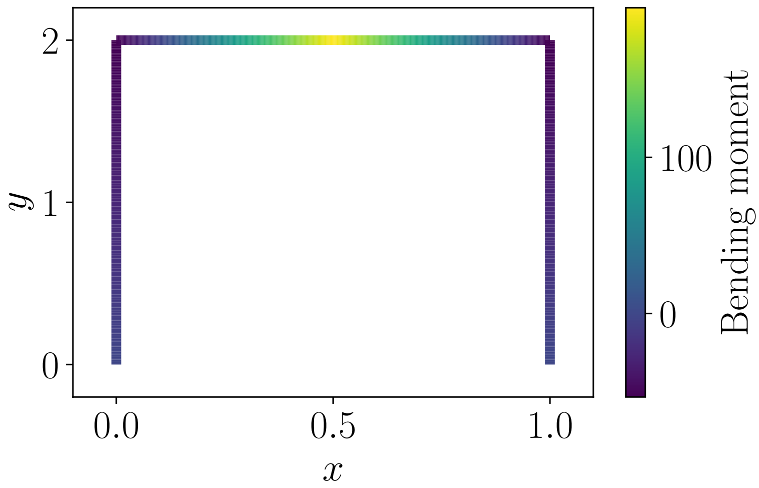

Figure 9: EngMod - Bending moment

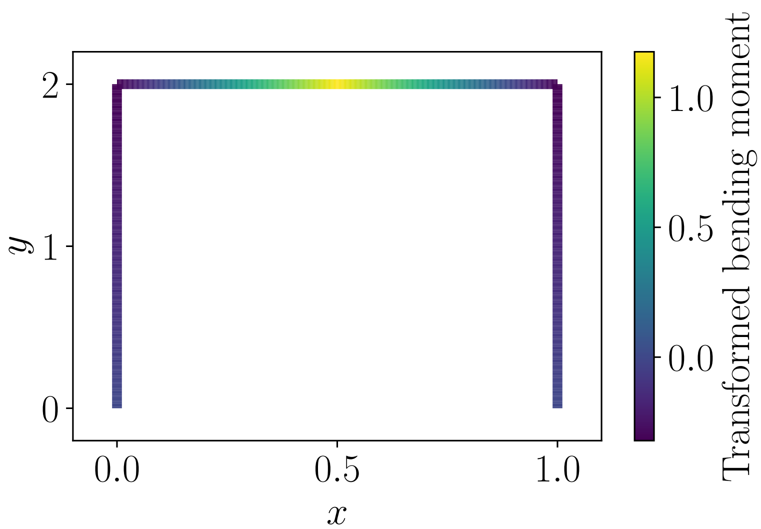

Figure 10: EngMod - Stress in MPa

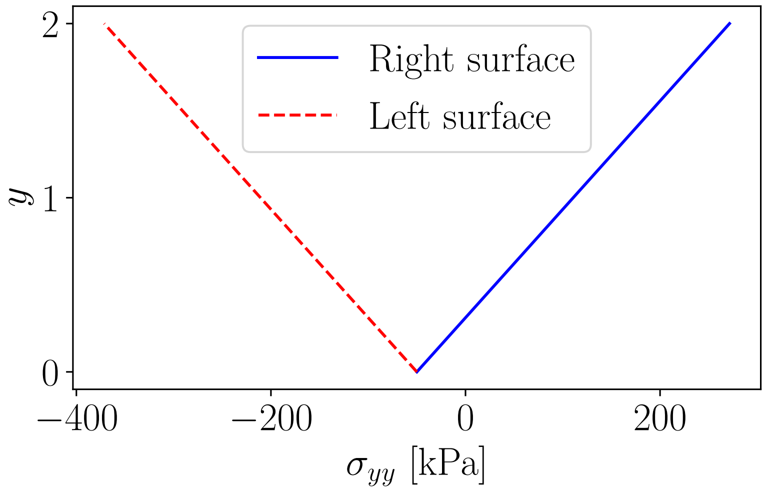

Figure 11: EngMod - Stress

(c) Perform a dynamic analysis by assuming the damping is zero and assuming that the particular solution is equal to the steady-state solution. Calculate the (i) largest absolute bending moment in the structure and (ii) the vertical displacement at node 2[2] for the following excitation forces:

(a) $F_2 = 1000 \cdot \cos(\omega \cdot t)$ [N], $\omega = 1$ rad/s.
(b) $F_2 = 1000 \cdot \cos(\omega \cdot t)$ [N], $\omega = 2 \cdot \pi \cdot 10$ rad/s.
(c) $F_2 = 1000 \cdot \cos(\omega \cdot t)$ [N], $\omega = 2 \cdot \pi \cdot 50$ rad/s.
(d) $F_2 = 1000 \cdot \cos(\omega \cdot t)$ [N], $\omega = 2 \cdot \pi \cdot 81$ rad/s.
(e) $F_2 = 1000 \cdot \cos(\omega \cdot t)$ [N], $\omega = 2 \cdot \pi \cdot 120$ rad/s.

```
Mff,Kff = fmn.dynamic_make_matrices()
_,Fss,_ = fmn.make_force_global(2,1,-1000)
Us = np.linalg.solve(Kff - Mff*w**2,Fss)
Us_global = fmn.dynamic_make_global_displacement(Us)
fmn.post_print_nodal_data(Us_global*1E3)
fmn.post_plot_bending_moment(displacement=Us_global)
```

- $\omega = 1$ rad/s,
  Displacement:- 0.008983 mm,
  $-53.56856532793211 N.m < M < 196.44152986283513$ N.m
- $\omega = 2 \cdot \pi \cdot 10$ rad/s,
  Displacement: -0.009038 mm,
  -53.76035578350623 N.m $< M < 197.2611290919746$ N.m
- $\omega = 2 \cdot \pi \cdot 50$ rad/s,
  Displacement: -0.011072 mm,
  -60.05943822449147 N.m $< M < 226.8630288259076$ N.m
- $\omega = 2 \cdot \pi \cdot 81$ rad/s,
  Displacement: -0.108276 mm,
  -1450.4886845370306 N.m $< M < 1588.9774624176428$ N.m

[2]You can access this with $U[7]$ where U is the global displacement vector.

- $\omega = 2 \cdot \pi \cdot 120$ rad/s,
  Displacement: -0.004027 mm,
  -32.1955878709017 N.m $< M <$ 135.274557961933 N.m

(d) Repeat the same process where the magnitude of F_1 is equal to 50 N and $F_2 = 0$.

```
_,Fss,_ = fmn.make_force_global(1,0,50)
Us = np.linalg.solve(Kff - Mff*w**2,Fss)
Us_global = fmn.dynamic_make_global_displacement(Us)
fmn.post_print_nodal_data(Us_global*1E3)
fmn.post_plot_bending_moment(displacement=Us_global)
```

(e) Repeat the step (c), but use proportional damping of the form $\boldsymbol{C}_{ff} = 0.1 \cdot \boldsymbol{K}_{ff}$.

```
_,Fss,_ = fmn.make_force_global(2,1,-1000)
```

- $\omega = 1$ rad/s,
  Displacement: -0.008894 mm,
  -194.4886704279263 N.m $< M <$ 53.03633018759045 N.m
- $\omega = 2 \cdot \pi \cdot 10$ rad/s,
  Displacement: -0.000221 mm,
  -1.3358029970735716 N.m $< M <$ 4.833577071632606 N.m
- $\omega = 2 \cdot \pi \cdot 50$ rad/s,
  Displacement: -0.000008 mm,
  -0.06762142549029837 N.m $< M <$ 0.17816612637846646 N.m
- $\omega = 2 \cdot \pi \cdot 81$ rad/s,
  Displacement: -0.000002 mm,
  -0.03409671598598853 N.m $< M <$ 0.05511240241734876 N.m
- $\omega = 2 \cdot \pi \cdot 120$ rad/s,
  Displacement: 0 mm,
  -0.022847261530434173 N.m $< M <$ 0.01590726199601475 N.m

Damping value not realistic. This shows the influence of damping in a more extreme case.

3 Compliance minimization

Consider the following beam:

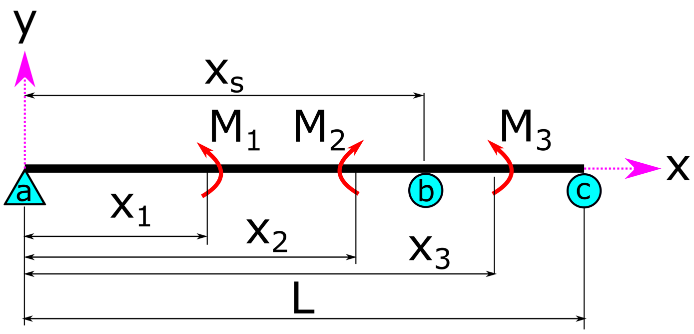

Figure 12: Problem 3

The beam has the following properties:

- Young's modulus: 70 GPa. $\rho = 2700$ kg/m^3 .
- Length of the beam: 1m.
- Cross-sectional area: Rectangular. Cross-sectional dimensions: $y \times z = 0.020 \times 0.010$m^2.
- $M_1 = 50$ N.m is applied to Node 4 with the same direction shown in the figure.
- $M_2 = 100$ N.m is applied to Node 10 with the same direction shown in the figure.
- $M_3 = 100$ N.m is applied to Node 15 with the same direction shown in the figure.

Twenty-one node positions are defined: Node 0 has an x-position of 0m and the last node, Node 20, has a x position of L.

(a) Calculate the compliance of the structure for $x_s = 0.5 \cdot L$.

(b) Calculate the first five natural frequencies of the structure for $x_s = 0.5 \cdot L$.

(c) Perform the following calculation in a for loop: Calculate the compliance, the maximum absolute vertical displacement and the natural frequencies of the structure for the different node positions.

- Plot the compliance vs. the support position.
- Plot the natural frequencies vs. the support position.
- Plot the maximum absolute vertical displacement of the beam vs. the support position.

(d) What should x_s be to minimize the compliance?

(e) What benefits would there be to refine the mesh in this calculation?

Does this make sense? Look at the mode shapes. Explore to understand.

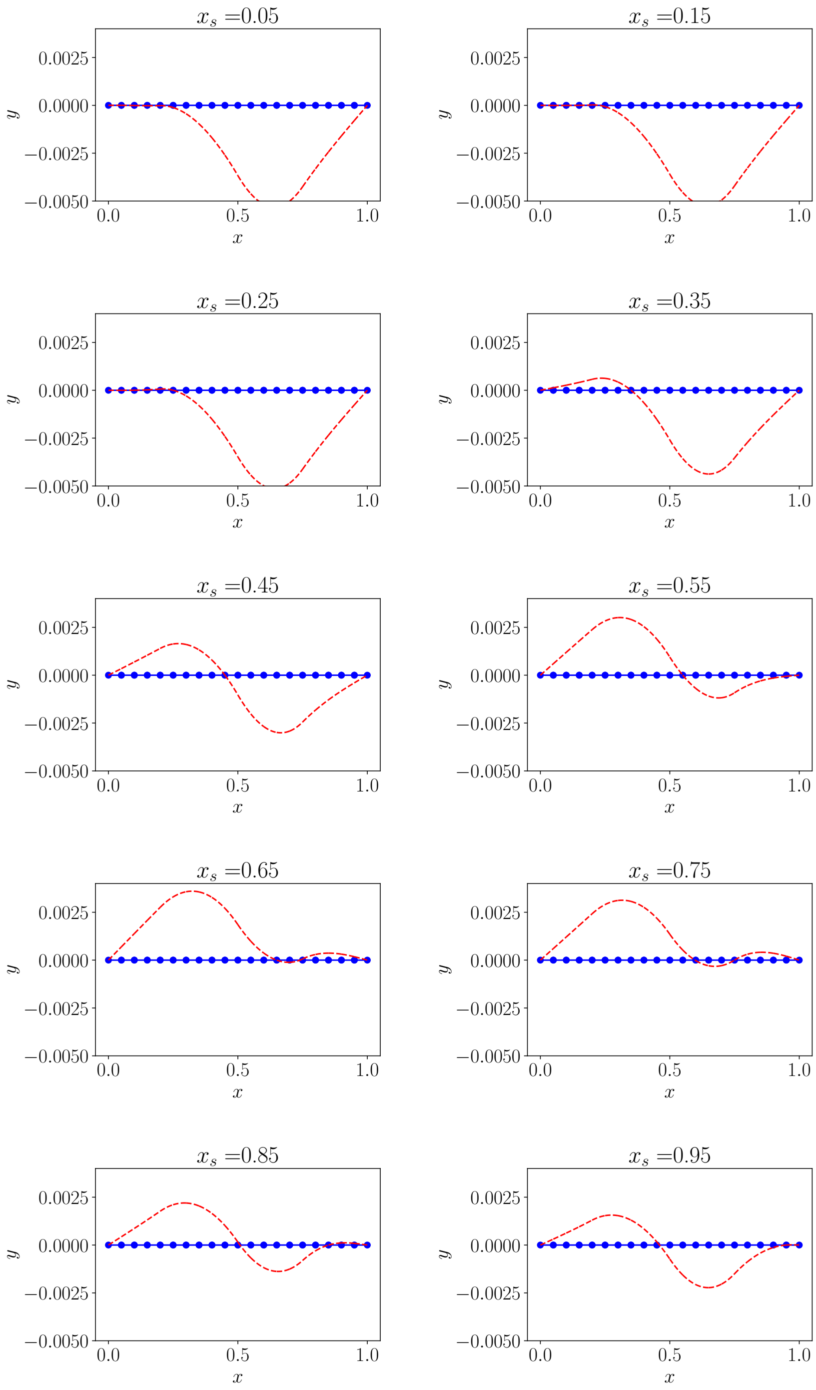

Figure 13: Displacement response

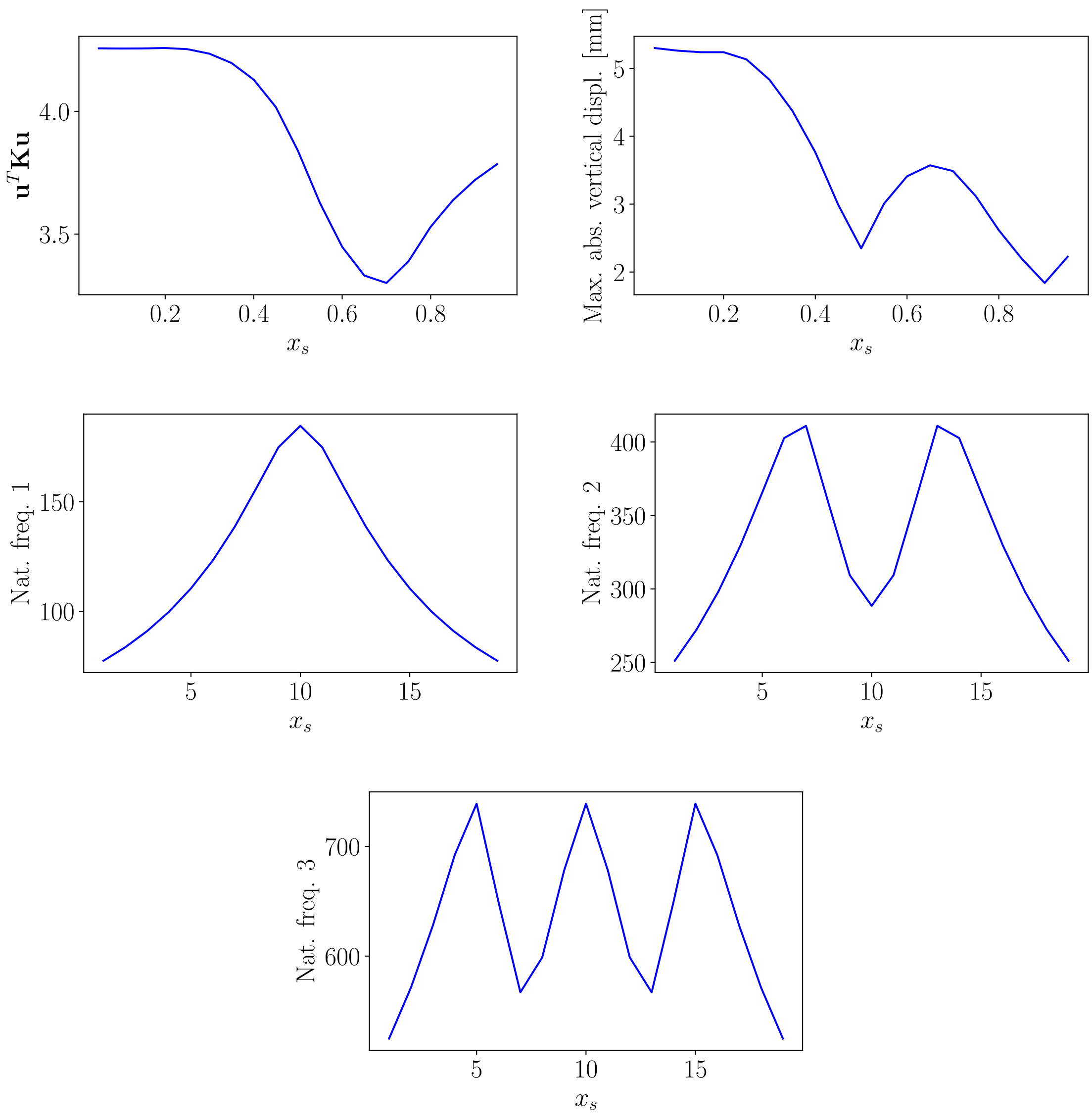

Figure 14: Results

4 Design

Consider the structure in Figure 15. Each element is connected via truss joints (i.e. cannot transfer moments). Each element should be made from the same cross-sectional geometry (e.g. the cross-sectional properties are the same). The parts will be machined.

The steel has a yield strength of 580MPa and an ultimate strength of 690MPa. $E = 200$ GPa. $\rho = 7800$ kg/m^3.

The following force is applied to the system:

$$f(t) = A_f \cdot \cos(\omega \cdot t) \text{ [N]} \tag{14}$$

The structure is operated in two load cases:

- $A_f = 3000$, $\omega = 2 \cdot \pi$ rad/s.
- $A_f = 2000$, $\omega = 2 \cdot \pi \cdot 100$ rad/s.

(a) Perform a modal analysis using $h = 0.01$m. Are the modes sensitive to h?

Change h and see whether this influences the natural frequencies. For example, change $h = 0.02$m and see how much the natural frequencies change.

(b) Use $h = 0.01$m, calculate the stresses in the structure for the two load cases.

Load case 1:

- Element 0: 15.001 MPa
- Element 1: -21.214 MPa
- Element 2: 30.0 MPa
- Element 3: 15.001 MPa
- Element 4: -21.214 MPa

Load case 2:

- Element 0: 14.863 MPa
- Element 1: -18.573 MPa
- Element 2: 22.038 MPa
- Element 3: 15.828 MPa
- Element 4: -17.587 MPa

(c) The system must be designed to adhere to the following specifications: The cross-section must be square, i.e. $A = h^2$. The structure must have a safety factor of five against buckling using Euler's theory, a safety factor of two against yielding and a safety factor of one against fatigue failure using the modified Goodman theory. All elements have a notch radius of 3mm and a stress concentration factor of 3.0 that needs to be incorporated. Assume that the temperature factor is $k_d = 1.0$ and use a reliability of 99%.

Does $h = 0.01$m satisfy the specifications? If it does not, choose a suitable h.

Load case 1:

- $k_a = 0.797777039378126$ (other version of textbook/MOW 227 spreadsheet - $k_a = 0.736$)
- $k_b = 1$
- $k_c = 0.85$

- $k_d = 1$
- $k_e = 0.814$
- S_e = 190.43376707327525 MPa (other version of textbook/MOW 227 spreadsheet - S_e = 175.68724799999995 MPa)
- $K_t = 3$
- $q = 0.8466$
- $k_f = 2.69335$

Effective stress $K_f \cdot \sigma_0$

- Element 0: 40.402 MPa
- Element 1: -57.136 MPa
- Element 2: 80.801 MPa
- Element 3: 40.402 MPa
- Element 4: -57.136 MPa

Safety factor: Reversed stress state, i.e. midrange stress is 0:

- Fatigue: Modified Goodman - 190.433767/80.8012931822966 = 2.35145604633606 (Good)
- Yielding: First cycle yield - 580/80.8012931822966 = 7.178102938172737 (Good)
- Critical buckling load for each element:
  - Element 0: 411.234 N
  - Element 1: 205.617 N
  - Element 2: 411.234 N
  - Element 3: 411.234 N
  - Element 4: 205.617 N
- Force in each element = $\sigma_0 \cdot A = 100mm^2 \cdot (15.001[MPa], -21.214[MPa], 30.0[MPa], 15.001[MPa], -21.214[MPa])$
- Force in each element $= \sigma_0 \cdot A = 1500.052N, -2121.37N, 3000.024N, 1500.062N, -2121.361N$
- Buckling: 0.09692640820361274 (Not good).

Load case 2: Effective maximum stress $K_f \cdot \sigma_0$

- Element 0: 40.032 MPa
- Element 1: -50.023 MPa
- Element 2: 59.357 MPa
- Element 3: 42.631 MPa
- Element 4: -47.368 MPa

- Fatigue: 190.43376707327525/59.35732236661826 = 3.208260741565603 (Good)
- Yielding: 580/59.35732236661827 = 9.771330256739882 (Good)
- Critical buckling load
  - Element 0: 411.234 N
  - Element 1: 205.617 N
  - Element 2: 411.234 N
  - Element 3: 411.234 N
  - Element 4: 205.617 N
- Force applied to the beam: $\sigma_0 \cdot A = 1486.32N, -1857.259N, 2203.843N, 1582.808N, -1758.704$
- Buckling: 0.11070982004548938 (Not good).

System does not conform to the design specifications.

We only have one design variable h. Increase h to improve the resistance against buckling:

$h = 0.027$m is adequate. Safety factor against buckling is 5.1510667302136195; safety factor against yielding is 52.32837041927927 and the safety factor against fatigue failures is 17.18118742.

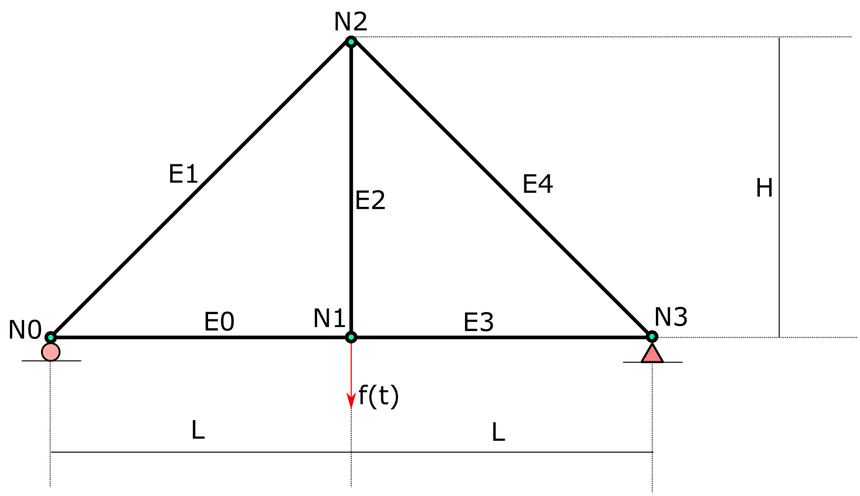

Figure 15: Truss system: $L = 2\text{m}$; $H = 2\text{m}$