Department of Mechanical and Aeronautical Engineering

Simulation-based design MOW 323

Exercises - Week 5

November 3, 2021

Shigley's:

- **Chapter 3** (Loads and stress)
- **Chapter 4** (Deflection and stiffness)
- **Chapter 5** (Failure modes for static loading)
- **Chapter 6** (Failure modes for dynamic loading)
- **Chapter 19** (Finite element analysis)

1 Dynamic analysis

Consider the bar-mass system shown in Figure 1.

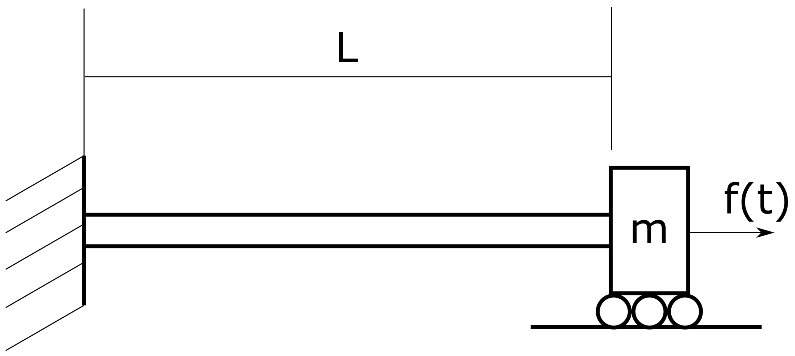

Figure 1: The slender bar is fixed to the wall on the left and a mass is fixed to the bar on the right end.

Assume $E = 200\text{GPa}$, $L = 2\text{m}$, $A = 100\text{mm}^2$, $m = 100\text{kg}$. Ignore the mass of the bar. Assume that there is viscous damping present with a damping coefficient of 10 N.s/m.

(a) What is the natural frequency of the system?

$$k = 10 \text{ MN/m} \tag{1}$$
$$\omega_n = 316.228 \text{ rad/s} \tag{2}$$
$$f_n = 50.329 \text{ Hz} \tag{3}$$

(b) What is the damping ratio of the system?

$$0.000158113883 \tag{4}$$

(c) Plot the magnitude of the frequency response function.

(d) Plot the steady state magnitude of the displacement of the mass as a function of frequency (in Hz) for a force magnitude of 1000N.

(e) Plot the steady state amplitude of the stress as a function of frequency (in Hz) for a force magnitude of 1000N.

(f) Determine the point where the quasi-static assumption exceeds a relative error of 5%. The relative error is defined as:

$$\frac{(\text{quasi-static stress} - \text{dynamic stress})}{\text{dynamic stress}} \tag{5}$$

For the following frequency ratio

$$\omega/\omega_r = 11.25395/50.329 = 0.2236 \tag{6}$$

we make a 5% error with a quasi-static approximation.

In this course, the quasi-static approximation is used to approximate the steady-state dynamic stresses with a static analysis. However, please note that the quasi-static approximation is only an **approximation** of the dynamic stresses. Unless $\omega = 0$, some error will be present. This is only allowed if ω/ω_r is small, e.g. at least $\omega/\omega_r < 0.3$.

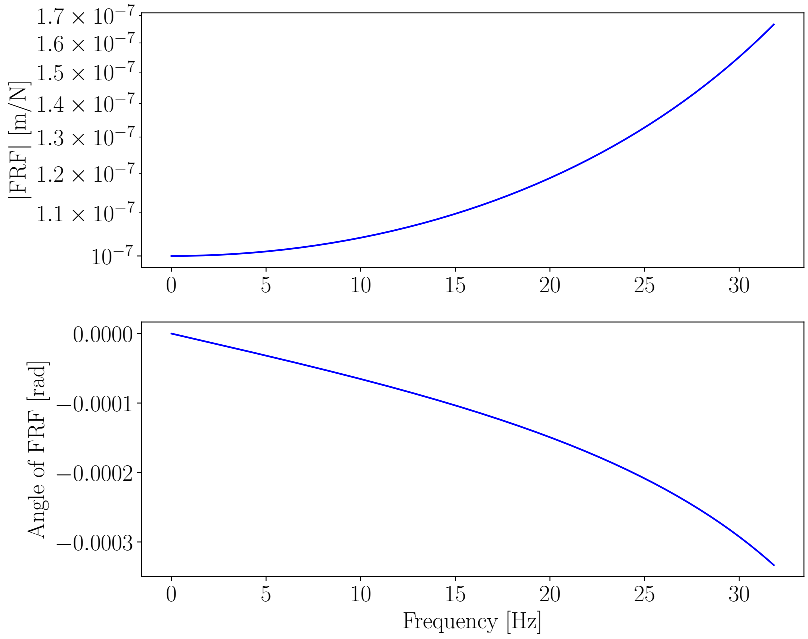

Figure 2: Answer for 1(c)

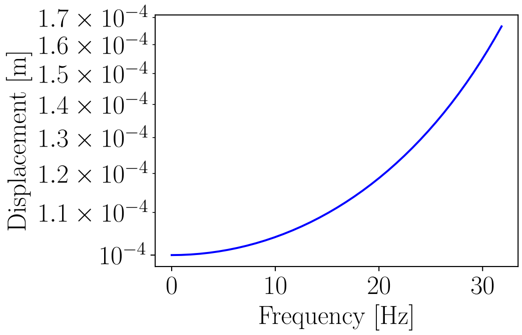

Figure 3: Answer for 1(d)

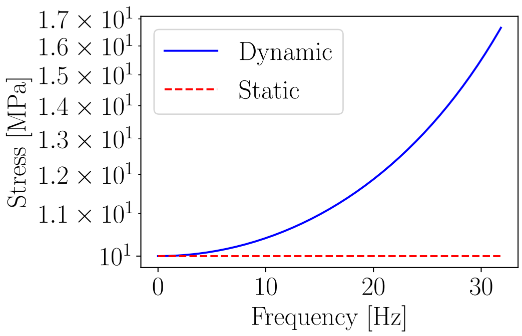

Figure 4: Answer for 1(e)

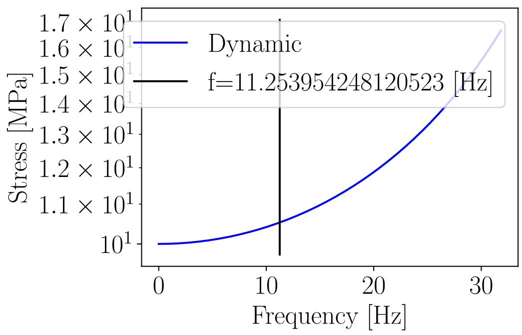

Figure 5: Answer for 1(f)

2 Fatigue analysis of a bar 1

Consider the following bar in Figure 6.

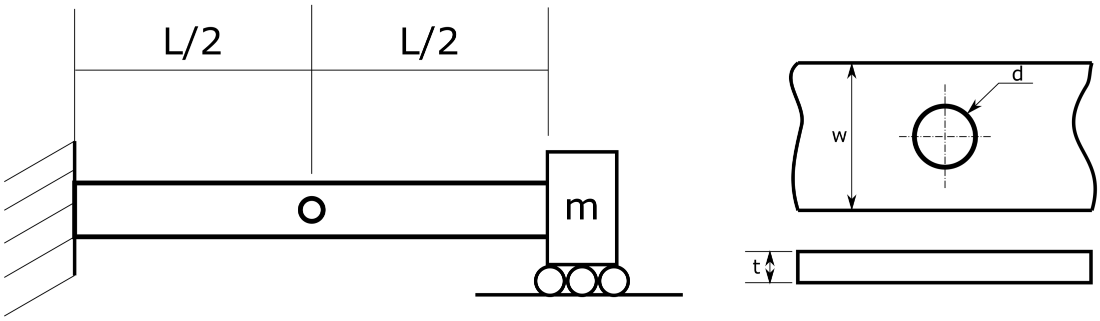

Figure 6: The slender bar is fixed to the wall on the left and a mass is fixed to the bar on the right end.

Approximate the bar as a slender bar with the following properties:

- Material of bar: Cold-drawn AISI 1040 steel
- Density: 7845kg/m^3
- Young's modulus: 200 GPa
- $m = 1000$kg
- L: 1m
- w: 25mm
- d: 6mm
- t: 10mm

Use a damping ratio of 10 N.s/m

The following axial force is applied to the structure:

$$f(t) = 20 \cdot \cos(100 \cdot t) \text{ [kN]} \tag{7}$$

(a) Calculate the amplitude of the nominal static stress at the hole, i.e. $x = 0.5$ for a force with an amplitude of $20kN$.

$$\sigma_0 = \frac{F}{A_n} = \frac{20}{(25-6)*10} = 105.2632 \text{ MPa} \tag{8}$$

(b) Calculate the amplitude of the nominal dynamic stress at the hole, i.e. $x = 0.5$.

$$k = 50\text{MN/m} \tag{9}$$
$$\omega_n = 223.606797 \text{ [rad/s]} \tag{10}$$
$$|U_{ss}| = 0.5 \text{ [mm]} \tag{11}$$

$$\sigma_0 = 131.578947 \text{ MPa} \tag{12}$$

(c) Calculate the safety factor against yielding using the load in equation (7).

$$
\begin{aligned}
S_{ut} &= 590 \text{ [MPa]} && (13)\\
S_y &= 490 \text{ [MPa]} && (14)\\
K_t &= 2.43650995 && (15)\\
q &= 0.8183927774340494 && (16)\\
K_f &= 2.1756293694289326 && (17)\\
\sigma_{min} &= -286.26702220382214 \text{ [MPa]} && (18)\\
\sigma_{max} &= 286.26702220382214 \text{ [MPa]} && (19)\\
\sigma_a &= 286.26702220382214 \text{ [MPa]} && (20)\\
\sigma_m &= 0 \text{ [MPa]} && (21)\\
SF_{yield} &= 1.7116886053718054 \text{ (O.K., ideally, this should be 2.)} && (22)\\
& && (23)
\end{aligned}
$$

(d) Calculate the safety factor against fatigue using the load in equation (7).

- The operating temperature of the machine is 60 degrees.
- Reliability of 98%.

$$
\begin{aligned}
k_a &= 0.8315737253764 \text{ (0.761 for a different version of the textbook)} && (24)\\
k_b &= 1 && (25)\\
k_c &= 0.85 && (26)\\
k_d &= 1.012 && (27)\\
k_e &= 0.8357000 && (28)\\
S_e &= 176.348861588565 \text{ (161.38254441378334 for a different version of the textbook)} && (29)
\end{aligned}
$$

Modified Goodman

$$N_{sf} = 1/(\sigma_a/S_e + \sigma_m/S_{ut}) = 0.6160292590845641 < 1, \text{ Finite life predicted.} \quad (30)$$

If you used different textbook version: 0.563950606399414

If limited life is predicted, calculate the number of cycles.

$$
\begin{aligned}
f &= 0.8659593004187649 && (31)\\
a &= 1480.2201935029436 && (32)\\
b &= -0.15399227736879778 && (33)\\
\sigma_r &= 286.26702220382214 && (34)\\
N &= 4.302356756063847 \cdot 10^4 \text{ cycles} && (35)
\end{aligned}
$$

If you used a different version of the textbook:

$$
\begin{aligned}
f &= 0.8659593004187649 && (36)\\
a &= 1616.9127911883313 && (37)\\
b &= -0.1667790347154593 && (38)\\
\sigma_r &= 286.26702220382214 && (39)\\
N &= 3.2244230856642115 \cdot 10^4 \text{ cycles} && (40)
\end{aligned}
$$

3 Fatigue analysis of a bar 2

Consider the bar in Figure 6. It has the same properties as the previous question. The bar is pre-loaded with a tensile load with a magnitude of 10kN.

The following cyclic axial load is subsequently applied to the bar:

$$f(t) = 20\cos(100 \cdot t) \text{ [kN]} \tag{41}$$

(a) Calculate the amplitude of the nominal static stress at the hole, i.e. $x = 0.5$ for a force with an amplitude of $20kN$.

This question should have been phrased as follows: What would be the nominal stresses be (ignoring the stress concentration factors) if we ignored the dynamics, i.e. we made a quasi-static approximation.

The nominal stress in Shigley's is defined as follows at th hole:

$$\sigma_0 = \frac{F}{A_n} = \frac{F}{t \cdot (w - d)} \tag{42}$$

$$\text{Stress due to pretension:} \frac{10000}{A_n} = 52.63157894736842\text{MPa} \tag{43}$$

$$\text{Stress due to varying load (ignoring fluctuation):} \pm 105.26315789473684\text{MPa} \tag{44}$$

$$\text{Stress:} \quad -52.631\text{MPa to } +157.895\text{MPa} \tag{45}$$

$$\tag{46}$$

(b) Calculate the amplitude of the nominal dynamic stress at the hole, i.e. $x = 0.5$.

$$\sigma_{0,min} = -78.9473683799342 \text{ MPa} \tag{47}$$

$$\sigma_{0,max} = 184.21052627467103 \text{ MPa} \tag{48}$$

(c) Calculate the safety factor against yielding.

$$\sigma_a = 286.29299156203234 \text{ MPa} \tag{49}$$

$$\sigma_m = 114.51719666059957 \text{ MPa} \tag{50}$$

$$N_{sf,yield} = \frac{S_y}{\sigma_a + \sigma_m} = \frac{490}{286.29299 + 114.51720} = 1.22\text{O.K., 2 would have been ideal.} \tag{51}$$

(d) Calculate the safety factor against fatigue. If limited life is predicted, calculate the number of cycles.

$$ka = 0.8315737253764363 \tag{52}$$
$$kb = 1 \tag{53}$$
$$kc = 0.85 \tag{54}$$
$$kd = 1.012 \tag{55}$$
$$ke = 0.8357000871494542 \tag{56}$$
$$S_e = 176.348861588565 \tag{57}$$
$$\text{Goodman:} \tag{58}$$
$$n_{sf,fatigue} = 0.55 \text{ Finite life.} \tag{59}$$
$$f = 0.8659593004187649 \tag{60}$$
$$a = 1480.2201935029436 \tag{61}$$
$$b = -0.15399227736879778 \tag{62}$$
$$\sigma_{rev} = 355.2449506802222[\text{MPa}] \tag{63}$$
$$N = 1.059 \cdot 10^4 \text{ cycles} \tag{64}$$

Other version of textbook

$$ka = 0.7613751482674179 \tag{65}$$
$$kb = 1 \tag{66}$$
$$kc = 0.85 \tag{67}$$
$$kd = 1.012 \tag{68}$$
$$ke = 0.8357000871494542 \tag{69}$$
$$S_e = 161.46210076322922 \tag{70}$$
$$\text{Goodman:} \tag{71}$$
$$n_{sf,fatigue} = 0.5083302451495754 \text{ Finite life.} \tag{72}$$
$$f = 0.8659593004187649 \tag{73}$$
$$a = 1616.6960840391637 \tag{74}$$
$$b = -0.16675963127225904 \tag{75}$$
$$\sigma_{rev} = 355.244[\text{MPa}] \tag{76}$$
$$N = 8.8389 \cdot 10^3 \text{ cycles} \tag{77}$$

4 Fatigue analysis of a bar 3

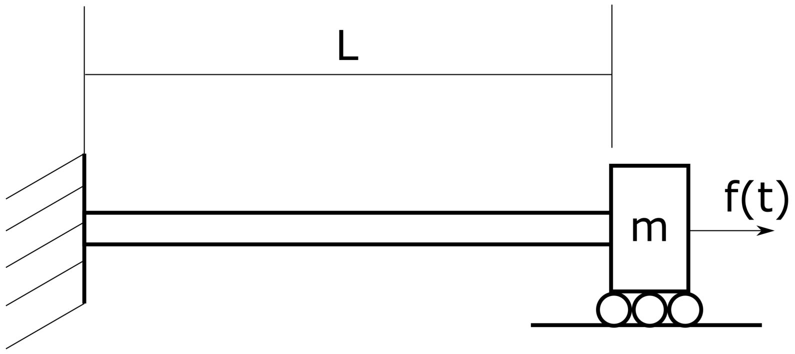

Figure 7: The slender bar is fixed to the wall on the left and a mass is fixed to the bar on the right end.

The following properties should be used: $S_{ut} = 530\text{MPa}$, $S_e = 200\text{MPa}$[1], $E = 200\text{GPa}$, $L = 1\text{m}$, $A = 200\text{mm}^2$, $m = 1000\text{kg}$, $c = 100\text{N.m/s}$.

The duty cycle of the machine is as follows:

$$f_1(t) = 40\cos(100 \cdot t) \text{ [kN]}, 50\% \text{ the time} \tag{78}$$
$$f_2(t) = 10\cos(250 \cdot t) \text{ [kN]}, 10\% \text{ the time} \tag{79}$$
$$f_3(t) = 60\cos(300 \cdot t) \text{ [kN]}, 40\% \text{ the time} \tag{80}$$

How many cycles N can the bar last?

The stresses and cycles are calculated for each operating cycle in a separate column:

$$\sigma_{min} = \begin{bmatrix} -266.66665185 & -88.88883402 & -239.9999568 \end{bmatrix} \text{ [MPa]} \tag{81}$$
$$\sigma_{max} = \begin{bmatrix} 266.66665185 & 88.88883402 & 239.9999568 \end{bmatrix} \text{ [MPa]} \tag{82}$$
$$\sigma_a = \begin{bmatrix} 266.66665185 & 88.88883402 & 239.9999568 \end{bmatrix} \text{ [MPa]} \tag{83}$$
$$\sigma_m = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \text{ [MPa]} \tag{84}$$
$$n_{sf} = \begin{bmatrix} 0.75000004 & 2.25000139 & 0.83333348 \end{bmatrix} \tag{85}$$
$$N_i = \begin{bmatrix} 9.65611440 \cdot 10^4 & \infty & 2.27304636 \cdot 10^5 \end{bmatrix} \tag{86}$$

$$\sum \frac{n_i}{N_i} = 1 \tag{87}$$
$$\sum \frac{0.5 \cdot N}{9.65611440 \cdot 10^4} + \lim_{x \to \infty} \frac{0.1 \cdot N}{x} + \frac{0.4 \cdot N}{2.27304636 \cdot 10^5} = 1 \tag{88}$$
$$N = 1.441375127640218 \cdot 10^5 \text{ cycles} \tag{89}$$

[1]This is the modified endurance strength

5 Fatigue analysis of a beam

Consider the cantilever beam in Figure 8. The machined steel beam has a circular cross-section with a diameter of 52mm and it has a length of 1m. The steel has a Young's modulus of 210MPa and $S_{ut} = 630$ MPa. Assume $K_f = 1$. Neglect the mass of the beam. Assume small deflections.

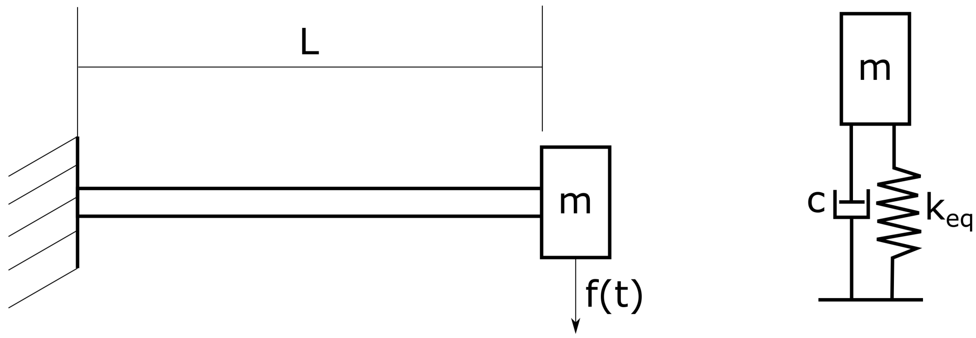

Figure 8: The slender beam is fixed to the wall on the left and a mass is fixed to the beam on the right end.

Use $m = 100$kg and $c = 20$ N.s/m.

(a) Model the beam-mass system as a single-degree-of-freedom system. Ignore the mass of the beam.

$$100 \cdot \ddot{y} + 20 \cdot \dot{y} + 226112.1100035287 \cdot y = f(t) \tag{90}$$

(b) Calculate the modified endurance strength.

$$d_e = 19.24\text{mm} \tag{91}$$

$$k_b = 1.24 \cdot d_e^{-0.107} \tag{92}$$

$$k_a \cdot k_b \cdot \cdot k_c \cdot k_d \cdot k_e = 0.8172431635323025 \cdot 0.9036739078924458 \cdot 1 \cdot 1.000 \cdot 1.0 = 0.738521323287621 \tag{93}$$

$$S_e = 232.63421683560063[\text{MPa}] \tag{94}$$

(c) Is a quasi-static approximation reasonable for a load of $1000 \cdot \cos(30 \cdot t)$ [N]?

$$\text{Natural frequency} = 47.55 \text{ rad/s} \tag{95}$$

$$\text{Excitation frequency} = 30\text{rad/s} \tag{96}$$

No, a quasi-static analysis is not appropriate as $\frac{\omega}{\omega_n} >> 0.1$

(d) Calculate the safety factor against first cycle yielding and fatigue for a load of $1000 \cdot \cos(30 \cdot t)$ [N].

Normal stress on the top of the beam due to the mass (Gravity assumed downwards):

$$\sigma_{mass} = -\frac{M \cdot r}{I} = 71.06554354717768 \text{ [MPa]} \tag{97}$$

As the presence of a gravity force was not specified/implied, you could have assumed that the mass is supported and does not result in a static stress, i.e. $\sigma_{mass} = 0$. However, in the next section, $\sigma_{mass} = 71.06554354717768$ [MPa] will be used to show another example of a stress where $\sigma_m \neq 0$.

Normal stress due to fluctuating load on the top of the beam:

$$-120.3408047744017 \leq \sigma_{dyn} = -\frac{M \cdot r}{I} \leq 120.3408047744017 \text{ [MPa]} \tag{98}$$

Total stress

$$-49.27526122722402 \leq \sigma_{tot,dyn} \leq 191.4063483215794 \text{ [MPa]} \tag{99}$$

In general the top of the beam will be in tension, which makes sense.

$$\sigma_a = 120.3408047744017 \text{ MPa} \tag{100}$$
$$\sigma_m = 71.0655435471777 \text{ MPa} \tag{101}$$
$$n_{sf,yield} = 2.5599986849796497 \text{ O.K.} \tag{102}$$
$$n_{sf,fatigue} = 1.59\text{, Infinite life is predicted.} \tag{103}$$

Other textbook version/Excel spreadsheet.

$$ka = 0.751 \tag{104}$$
$$kb = 0.9036739078924458 \tag{105}$$
$$kc = 1 \tag{106}$$
$$kd = 1.0 \tag{107}$$
$$ke = 1.0 \tag{108}$$
$$k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e = 0.6786591048272268 \tag{109}$$
$$S_e = 213.77761802057645 \tag{110}$$
$$n_{sf,yield} = 2.5599986849796497 \text{ O.K.} \tag{111}$$
$$n_{sf,fatigue} = 1.4798862009678415\text{, Infinite life is predicted.} \tag{112}$$