STAT 443: Lab 11

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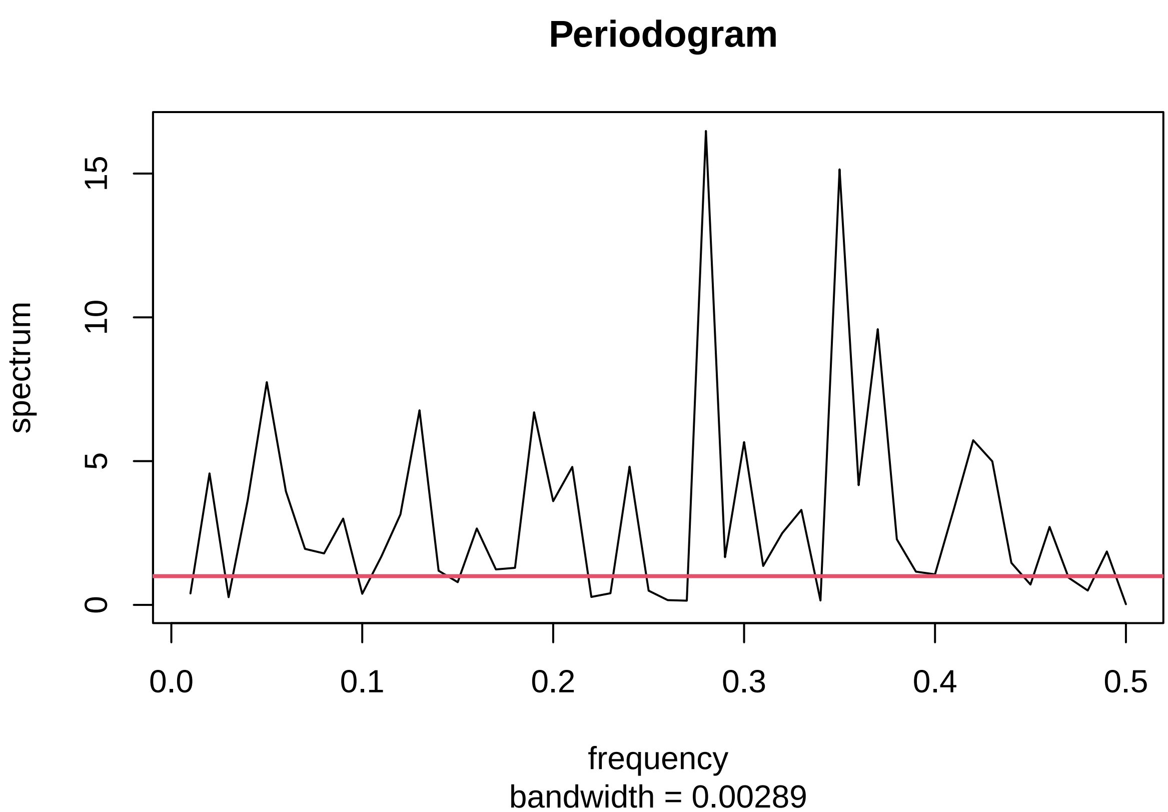
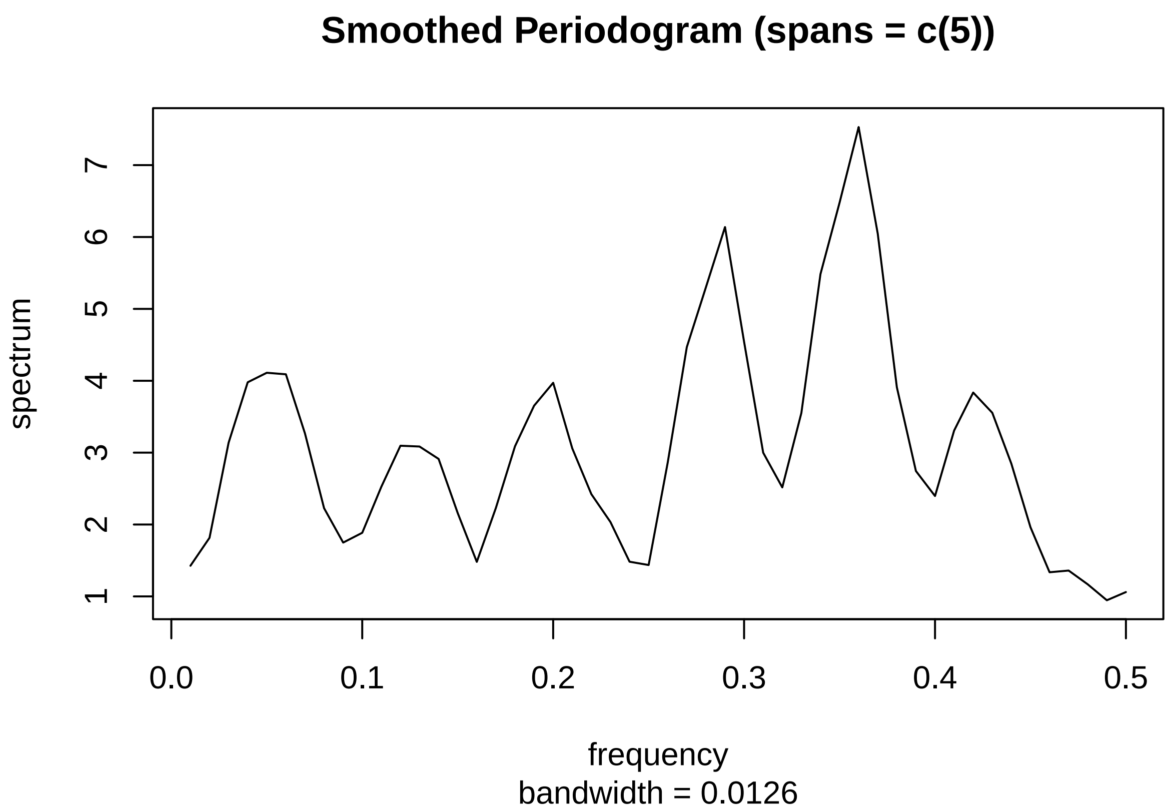
Question 1

(a)

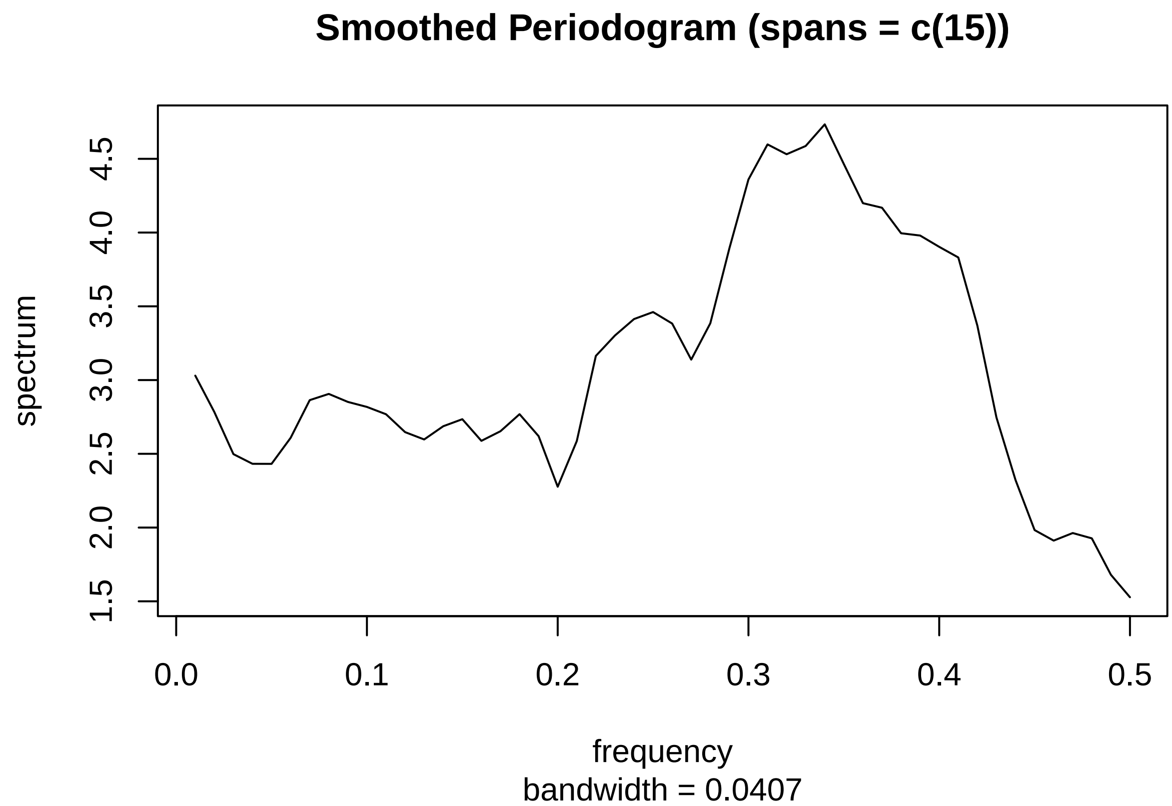
```r
set.seed(123)

wn <- arima.sim(model = list(order = c(0, 0, 0)), n = 100, sd = 2)

# Before Smoothing
spec.pgram(wn, log = "no", main = " Periodogram")
abline(h = 1, col = 2, lty = 1, lwd = 2)
```



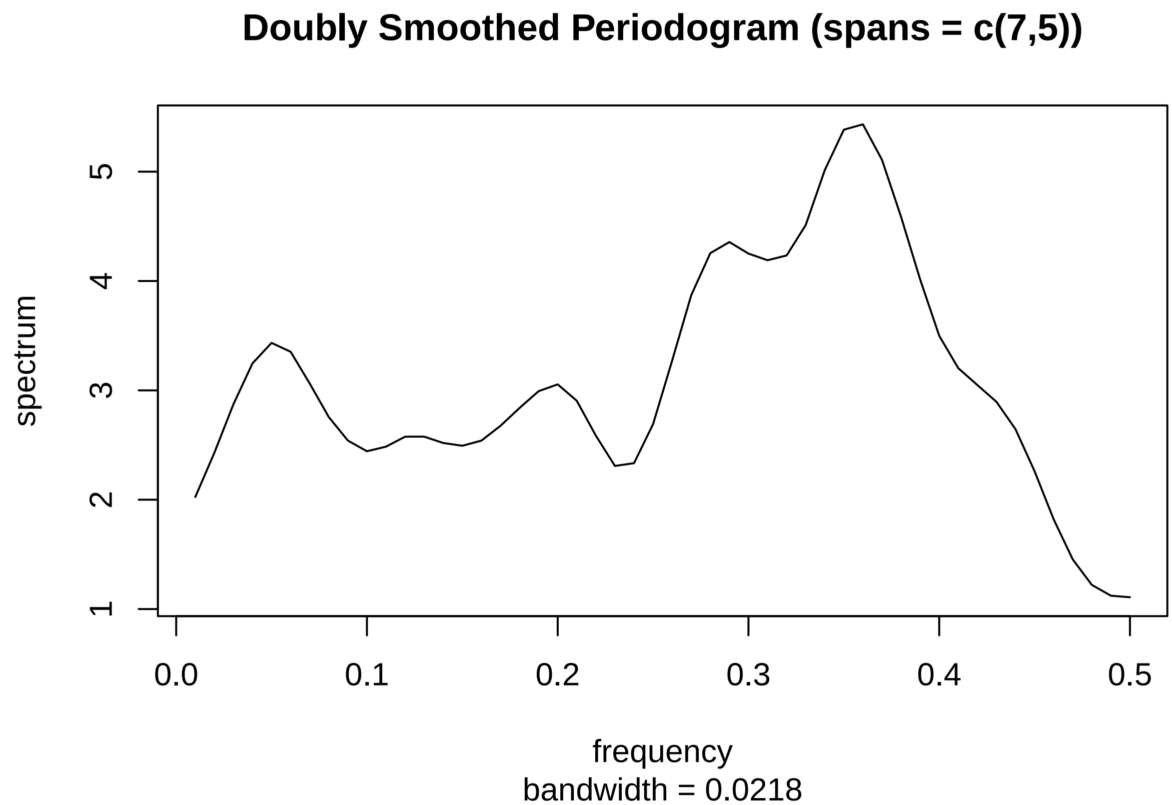
```r
# After Smoothing (From Lec 6 P32)
spec.pgram(wn, spans = c(5), log = "no", main = "Smoothed Periodogram (spans = c(5))")
```




```r
spec.pgram(wn, spans = c(15), log = "no", main = "Smoothed Periodogram (spans = c(15))")
```



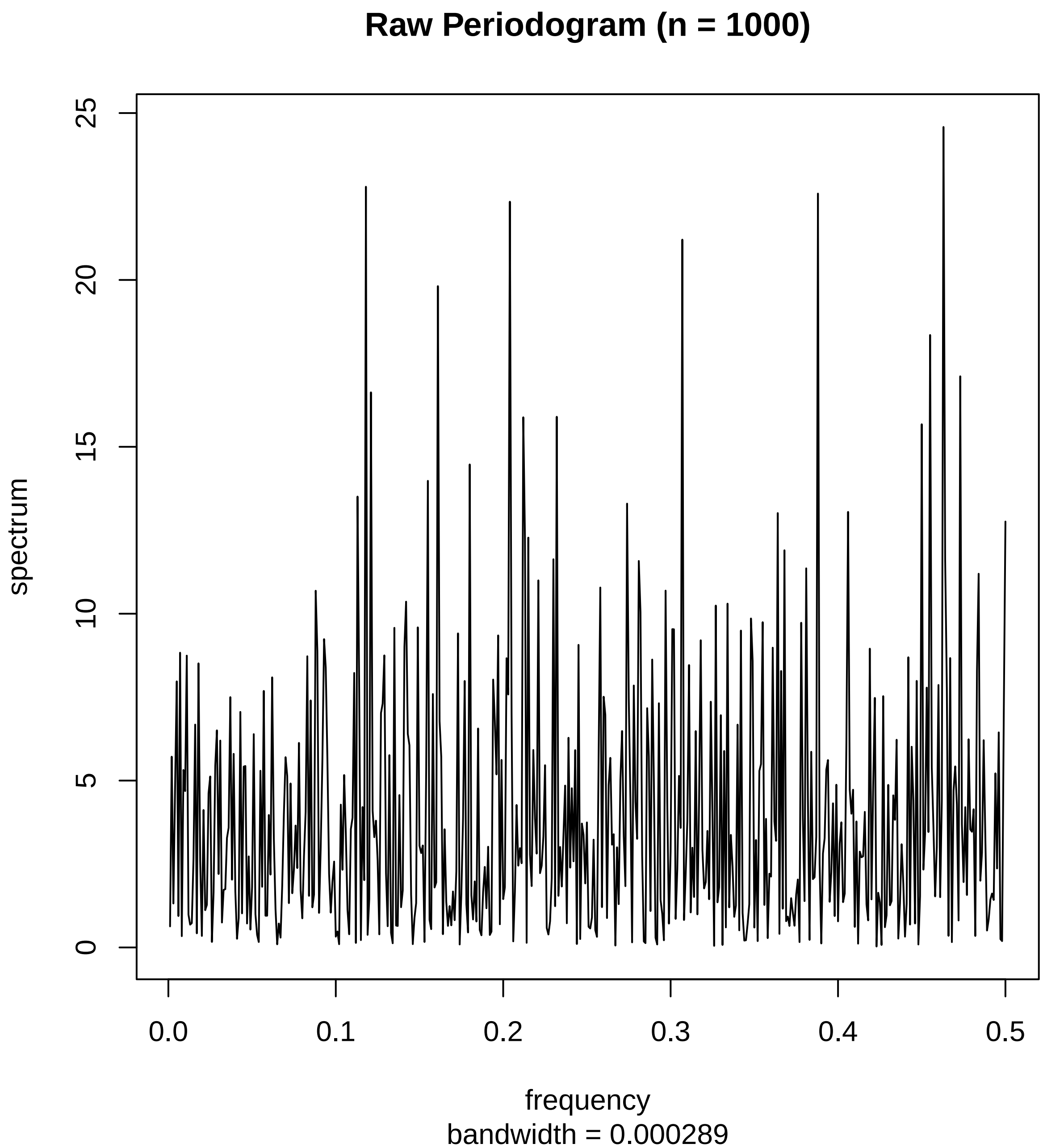

```
spec.pgram(wn, spans = c(7, 5), log = "no", main = "Doubly Smoothed Periodogram (spans = c(7,5))")
```



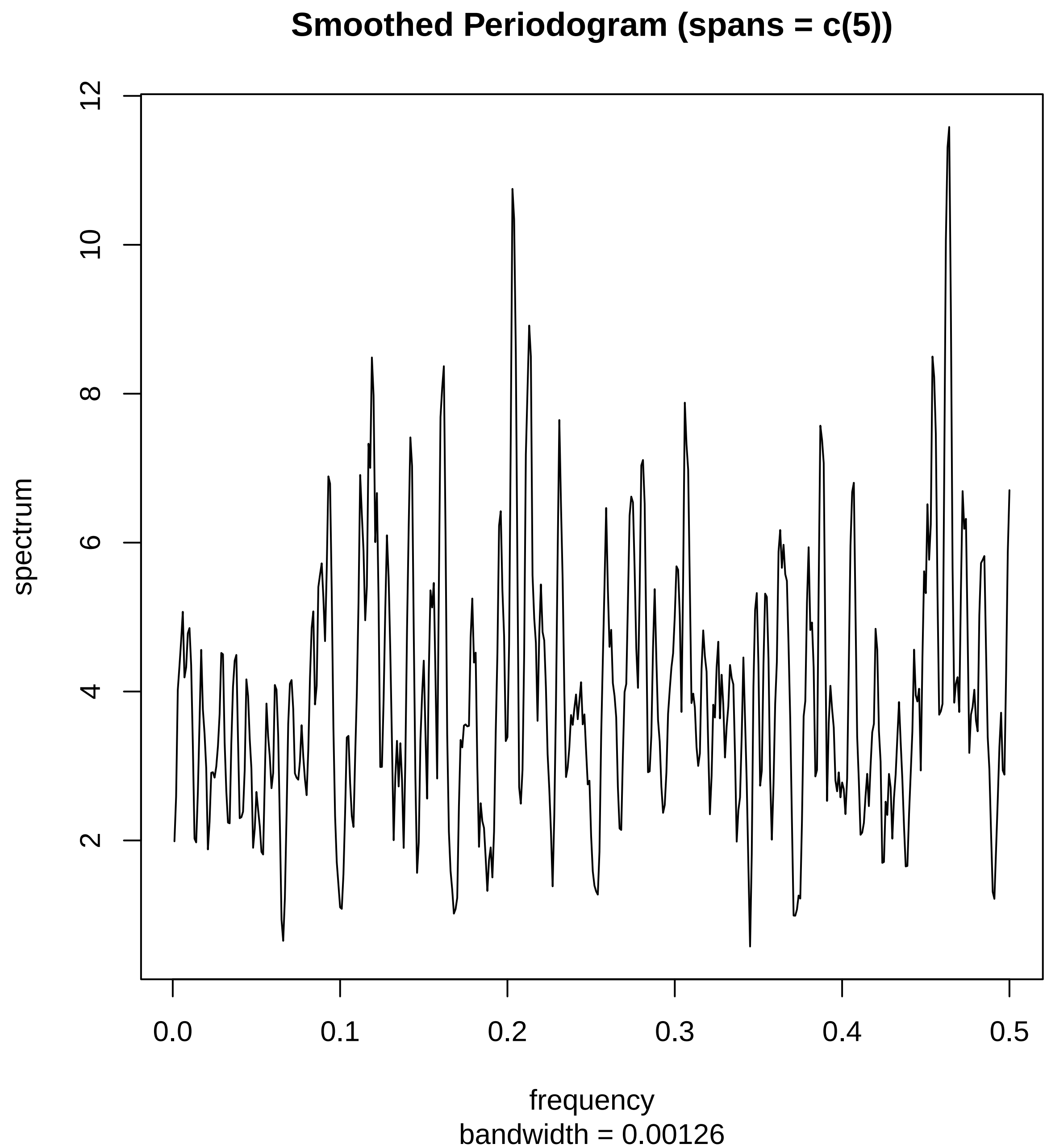
For the first periodogram, we observe a more 'noisy' graph. As we increase the length (m) of our smoothing filter from 5 to 15, we observe an increase in smoothing where the noise reduces. By applying the two step smoothing, we observe additional smoothing effects on several peaks. Although Larger spans could provide a more percise estimate of the spectral density, but we must consider for over smoothing.

(b)

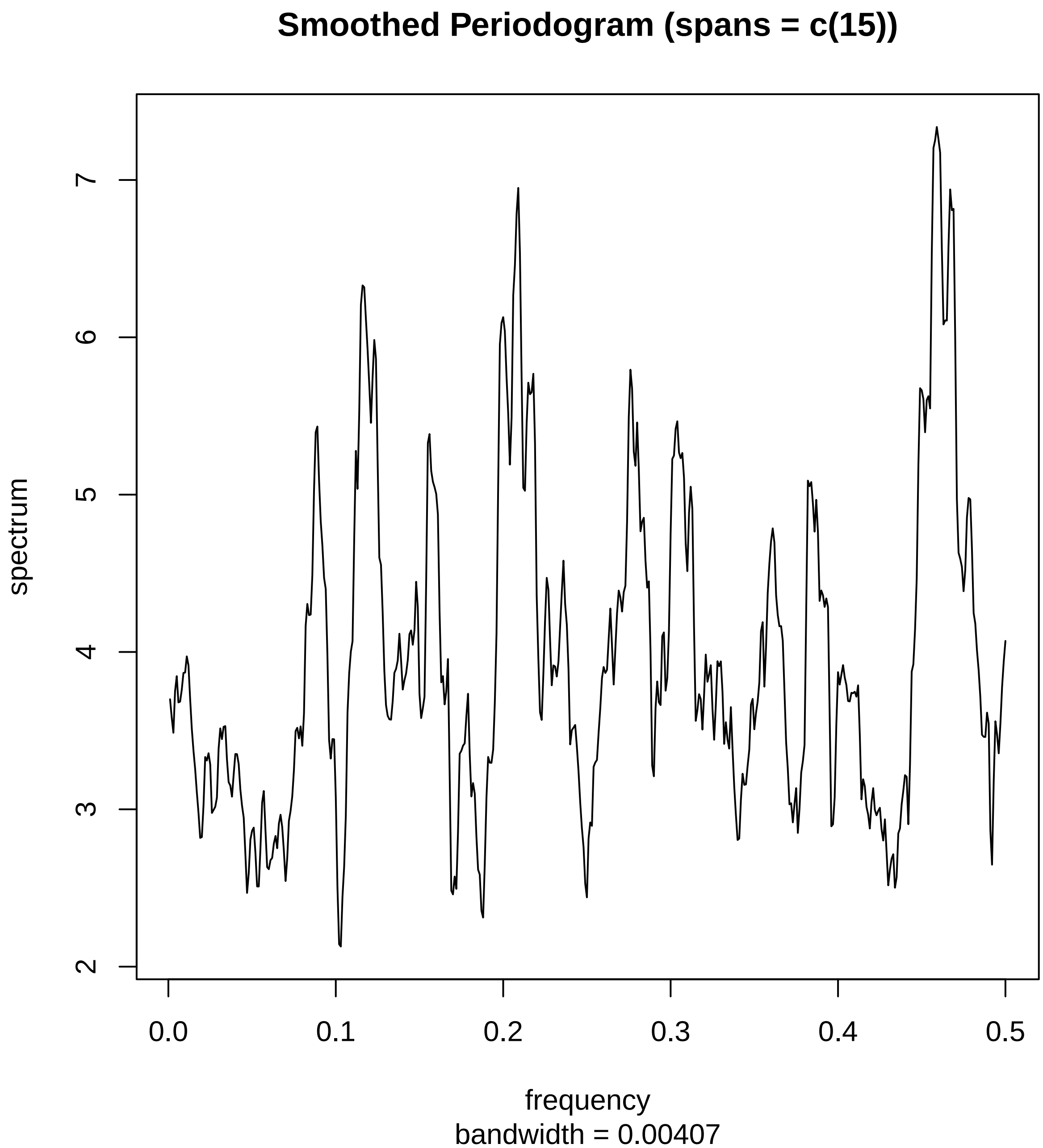
```r
set.seed(123)

wn_1000 <- arima.sim(model = list(order = c(0, 0, 0)), n = 1000, sd = 2)

# Before Smoothing / Raw Periodogram
spec.pgram(wn_1000, log = "no", main = "Raw Periodogram (n = 1000)")
```



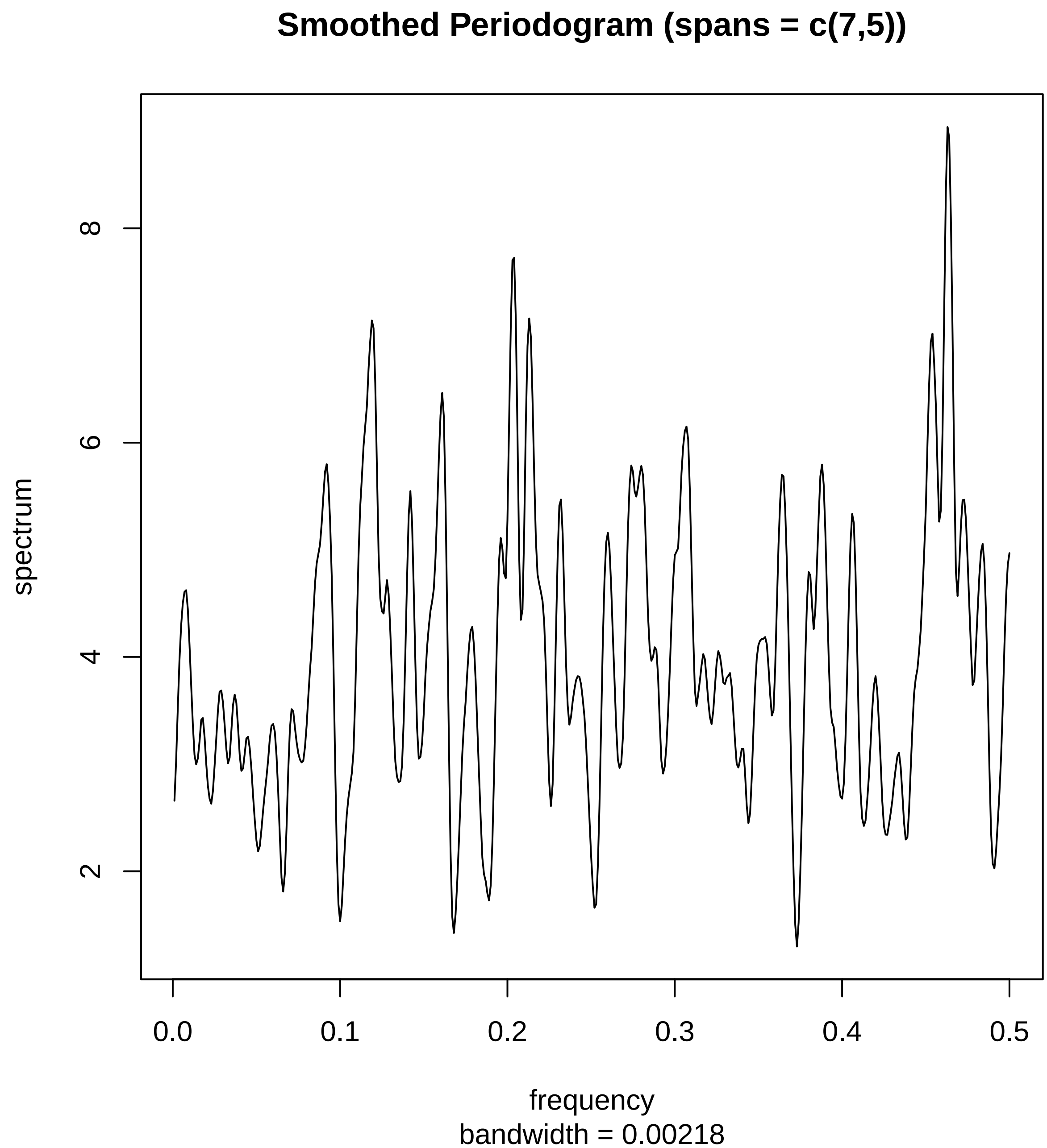

```
# After Smoothing Periodogram
spec.pgram(wn_1000, spans = c(5), log = "no", main = "Smoothed Periodogram (spans = c(5))")
```




```r
spec.pgram(wn_1000, spans = c(15), log = "no", main = "Smoothed Periodogram (spans = c(15))")
```




```
spec.pgram(wn_1000, spans = c(7,5), log = "no", main = "Smoothed Periodogram (spans = c(7,5))")
```

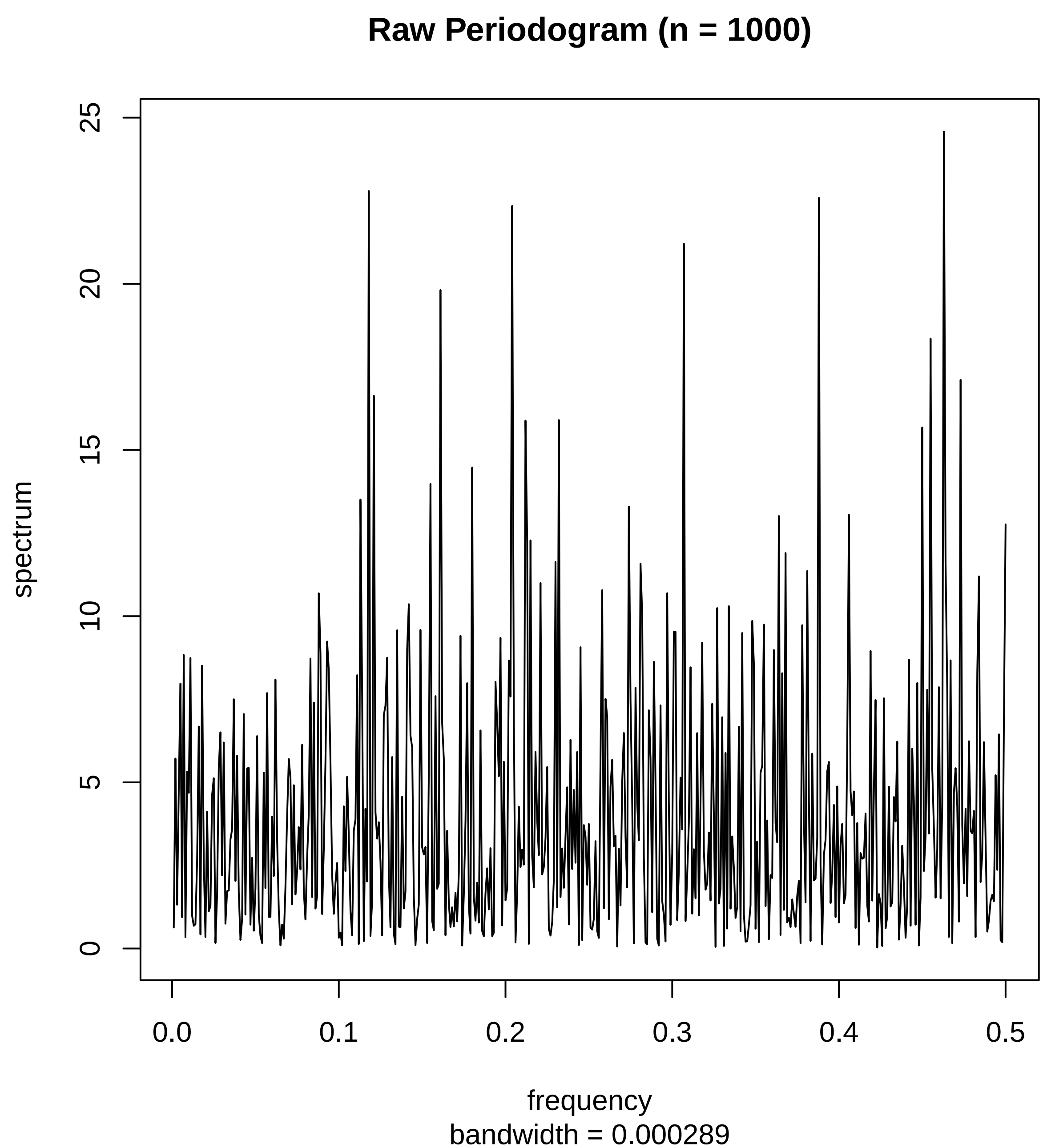


For our first periodogram before smoothing, we observe a 'noisy' fluctuating graph because of noise. And as we increase our span size, we see our periodogram smoothing out more with less variability for span c(15). We also see a doubly smoothing which gives a relatively 'smoother' look in the graph. Overall, we observe a smoothed periodogram with better approximation for our spectral density function by removing any extreme fluctuations.

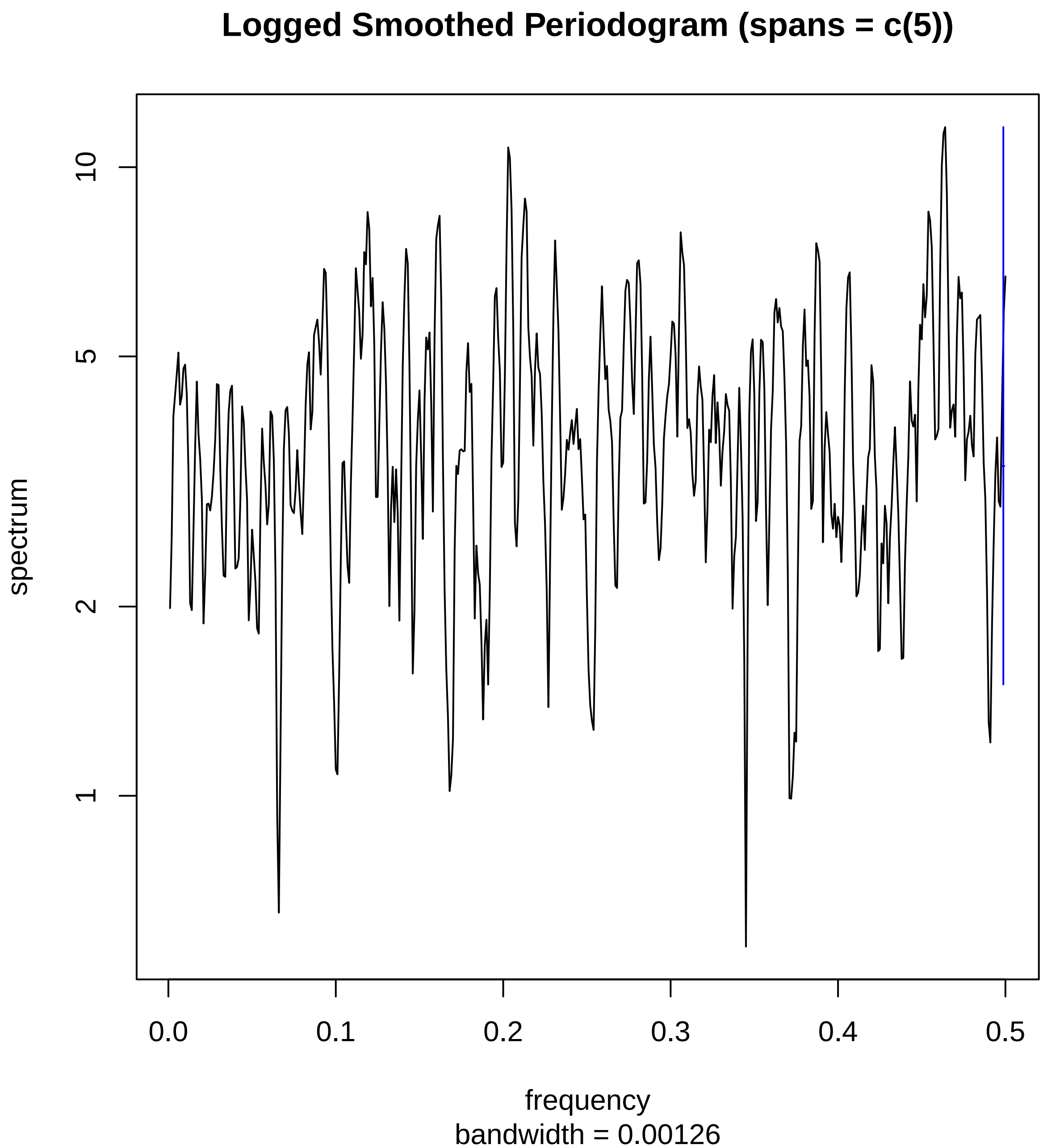
(c)

Setting logs = “yes” from question (b)

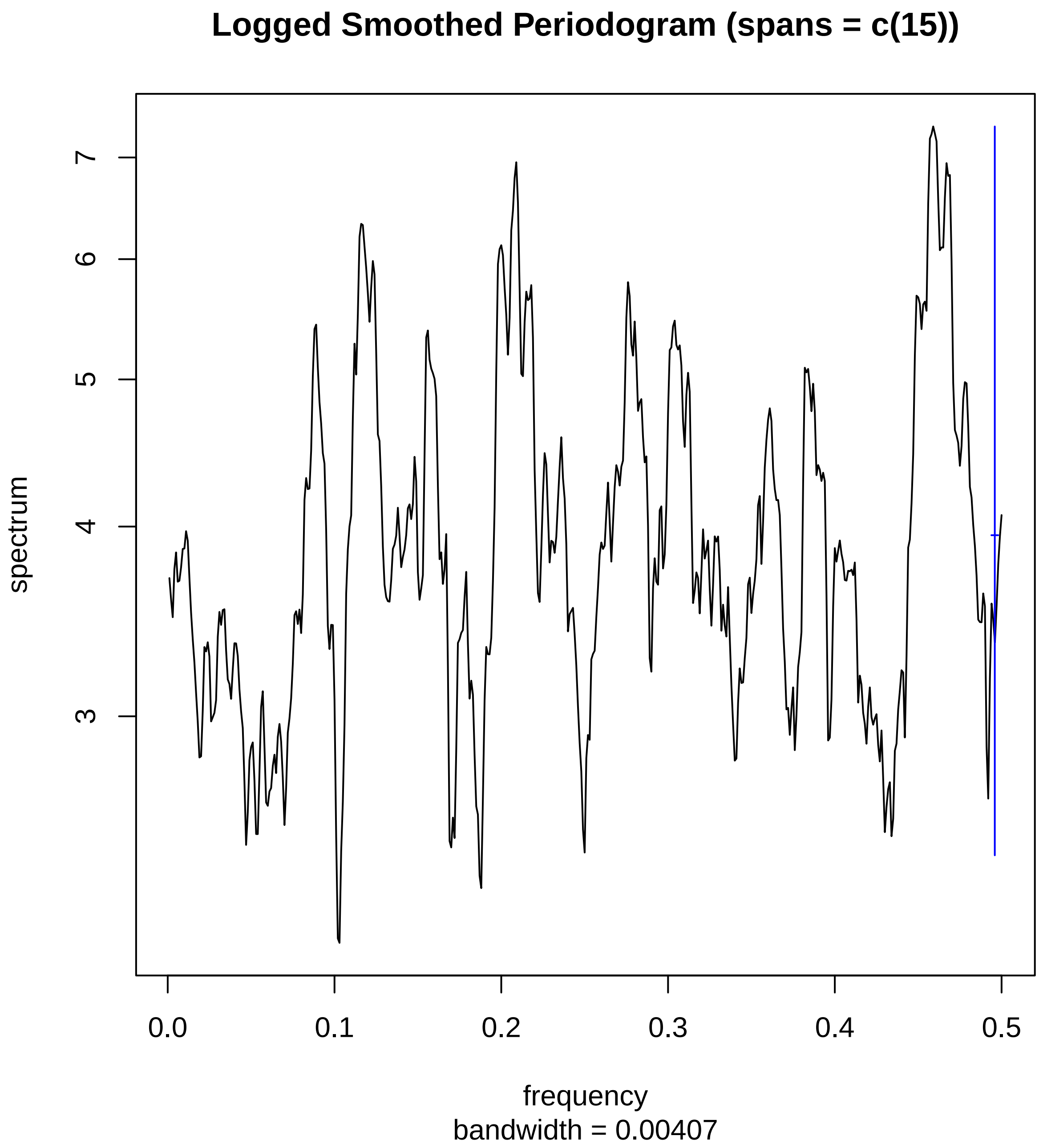
```r
set.seed(123)

wn_1000 <- arima.sim(model = list(order = c(0, 0, 0)), n = 1000, sd = 2)

# Before Smoothing / Raw Periodogram
spec.pgram(wn_1000, log = "no", main = "Raw Periodogram (n = 1000)")
```



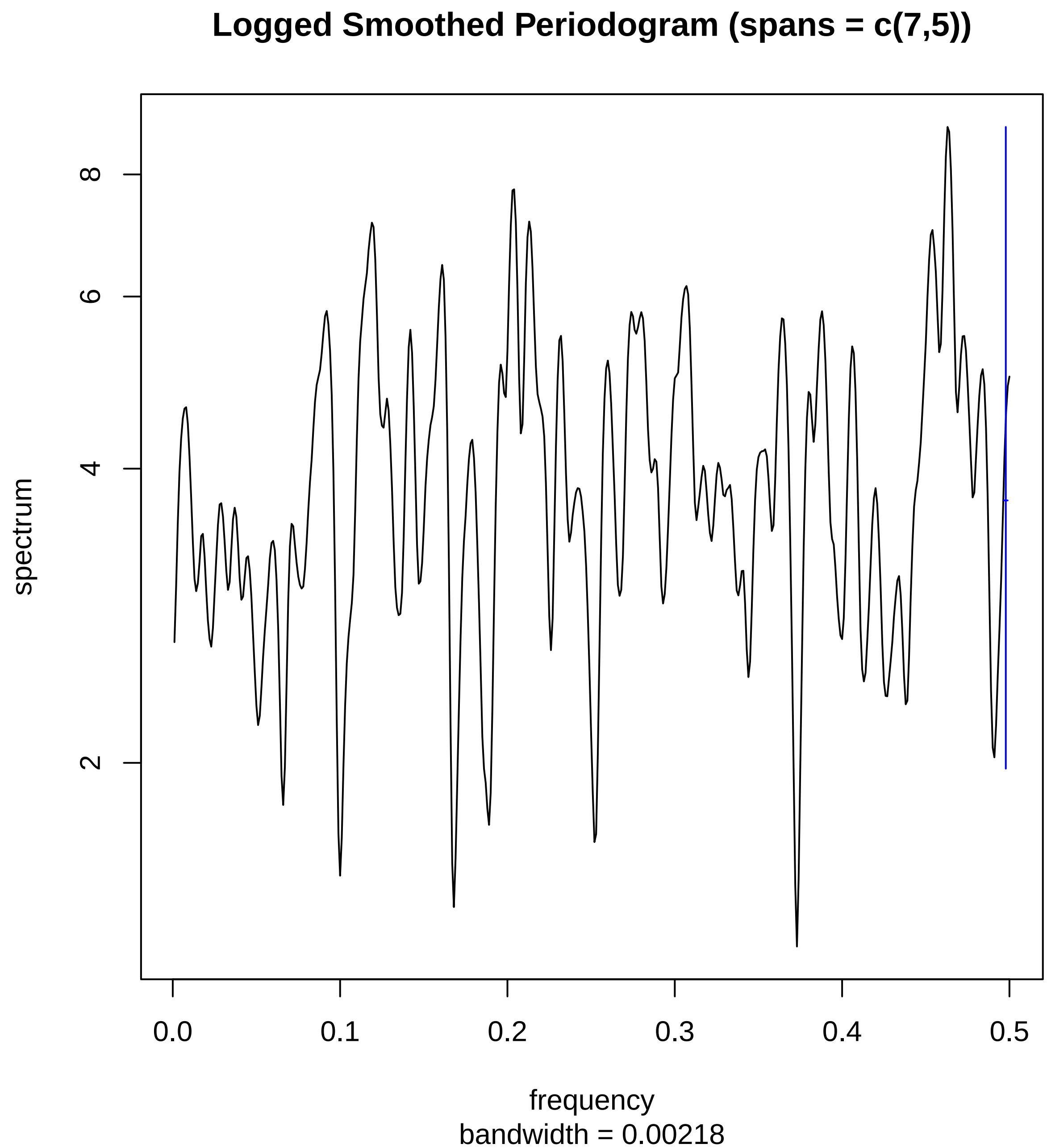
```r
# After Smoothing Periodogram
spec.pgram(wn_1000, spans = c(5), log = "yes", main = "Logged Smoothed Periodogram (spans = c(5))")
```




```r
spec.pgram(wn_1000, spans = c(15), log = "yes", main = "Logged Smoothed Periodogram (spans = c(15))")
```




```
spec.pgram(wn_1000, spans = c(7,5), log = "yes", main = "Logged Smoothed Periodogram (spans = c(7,5))")
```



While our periodogram without log and smoothing will maintain the same, our logged and smoothed periodogram shows more compressed higher values with out spectrum range decreasing. As we increase the smoothing, we observe less fluctuations and smoother curves. Our doubly logged and smoothed periodogram shows a relatively smoother and similar spectrum range.

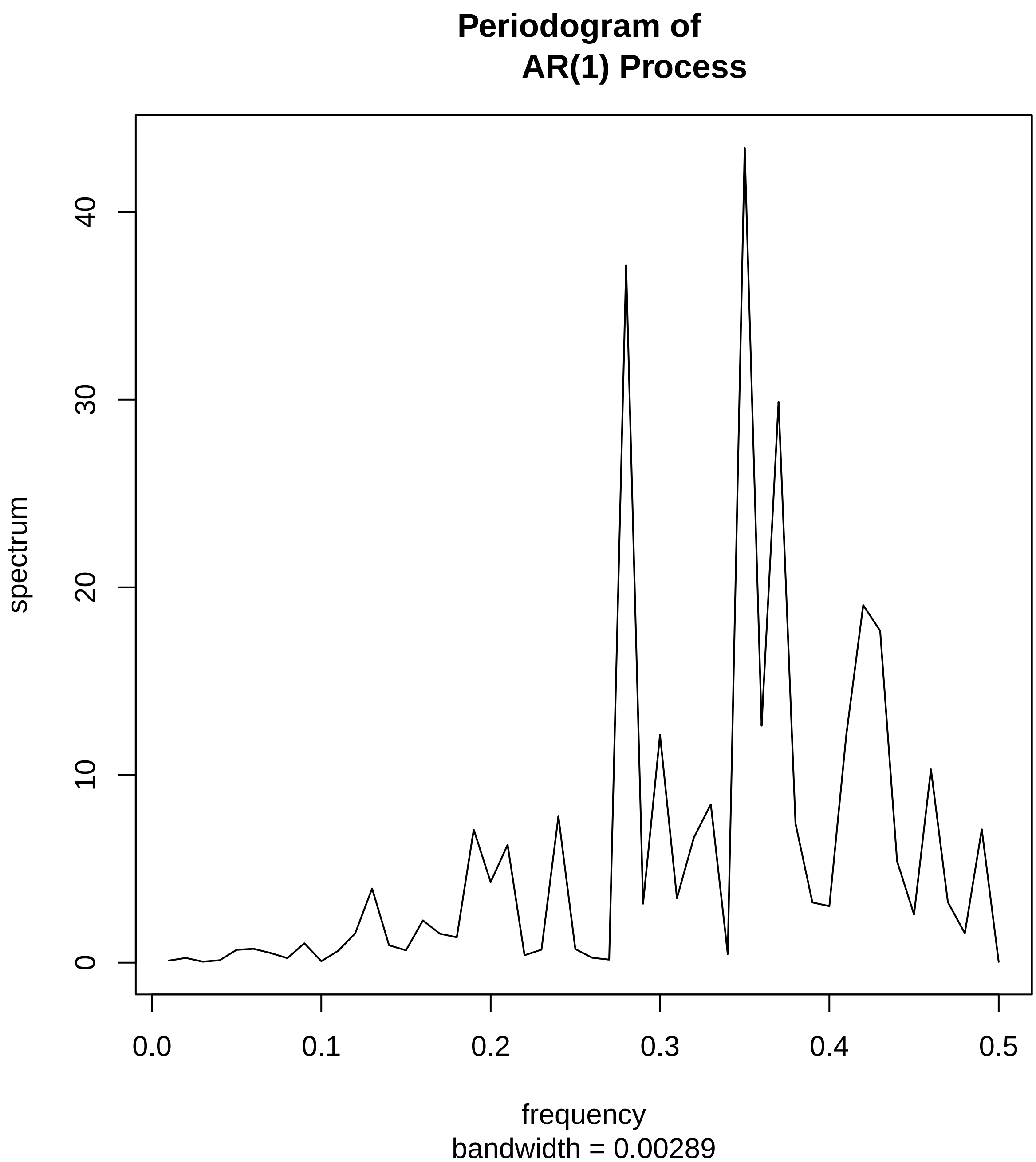
I would believe span c(7, 5) be the 'best' choice in criteria of consistent horizontal levels and relatively fewer dips and rises, meaning that it has a fair variance while also reducing noise well.

(d) From my findings in c), Taking the log helps reduce error by 'compressing' the higher values. Hence, enabling the log transformation reduced the variance in the spectrum.

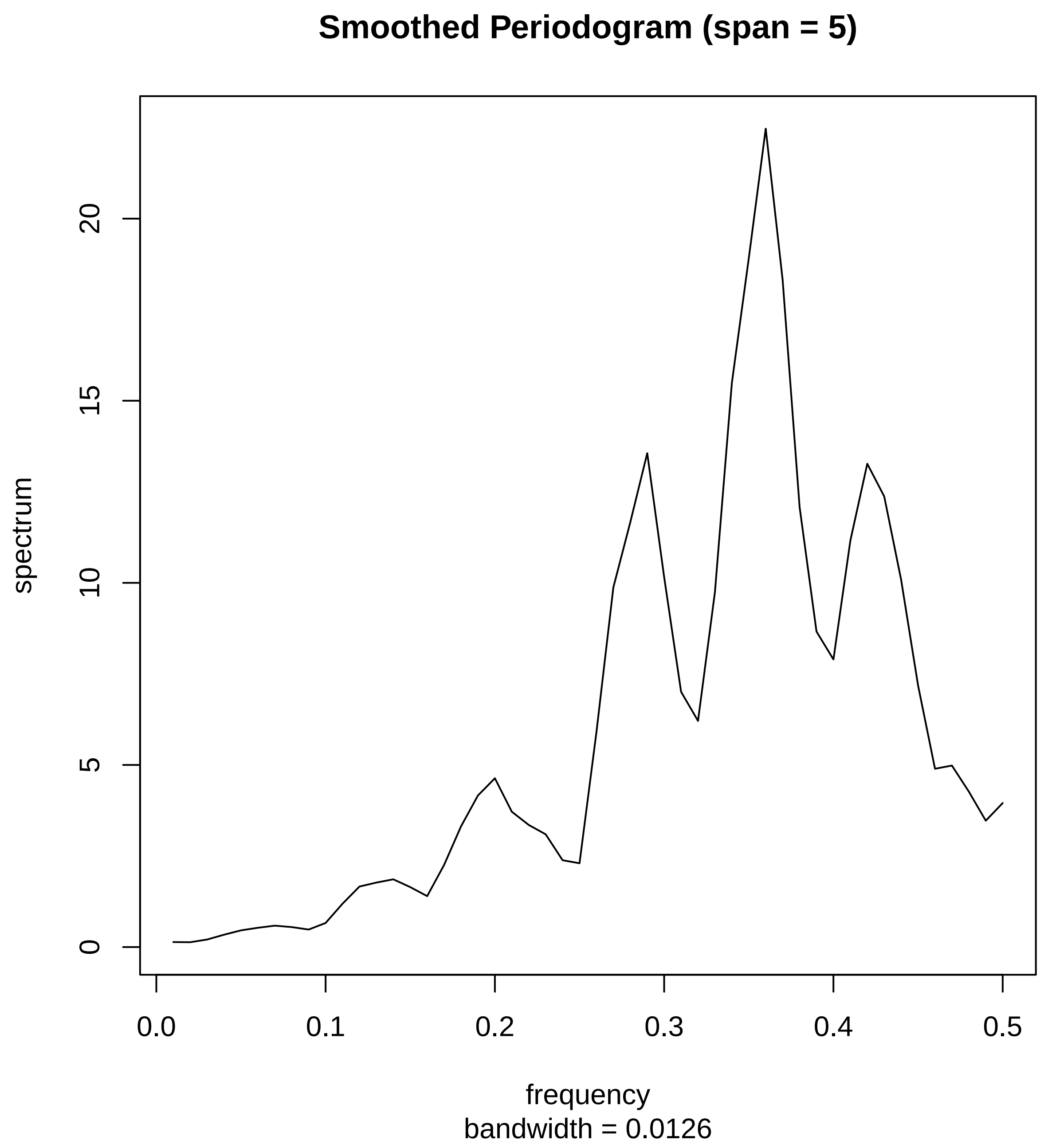
Question 2

(a)

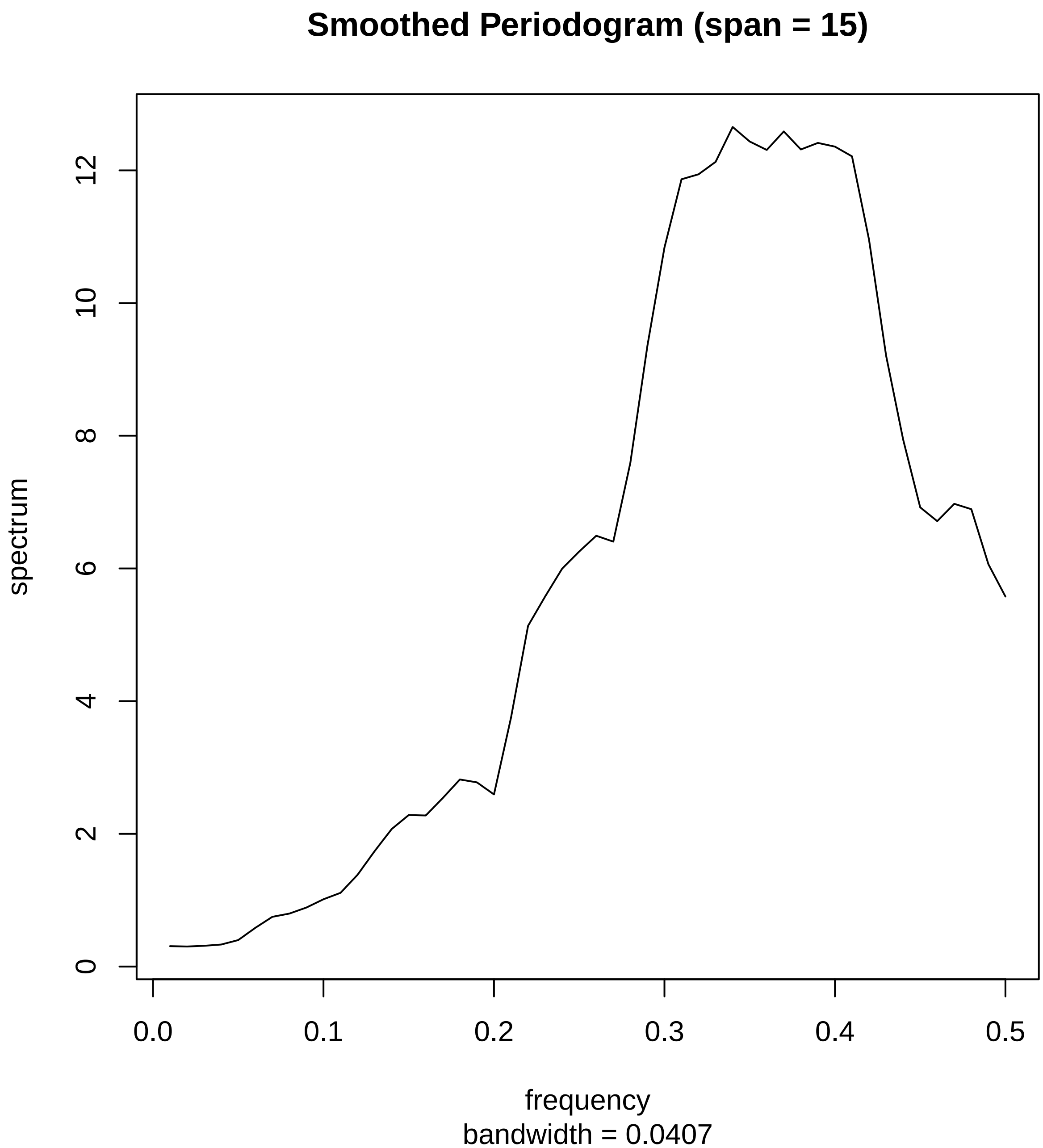
```
set.seed(123)

ma_model <- arima.sim(n = 100, model = list(ma = -0.9), sd = 2)

# Raw periodogram
spec.pgram(ma_model, log = "no",main = "Periodogram of
          AR(1) Process")
```



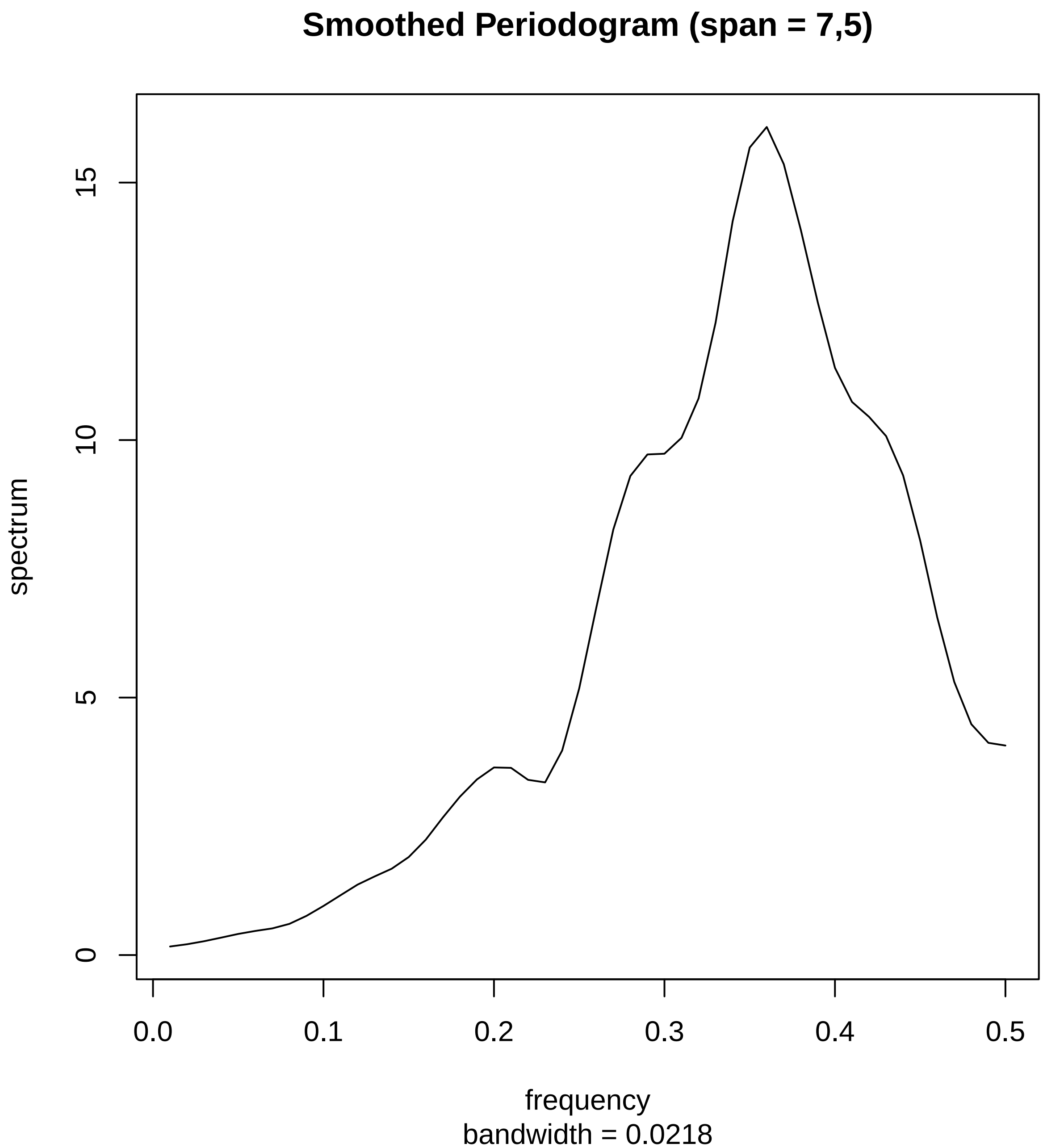
```r
# Smoothed Periodogram
spec.pgram(ma_model, log = "no", spans = c(5), main =
             "Smoothed Periodogram (span = 5)")
```




```r
spec.pgram(ma_model, log = "no", spans = c(15), main =
             "Smoothed Periodogram (span = 15)")
```



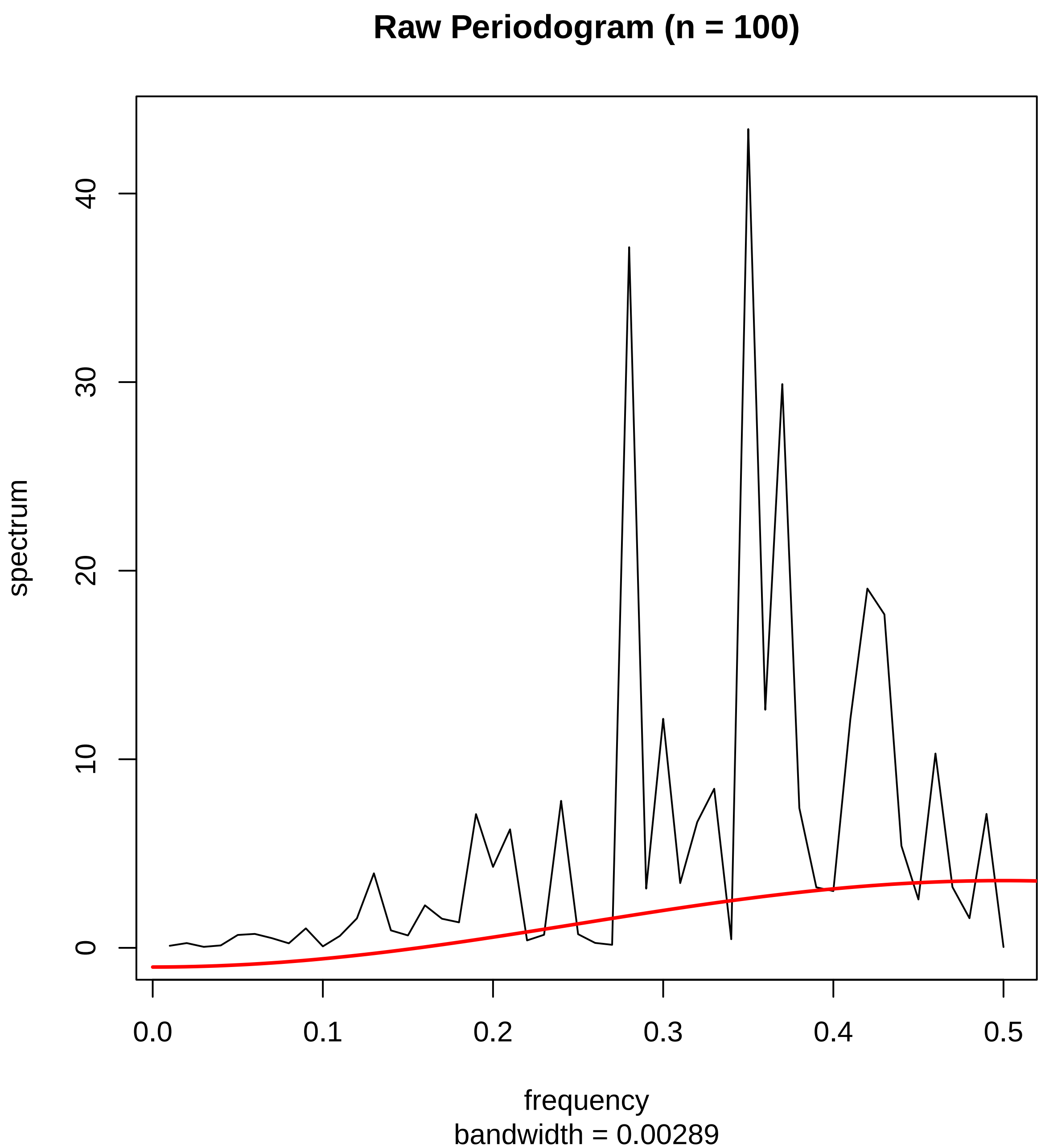

```
spec.pgram(ma_model, log = "no", spans = c(7,5), main =
                "Smoothed Periodogram (span = 7,5)")
```



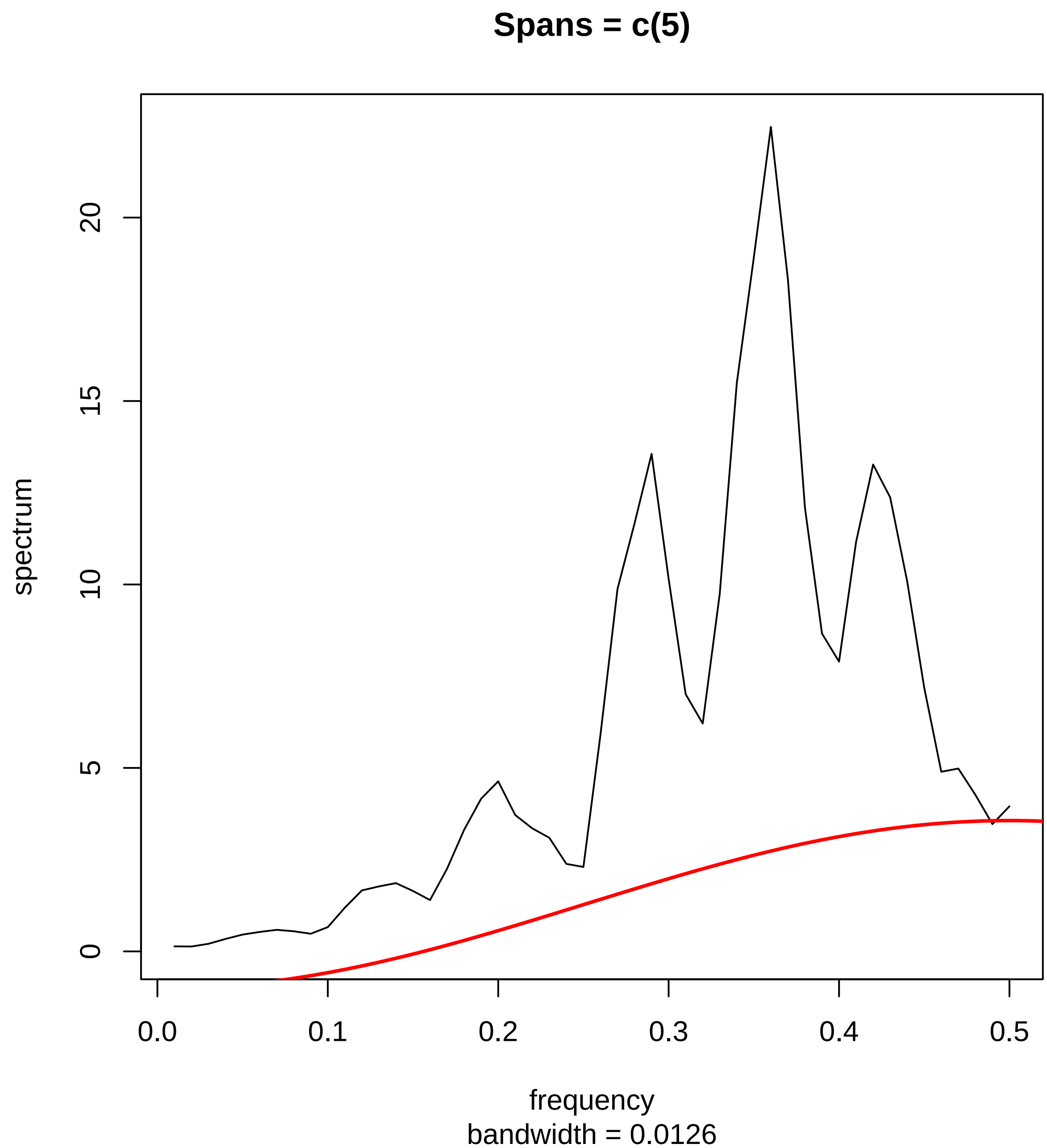
As we increase the span 5, 7, and 15, we observe a lot smoother with less fluctuations (rise & dips) with 15 having the least rise and dips. Overall, we can observe a notice-able increase trend. (b)

```
set.seed(123)

true_spectrum <- function(omega) {
  (7.24 / pi) * ((1 - 1.8 * cos(omega)) / 1.81)
}

omega_vals <- seq(0, pi, length.out = 500)
theoretical_vals <- true_spectrum(2 * pi * omega_vals)
```

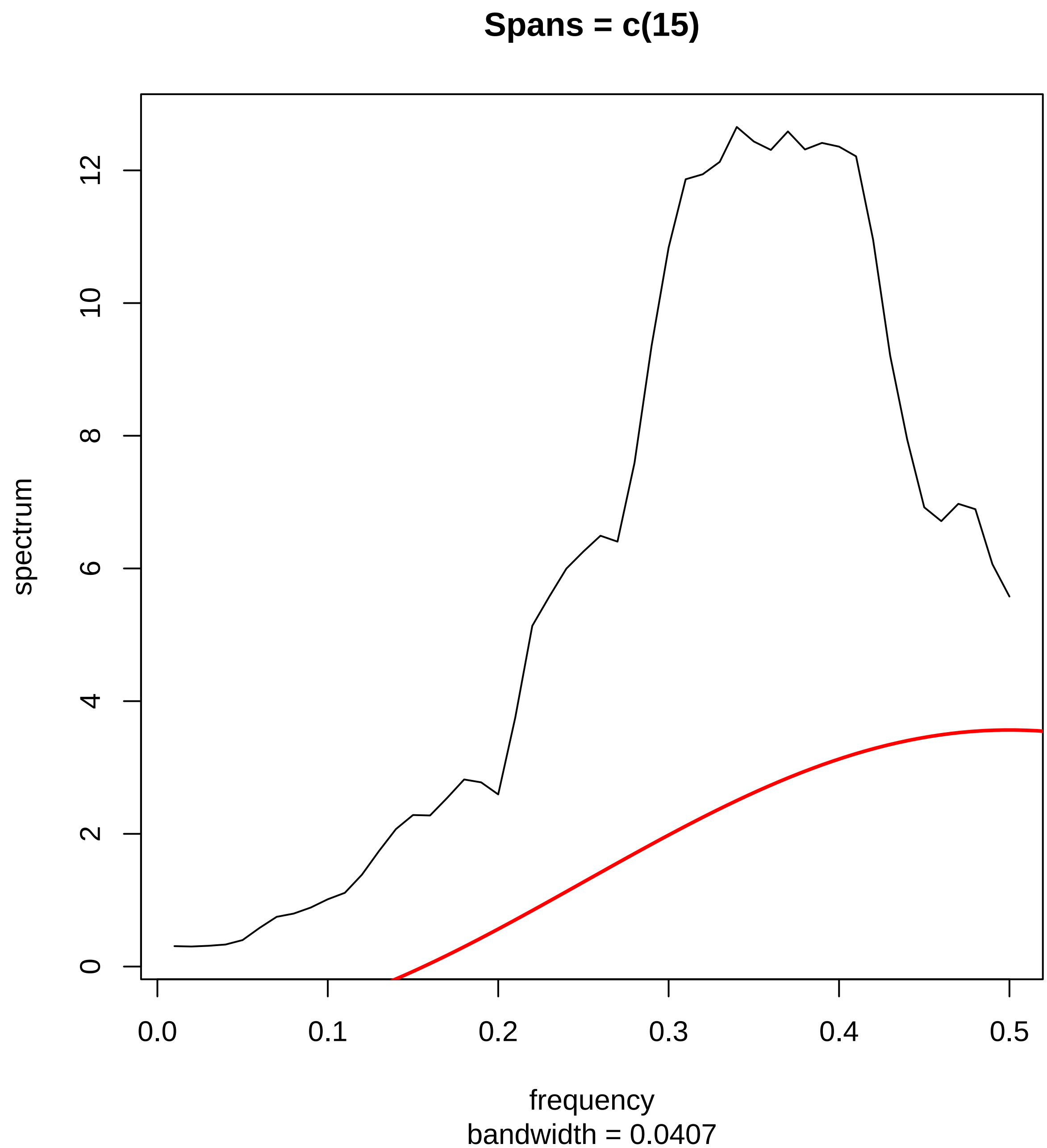
```r
spec.pgram(ma_model, log = "no", main = "Raw Periodogram (n = 100)")
lines(omega_vals, theoretical_vals, col = "red", lwd = 2)
```



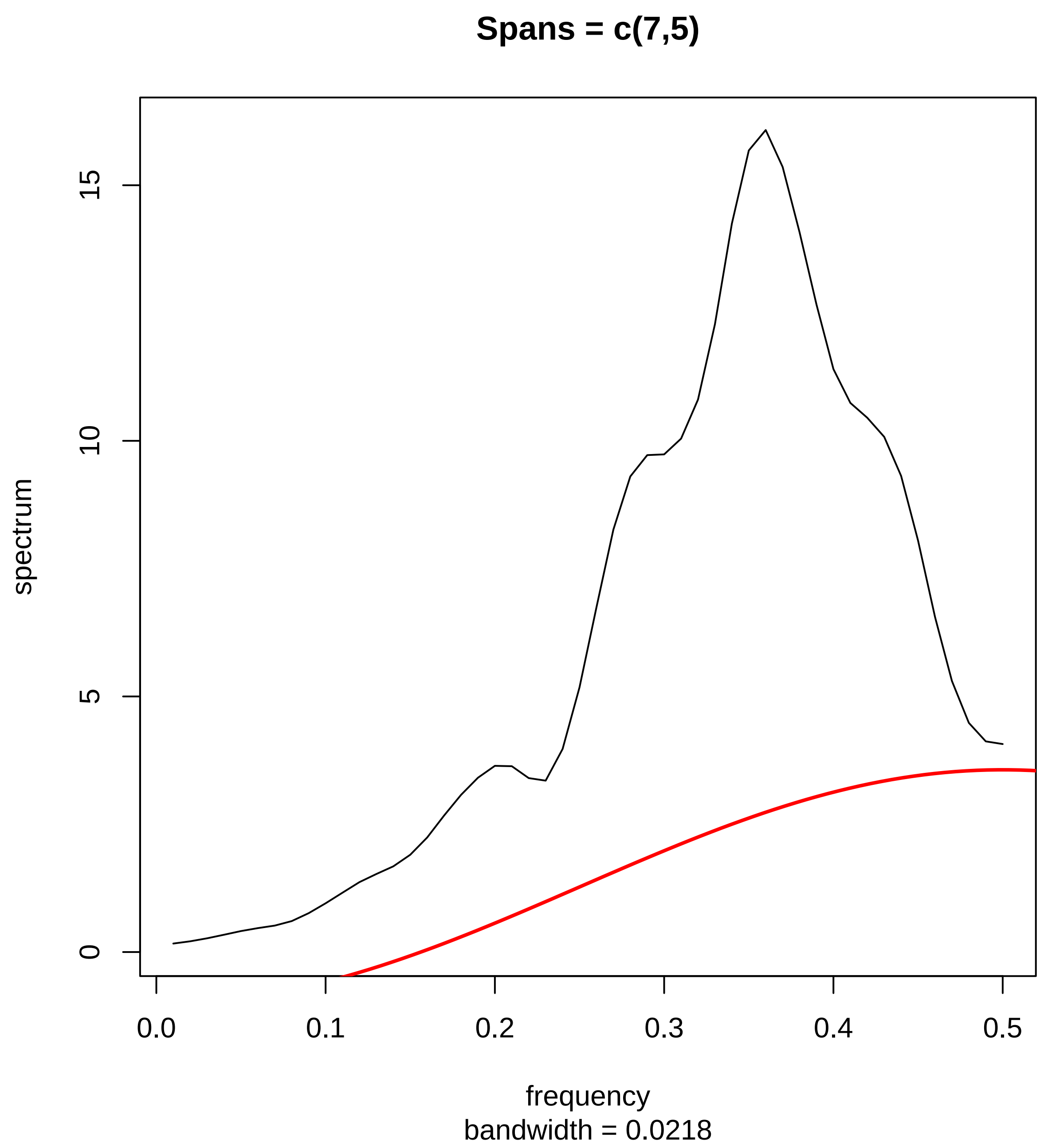

```r
spec.pgram(ma_model, spans = c(5), log = "no", main = "Spans = c(5)")
lines(omega_vals, theoretical_vals, col = "red", lwd = 2)
```




```r
spec.pgram(ma_model, spans = c(15), log = "no", main = "Spans = c(15)")
lines(omega_vals, theoretical_vals, col = "red", lwd = 2)
```

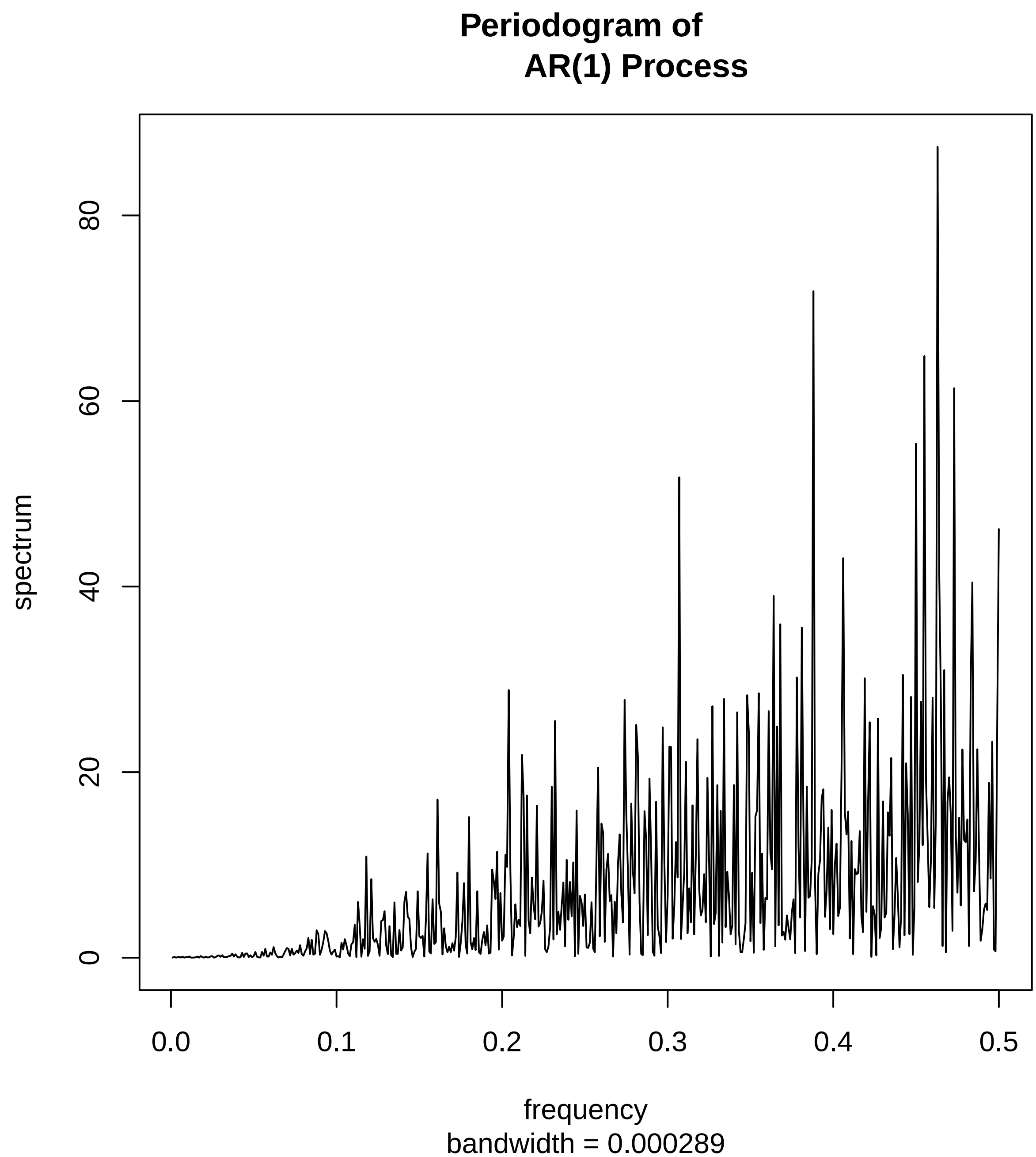



```r
spec.pgram(ma_model, spans = c(7, 5), log = "no", main = "Spans = c(7,5)")
lines(omega_vals, theoretical_vals, col = "red", lwd = 2)
```

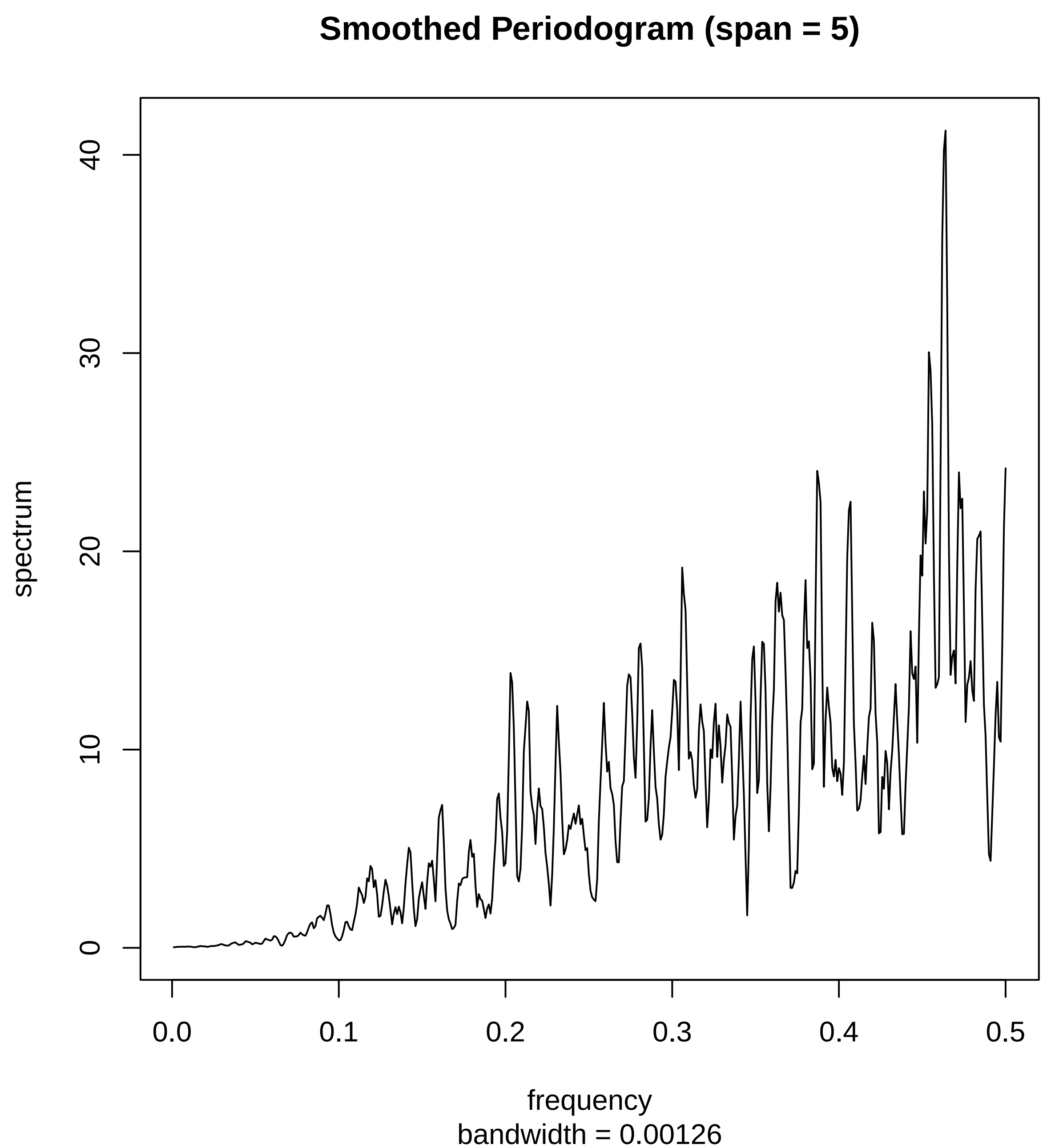


(c)

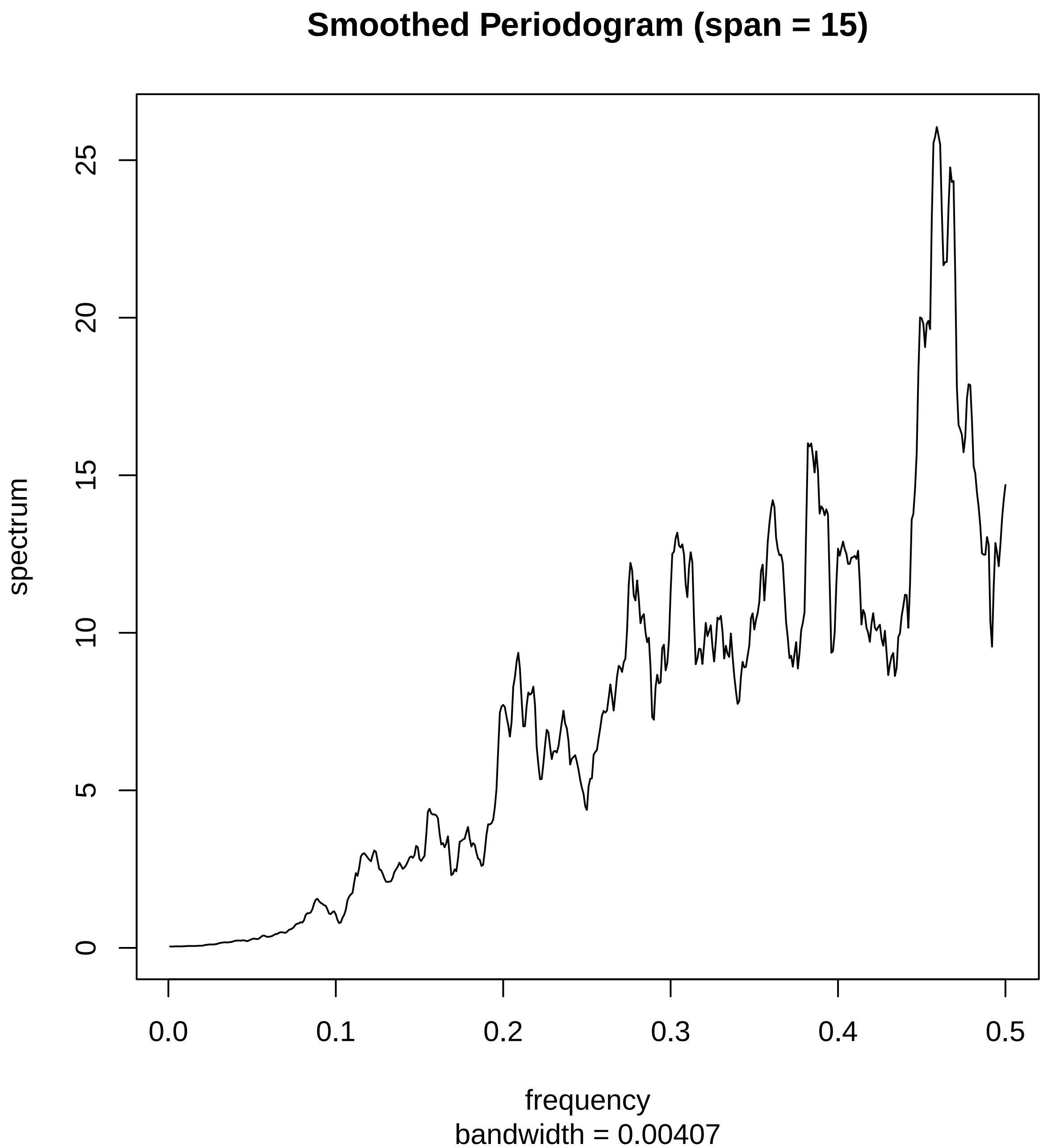
```
set.seed(123)

ma_model2 <- arima.sim(n = 1000, model = list(ma = -0.9), sd = 2)

# Raw periodogram
spec.pgram(ma_model2, log = "no",main = "Periodogram of
           AR(1) Process")
```



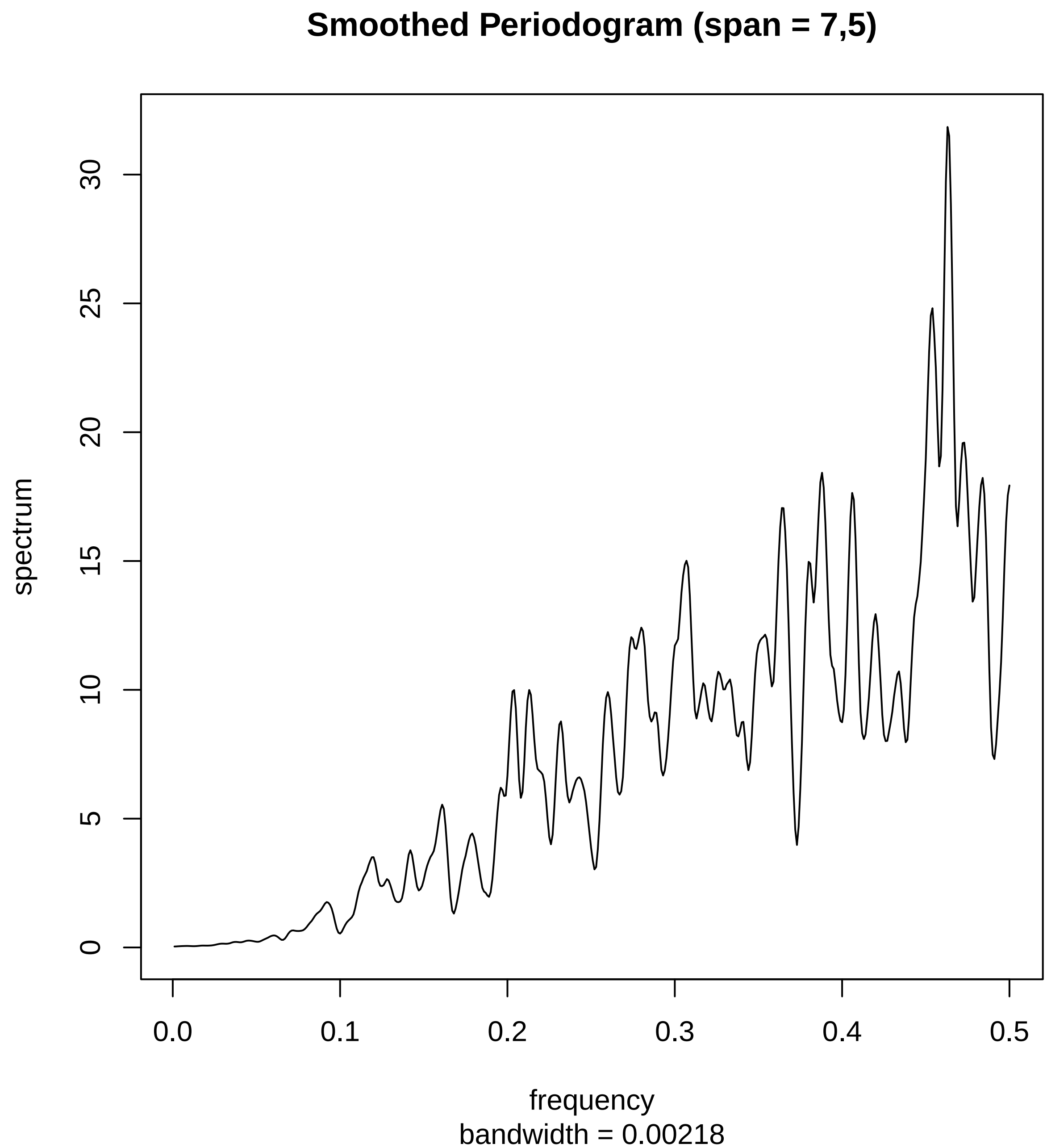

```
# Smoothed Periodogram
spec.pgram(ma_model2, log = "no", spans = c(5), main =
              "Smoothed Periodogram (span = 5)")
```




```
spec.pgram(ma_model2, log = "no", spans = c(15), main =
             "Smoothed Periodogram (span = 15)")
```




```
spec.pgram(ma_model2, log = "no", spans = c(7,5), main =
               "Smoothed Periodogram (span = 7,5)")
```

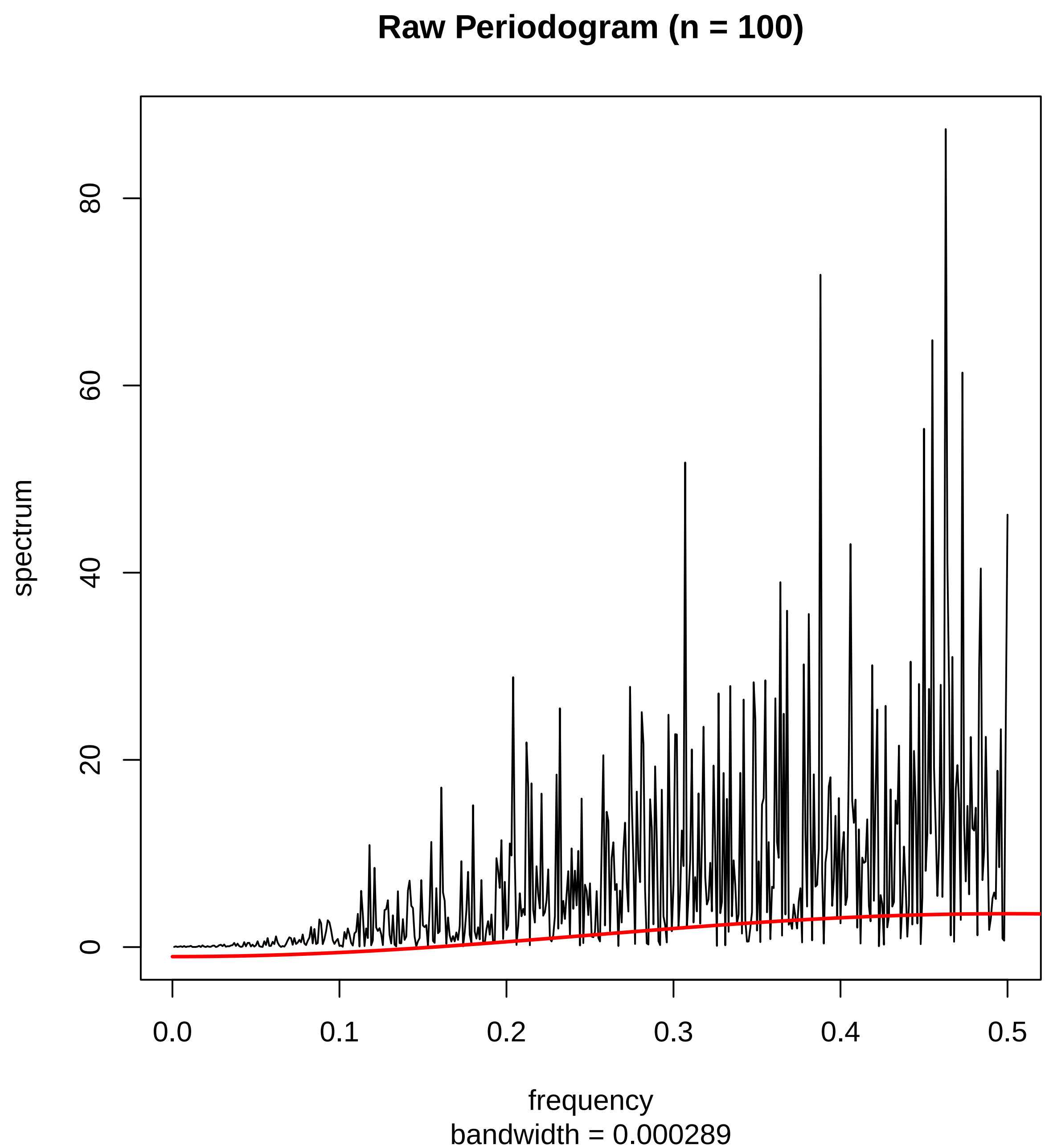


As we increase our smoothing span from 5 to 15, we start to observe a reduction in noise where it becomes much more smoother with less fluctuations (rise & dips). We can see the transition from span 5 to 15 where more high-frequency noises are reduced.

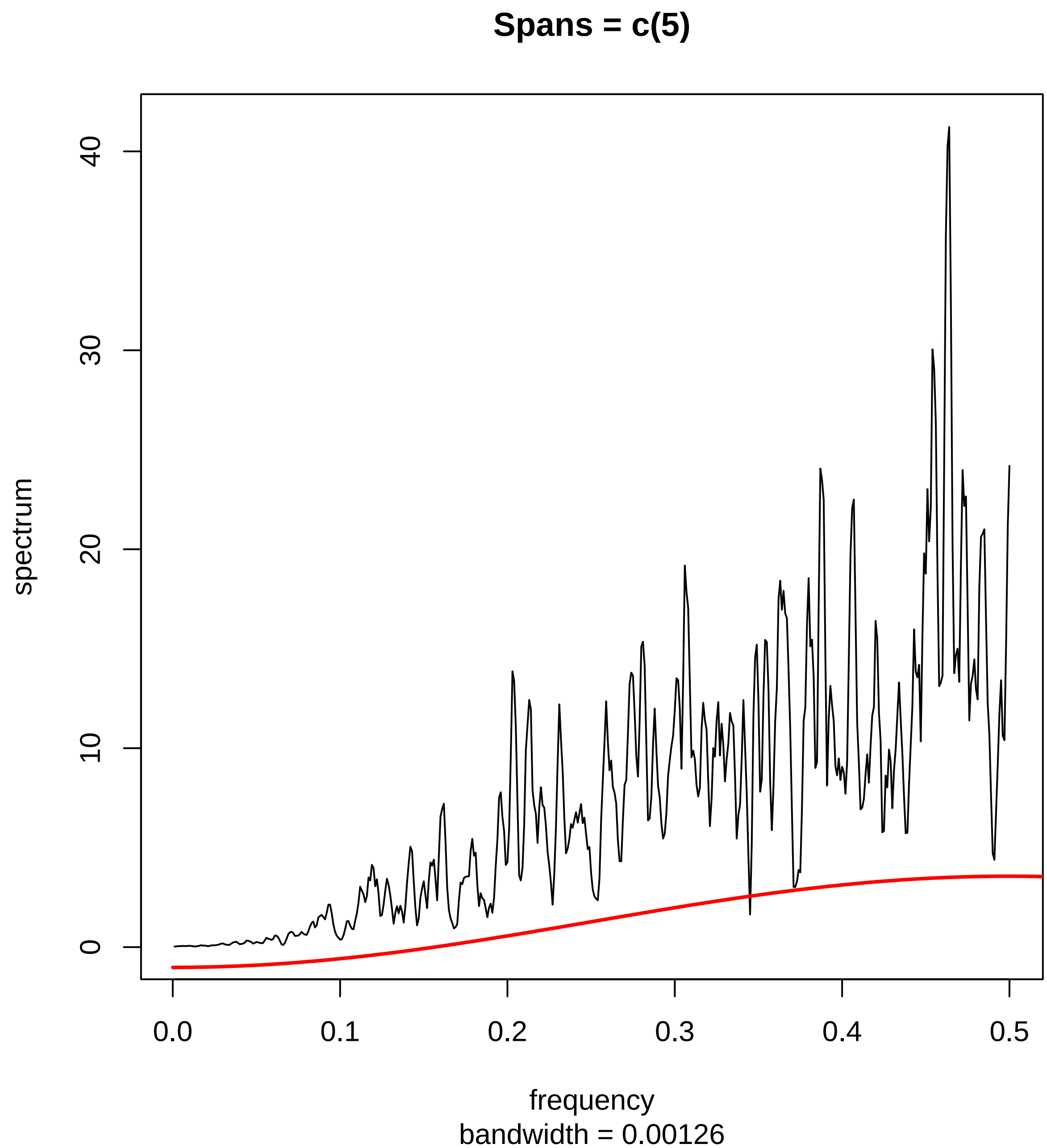
(d)

```
set.seed(123)

omega_vals <- seq(0, pi, length.out = 500)
theoretical_vals <- true_spectrum(2 * pi * omega_vals)
```

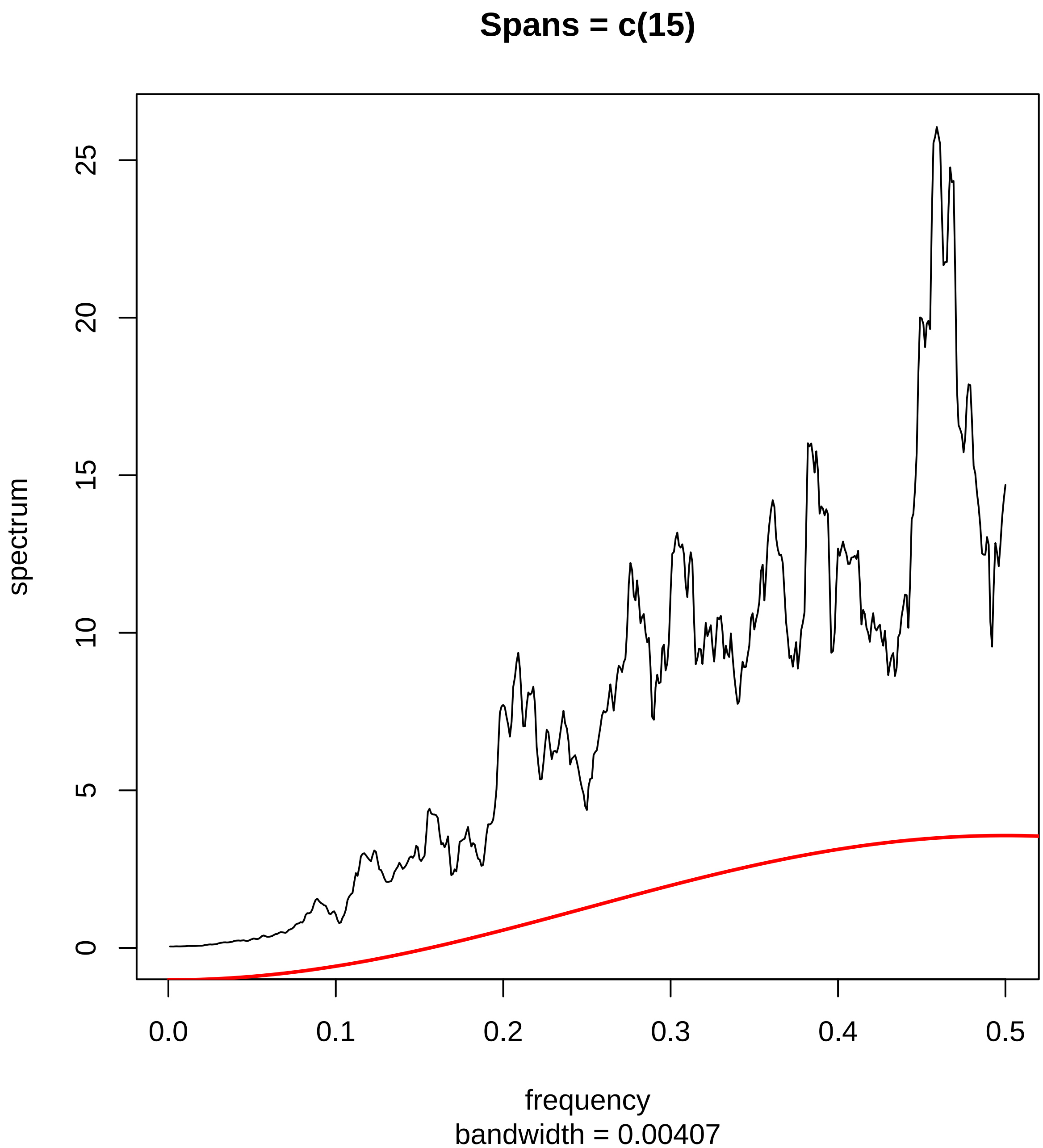
```r
spec.pgram(ma_model2, log = "no", main = "Raw Periodogram (n = 100)")
lines(omega_vals, theoretical_vals, col = "red", lwd = 2)
```



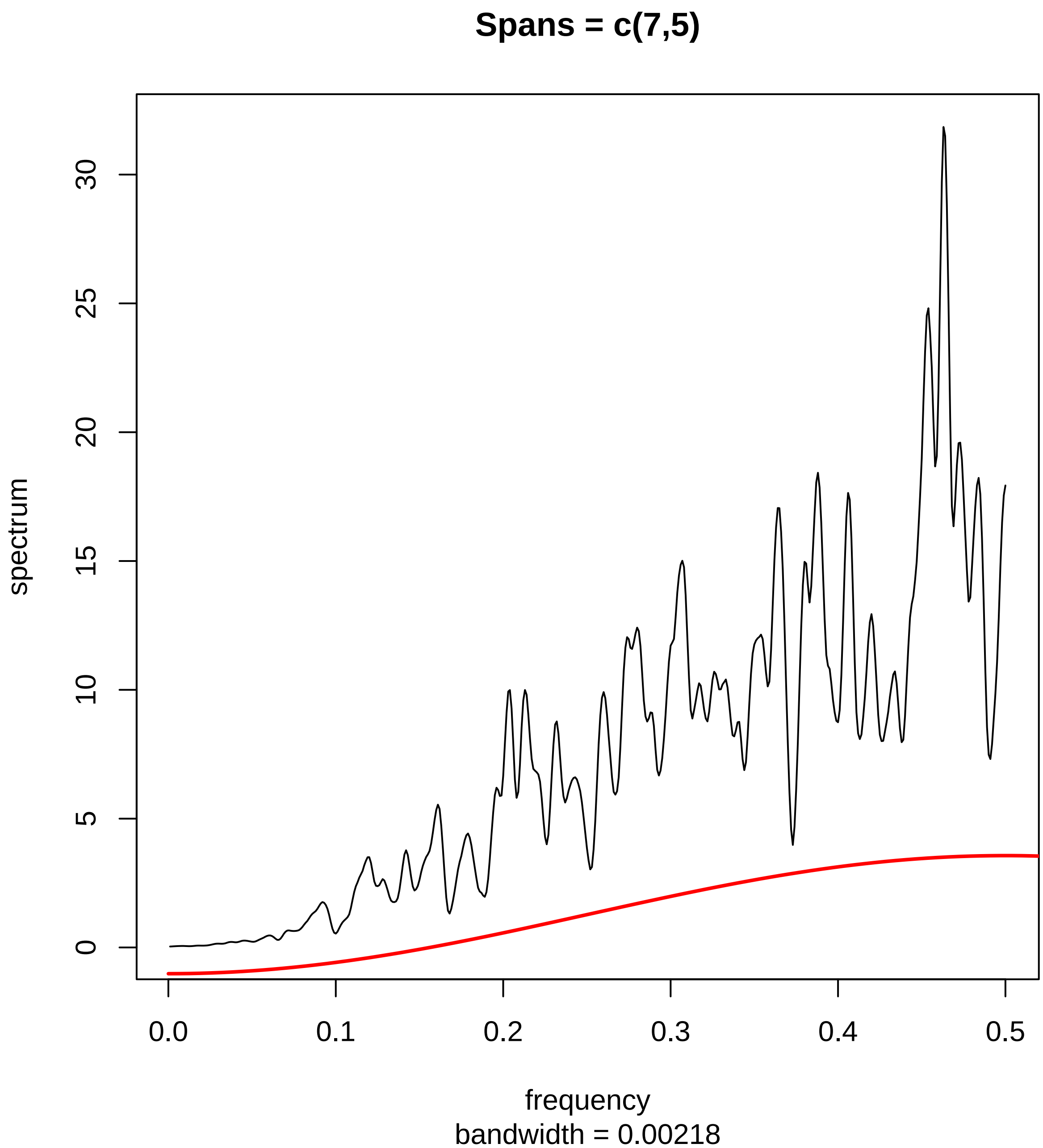

```r
spec.pgram(ma_model2, spans = c(5), log = "no", main = "Spans = c(5)")
lines(omega_vals, theoretical_vals, col = "red", lwd = 2)
```




```r
spec.pgram(ma_model2, spans = c(15), log = "no", main = "Spans = c(15)")
lines(omega_vals, theoretical_vals, col = "red", lwd = 2)
```




```r
spec.pgram(ma_model2, spans = c(7, 5), log = "no", main = "Spans = c(7,5)")
lines(omega_vals, theoretical_vals, col = "red", lwd = 2)
```



Since we used n = 1000, we expect the periodogram to be closer to the true spectral density than when n = 100. Observing our graphs, while increasing the span of the periodogram reduces variability for both n's, increasing n improves our periodogram's ability to capture the spectral density better.