STAT 443: Lab 5

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Question 1

The process in equation 1 is an AR(3) Process with an order p =

p(3) = alpha^(k) for k = 1, 2, 3 alpha^(1) = 0.8 alpha^(2) = -1/3 alpha^(3) = 0.6/sqrt(3)

```
# Process AR(3) order in R studio
alpha1 <- 0.8
alpha2 <- -1/3
alpha3 <- 0.6 / sqrt(3)

variance <- 0.8^2
```

alpha^(1) = 0.8 alpha^(2) = -1/3 alpha^(3) = 0.6 / sqrt(3)

Question 2

Explain how to recognize this process based on an observed time series and how to determine its order.

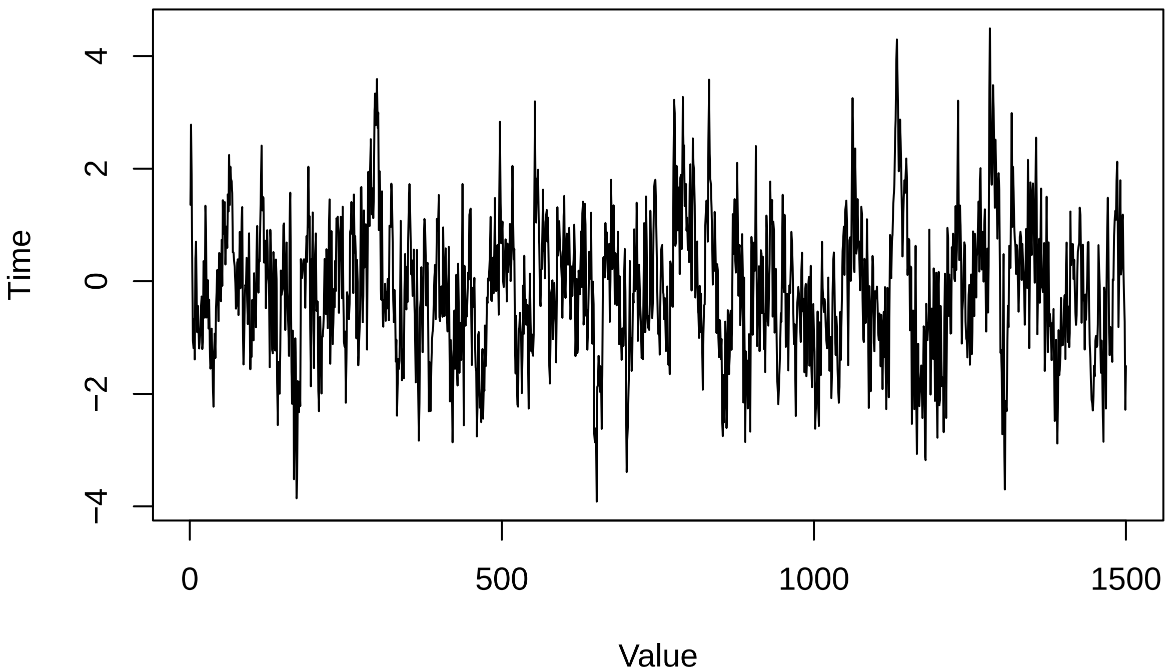
We need to examine for stationarity by its constant mean and invertibility along with it's invertibility. We can also analyze through the ACF plot, where a AR(3) process should show a gradual / smooth decaying pattern. In addition, we need to plot its PACF when it cuts off at lag 3 and ensure that there are no autocorrelation.

Question 3

Use the command set.seed(23456) to set the random seed for reproducibility and then use function arima.sim() to generate 1500 observations from the model in (1). Plot the simulated time series.

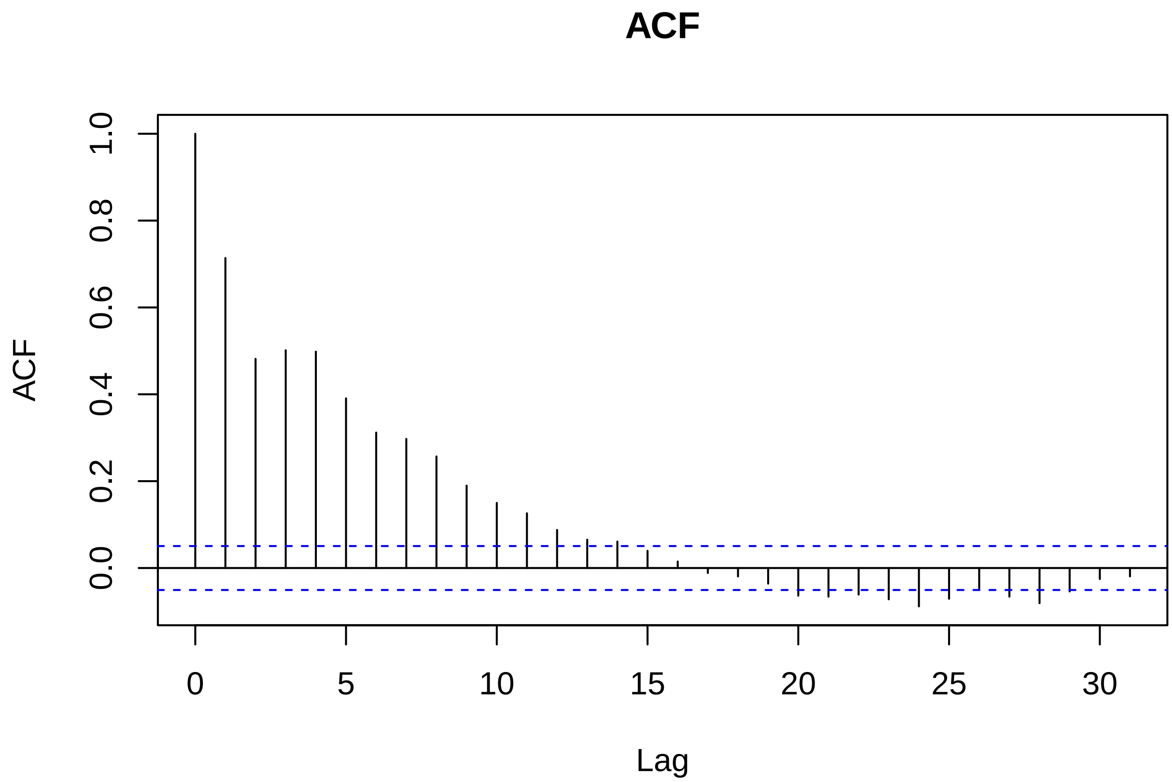
```
set.seed(23456)

q3 <- arima.sim(model = list(ar = c(alpha1, alpha2, alpha3)), n = 1500, sd = sqrt(variance))
plot(q3, xlab = "Value", ylab = "Time")
```



Question 4

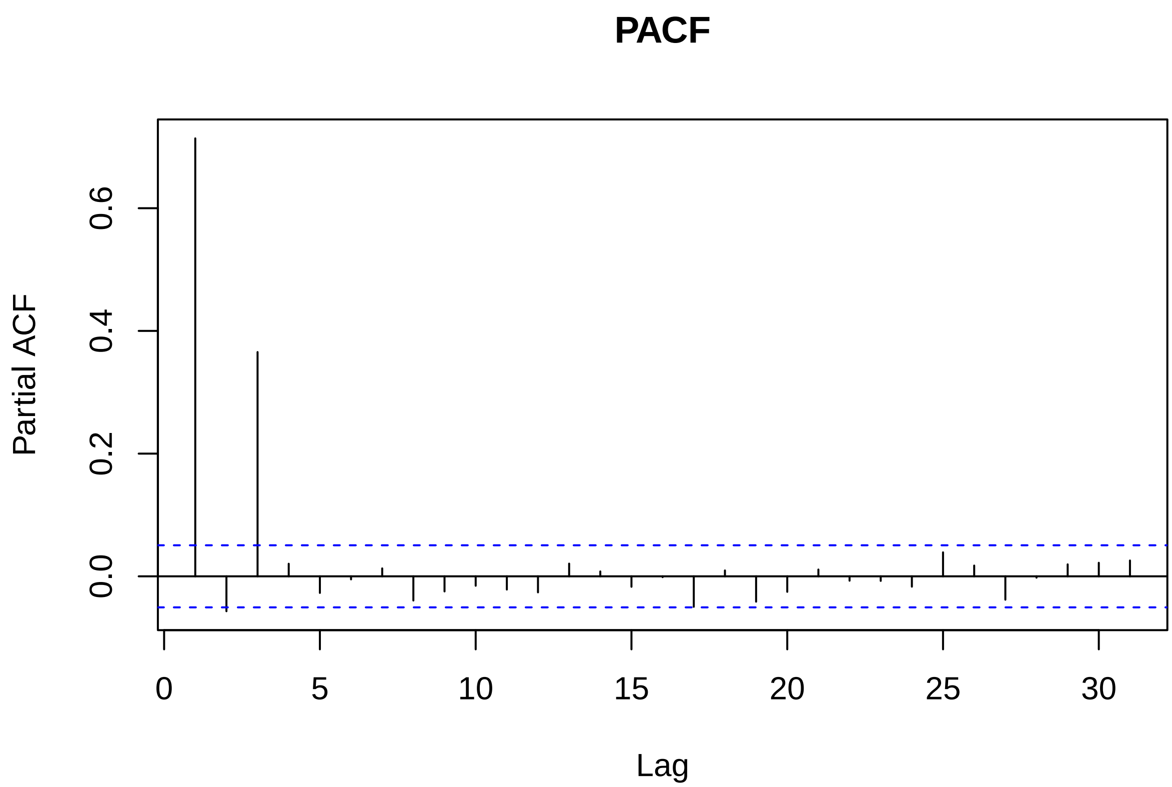
```
q3_ts <- ts(q3)
acf(q3_ts, main = "ACF")
```



The sample ACF plot shows a gradual decay pattern starting at lag 0 with some osciliation after lag 16. After lag 14, we start to see insignificant results within the boundary line. I believe that our results align with our AR(3) model because we see alternating signs which would cause osciliation and the decaying pattern is also exhibited in an AR model.

Question 5

```
pacf(q3_ts, main = "PACF")
```



Based on the sample pcaf plot, we can observe a strong positive spike at lag 1, a negative small spike at lag 2, a smaller positive spike at lag 3, and all values fall within the significant boundary line beyond lag 3. As it cuts off at lag 3, this suggest it is an AR(3) process. Furthermore, it is as we expected because for our AR(3) process we see our alphas 1-3 at 0.8, -1/3, and ~0.346 respectively which corresponds to our PACF plot lags 1-3.

Question 6

```
set.seed(23456)

# Simulate AR(3) process
Z <- rnorm(1500, mean = 0, sd = sqrt(variance))  # White noise
X <- numeric(1500)  # Initialize time series

# Generate time series data
X[1:3] <- rnorm(3)
for (t in 4:1500) {
  X[t] <- alpha1 * X[t-1] + alpha2 * X[t-2] + alpha3 * X[t-3] + Z[t]
}

# Fit AR(3) model with different estimation methods
model_css_ml <- arima(X, order = c(3, 0, 0), method = "CSS-ML")
model_ml <- arima(X, order = c(3, 0, 0), method = "ML")
model_css <- arima(X, order = c(3, 0, 0), method = "CSS")
```

```r
# Extract parameter estimates
results <- data.frame(
  Estimation = c("True Values", "CSS-ML", "ML", "CSS"),
  alpha1 = c(alpha1, coef(model_css_ml)[1], coef(model_ml)[1], coef(model_css)[1]),
  alpha2 = c(alpha2, coef(model_css_ml)[2], coef(model_ml)[2], coef(model_css)[2]),
  alpha3 = c(alpha3, coef(model_css_ml)[3], coef(model_ml)[3], coef(model_css)[3]),
  Variance = c(variance, model_css_ml$variance, model_ml$variance, model_css$variance)
)

# Rounding
results[, -1] <- round(results[, -1], 3)

# Build table
kable(results,
      caption = "Comparison of AR(3) Parameter Estimates Using Different Estimation Methods")
```

Table 1: Comparison of AR(3) Parameter Estimates Using Different Estimation Methods

Estimation	alpha1	alpha2	alpha3	Variance
True Values	0.800	-0.333	0.346	0.64
CSS-ML	0.763	-0.320	0.367	0.64
ML	0.763	-0.320	0.367	0.64
CSS	0.763	-0.321	0.368	0.64

CSS-ML, ML, and CSS all have similar first and second lag coefficient (alpha 1 & 2) which is slightly lower than the true value while their third lag coefficient (alpha 3) is slightly larger than the true value. The estimated variance is consistently 0.64 across all methods, exactly matching the true variance.