STAT 443: Lab 3

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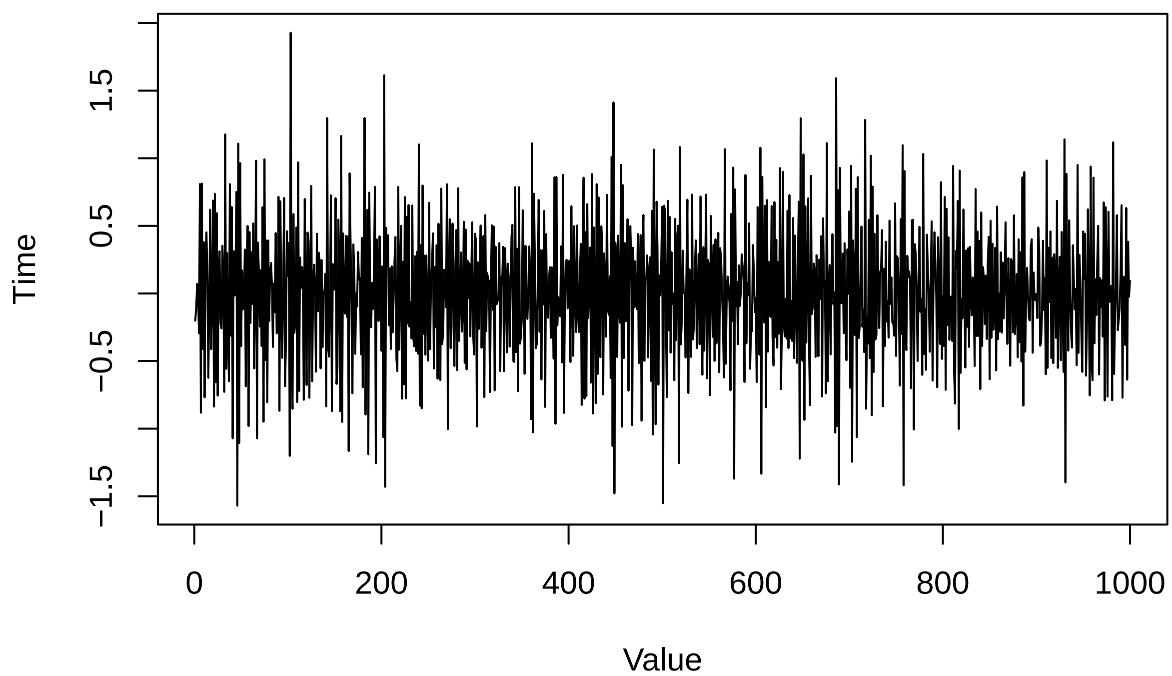
Question 1

(a)

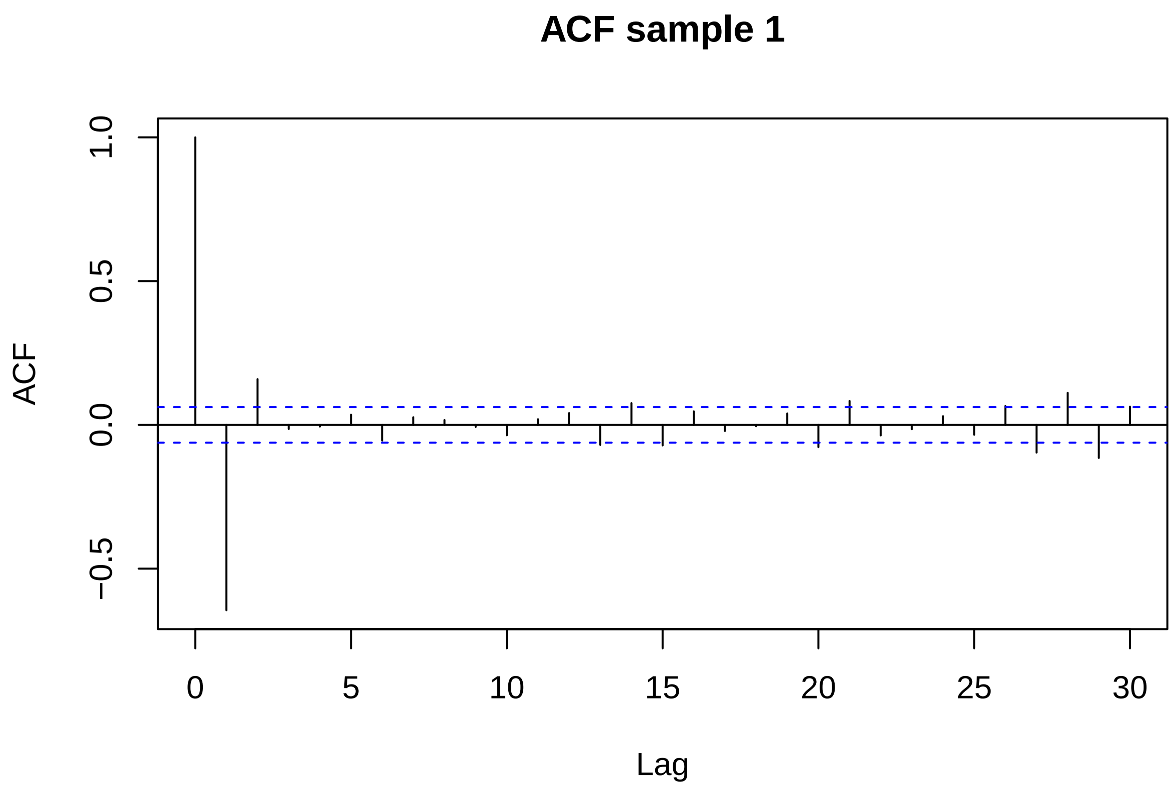
```
# this is where your R code goes

q1a <- arima.sim(model = list(ma = c(-1.3, 0.4)), n = 1000, sd = sqrt(0.1))
plot(q1a, xlab = "Value", ylab = "Time")
```



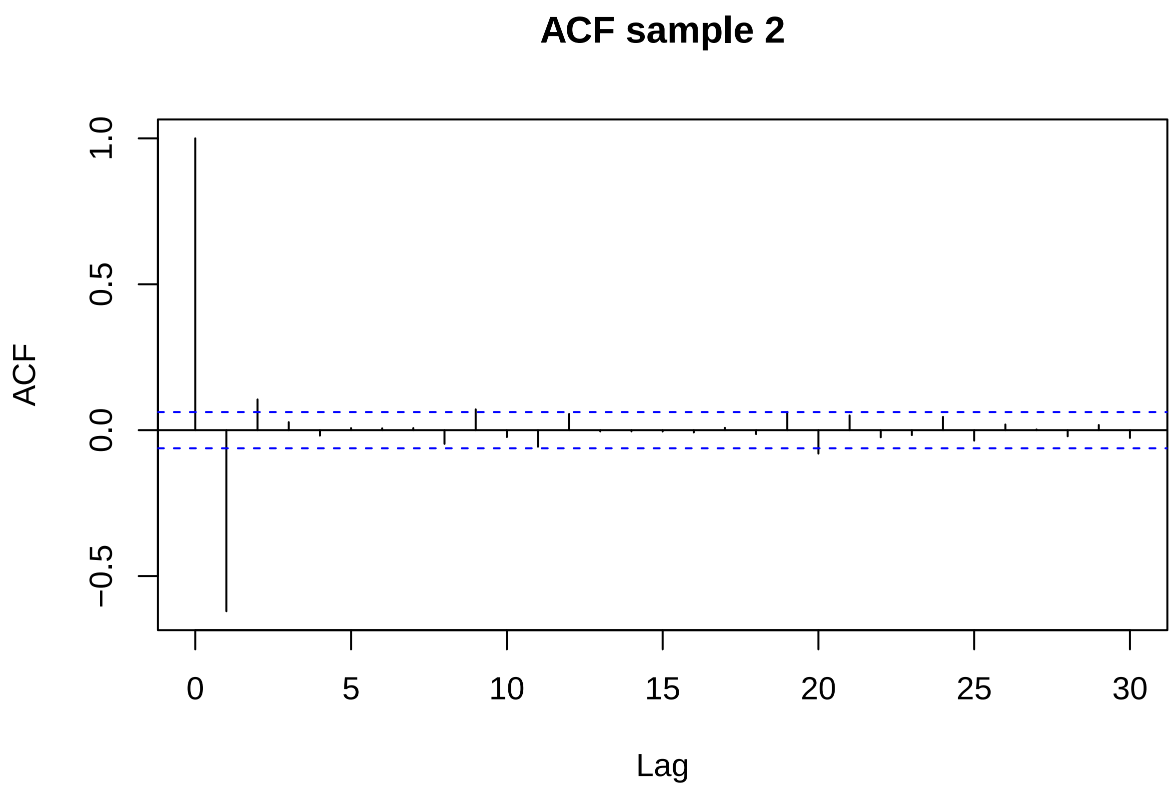
The simulated data displays a stationary process that fluctuates around the mean = 0.0

(b)

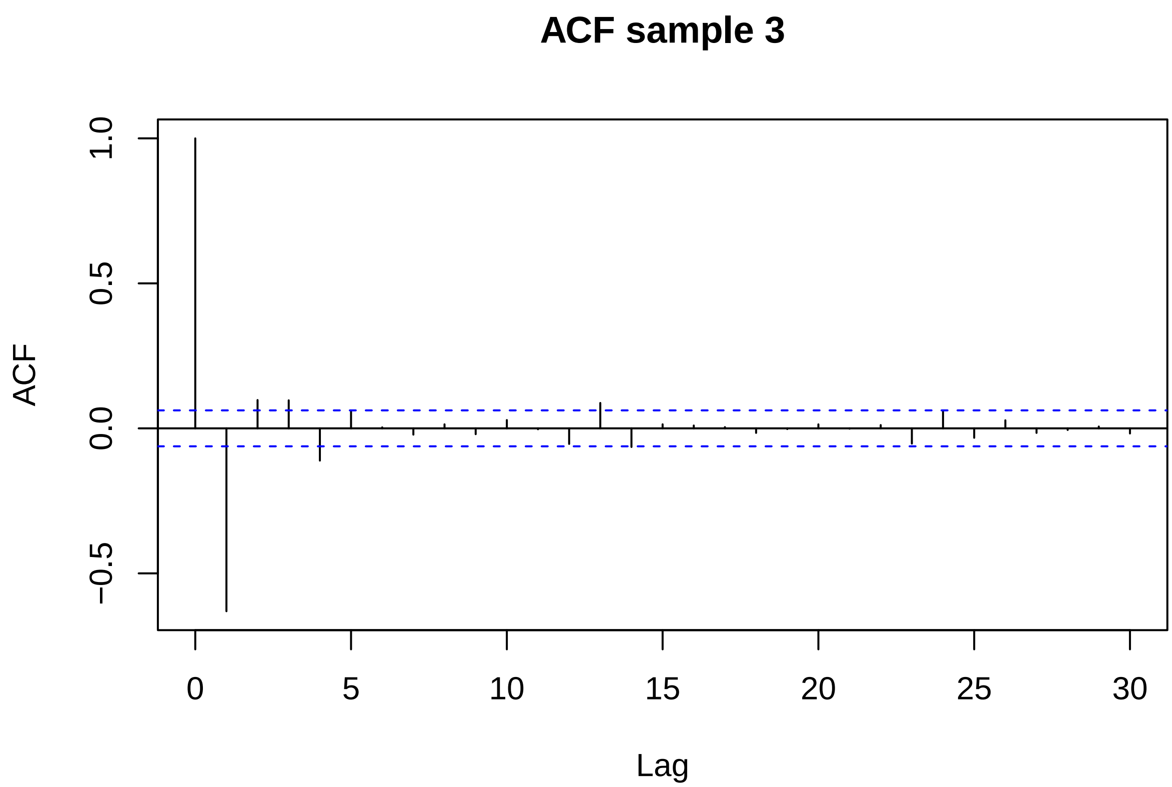
```r
# this is where your R code goes
q1a_ts <- ts(q1a)
acf(q1a, main = "ACF sample 1")
```



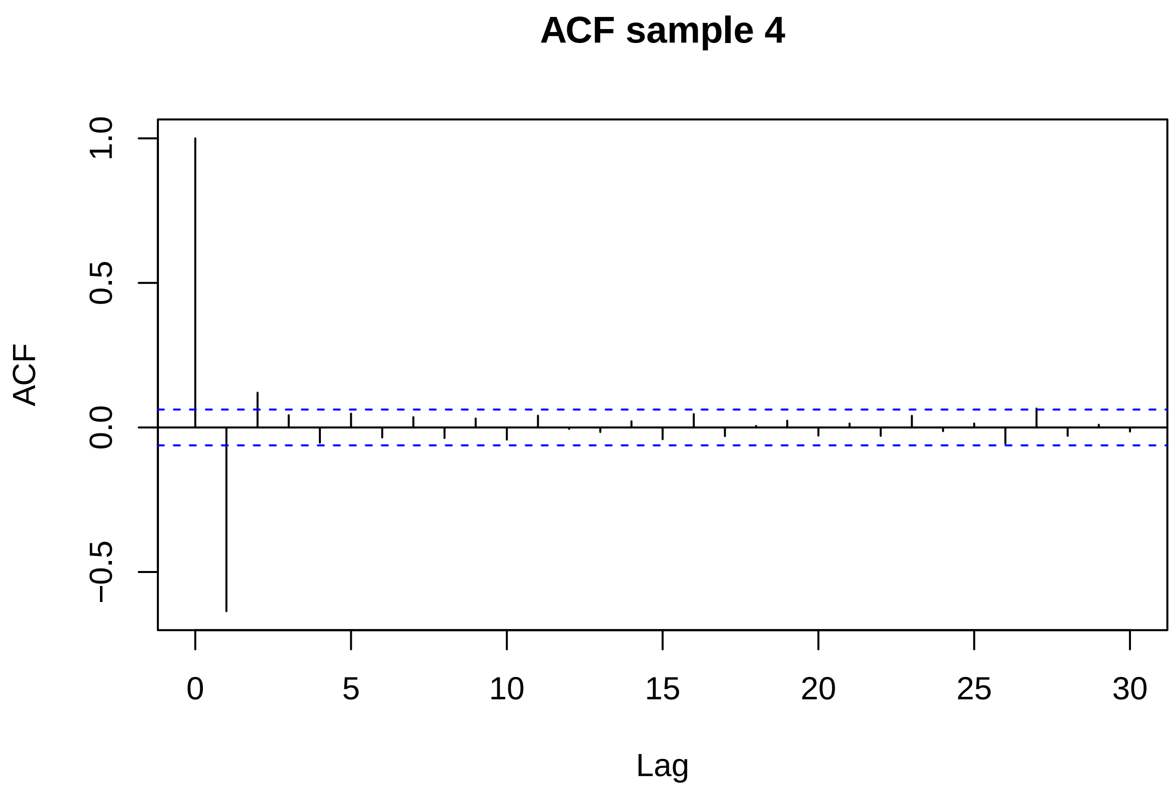

```r
sample2 <- arima.sim(model = list(ma = c(-1.3, 0.4)), n = 1000, sd = sqrt(0.1))
sample2_ts <- ts(sample2)
acf(sample2_ts, main = "ACF sample 2")
```




```r
sample3 <- arima.sim(model = list(ma = c(-1.3, 0.4)), n = 1000, sd = sqrt(0.1))
sample3_ts <- ts(sample3)
acf(sample3_ts, main = "ACF sample 3")
```




```r
sample4 <- arima.sim(model = list(ma = c(-1.3, 0.4)), n = 1000, sd = sqrt(0.1))
sample4_ts <- ts(sample4)
acf(sample4_ts, main = "ACF sample 4")
```



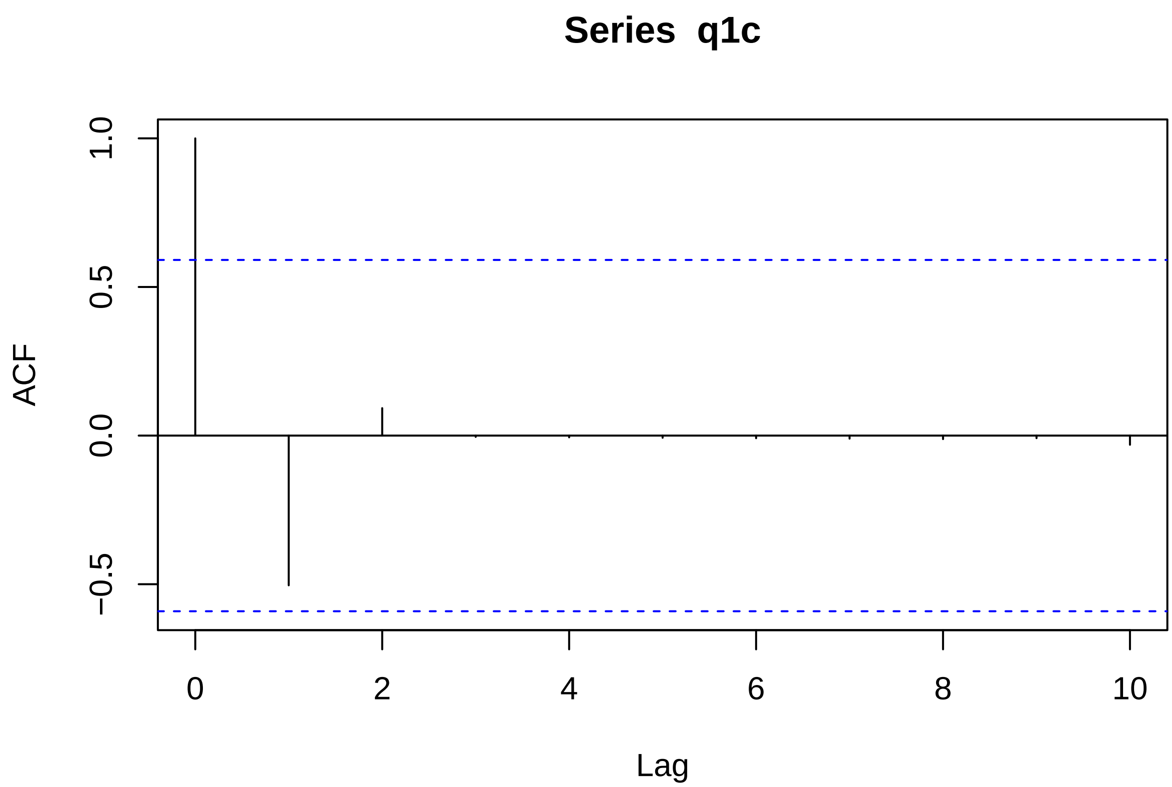
Repeating the simulation 4 times, we can observe a MA(2) process with significant decrease to a negative ACF at lag 1 then we can see a steady increase and decrease of ACF values close to 0 after lag 2.

(c)

```
# this is where your R code goes
q1c <- ARMAacf(ma = c(-1.3, 0.4), lag.max = 10)
q1c
```

```
##         0          1          2          3          4          5          6
##  1.0000000 -0.6385965  0.1403509  0.0000000  0.0000000  0.0000000  0.0000000
##         7          8          9         10
##  0.0000000  0.0000000  0.0000000  0.0000000
```

```
acf(q1c)
```



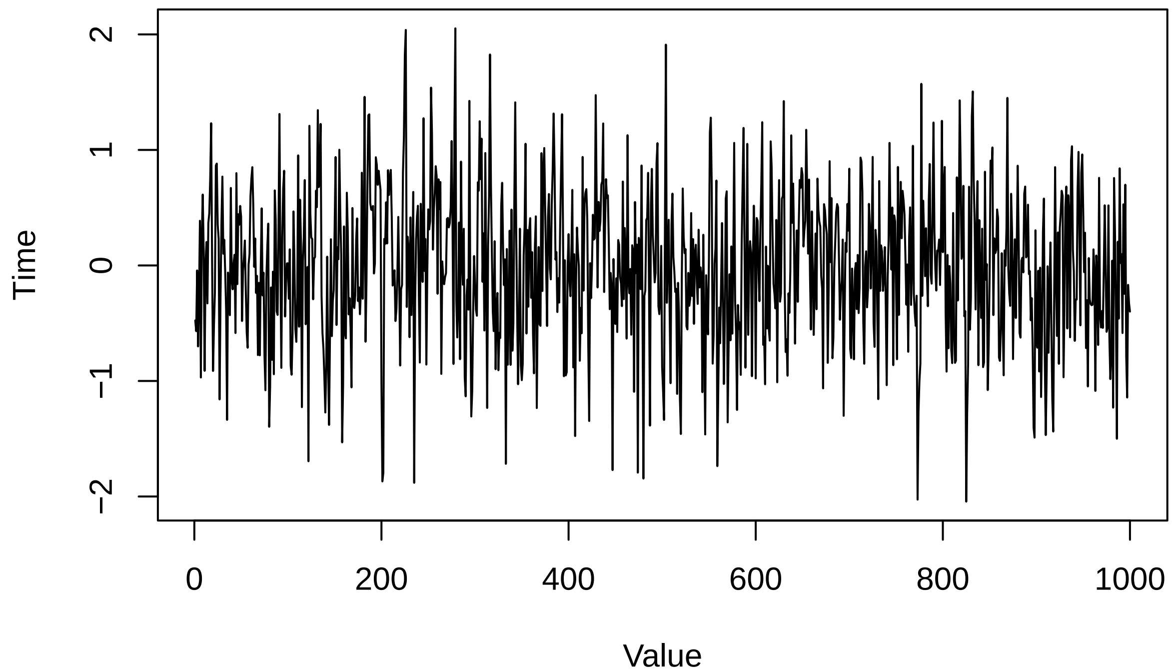
The ARMAacf function calculates the theoretical autocorrelation function for our MA(2) process up to lag.max = 10. The output shows the correlation at different lags, especially with non-zero correlation at lags 1 and 2, showing a MA(2) process.

(d) The sample ACF has non-zero and significant 'spiked' correlations at lags 1 and 2 with non-significant values for the other lags. As expected, the auto correlations drop close to zero after lag 2 which is similar with the theoretical ACF for an MA(2) model. The differences between our simulated data and the theoretical pattern could be due to sampling error

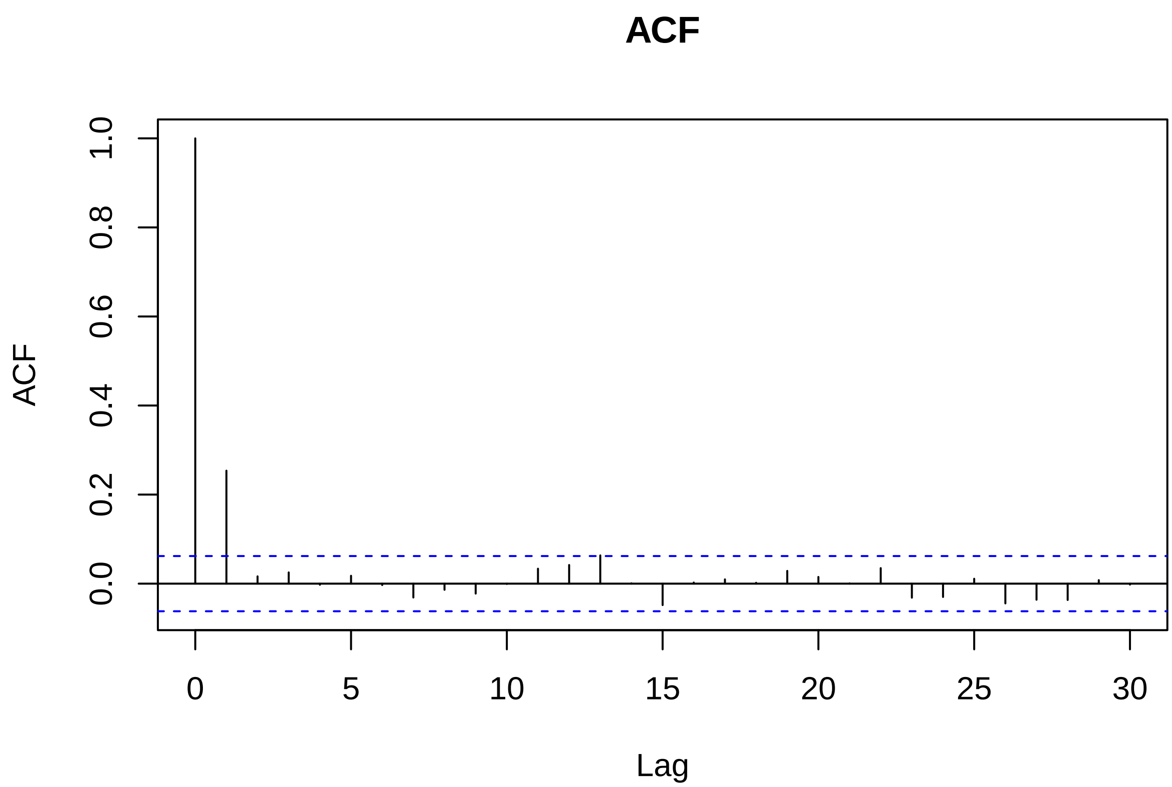
Question 2

(a)

```
# this is where your R code goes
q2a <- arima.sim(model = list(ma = c(0.25)), n =  1000, sd = sqrt(0.4))
plot(q2a, xlab = "Value", ylab = "Time")
```



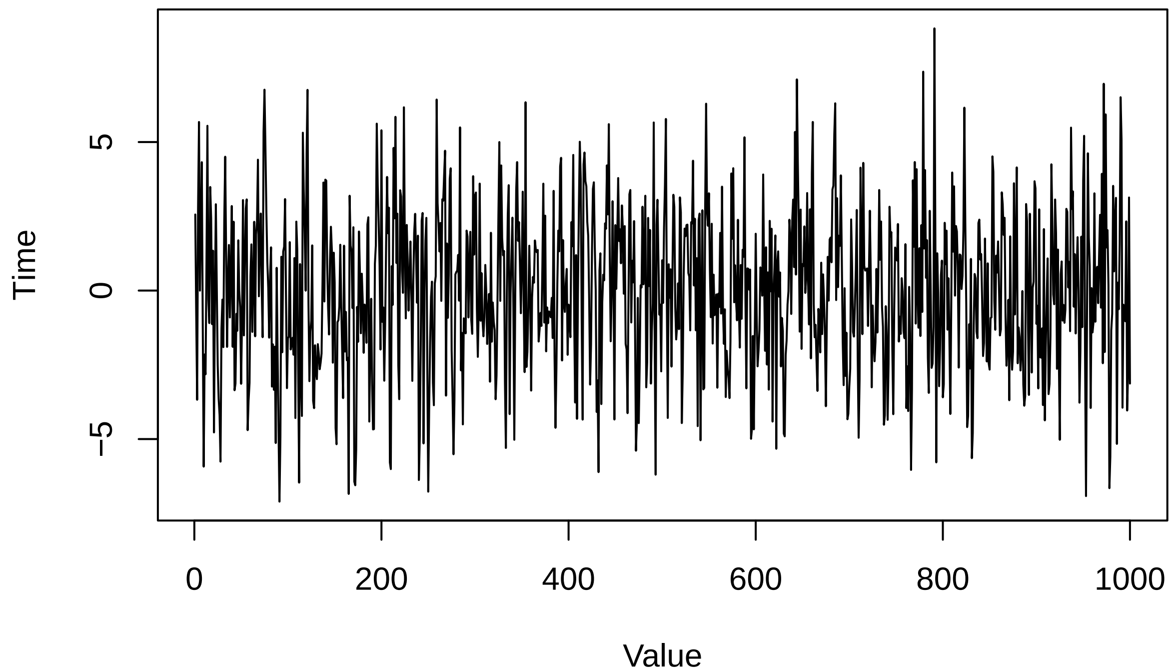

```
q2a_ts <- ts(q2a)
acf(q2a_ts, main = "ACF")
```



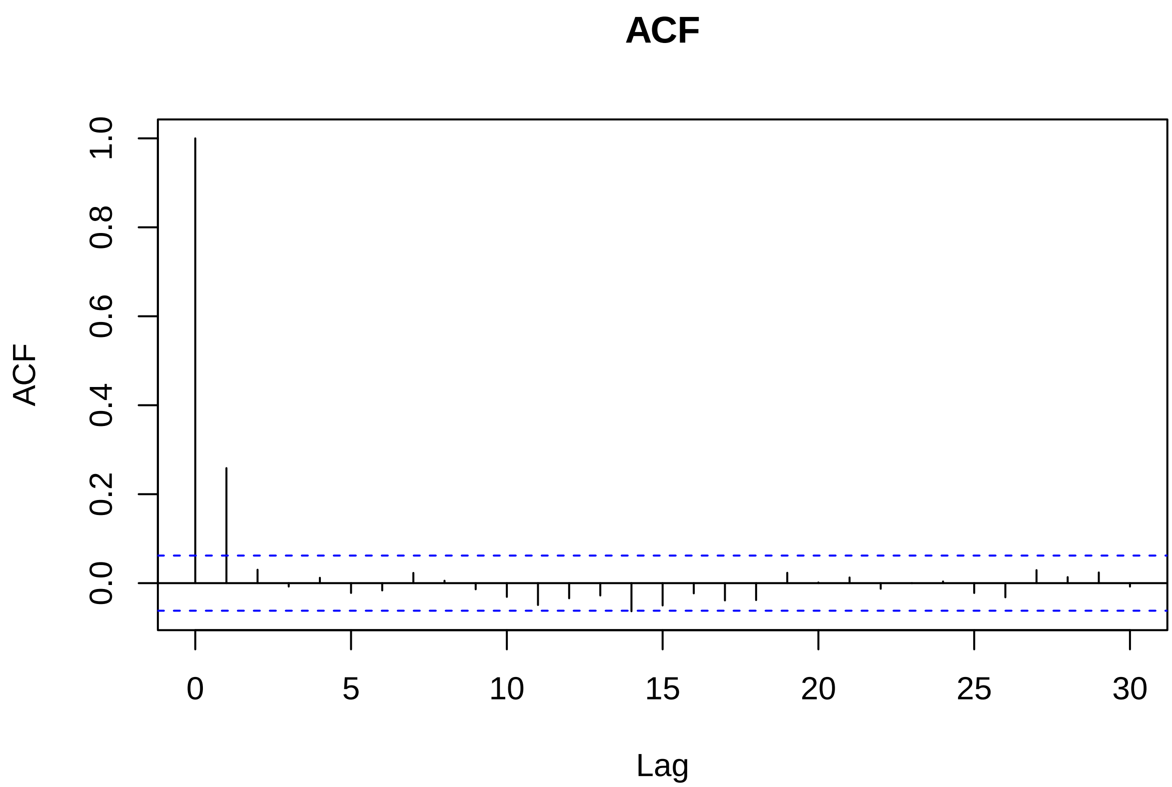
The sample ACF shows significant ‘spike’ autocorrelation at lag 0 and 1, with higher lags beyond 1 dropping close to 0 and insignificant. I believe this exhibits a MA(1) process.

(b)

```
# this is where your R code goes
q2b <- arima.sim(model = list(ma = c(4)), n =  1000, sd = sqrt(0.4))
plot(q2b, xlab = "Value", ylab = "Time")
```




```r
q2b_ts <- ts(q2b)
acf(q2b_ts, main = "ACF")
```



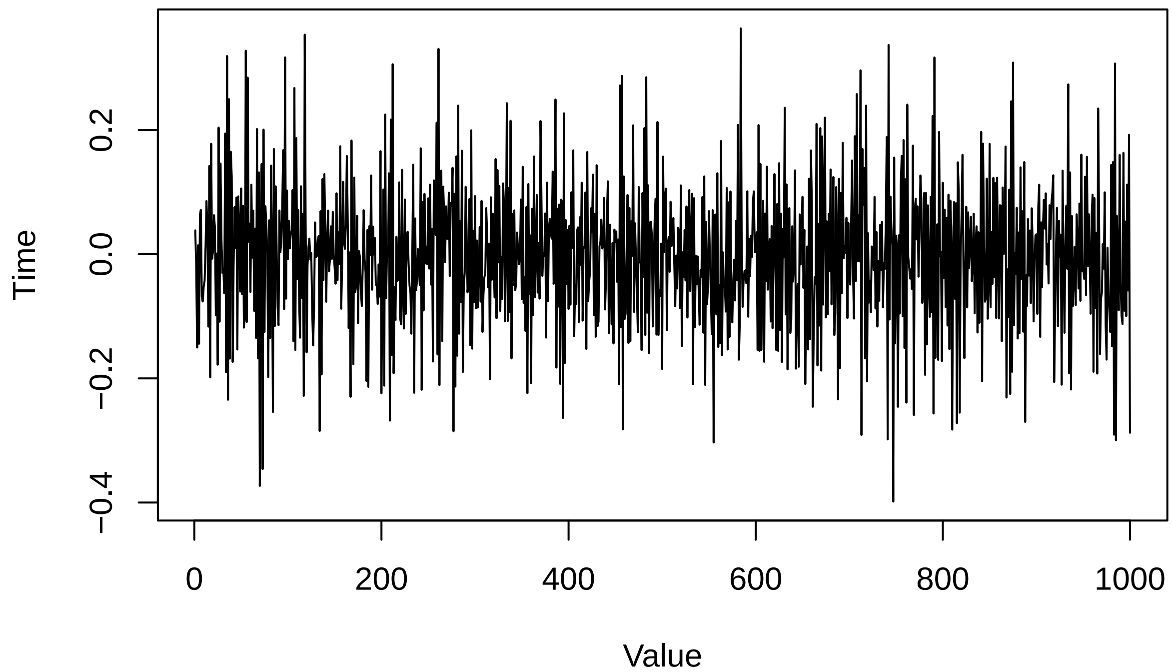
Similar to Q2a's sample ACF plot, we see a clear significant positive autocorrelation at lag 1 and the ACF values drop close to 0 beyound lag 1 meaning that there is no further significant autocorrelation. Thus, exhibiting a MA(1) process as well.

(c) Repeating both Q2a and b's simulation, they all will exhibit a general pattern of a MA(1) process with significant peak at lag 1 followed by a values close to 0. We will notice slight differences between samples because they produce different white noises which explains the slight variation between samples.

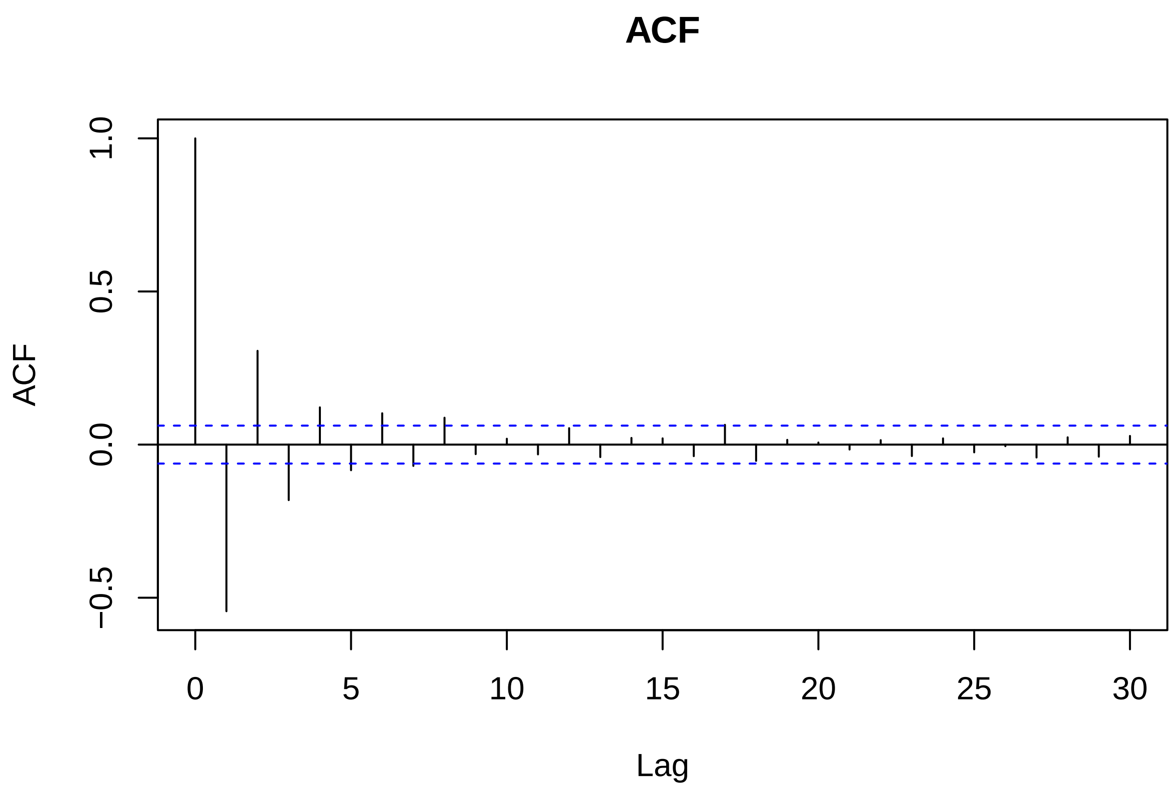
Question 3

(a)

```r
# this is where your R code goes
q3a <- arima.sim(model = list(ar = c(-0.5)), n =  1000, sd = 0.1)
plot(q3a, xlab = "Value", ylab = "Time")
```



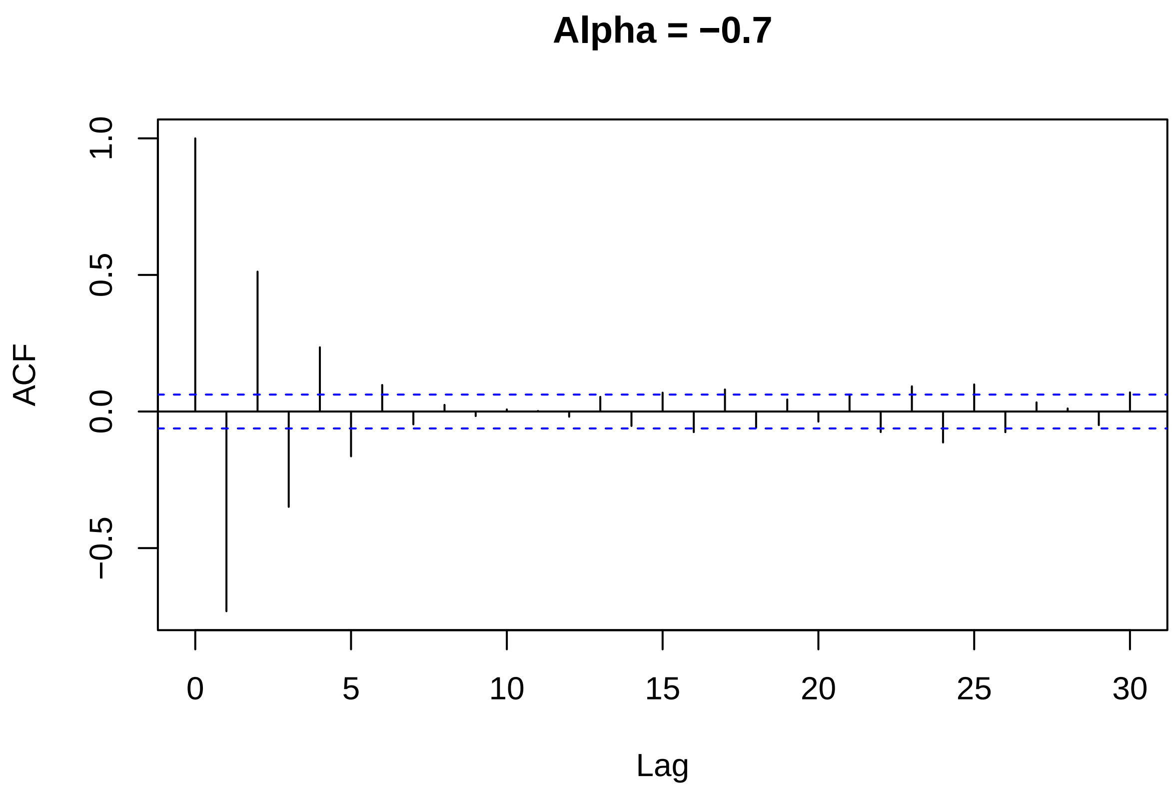

```r
q3a_ts <- ts(q3a)
acf(q3a_ts, main = "ACF")
```



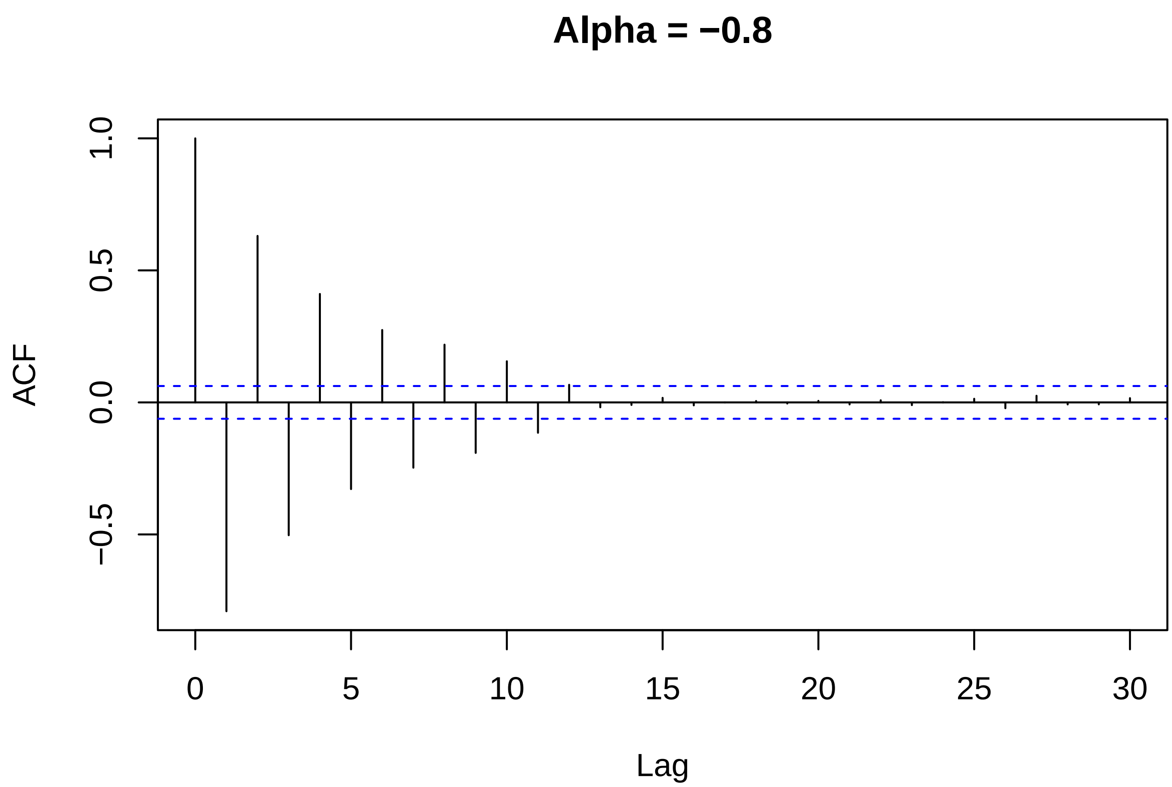
The sample ACF plot shows a significant negative autocorrelation at lag 1 and significant positive autocorrelation at lag 2. As we go for lags beyond 2, we can observe an oscillating pattern with insignificant acf values.

(b)

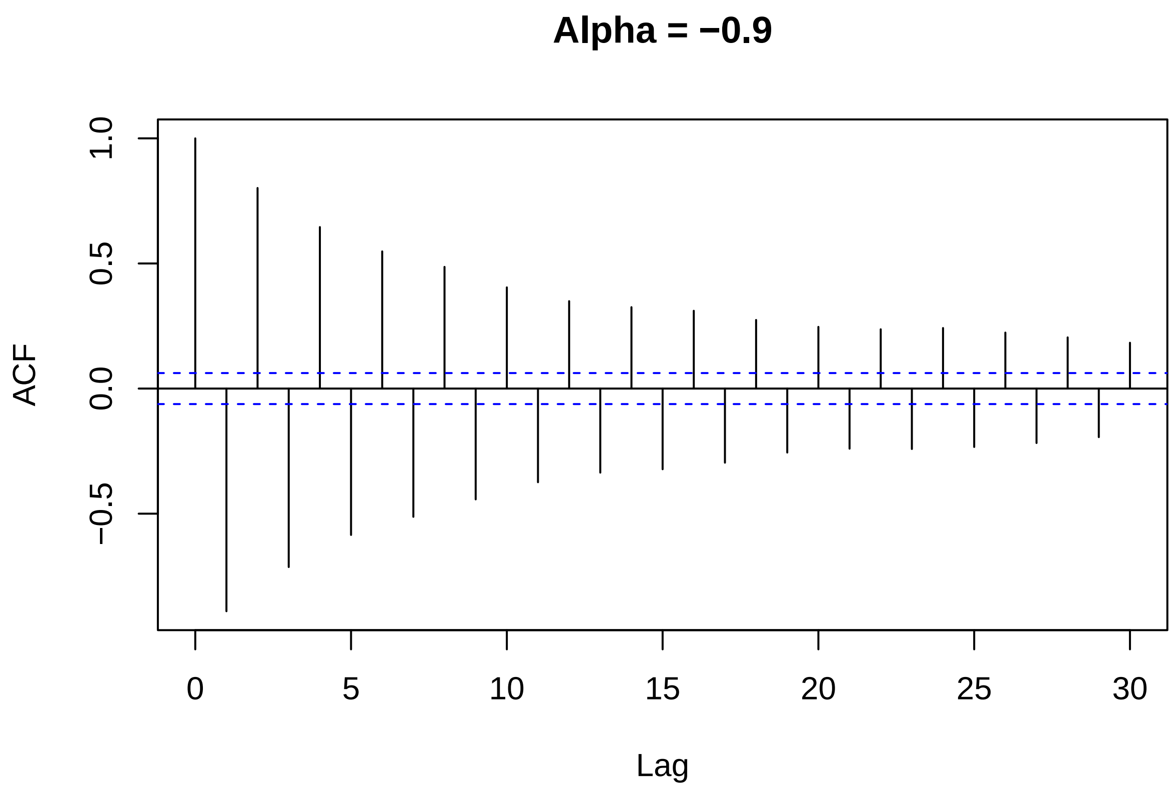
```
# this is where your R code goes
# At -0.7
q3b_s1 <- arima.sim(model = list(ar = c(-0.7)), n =  1000, sd = 0.1)
q3b_s1_ts <- ts(q3b_s1)
acf(q3b_s1_ts, main = "Alpha = -0.7")
```




```r
# At -0.8
q3b_s2 <- arima.sim(model = list(ar = c(-0.8)), n =  1000, sd = 0.1)
q3b_s2_ts <- ts(q3b_s2)
acf(q3b_s2_ts, main = "Alpha = -0.8")
```




```r
# At -0.9
q3b_s3 <- arima.sim(model = list(ar = c(-0.9)), n =  1000, sd = 0.1)
q3b_s3_ts <- ts(q3b_s3)
acf(q3b_s3_ts, main = "Alpha = -0.9")
```

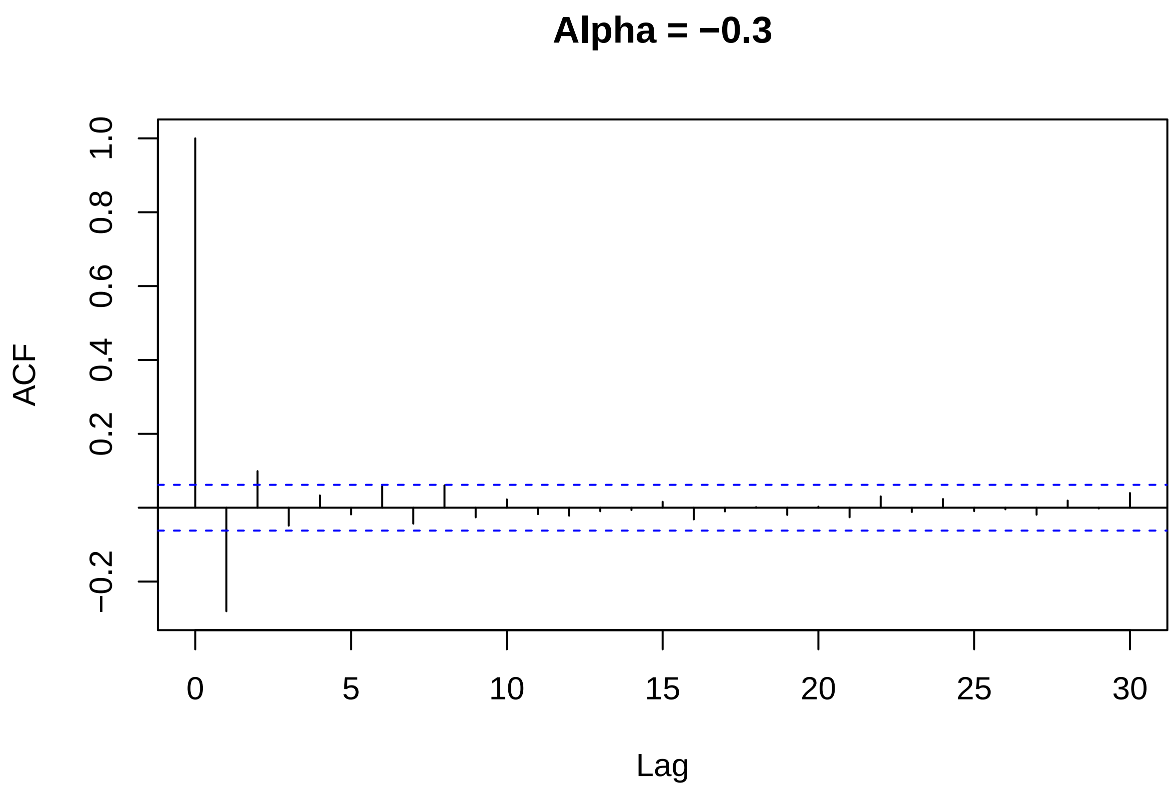


The sample ACF plot reflects an auto-regressive function, and the acf values is oscillating and its negative ACF values decreases less gradually as our alpha value approach -1.

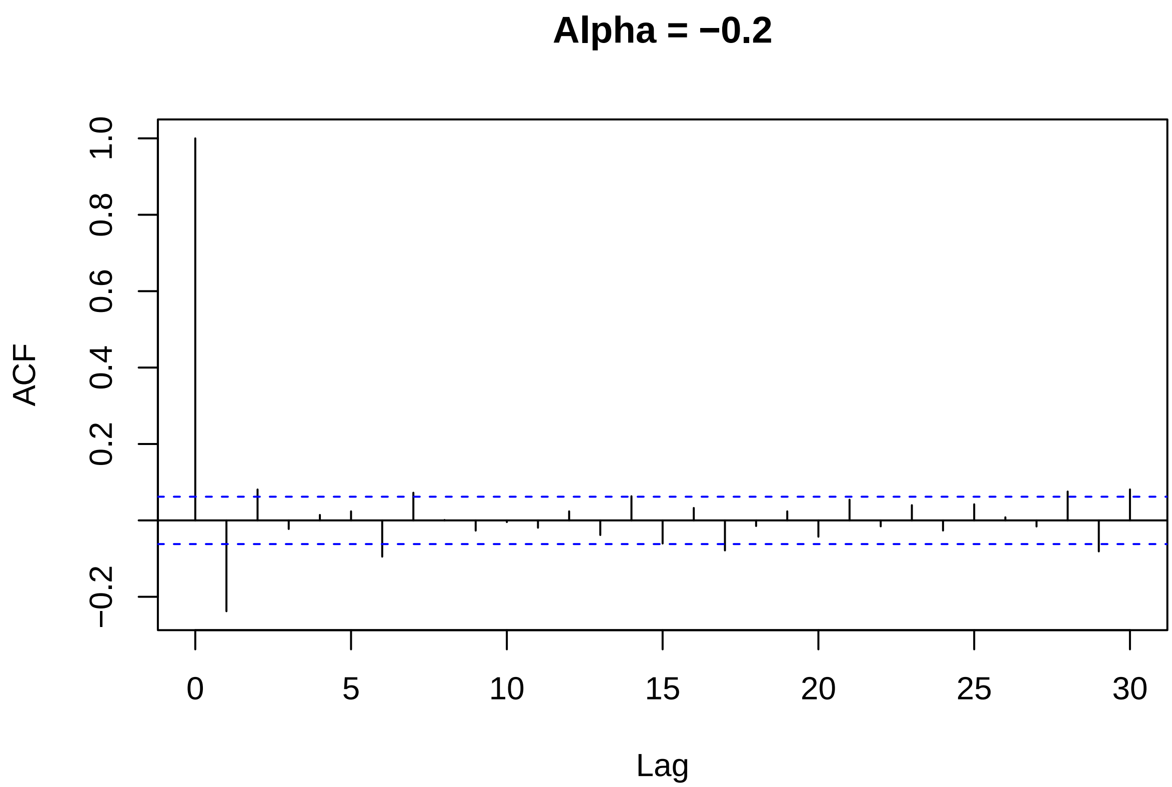
Furthermore, we get positive ACF values for even numbers of lag values and negative ACF values for odd numbers.

(c)

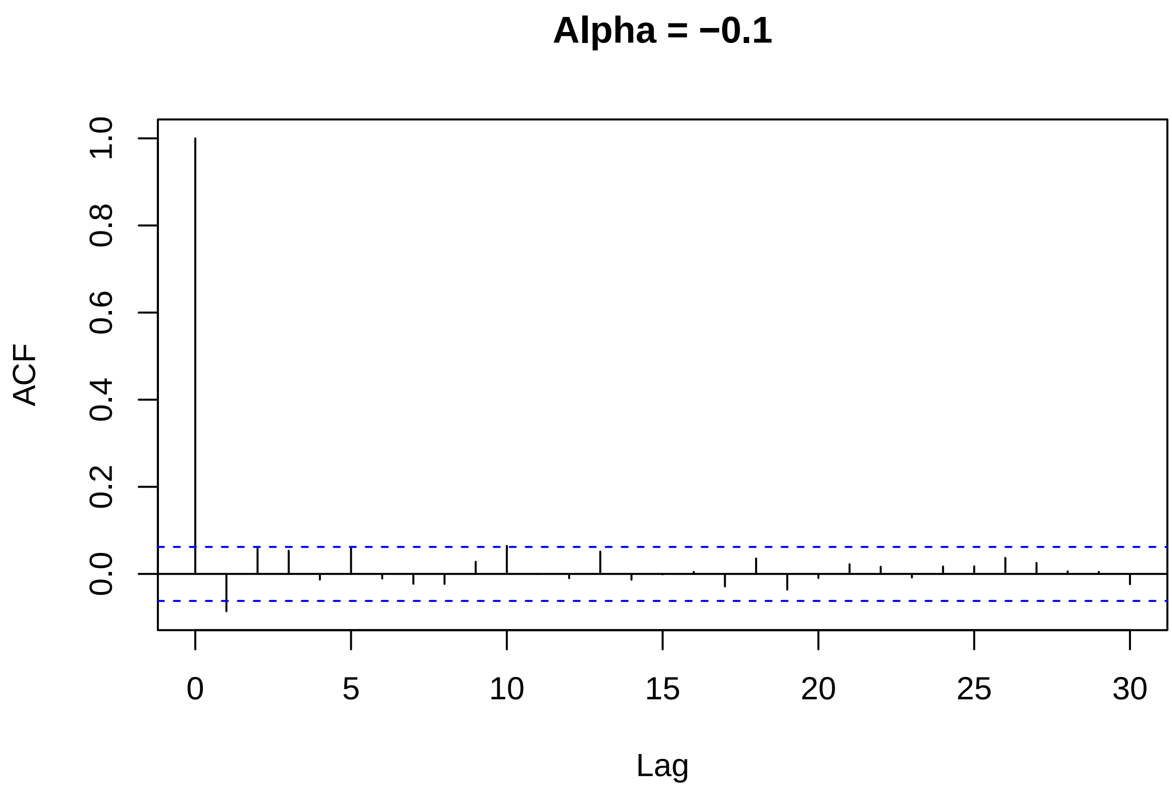
```r
# this is where your R code goes

# At -0.3
q3b_s1 <- arima.sim(model = list(ar = c(-0.3)), n =  1000, sd = 0.1)
q3b_s1_ts <- ts(q3b_s1)
acf(q3b_s1_ts, main = "Alpha = -0.3")
```




```r
# At -0.2
q3b_s2 <- arima.sim(model = list(ar = c(-0.2)), n =  1000, sd = 0.1)
q3b_s2_ts <- ts(q3b_s2)
acf(q3b_s2_ts, main = "Alpha = -0.2")
```



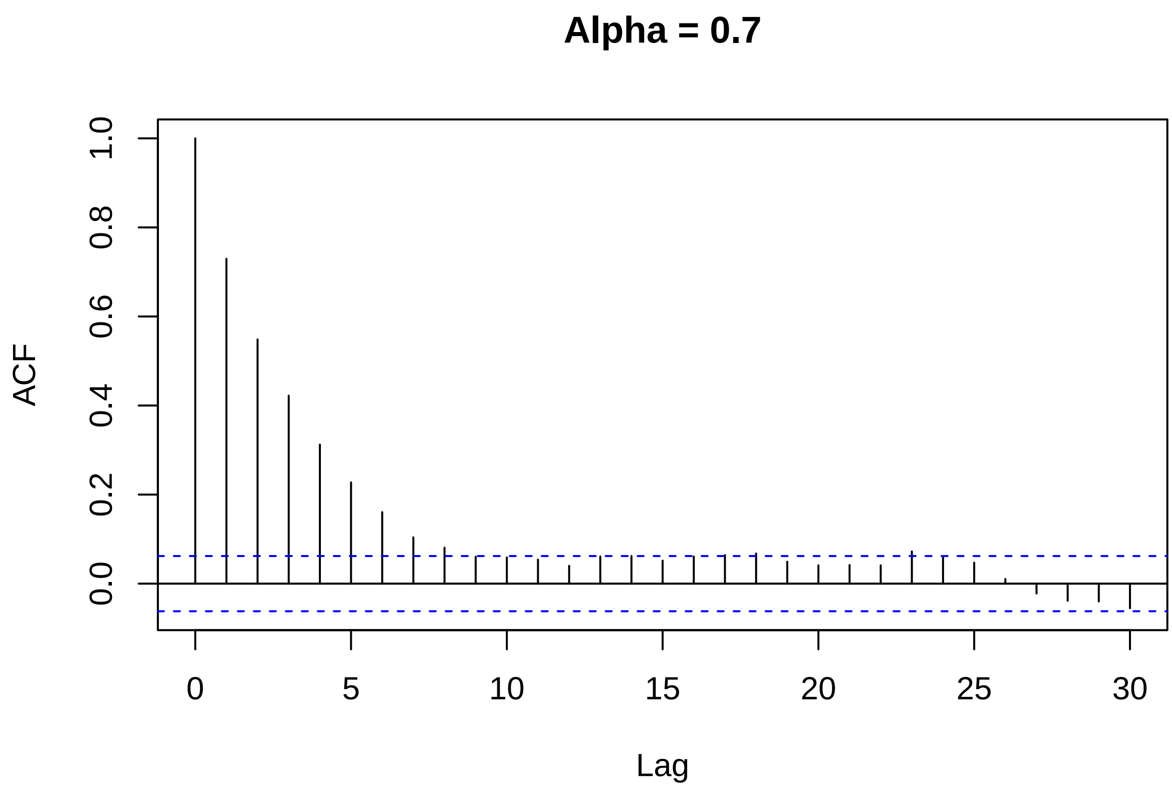

```
# At -0.1
q3b_s3 <- arima.sim(model = list(ar = c(-0.1)), n =  1000, sd = 0.1)
q3b_s3_ts <- ts(q3b_s3)
acf(q3b_s3_ts, main = "Alpha = -0.1")
```



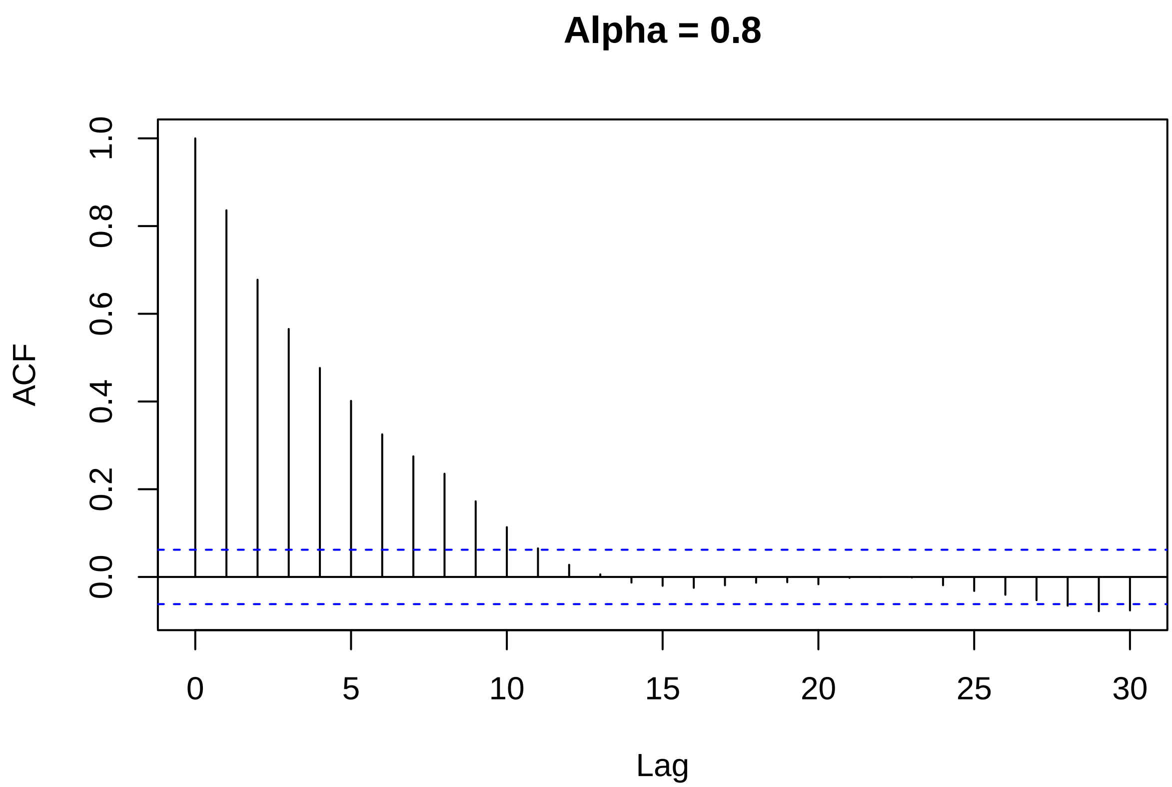
We can still see a slight osciliating behaviour, with the ACF values at higher lags will remain close to 0. So as alpha approaches 0 from the negative side, the sample ACF will transition from showing a significant negative correlation at lag 1 to having values close to 0 across all lags.

(d)

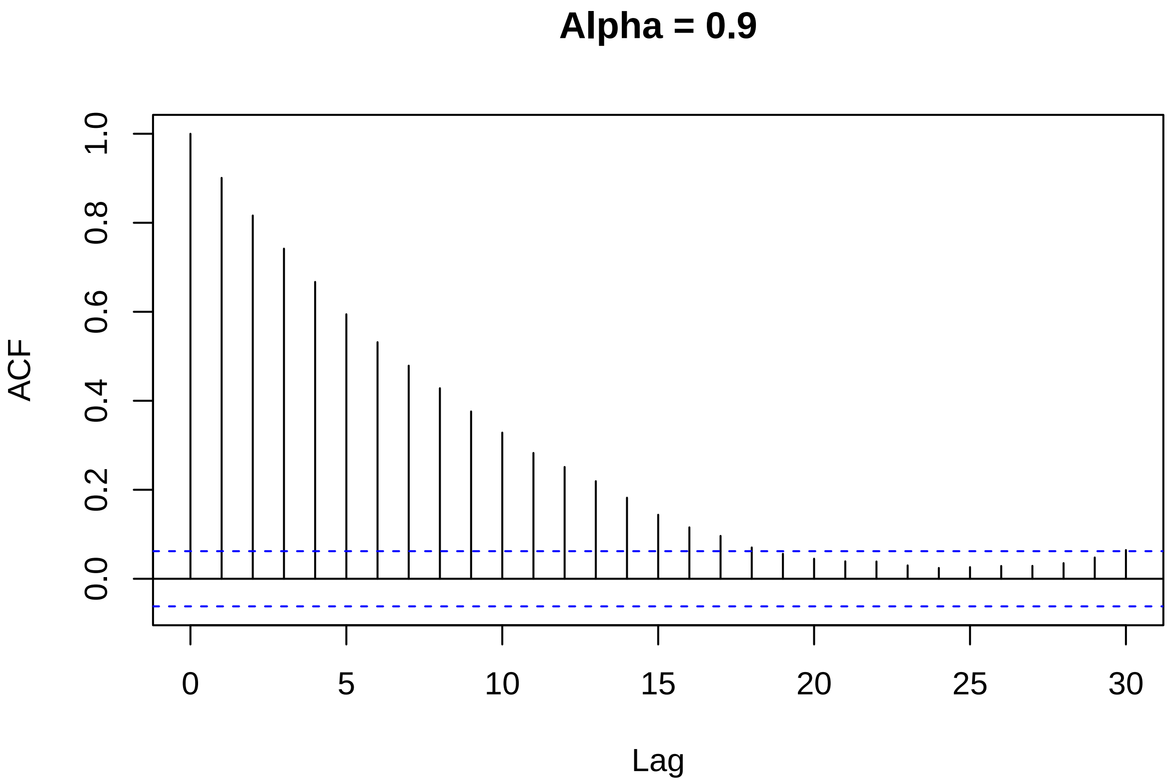
```
# this is where your R code goes

# At 0.7
q3b_s1 <- arima.sim(model = list(ar = c(0.7)), n =  1000, sd = 0.1)
q3b_s1_ts <- ts(q3b_s1)
acf(q3b_s1_ts, main = "Alpha = 0.7")
```




```r
# At 0.8
q3b_s2 <- arima.sim(model = list(ar = c(0.8)), n =  1000, sd = 0.1)
q3b_s2_ts <- ts(q3b_s2)
acf(q3b_s2_ts, main = "Alpha = 0.8")
```




```r
# At 0.9
q3b_s3 <- arima.sim(model = list(ar = c(0.9)), n =  1000, sd = 0.1)
q3b_s3_ts <- ts(q3b_s3)
acf(q3b_s3_ts, main = "Alpha = 0.9")
```



As alpha approach 1 (opposite sign of -1), the sample acf decreases less gradually as our lag increases. We notice that when alpha is positive, it does not exhibit the oscillating behaviour similar to the above plots. Furthermore, we notice that the alpha sign determines the direction of the initial autocorrelation. Positive alpha values leads to positive correlation, while negative alpha values in negative correlation at lag 1.