Constant Bounded Approximation Algorithms for Stochastic Inventory Control Models

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Overview

Purpose

- Inventory models are typically based on MDPs.
  - solved through dynamic programming.
  - often results in uncomputable solutions.
- Plenty of theory behind approximating MDPs (Reinforcement Learning).
- Alternatively, approximate optimal policies through specialized methods.
- Several metrics to predict/evaluate performance:
  - Average cost relative to optimal
  - Worst case cost relative to optimal

New Ideas from Levi et al. [2007]

- Marginal Cost Accounting: Given a forecast, costs of a decision over entire horizon are estimated and attributed to that decision.
  - Knowledge of future decisions/policy are not required.
  - Can be applied online without dynamic programming.

- Cost Balancing: All costs (holding, backordering, etc) are set equal to eachother.
  - Generally not optimal policy.
  - Provides constant worst case performance guarantees.

Models Developed

- Unconstrained Periodic Review with $C = 2$ [Levi et al., 2007]
- Lot Sizing with $C = 3$ [Levi et al., 2007]
- Constrained Periodic Review with $C = 2$ [Levi et al., 2008]
- n-Stage Multi-Echelon with $C = 2$ [Levi et al., 2016]

Unconstrained Periodic Review

Given per unit holding costs h_s, Per unit lost sales costs p_s, demand sequence $\{d_t\}_t$, a starting inventory X_t, and a forecast f_s.

Holding costs $H_s^B(q_s)$ are:

$$H_s^B(q_s) = \sum_{j=s}^{T} h_j \big[q_s - \big(\sum_{i=s}^{j} d_i - X_s \big)^+ \big]^+$$

Penalty costs $\Pi_s^B(q_s)$ are:

$$\Pi_s^B(q_s) = p_s[d_s - (X_s^B + q_s)]^+$$

Balancing Algorithm seeks the order size q_s such that:

$$l_s^B(q_s) = \pi_s^B(q_s)$$

where $l_s^B(q_s) = \mathbb{E}[H_s^B(q_s) \,|f_s]$ and $\pi_s^B(q_s) = \mathbb{E}[\Pi_s^B(q_s) \,|f_s]$.

Unconstrained Periodic Review (cont)

Theorem (Levi et al. [2007])

The dual balancing policy for uncapacitated periodic-review stochastic inventory control problem has a worst-case performance guarantee of two.

Let $Z_t = \mathbb{E}[H_t^B \mid F_t] = \mathbb{E}[\Pi_t^B \mid F_t]$, and $\mathbb{E}[\mathcal{C}(P)] = \sum_t \mathbb{E}[H_t^P + \Pi_t^P]$.

It can be shown that:

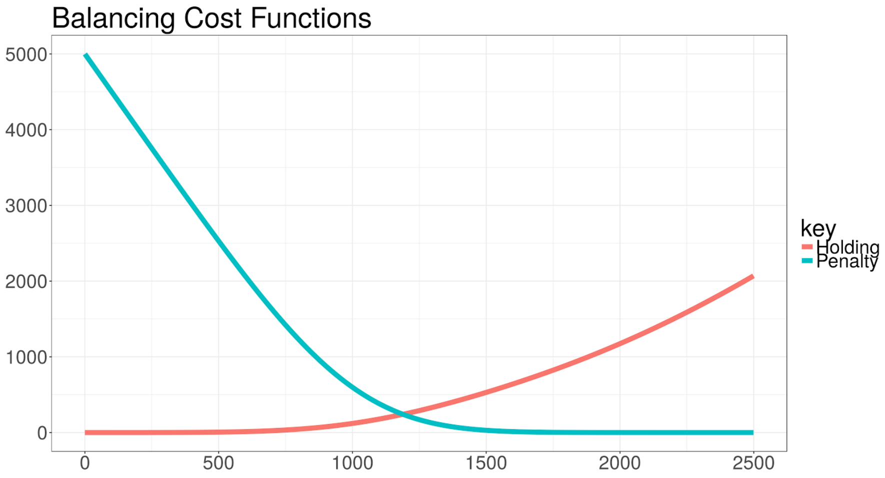
$$\mathbb{E}[\mathcal{C}(B)] = 2\sum_t \mathbb{E}[Z_t]$$

Partition the horizon into two sets, $\mathcal{J}_H$ and $\mathcal{J}_\Pi$, where inventory held is above and below optimal respectively. It can be shown that:

$$H^{opt} + \Pi^{opt} \geq \sum_{t \in \mathcal{J}_H} H_t^B + \sum_{t \in \mathcal{J}_\Pi} \Pi_t^B$$

Therefore:

$$\mathbb{E}[\mathcal{C}(opt)] \geq \sum_t \mathbb{E}[Z_t]$$


Balancing Cost Functions
5000
4000
3000
2000
1000
0
0
500
1000
1500
2000
2500
key
Holding
Penalty

Computing the Balancing Point

Newton-Raphson begins with an initial estimate x_0, and then iterates the following equation:

$$x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$$

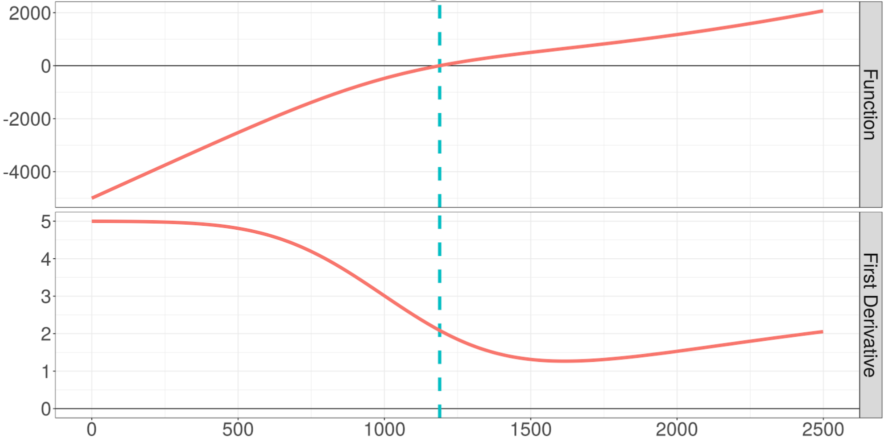
For the dual balancing policy, f and f' would be defined as follows:

$$f(x) = l_s^B(x) - \pi_s^B(x)$$

$$f'(x) = \frac{d}{dx} l_s^B(x) - \frac{d}{dx} \pi_s^B(x)$$

Requires good starting point, $x_0 = \mu$ works well.

Loss Function for Root-Finding



Efficiently Computing the Balancing Point

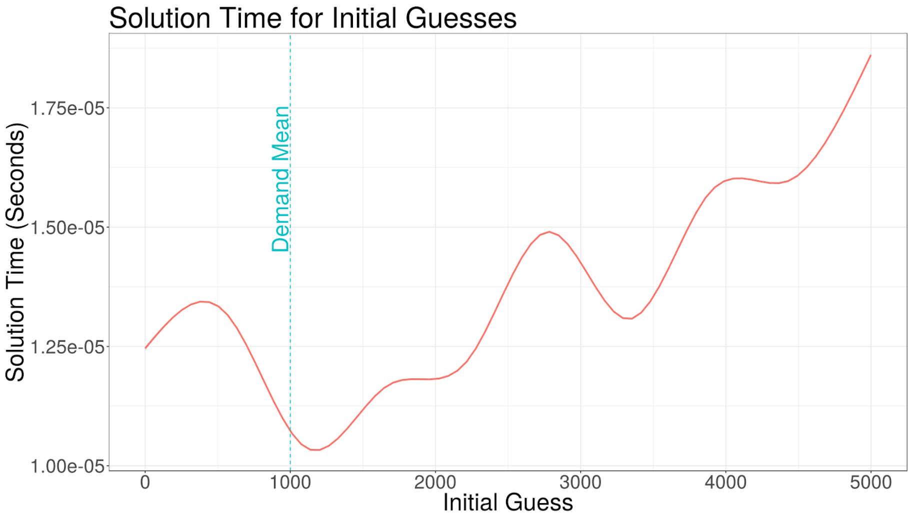
Computing holding costs requires a very large integral:

$$
\begin{aligned}
I_s^B(q_s) &= \mathbb{E}\left\{ \sum_{j=s}^{T} H_{[s,j]}^B(\vec{D}) \mid f_s \right\} \\
&= \int_{z_s=0}^{\infty} \int_{z_{(s+1)}=0}^{\infty} \cdots \int_{z_T=0}^{\infty} \sum_{j=s}^{T} H_{[s,j]}^B(\vec{z}) d\Phi(z_s, z_{s+1}, \dots z_T) \\
&= \sum_{j=s}^{T} \int_{z_s=0}^{\infty} \int_{z_{(s+1)}=0}^{\infty} \cdots \int_{z_j=0}^{\infty} H_{[s,j]}^B(\vec{z}) d\Phi(z_s, z_{s+1}, \dots z_j)
\end{aligned}
$$

Let $\psi_{[s,j]}$ be the cumulative demand distribution.

$$
I_s^B(q_s) = \sum_{j=s}^{T} h_j \int_{z_j=X_s}^{X_s+q_s} \left(q_s + X_s - z_j \right) \psi_{[s,j]}(z_j) dz_j
$$

For nonincreasing h_j, the series in $I_s^B(q_s)$ is nonincreasing with j, so series can be truncated.


Solution Time for Initial Guesses
Demand Mean
Solution Time (Seconds)
1.75e-05
1.50e-05
1.25e-05
1.00e-05
0
1000
2000
3000
4000
5000
Initial Guess

Retsef Levi, Martin Pl, Robin O. Roundy, and David B. Shmoys. Approximation algorithms for stochastic inventory control models. *Mathematics of Operations Research*, 32(2):284–302, 2007. doi: 10.1287/moor.1060.0205.

Retsef Levi, Robin O. Roundy, David B. Shmoys, and Van Anh Truong. Approximation algorithms for capacitated stochastic inventory control models. *Operations Research*, 56(5):1184–1199, 2008. doi: 10.1287/opre.1080.0580.

Retsef Levi, Robin Roundy, Van Anh Truong, and Xinshang Wang. Provably near-optimal balancing policies for multi-echelon stochastic inventory control models. *Mathematics of Operations Research*, 42(1): 256–276, 2016.