Computing $l_s^B(q_s)$ in the general case requires the computation of a $T-s$ dimensional integral. Traditional quadrature algorithms have exponential complexity in dimensions, making such an integral unreasonable to compute. However, some conditions allow it to be reduced to $T-s$ integrals of fewer dimensions. Letting Φ be the (generally multivariate) distribution of forecast f_s:

$$
\begin{aligned}
l_s^B(q_s) &= \mathbb{E}[H_s^B(q_s) \mid f_s] \\
&= \mathbb{E}\Bigg\{ \sum_{j=s}^{T} h_j \Bigg[q_s - \Bigg(\sum_{i=s}^{j} u_i - X_s \Bigg)^+ \Bigg]^+ \mid f_s \Bigg\} \\
&= \int_{u_s=0}^{\infty} \int_{u_{(s+1)}=0}^{\infty} \cdots \int_{u_T=0}^{\infty} \sum_{j=s}^{T} h_j \Bigg[q_s - \Bigg(\sum_{i=s}^{j} u_i - X_s \Bigg)^+ \Bigg]^+ d\Phi(u_s, u_{s+1}, \dots u_T) \\
&= \sum_{j=s}^{T} \int_{u_s=0}^{\infty} \int_{u_{(s+1)}=0}^{\infty} \cdots \int_{u_j=0}^{\infty} h_j \Bigg[q_s - \Bigg(\sum_{i=s}^{j} u_i - X_s \Bigg)^+ \Bigg]^+ d\Phi(u_s, u_{s+1}, \dots u_T)
\end{aligned}
$$

If the conditions allow for $D_{[s,j]} = \sum_{i=s}^{j} D_s$ to be expressed as a single variate distribution $\Phi_{[s,j]}$, then $l_s^B(q_s)$ can be expressed as $T-s$ integrals in one dimension:

$$
l_s^B(q_s) = \sum_{j=s}^{T} \int_{u_j=X_s}^{X_s+q_s} h_j \Big(q_s + X_s - u_j \Big) d\Phi_{[s,j]}(u_j) \tag{1}
$$

Under the same conditions, $\frac{d}{dq_s} l_s^B(q_s)$ can be expressed:

$$
\frac{d}{dq_s} l_s^B(q_s) = \sum_{j=s}^{T} h_j(\Phi_{[s,j]}(X_s + q_s) - \Phi_{[s,j]}(X_s)) \tag{2}
$$

This condition would hold for a series of independent random variables.