HUDSON
AND THAMES

Learning Side and Size

Advanced Topics in Financial Machine Learning

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Overview

Often we have the side of the position but are left asking how large should the size of our bet be, or if we should bet at all.

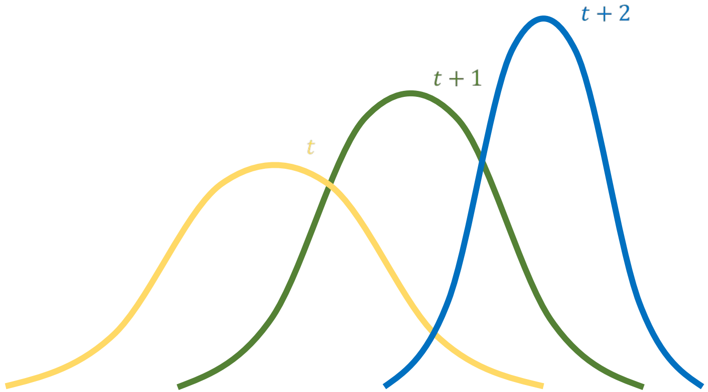
Meta-Labeling helps us to address situations where the primary model is likely to fail and thus reduce our position size, however in an event with a high probability of success it up weights position sizes. In this way it helps to both filter out false positives and size positions.

Problem: non-stationarity

Probability distribution shifts in time, consequently parameters such as mean, variance, and covariance also change over time.

The underlying data generating process f(x) is changing through time.

Most ML models assume that the data is generated by an independent and identically distributed (IID) processes.



Stuart Reid, Dynamic Systems, IndabaX, 2018

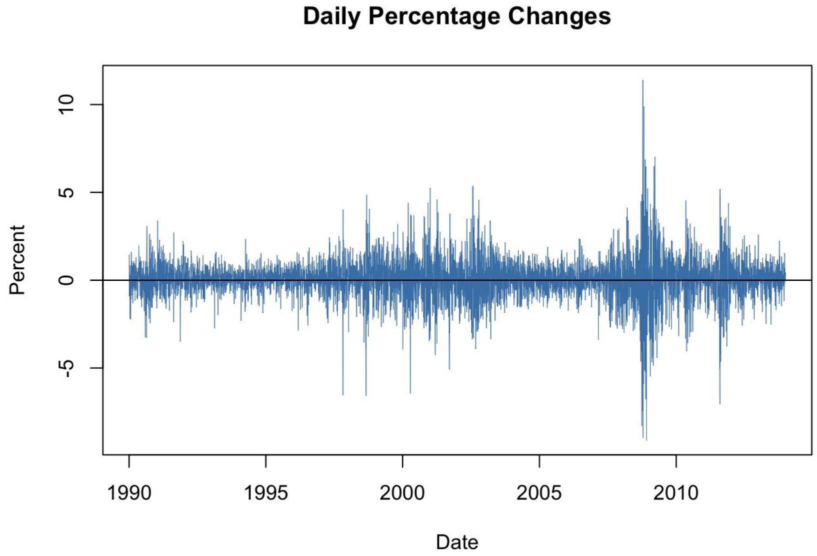
Problem: non-stationarity

Fundamentals change:

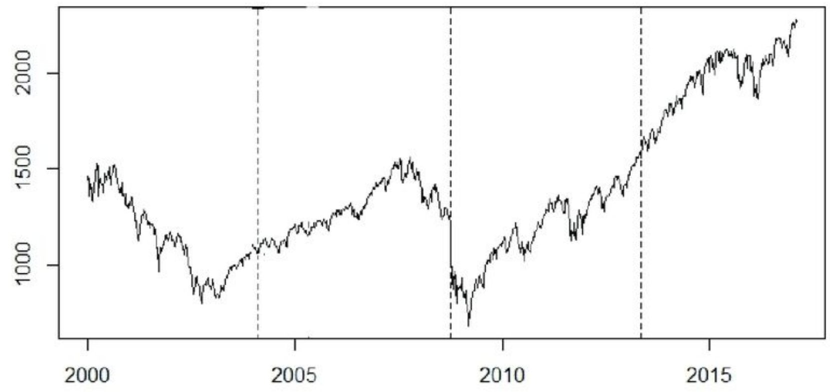
- Open outcry -> electronic
- A single exchange -> multiple across various locations.
- Introduction of HFT

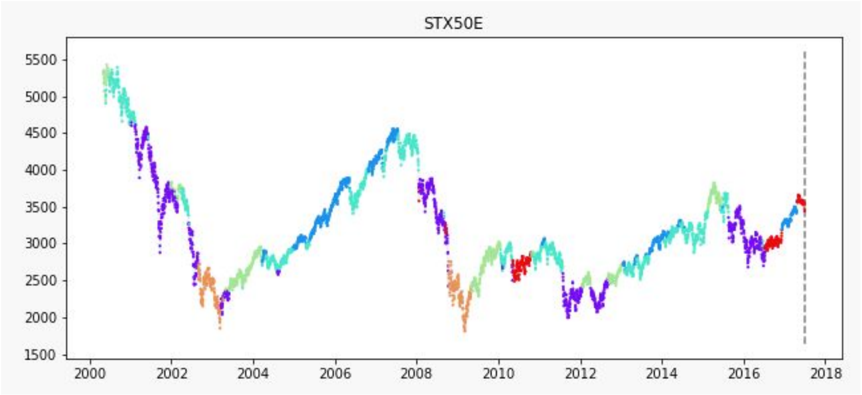
Regimes:

- Trending
- Mean reverting
- Volatility clustering
- Random walk
- Recession

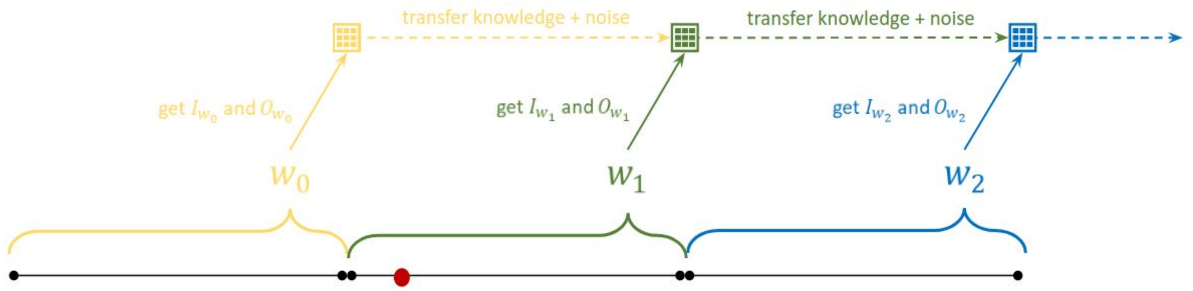


Problem: Structural Break / Regime Shift



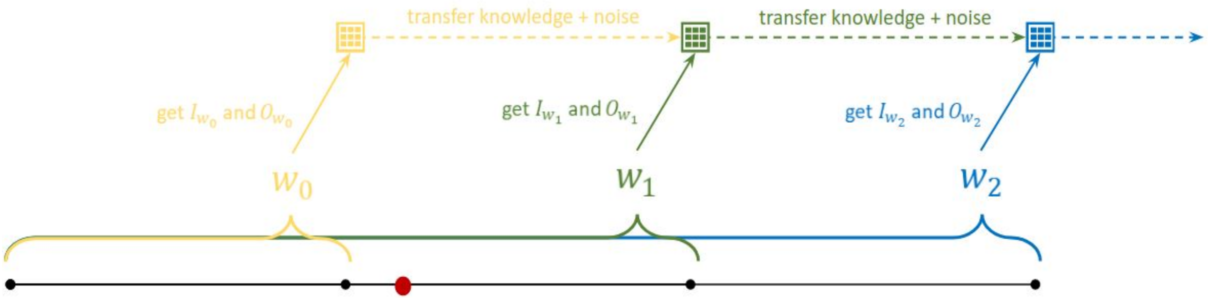


Solution 1: Online Machine Learning



Fixed window size

- Pessimistic data sampling
+ Adapts to change quickly
- Less data to train on
+ Faster (less data)
- Inefficient data usage
- Hard with large models



Increasing window size

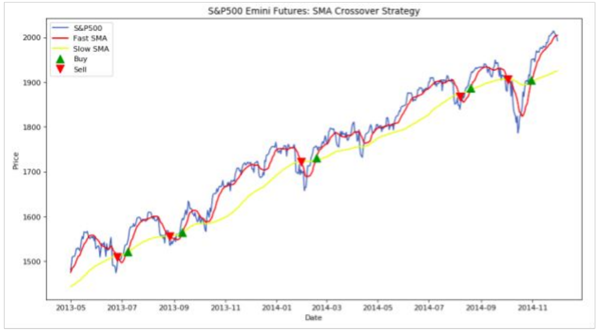
- Optimistic data sampling
- Adapts to change slowly
+ More data to learn from
- Slower (more data)
- Most data is *irrelevant* ~ poor model performance

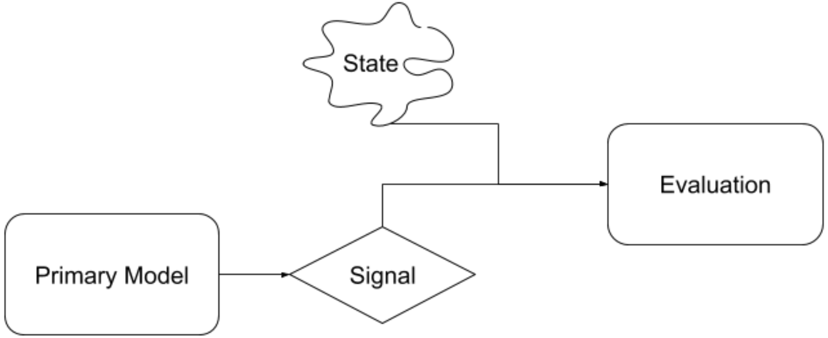
[Stuart Reid, Dynamic Systems, IndabaX, 2018]

Solution 2: Meta Labeling

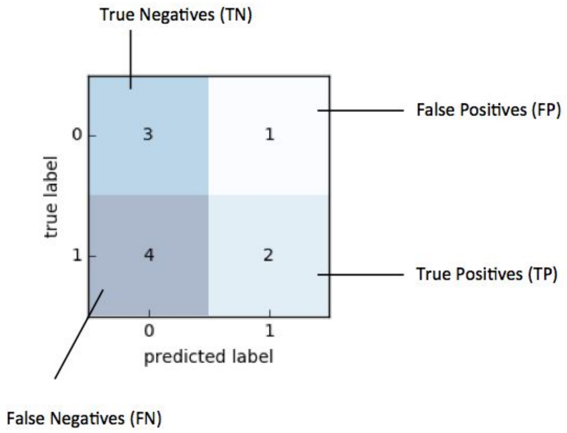
To trade or not to trade!

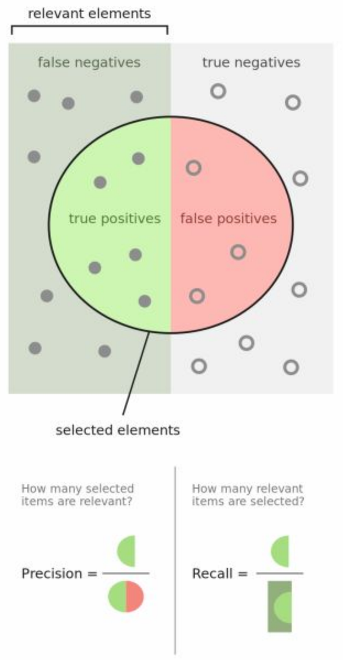
- Meta-Labels: Takes the side from the primary model [-1, 1] and labels it as correct or incorrect.
- Train a secondary model to determine if we should trade the signal or not.
- Features:
    - Primary model features (Market state)
    - Features indicative of false positives
    - Additional market information
- Primary model can be, discretionary trader, technical rules, classic quant, ml model.
- Trade off between recall and precision. (Want more correct trades).

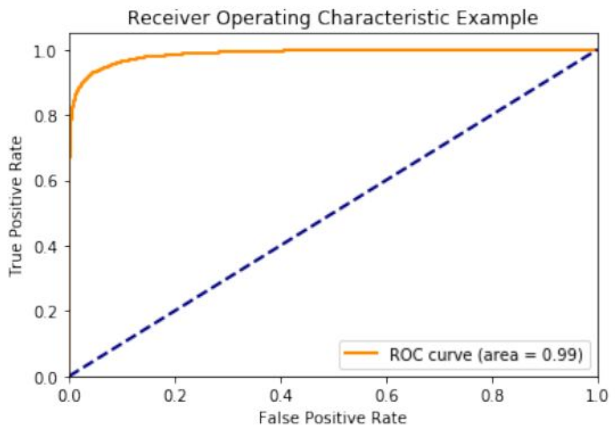




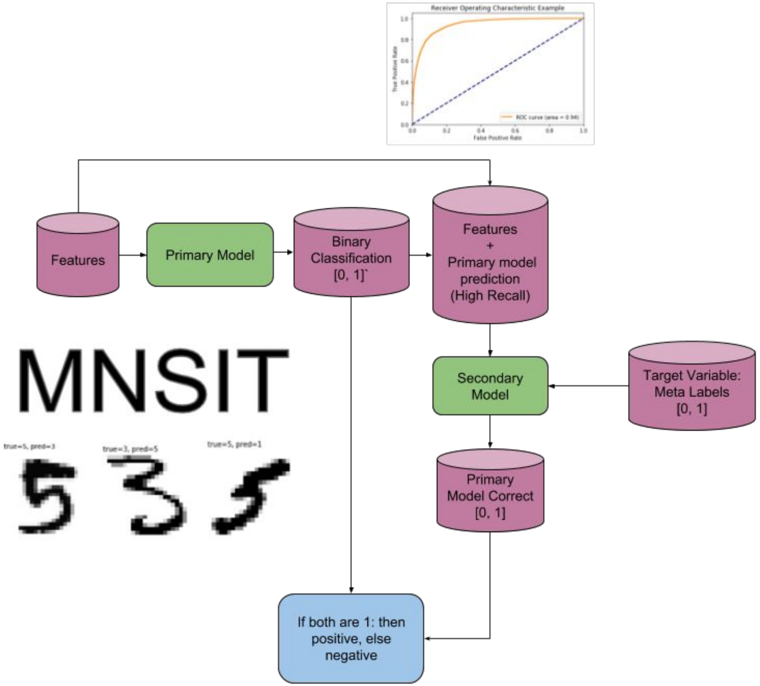
Important Classification Metrics





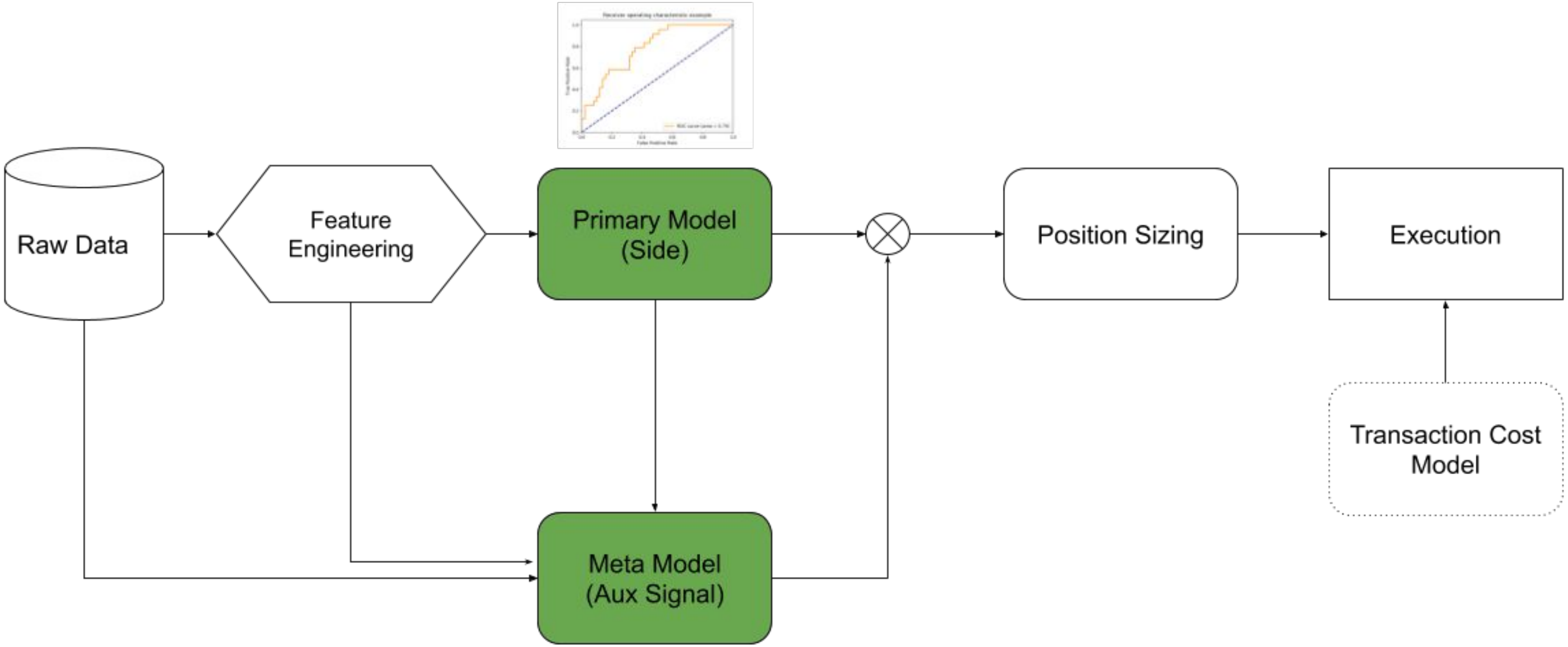


Toy Example: MNIST



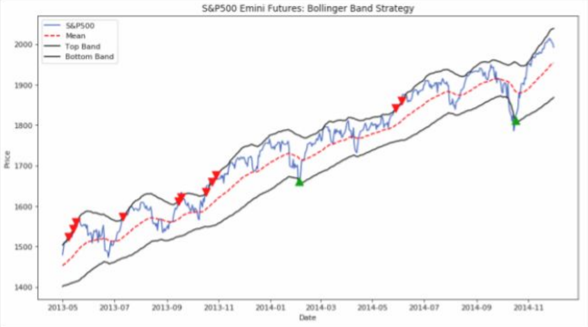

```
Base Model Metrics:
              precision    recall  f1-score   support

       False       0.95      0.94      0.94       892
        True       0.94      0.96      0.95      1010

   micro avg       0.95      0.95      0.95      1902
   macro avg       0.95      0.95      0.95      1902
weighted avg       0.95      0.95      0.95      1902

Confusion Matrix
[[700 192]
 [ 11 999]]
Accuracy:  0.8933

Meta Label Metrics:
              precision    recall  f1-score   support

       False       0.95      0.96      0.95       892
        True       0.96      0.95      0.96      1010

   micro avg       0.96      0.96      0.96      1902
   macro avg       0.96      0.96      0.96      1902
weighted avg       0.96      0.96      0.96      1902

Confusion Matrix
[[857  35]
 [ 47 963]]
Accuracy:  0.9569
```

Strategy Framework



Meta Labeling: Trading Example

```
              precision    recall  f1-score   support

           0       0.00      0.00      0.00       749
           1       0.17      1.00      0.29       151

   micro avg       0.17      0.17      0.17       900
   macro avg       0.08      0.50      0.14       900
weighted avg       0.03      0.17      0.05       900

Confusion Matrix
[[  0 749]
 [  0 151]]

Accuracy
0.16777777777777778
```




```
              precision    recall  f1-score   support

           0       0.85      0.68      0.75       749
           1       0.20      0.41      0.27       151

   micro avg       0.63      0.63      0.63       900
   macro avg       0.53      0.54      0.51       900
weighted avg       0.74      0.63      0.67       900

Confusion Matrix
[[506 243]
 [ 89  62]]

Accuracy
0.6311111111111111
```

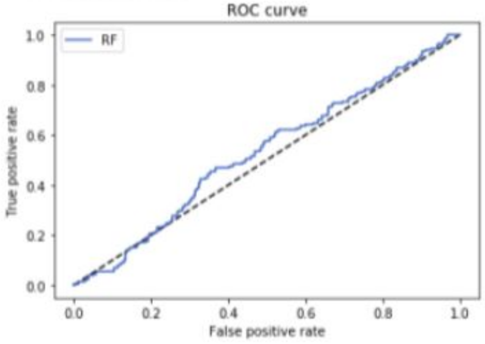


Table 1: Out-of-sample (2018-01-04 : 2019-01-28)

	Primary Model	Meta Model
Annual return		
Cumulative returns	19.7%	39.6%
Annual volatility	95.0%	56.7%
Sharpe ratio	0.65	0.82
Calmar ratio	0.29	0.96
Max drawdown	-61.9%	-36.8%
Daily value at risk	-11.7%	-7.0%

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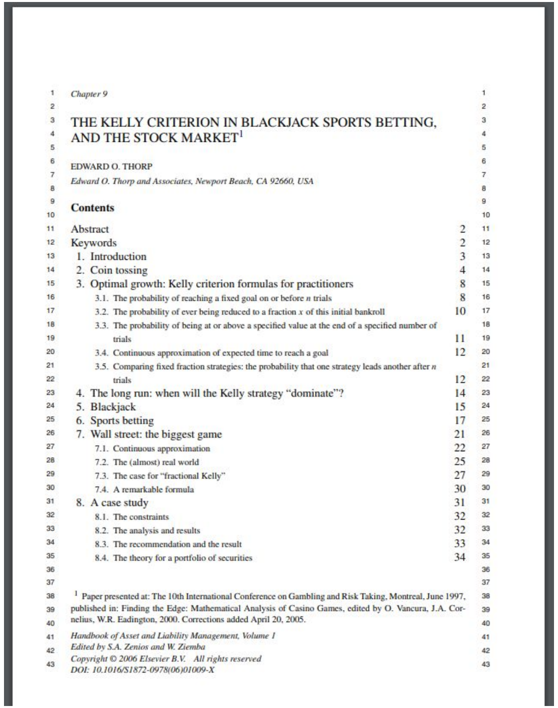
Position Sizing: Kelly Criterion



Chapter 9

THE KELLY CRITERION IN BLACKJACK SPORTS BETTING, AND THE STOCK MARKET[1]

EDWARD O. THORP

Edward O. Thorp and Associates, Newport Beach, CA 92660, USA

Contents

[1] Paper presented at: The 10th International Conference on Gambling and Risk Taking, Montreal, June 1997, published in: Finding the Edge: Mathematical Analysis of Casino Games, edited by O. Vancura, J.A. Cornelius, W.R. Eadington, 2000. Corrections added April 20, 2005.

Handbook of Asset and Liability Management, Volume 1
Edited by S.A. Zenios and W. Ziemba

DOI: 10.1016/S1872-0978(06)01009-X

$$f^* = \frac{\beta p - q}{\beta}$$

$$f^* = p - q$$
$$f^* = 2p - 1$$

Where:

- f^* = optimal bet size
- Beta = odds (win amount / lose amount)
- p = probability of success
- q = probability of failure (1-p)

Additional papers:

- A New Interpretation of Information Rate
- Understanding the Kelly Capital Growth Investment Strategy
- How Does the Fortune's Formula Kelly Capital Growth Model Perform?
- A Response to Professor Paul A. Samuelson's Objections to Kelly Capital Growth Investing
- Good and bad properties of the Kelly criterion

The Investment Opportunities

Win Probability	Odds	Prob. of Selection in Simulation	Kelly Bets
0.570	1−1	0.1	0.140
0.380	2−1	0.3	0.070
0.285	3−1	0.3	0.047
0.228	4−1	0.2	0.035
0.190	5−1	0.1	0.028

Final Wealth Statistics by Kelly Fraction: Ziemba-Hausch [1986] Model

	Kelly Fraction				
Statistic	1.0k	0.75k	0.50k	0.25k	0.125k
Max	318854673	4370619	1117424	27067	6330
Mean	524195	70991	19005	4339	2072
Min	4	56	111	513	587
St. Dev.	8033178	242313	41289	2951	650
Skewness	35	11	13	2	1
Kurtosis	1299	155	278	9	2
$> 5 \times 10$	1981	2000	2000	2000	2000
10^2	1965	1996	2000	2000	2000
$> 5 \times 102$	1854	1936	1985	2000	2000
$> 10^3$	1752	1855	1930	1957	1978
$> 10^4$	1175	1185	912	104	0
$> 10^5$	479	284	50	0	0
$> 10^6$	111	17	1	0	0

How Does the Fortune's Formula Kelly Capital Growth Model Perform?

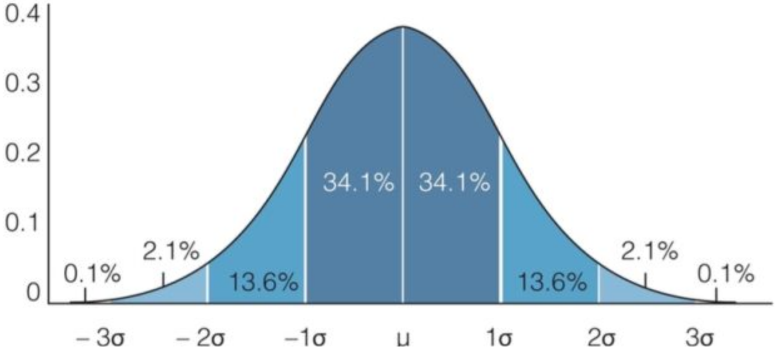
Position Sizing with Meta Labeling

1

$$z = \frac{p[x=1] - \frac{1}{2}}{\sqrt{p[x=1](1 - p[x=1])}}$$

Where:

- p[x] = probability that label x takes place.
- z = test statistic
- x is element of {-1, 1}



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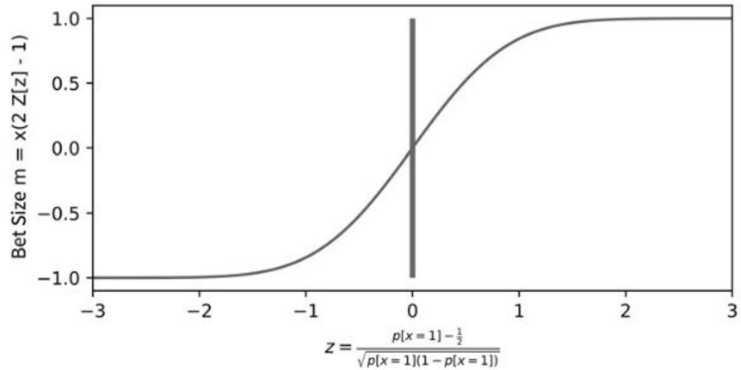


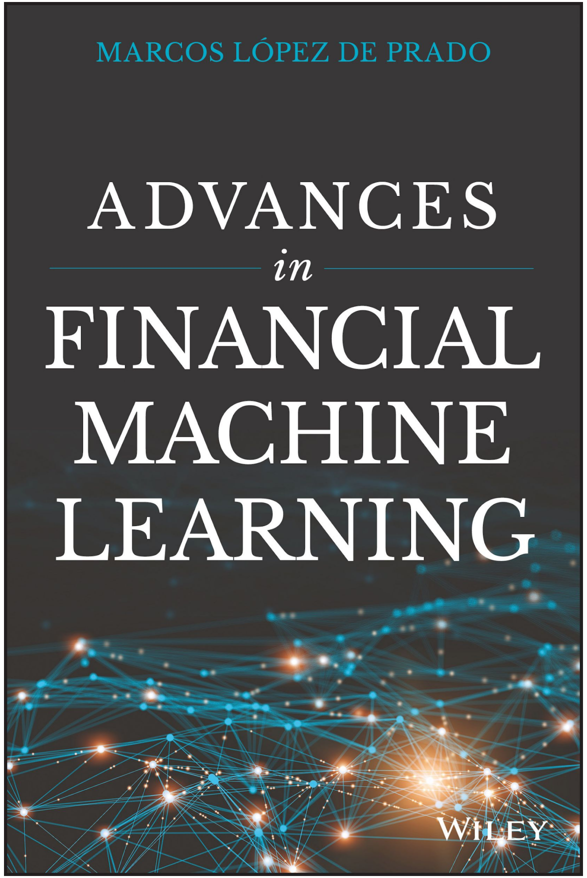
3

$$f = side(2Z[z] - 1)$$
$$f^* = 2p - 1$$

Where:
Z[.] = is the CDF of Z.

Next Steps

- The concepts of position sizing and meta-labeling are both addressed in the textbook Advances in Financial Machine Learning.
- A great additional resource is the Journal of Financial Data Science and the Journal of Portfolio Management.
- If these concepts interest you, there is room available in our research group. You will get to work with high quality tick data and contribute to open-source.



Conclusion

- Meta-Labeling helps to address the problem of non-stationarity and structural breaks by down weighting position sizes in strategies that have a low probability of success in a given market state.
- The Kelly Criterion can be used to determine optimal position sizes.

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Thank You