Task 3 **Convolution** (17 Points)

(a) Consider the a 2x4 input X with one channel and a 2x2 filter K.

$$X = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 2 & -1 \\ 1 & 0 & -1 \\ 2 & 1 & 1 \\ 3 & -1 & 2 \end{array} \qquad K = \begin{array}{|c|c|} \hline K_{00} & K_{01} \\ \hline K_{10} & K_{11} \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline -1 & -2 \\ \hline \end{array}$$

i. Apply the 2D-convolution with kernel K to X, using a stride of (1,1) and no padding to obtain the output H, i.e., $H = X \star K$. Note: You do not have to flip the kernel, i.e., technically speaking, we are interested in cross-correlation here. (2P)

$H_0 = 2+2=4$
$H_1 = -1-2=-3$
$H_2 = 1+1-4=-2$

$$H = \begin{array}{c|c} & 0 \\ \hline 0 & 4 \\ 1 & -3 \\ 2 & -2 \end{array}$$

✓ 1.5

ii. Calculate $\frac{\partial H}{\partial K}$ in the following format: $\frac{\partial H}{\partial K} =$ (2P)

$\frac{\partial H_0}{\partial K_{00}}$	$\frac{\partial H_0}{\partial K_{01}}$	$\frac{\partial H_0}{\partial K_{10}}$	$\frac{\partial H_0}{\partial K_{11}}$
$\frac{\partial H_1}{\partial K_{00}}$	$\frac{\partial H_1}{\partial K_{01}}$	$\frac{\partial H_1}{\partial K_{10}}$	$\frac{\partial H_1}{\partial K_{11}}$
$\frac{\partial H_2}{\partial K_{00}}$	$\frac{\partial H_2}{\partial K_{01}}$	$\frac{\partial H_2}{\partial K_{10}}$	$\frac{\partial H_2}{\partial K_{11}}$

$$\frac{\partial H}{\partial K} = \begin{bmatrix} X_{00}, & X_{01}, & X_{10}, & X_{11} \\ X_{10}, & X_{11}, & X_{20}, & X_{21} \\ X_{20}, & X_{21}, & X_{30}, & X_{31} \end{bmatrix} = \begin{bmatrix} 2, & -1, & 0, & -1 \\ 0, & -1, & 1, & 1 \\ 1, & 1, & -1, & 2 \end{bmatrix}$$

✓ 2

iii. Given that $\frac{\partial \mathcal{L}}{\partial H} = \begin{array}{|c|c|c|} \hline 0 & 1 & 2 \\ \hline \end{array}$,
compute the gradient $\frac{\partial \mathcal{L}}{\partial K}$ in the following format: $\frac{\partial \mathcal{L}}{\partial K} = \begin{array}{|c|c|c|c|} \hline \frac{\partial \mathcal{L}}{\partial K_{00}} & \frac{\partial \mathcal{L}}{\partial K_{01}} & \frac{\partial \mathcal{L}}{\partial K_{10}} & \frac{\partial \mathcal{L}}{\partial K_{11}} \\ \hline \end{array}$
(3P)

$$\frac{\partial L}{\partial K} = \frac{\partial L}{\partial H} \cdot \frac{\partial H}{\partial K} = [0, 1, 2] \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & 2 \end{bmatrix}$$

$$= [2, -1+2, 1-2, 1+4]$$

$$= [2, 1, -1, 5]$$

✓ 3

卷积后的结果 Input

iv. Specify a term $\mathcal{L}$ such that $\frac{\partial \mathcal{L}}{\partial H} = \boxed{\frac{\partial \mathcal{L}}{\partial H_{00}} \mid \frac{\partial \mathcal{L}}{\partial H_{10}} \mid \frac{\partial \mathcal{L}}{\partial H_2}} = \boxed{0 \mid 1 \mid 2}$ (1P).

kernel

~~L is a pooling kernel~~

L = [0, 3, 2].

v. Now assume $\frac{\partial \mathcal{L}}{\partial K_{00}}$ is a large negative value (e.g., -100). How will small changes to the value of K_{00} affect $\mathcal{L}$? When will $\mathcal{L}$ increase, when will it decrease? (2P)

When Koo increase, L will decrease ✓

When koo decrease, L will increase.

✓ sign