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Deep Learning and Artificial Intelligence

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Exercise 4: Convolutional Neural Networks

Exercise 4-1 Convolutions

Given the following 5x5 input image with one channel:

5	5	2	5	5
5	5	2	5	5
7	7	5	7	7
5	5	2	5	5
5	5	2	5	5

Let's assume we have the following 3x3 filters:

1	0	-1
2	0	-2
1	0	-1

1	2	1
0	0	0
-1	-2	-1

0	1	0
1	-4	1
0	1	0

(a) Apply the given filters (by *cross-correlation*) to the above dataset, i.e.:

$$Y_{i,j} = (K \star X)_{i,j} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} K_{m,n} X_{i+m,j+n}.$$

where Y is the output and $M = N = 3$ are the kernel sizes. Use 'valid' padding and a stride of one. You can also write a small program (e.g. using the method scipy.signal.convolve2d()) for that purpose.

(b) Look at the structure of the filters. What do they do?

Exercise 4-2 Backpropagtion through Convolutional Layers

Let the output of a convolutional layer with weights $W \in \mathbb{R}^{k \times k}$ and an input image $X \in \mathbb{R}^{d \times d}$ be given by the cross-correlation $Y = W \star X$ (stride = 1, valid padding).

(a) Derive the quantity $\frac{\partial Y_{i,j}}{\partial W_{u,v}}$!

(b) Assume we have the following input image (640x428 pixels):

This image is an input to a convolutional neural network that has 10 convolutional layers, each with one channel and a 5x5 filter (padding: same, stride: 1x1).

Compute whether the top right neuron of the last layer in the CNN contains any information about the eye of the cat (assume the eye is at pixel 255x210)?

Exercise 4-3 Equivariance of Convolutional Layers

Let X be an input image and K be a filter. For all $(x, y) \in \mathbb{Z}^2$ we define $T_{x,y}$ to be the translation operator that moves every point $X_{i,j}$ in the image by x in the x-direction and by y in the y-direction, i.e.:

$$T_{x,y}X_{i,j} = X_{i-x,j-y}.$$

Show that the *convolution* operator $*$ is translation-equivariant, i.e. commutes with translations:

$$T_{x,y}X * K = T_{x,y}(X * K).$$

Exercise 4-4 Convolutional Neural Network in PyTorch

In Uni2Work / Moodle you find a Jupyter notebook asking you to implement CNNs in PyTorch.