Lecture Notes for

Deep Learning and Artificial Intelligence

Winter Semester 2024/2025

Model-Free Reinforcement Learning

Why model-free Reinforcement Learning?

situations where MDPs are challenging to apply:

- we do not know the exact mechanics (i.e., states, transition function,..) of the environment
- computing all possible outcomes of an action is not scale (too many states, actions, possible transitions..)

$\Rightarrow$ model-free approaches learn while acting without explicitly modelling the environment

$\Rightarrow$ requires an environment reacting to the agent

in this lecture

- model free approaches on discrete state spaces
  - $\Rightarrow$ we still learn an action for every state
  - $\Rightarrow$ large state spaces still yield problems
- model free prediction:
  - learning from complete episodes (Monte Carlo Learning)
  - learning based on bootstrapping (Temporal Difference Learning)
  - combinations (TD(λ) approach)
- model free control:
  - exploration vs. exploitation
  - on-policy learning (SARSA)
  - off-policy learning (Q-Learning)

Model free Prediction

Given policy π, we want to predict $U^{\pi}(s)\ \forall\ s \in S$.

Without a model, we cannot compute the expectation directly.

We have to sample from the environment.

Two ways to do this:

- sample complete episodes and afterwards update the empirical mean of all states (Monte-Carlo Learning)

- sample one step and update the estimate by the direct reward and the current estimate of the next step (temporal difference learning)

Monte-Carlo Learning

- learn directly on complete episodes $S_1, A_1, R_1, S_2, .. O_t, R_t$
  $\Rightarrow$ allows for seeing all future rewards
- does not need a model/MDP transitions or the exact distribution of rewards
- no estimate about the future is involved (no bootstrapping)
- estimate the expected reward by the empirical mean of future rewards when following policy π
- BUT: can only be applied to episodic problems (episodes must terminate to be complete)

Monte-Carlo Policy Evaluation

Given policy π, we want to predict $U^{\pi}(s)\ \forall\ s \in S$ and a set of episodes of experience x ∈ X:

$$x = s_1, r_1, a_1, s_2, r_2, a_2, \dots, a_{t-1}, s_t, r_t \sim \pi$$

remember: $G_t(x) = \sum_{i=t}^{l} \gamma^i r_i^x \mathit{with}\ 0 < \gamma \leq 1$

$$\mathit{and}\ U^{\pi}(s_i) = \mathbb{E}[G_t | s_t = s_i, \pi]$$

⇒ Monte-Carlo Learning uses the empirical mean over all episodes X to estimate the utility.

Monte-Carlo Policy Evaluation (MCPE)

for a known policy π and a set of complete sample episodes X following π:

- let X(s) be the set of (sub-)episodes starting with s
- to estimate utility $U^{\pi}(s)$ average over the expected reward:

$$\mathrm{U}(s) = \sum_{x \in X(s)} \frac{\sum_{i=1}^{l} \gamma^i r_i}{|X(s)|}$$

- if |X(s)| is sufficiently large for all $s \in S$:

  $$U(s) \rightarrow U^{\pi}(s)$$

  (c.f., the law of large numbers)

First Visit and Every Visit MCPE

Sometimes an episode x might visit state s more than once:

- ***First Visit Monte-Carlo Policy Evaluation*** only considers the first time s is visited in x

  $\Rightarrow$ in case of costs, too pessimistic

  $\Rightarrow$ in case of rewards, too optimistic

- ***Every Visit Monte-Carlo Policy Evaluation*** considers every time s is visited in x

  $\Rightarrow$ considers postfixes of the same episode multiple times

Incremental Monte-Carlo updates

- when training on an environment, we usually sample until the result is stable ⇒ incremental training
- compute incremental mean:

$$\mu_k = \frac{1}{k}\sum_{j=1}^{k} x_j = \frac{1}{k}\left(x_k + \sum_{j=1}^{k-1} x_j\right) = \frac{1}{k}(x_k + (k-1)\mu_{k-1})$$

$$= \mu_{k-1} + \frac{1}{k}(x_k - \mu_{k-1})$$

- if the environment is non-stationary, we should limit the weight of older episodes:

$$\mathrm{U}(s_t) \leftarrow \mathrm{U}(s_t) + \alpha\big(G_t - \mathrm{U}(s_t)\big)$$

Temporal Difference Learning

problem: Can we still learn if episodes are incomplete?

- the later part of $\sum_{i=1}^{l} \gamma^i r_i$ is missing
- in the extreme case, we just have 1 Step: s_t, a, r, s_{t+1}

Temporal Difference Learning

- idea similar to incremental Monte-Carlo learning:

$$\mathrm{U}(s_t) \leftarrow \mathrm{U}(s_t) + \alpha\big(G_t - \mathrm{U}(s_t)\big)$$

- Policy Evaluation with Temporal Difference (TD) Learning:

$$\mathrm{U}(s_t) \leftarrow \mathrm{U}(s_t) + \alpha\big(R(s_{t+1}) + \gamma \mathrm{U}(s_{t+1}) - \mathrm{U}(s_t)\big)$$

- TD target: $R(s_{t+1}) + \gamma \mathrm{U}(s_{t+1})$
- TD error: $R(s_{t+1}) + \gamma \mathrm{U}(s_{t+1}) - \mathrm{U}(s_t)$
- each step estimates the mean utility incrementally

Properties of TD-Learning

- TD-learning can learn online after every step
  - $\Rightarrow$ TD learning does not have to wait until the episode ends to update U(s)
  - $\Rightarrow$ TD learning works for continuing (non-terminating) environments

- TD learning learns based on an estimate of the future development of the following state
  - $\Rightarrow$ initial estimate influences convergence
  - $\Rightarrow$ may add bias during training

Bias and Variance

To further compare MC and TD learning, we examine the properties of the way $\mathrm{U}(s_t)$ is computed.

- $G_t(x) = \sum_{i=t}^{l} \gamma^i r_i^x$ is an unbiased estimate of $U_\pi(S_t)$
- the true TD target : $R(s_{t+1}) + \gamma U_\pi(s_{t+1})$ is also an unbiased estimate of $U_\pi(S_t)$
- given the TD estimate $\widehat{U}_\pi(S_t)$ of the $U_\pi(S_t)$, the TD target $R(s_{t+1}) + \gamma \widehat{U}_\pi(s_{t+1})$ is biased

$\Rightarrow$ TD has a smaller variance than G_t:

- G_t depends on many random actions, transitions, rewards
- TD only depends on one action, transition and reward

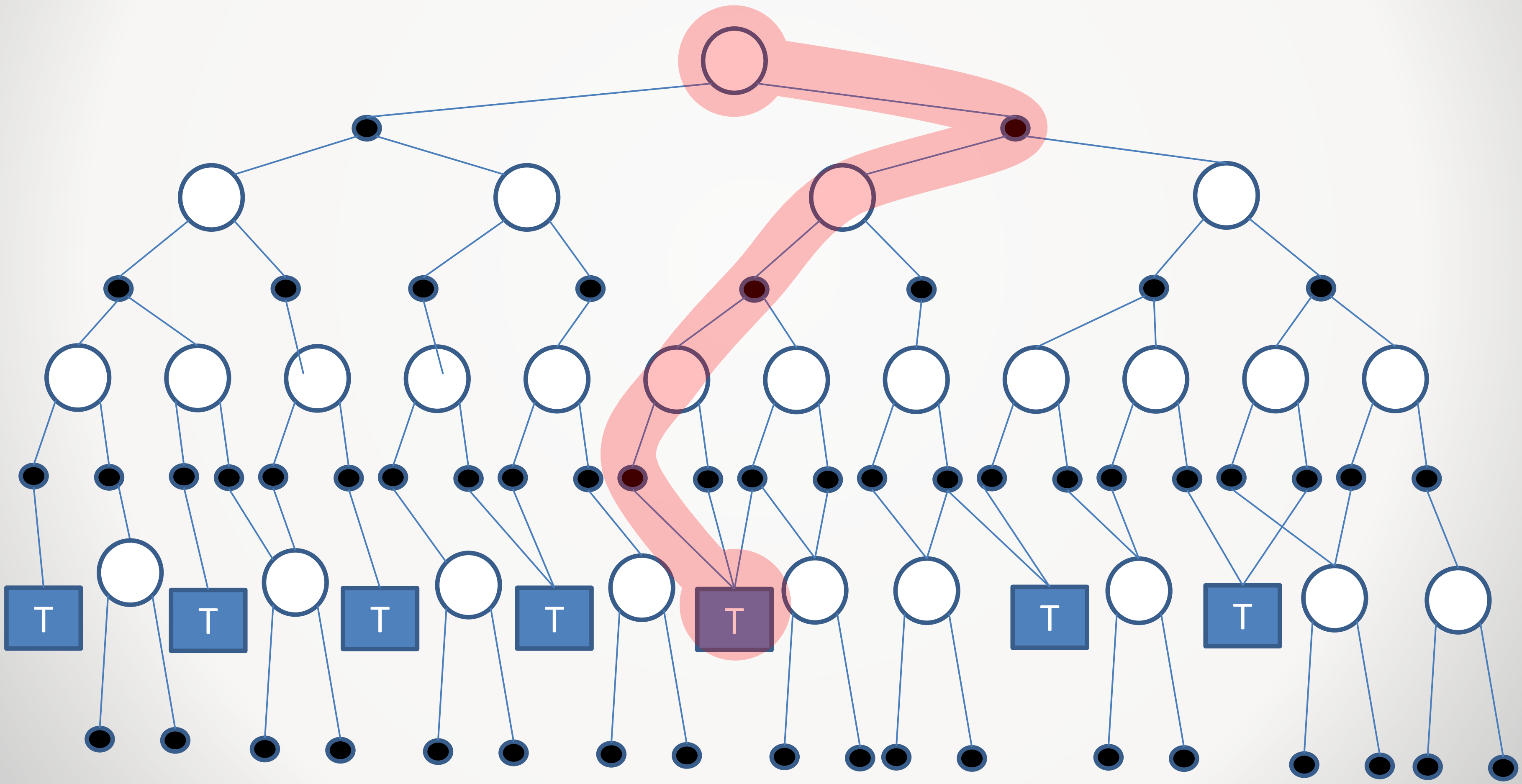
TD vs. MC Summary

- MC has high variance but no bias
  - stable convergence against $U\pi(s)$
  - not very sensitive to initial estimate
  - simple to use
  - might suffer from insufficient samples

- TD has low variance, but is biased
  - usually faster convergence than MC
  - sensitive to initial models
  - TD(0) still converges to $U\pi(s)$ in most cases (function approximations might cause problems)
  - copes well with limited samples due to exploiting the Markov property

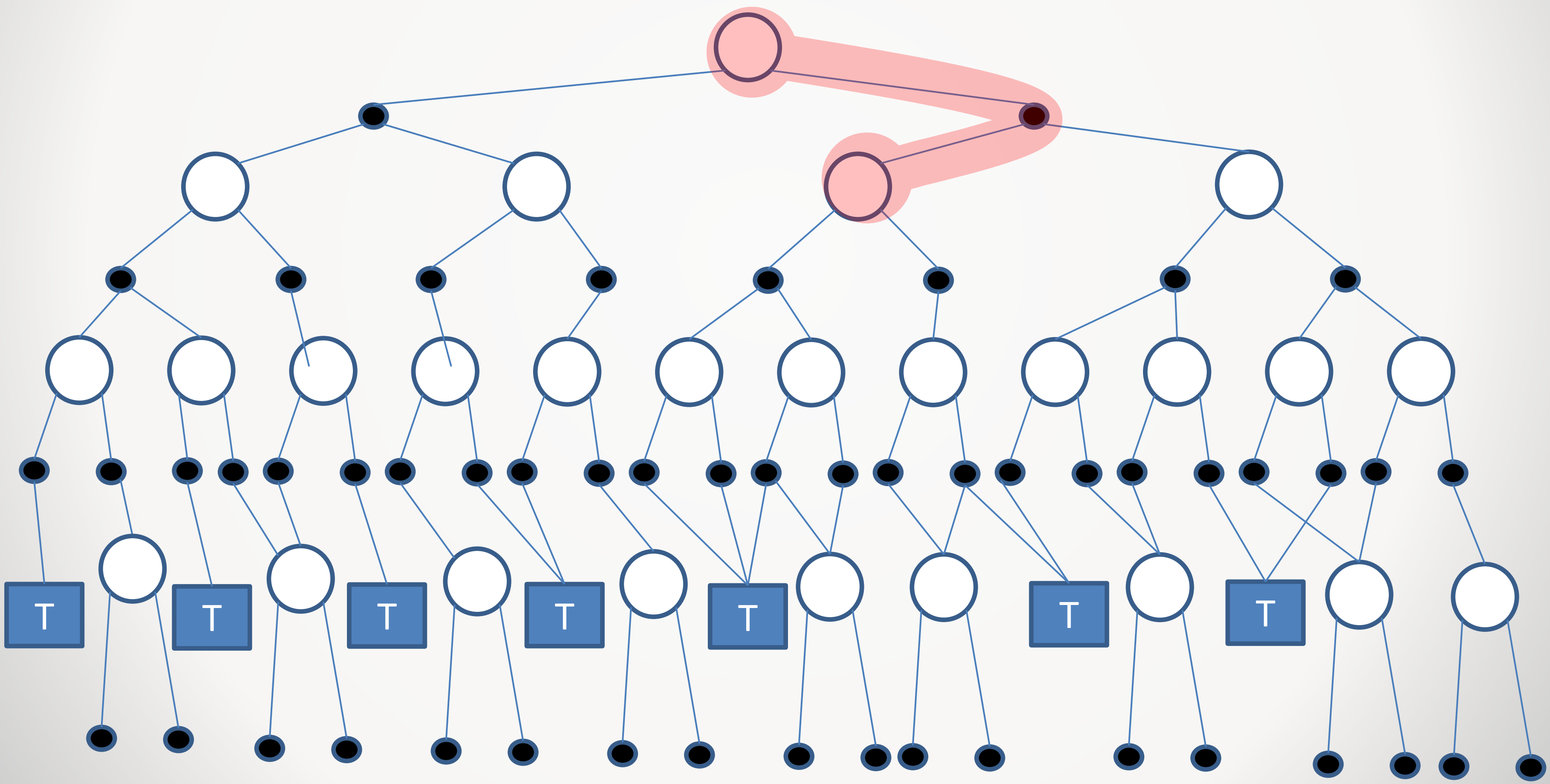
Recap Backup Strategies

Monte Carlo Backup: $U(S_t) \leftarrow U(S_t) + \alpha\big(G_t - U(S_t)\big)$



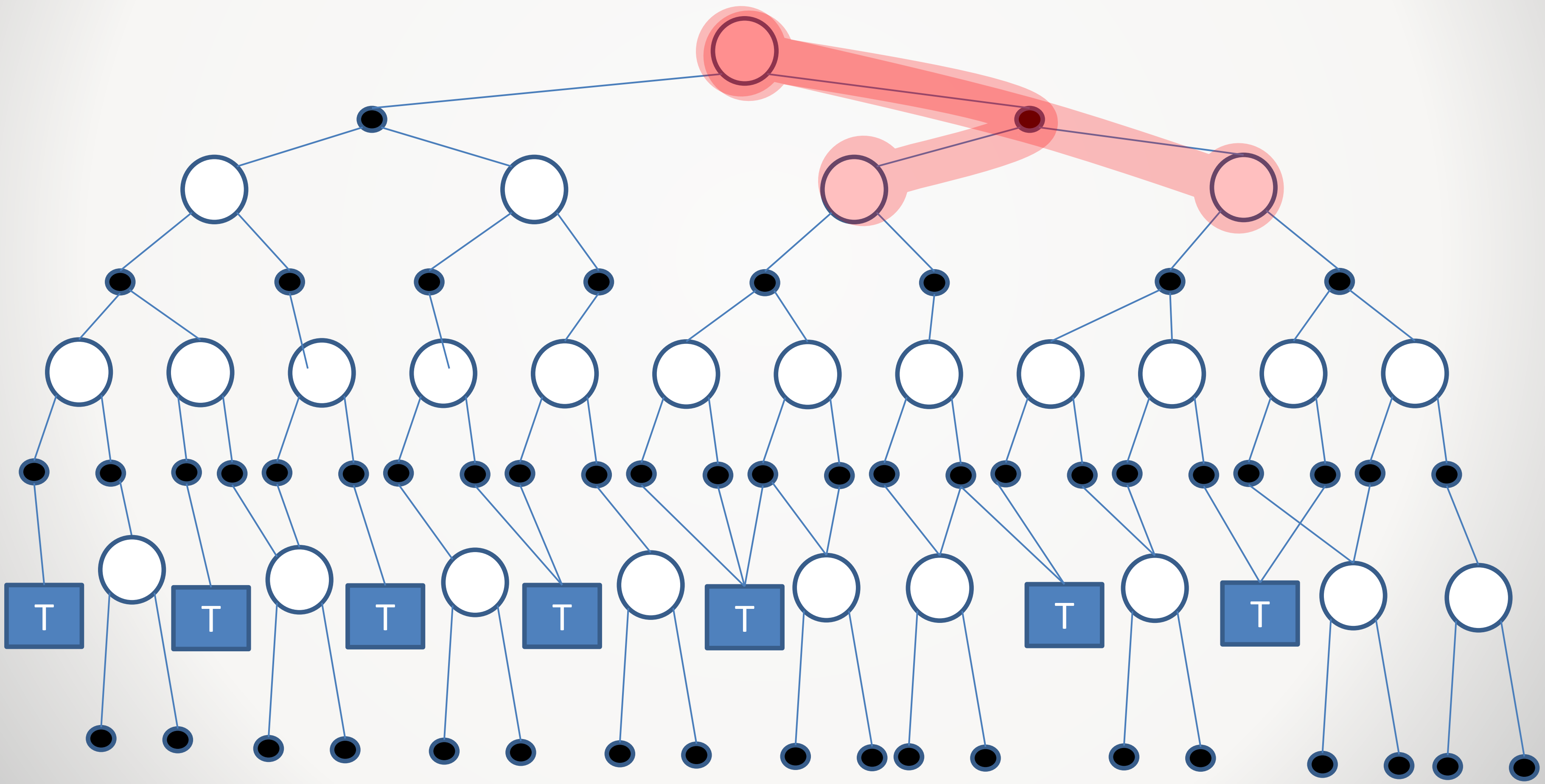
Recap Backup Strategies

Temporal Difference Backup: $U(S_t) \leftarrow U(S_t) + \alpha\big(R(s_{t+1}) + \gamma \mathrm{U}(s_{t+1}) - \mathrm{U}(s_t)\big)$



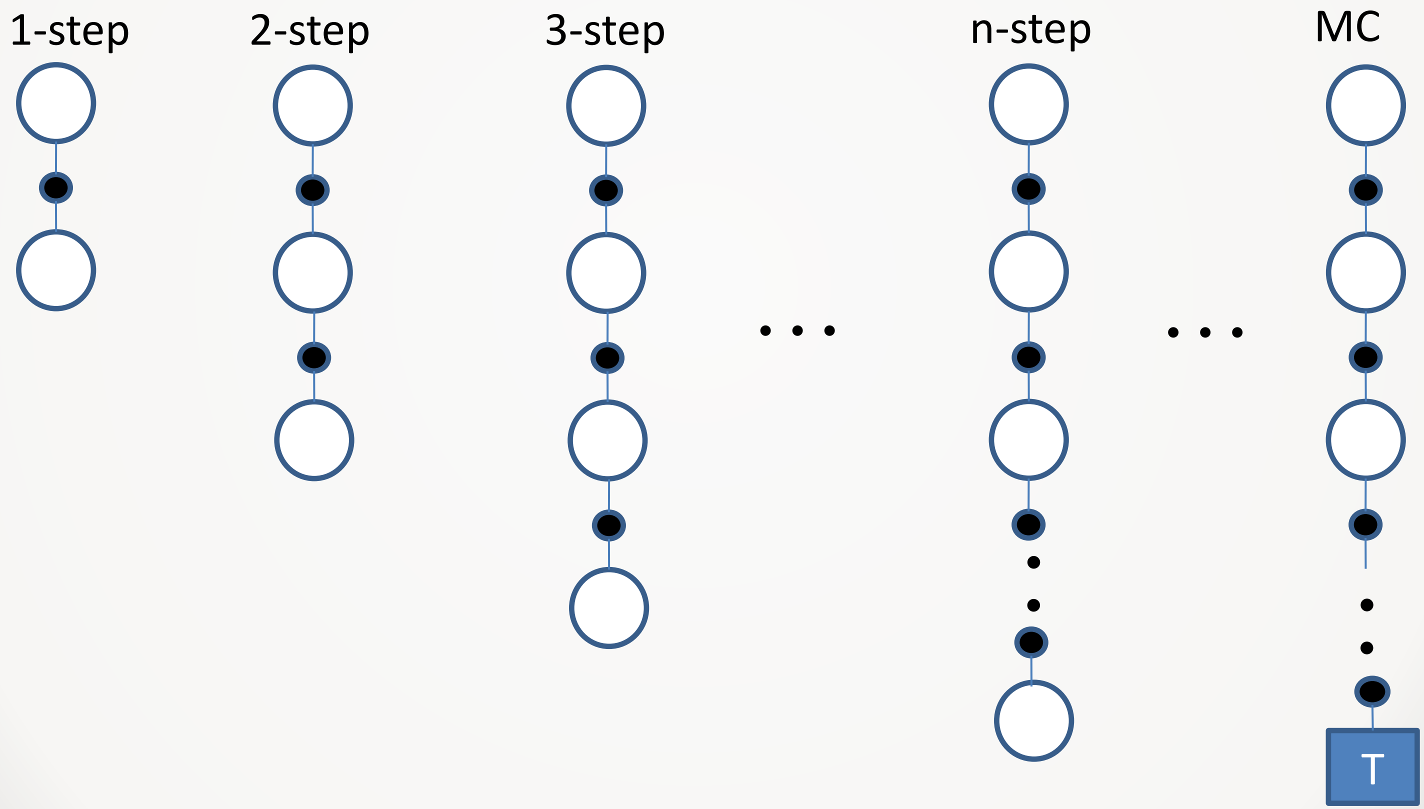
Recap Backup Strategies

Dynamic Programming Backup: $U(S_t) \leftarrow \mathbb{E}[R(s_{t+1}) + \gamma \mathrm{U}(s_{t+1})]$



n-step Prediction

idea: TD looks 1 step ahead and MC looks until the end of the episode. Why not combine and look n steps ahead?



n-step Return

- return for n = 1,2,..,∞:
  n=1 : $G_t^{(1)} = R_{t+1} + \gamma U(S_{t+1})$ (TD)
  n=2 : $G_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma^2 U(S_{t+2})$
  $\vdots$
  n=∞ : $G_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{T-1} R_T$ (MC)

- general n-step return:

$$G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n U(S_{t+n})$$

- n-step temporal difference learning:

$$U(S_t) \leftarrow U(S_t) + \alpha \left(G_t^{(n)} - \mathrm{U}(s_t) \right)$$

properties of n-step TD learning

Pro:

- less bias than TD learning and converges faster than MC

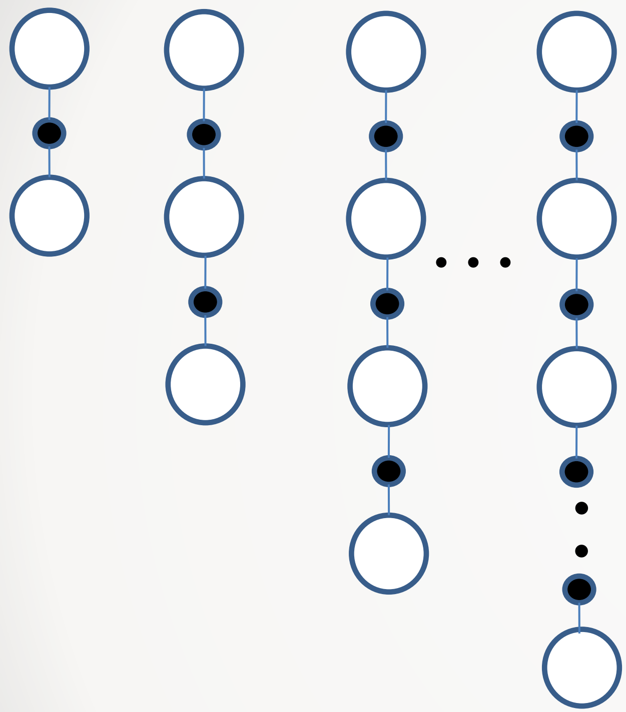
Con:

- episodes might strongly vary in length
- choice of *n* might influence convergence speed

$\Rightarrow$ combine multiple values for *n*

$\Rightarrow$ average over all values for *n*

TD(λ) and λ Returns



1-λ (1-λ) λ (1-λ) λ^2 λ^{T-t-1}

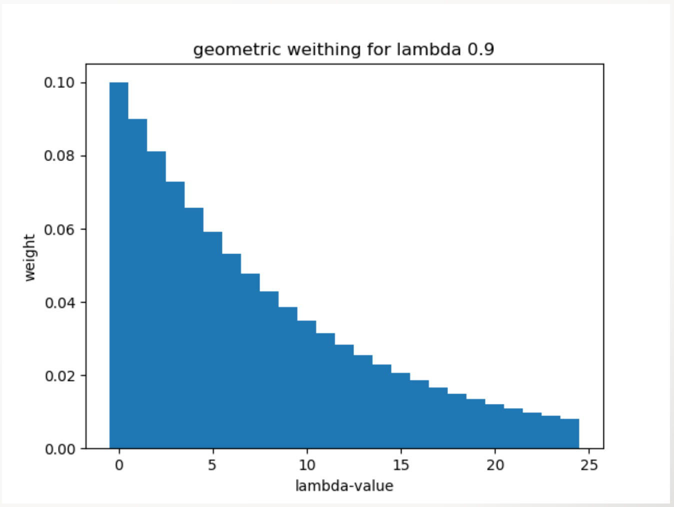
- λ-return G_t^{λ} combines all n-step returns $G_t^{(n)}$
- Using weight (1-λ) λ^{n-1} :

$$G_t^{\lambda} = (1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1} G_t^{(n)}$$

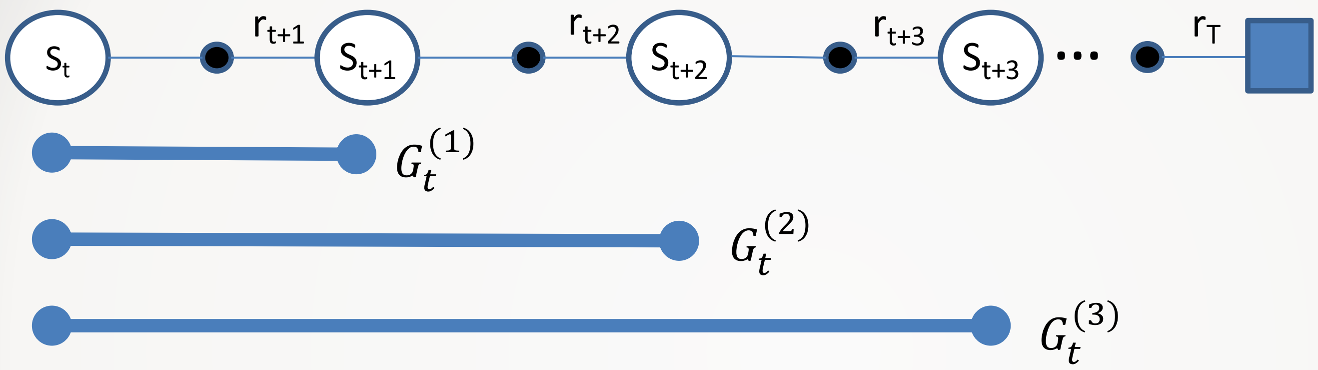
(geometric weighting)

- Forward-view TD(λ):

$$U(S_t) \leftarrow U(S_t) + \alpha\left(G_t^{\lambda} - \mathrm{U}(s_t)\right)$$



Forward TD(λ)



- update utility towards λ-return G_t^{λ}
- forward-view looks into the future to compute G_t^{λ}
- suffers from the same problems as MC

  ⇒ must compute complete episodes

Backward View

To practically apply *TD(λ)*, we want a method which does not have to wait until the end of the episode.

$\Rightarrow$ look backward and update return of states visited so far

Idea:

- in the forward approach, we consider each state and update $U(S_t)$ for the remaining episode
- in the backward approach, we update the $U(S_{t-i})$ for previous states S_i after observing reward R_t
  - each R_t influences the utility of all previous states S_i
  - part of the TD error for S_i relates to the later R_t

$\Rightarrow$ update the $U(S_{t-i})$ with a part of the recent TD error:

TD-error: $\delta_t = R_{t+1} + \gamma U(S_{t+1}) - U(S_t)$

TD-update: $U(s) \leftarrow U(s) + \alpha\delta_t E_t(s)$

where $E_t(s)$ is a measure describing the eligibility of δ_t for state s.

Eligibility Traces

- We need a measure to describe how strong the current TD error influences the utility of state s : U(s)
- general heuristics: frequency and recency
  - The more recent a state s was visited, the more U(s) is influenced by δ_t
  - The more often a state s occurred, the more U(s) is influenced by δ_t

$\Rightarrow$ eligibility traces combine both heuristics

$$E_o(s) = 0$$

$$E_t(s) = \underbrace{\gamma\lambda E_{t-1}(s)}_{\text{decayed old eligibility}} + \underbrace{1(S_t = s)}_{\text{increment when observing state s by 1}}$$

TD(λ) and TD(0)

For λ=0, $E_t(s) = 1(S_t = s)$:

$$\begin{aligned} U(s) \leftarrow U(s) + \alpha\delta_t E_t(s) \\ = U(s) + \alpha\delta_t \\ = U(s) + \alpha\big(R_{t+1} + \gamma U(S_{t+1}) - U(s)\big) \end{aligned}$$

- λ=0 means only the previous state is updated
- this corresponds to the original 1-step TD learning
- In the forward view:

$$G_t^0 = (1-0)\sum_{n=1}^{\infty} 0^{n-1}\, G_t^{(n)} = (1-0)\sum_{n=1}^{\infty} 0^{n-1}\left(U(S_{t+n}) + \sum_{j=1}^{n} R_{t+j}\right)$$

$$= 0^0\big(U(S_{t+1}) + \textstyle\sum_{j=1}^{1} R_{t+j}\big) + \sum_{n=2}^{\infty} 0^{n-1}\big(U(S_{t+n}) + \sum_{j=1}^{n} R_{t+j}\big)$$

$$= U(S_{t+1}) + R_{t+1}$$

TD(λ) and TD(0)

- for λ=1 (credit is kept until end of the episode)
- in an episodic environment (finite episodes)

$\Rightarrow$ over the course of an episode, the total updates w.r.t. TD(1) are the same as the total updates for MC

Theorem: The sum of offline updates is identical for forward-view and backward-view TD(λ):

$$\sum_{t=1}^{T} \alpha \delta_t E_t = \sum_{t=1}^{T} \alpha \left(G_t^{\lambda} - U(S_t) \right) 1(S_t = s)$$

TD(1) and MC Learning

- TD(1) is roughly equivalent to every-visit Monte-Carlo
- error is accumulated online, step-by-step
- If the utility is only updated offline at the end of the episode, then the total update is exactly the same as for MC Learning.

Model Free Control

so far: We can evaluate policy π based on observing it in an environment.
But what is a good policy?

Example tasks for model-free control:

- control robots
- play games
- control automatic systems
- ..

Apply Reinforcement Learning if MDP is unknown or too big to solve:
$\Rightarrow$ sample experience from an environment and employ model-free control

Policy Optimization

Idea: adapt Policy Iteration
(evaluate policy and update greedily)

- greedy policy update of U(s) requires MDP:

$$\pi'(s) = R(s) + argmax_{a \in A(s)} P(s'|s,a) U(s')$$

- Q-Value ***Q(s,a)***: If we choose action a in state s, what is the expected reward?
  ⇒ We do not need to know where action ***a*** will take us!

- Improving Q(s,a) is model free:

$$\pi'(s) = argmax_{a \in A(s)} Q(s,a)$$

- Adapt the idea of Policy Iteration:
  - Start with a default policy
  - evaluate policy (previous slide)
  - update policy: e.g., with greedy strategy

Samples and Policy Updates

Problem: After updating a policy, we need enough samples following the policy.

- actual observed episodes usually do not cover enough policies (episodic samples are policy-dependent)

$$s_1, a_1, r_1, s_2, a_2, r_2, s_3, a_3, r_3, s_3 \ldots, s_l, a_l, r_l, s_{l+1}$$

- we need to sample from an environment dynamically:
  - measure the reaction of the physical world (e.g., robotics..)
  - build simulations that mimic the physical world
  - In games, let the agent play and learn !!!
- we need a strategy for sampling these (s,*a*) pairs.
- the environment often determines *s* as a result of the last action *a*.

Learning from a Queryable Environment

- we can generate an infinite amount of samples
- the environment might be non-deterministic:
  - the same state *s* and action *a* might cause different outcomes *s'* and *R(s')*
  - multiple samples for the same (s,*a*) might be necessary to estimate *Q(s,a)*

- How to sample over the state-action space?
  - **exploit**: If we find a good action, keep it and improve the estimate of Q(s,*a*). Usually, it's a waste of time to optimise Q(s,*a*) for bad actions.

  - **explore**: Select unknown or undersampled actions:
    - a low estimate of Q(s,*a*) does not mean that the option is bad. Maybe, (s,*a*) is just underexplored.
    - try out new things might lead to an even better solution

ε-Greedy Exploration

- makes sure that sampling considers new actions
- when sampling:
  - with probability 1-ε choose greedy action
  - with probability ε chose random action
- Sampling policy:

$$\pi(a|s) = \begin{cases} \dfrac{\varepsilon}{m} + (1-\varepsilon) & if\ a = argmax_{a \in A(s)} Q(s,a) \\ \dfrac{\varepsilon}{m} & otherwise \end{cases}$$

- achieves that Q-values improve and guarantees that all actions are explored if optimized long enough

ε-Greedy improvement

Theorem: For any ε-greedy policy π, the ε-greedy policy π' with respect to q_π is an improvement, i.e., $U_{\pi'}(s) \geq U_\pi(s)$.

Proof:

$$q_\pi\big(s, \pi'(s)\big) = \sum_{a \in A} \pi'(a|s)\, q_\pi(s,a)$$

$$= \frac{\varepsilon}{m} \sum_{a \in A} q_\pi(s,a) + (1-\varepsilon) \max_{a \in A} q_\pi(s,a)$$

$$\geq \frac{\varepsilon}{m} \sum_{a \in A} q_\pi(s,a) + (1-\varepsilon) \sum_{a \in A} \frac{\pi(a|s) - \frac{\varepsilon}{m}}{1-\varepsilon} q_\pi(s,a)$$

$$= \sum_{a \in A} \pi(a|s)\, q_\pi(s,a) = U_\pi(s)$$

Monte-Carlo Policy Iteration

- sample episode(s) from the environment
- do MC policy evaluation to update q-values *q(s,a)*
- update $\pi'(s)$ based on the observed episodes and so on

remarks:

- it might not be necessary to update all *q(s,a)*
- using ε-greedy policy π' makes sure that we explore underexplored action sufficiently at some point in time
- optimising π' is not the same as the optimal policy π*

⇒ How can we still find π*?

Greedy in the Limit with Infinite Exploration (GLIE)

Definition:

Greedy in the Limit with Infinite Exploration (GLIE)

- All state-action pairs are explored infinitely many times

$$\lim_{k \to \infty} N_k(s, a) = \infty$$

- The policy converges to a greedy policy,

$$\lim_{k \to \infty} \pi_k(a|s) = 1\left(a = \operatorname*{argmax}_{a\prime \in A} Q_k(s, a')\right)$$

GLIE Monte-Carlo Control

- Sample k^{th} episode using π:
  x=$S_1, R_1, A_1,\ldots, A_{t-1}, S_t, R_t \sim \pi$
- For each state S_t and action A_t in x update:
  $N_t(S_t,\ A_t) \leftarrow N_t(S_t,\ A_t)$+1
  $Q_t(S_t,\ A_t) \leftarrow Q_t(S_t,\ A_t) + \frac{1}{N_t(S_t,\ A_t)}\big(G_t - Q(S_t,\ A_t)\big)$
- Improve policy based on new action-value faction Q
  $\epsilon \leftarrow \frac{1}{k}$
  $\pi \leftarrow \epsilon - greedy(Q)$

Theorem: GLIE Monte-Carlo control converges to the optimal action-value function *Q(s,a)*$\rightarrow$*q*(s,a).*

TD and Monte-Carlo Control

In general TD has several advantages over MC:

- lower variance
- online learning (no waiting until end of episode)
- learning from incomplete episodes

How to control based on TD?

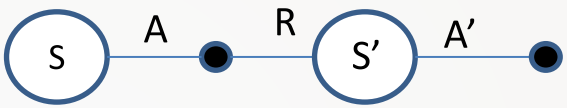
- Update Q(S,A) based on TD
- Use ε-greedy policy improvement
- Update every time step

SARSA

TD applied to q-function Q(S,A):

$$Q(S,A) \leftarrow Q(S,A) + \alpha\big(R + \gamma Q(S',A') - Q(S,A)\big)$$

requires a sample of the form:



Basic idea behind SARSA (State Action Reward State Action):

For every step we do:

- evaluate policy based on the above formula (policy evaluation)
- update policy based on ε-greedy policy (policy evaluation)

SARSA Algorithm

```
init Q(s,a)∀s ∈ S,a ∈ A and Q(terminal,)=0
for n episodes:
  init s
  choose a from A(s) based on Q(e.g.,ε-greedy)
  repeat until episode is finished:
    s’,r = query_Env(s,a)
    choose a’from A(s’)based on Q(e.g.,ε-greedy)
    Q(s,a)← Q(s,a)+α(r+γQ(S’,a)-Q(s,a))
    s ← s’, a ← a’
  until s is terminal
```

SARSA Convergence

Theorem: SARSA converges to the optimal action-value function q*, Q(S,A)→ q*(s,a), if the following conditions hold:

- GLIE sequence of policies $\pi_t(a|s)$
- Robbins-Monroe sequence of step-sizes α_t

$$\sum_{t=1}^{\infty} \alpha_t = \infty$$
$$\sum_{t=1}^{\infty} \alpha_t^2 < \infty$$

n-step SARSA

- consider the following n-step q-values for n = 1,2,..,∞:

  n=1 : $q_t^1 = R_{t+1} + \gamma Q(S_{t+1}, A_{t+1})$

  n=2 : $q_t^1 = R_{t+1} + \gamma R_{t+2} + \gamma^2 Q(S_{t+2}, A_{t+2})$

  …

  n=∞ :$q_t^\infty = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{T-1} R_T$

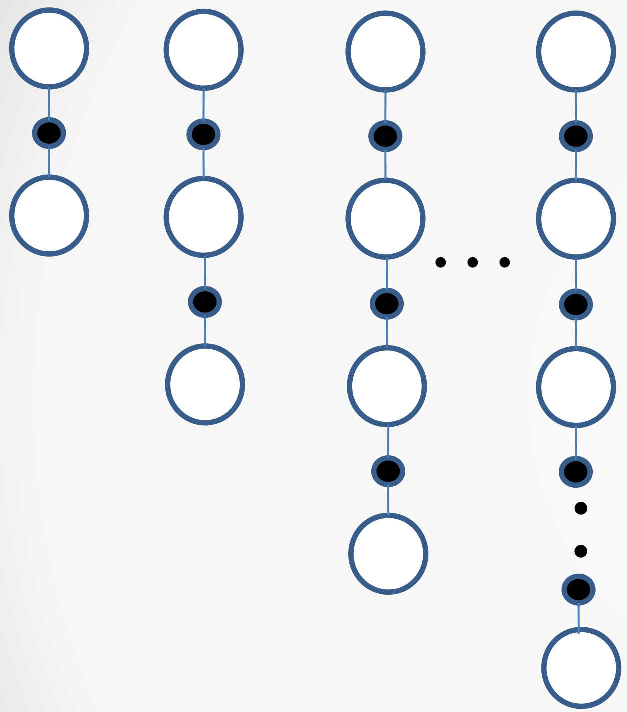
- general n-step Q-return:

$$q_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n Q(S_{t+n}, A_{t+n})$$

- n-step temporal difference learning:

$$\mathrm{Q}(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\left(q_t^n - \mathrm{Q}(S_t, A_t)\right)$$

Forward View SARAS(λ)



1-λ (1-λ) λ (1-λ) λ^2 λ^{T-t-1}

- The q^λ-return combines all n-step Q-returns q_t^n
- Using weight (1-λ) λ^{n-1} :

$$q_t^\lambda = (1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1}\, q_t^n$$

- Forward-view SARSA(λ):

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\left(q_t^\lambda - Q(S_t, A_t)\right)$$

Backward View SARSA(λ)

- Just as for TD(λ), we can employ eligibility traces
- However, SARSA(λ) needs an eligibility trace for each state-action pair instead for each state:

$$E_o(s,a) = 0$$

$$E_t(s,a) = \gamma\lambda E_{t-1}(s,a) + 1(\mathrm{S}_t = s, \mathrm{A}_t = \mathrm{a})$$

- Q(s,a) is updated for every state s and action a
- Computing the error δ_t and eligibility trace $E_t(s,a)$ we can adapt the backward updates:

TD-error: $\delta_t = R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)$

TD-update: $Q(s,a) \leftarrow Q(s,a) + \alpha\delta_t E_t(s,a)$

n-step SARSA Algorithm

```
init Q(s,a) ∀s ∈ S, a ∈ A(s)
for n episodes:
  E(s,a)=0, ∀s ∈ S, a ∈ A(s)
  init S,A
  repeat until episode is finished:
    Take action A, observe R,S'
    choose A' from A(S') based on Q(e.g.ε-greedy)
    δ ← R + γQ(S',A') − Q(S,A)
    E(S,A) ← E(S,A)+1
    For ∀s ∈ S, a ∈ A(s):
        Q(s,a) ← Q(s,a) + αδ_t E_t(s,a)
        E_t(s,a) ← γλE_{t−1}(s,a)
    S←S',A ←A'
  until S is terminal
```

Off-Policy Learning

On-policy Learning: Learn a policy by sampling from the same policy.

But: Sometimes, it is better to observe the behaviour of another policy to find a better policy.

$\Rightarrow$ **Off-Policy Learning**

- learn based on the experience of other agents
  (or humans)
- Re-use experience generated from old policies
- Learn the optimal policy by following an exploratory policy
- Learn about multiple policies while following one policy

Importance Sampling

Can we compute the expectation of our returns when following another policy?

$\Rightarrow$ Rewards stay the same, but distribution over the states changes

$\Rightarrow$ We need to correct the likelihoods to adjust for the different visiting probabilities.

Importance sampling:

$$\mathbb{E}_{X\sim P}[f(X)] = \sum P(X) f(X) =$$

$$\sum Q(X) \frac{P(X)}{Q(X)} f(X) = \mathbb{E}_{X\sim Q}\left[\frac{P(X)}{Q(X)} f(X)\right]$$

Importance Sampling for Off-Policy MC

- Consider the observed policy μ to evaluate target policy π
- Weight return G_t w.r.t. similarity between these policies
- Multiply importance sampling corrections throughout the whole episode

$$G_t^{\pi/\mu} = \frac{\pi(A_t|S_t)\pi(A_{t+1}|S_{t+1})}{\mu(A_t|S_t)\mu(A_{t+1}|S_{t+1})} \dots \frac{\pi(A_T|S_T)}{\mu(A_T|S_T)} G_t$$

- Update value with corrected return:

$$U(S_t) \leftarrow U(S_t) + \alpha\left(G_t^{\pi/\mu} - U(S_t)\right)$$

- Does not work if μ=0 and π >0
- Can significantly increase variance

Importance Sampling for Off-Policy TD

- Use TD targets generated from μ to evaluate π
- Weight TD target R+γU(S') by importance sampling
- For TD only a single step correction is needed:

$$U(S_t) \leftarrow U(S_t) + \alpha\left(\frac{\pi(A_t|S_t)}{\mu(A_t|S_t)}\left(R_{t+1} + \gamma U(S_{t+1})\right) - U(S_t)\right)$$

- Much lower variance than MC importance sampling
- Policies need to be similar over a single step

Q-Learning

- Off-policy learning of action-value pairs Q(s,a)
- No importance sampling is necessary
- Next action selected based on behaviour policy

$$A_{t+1} \sim \mu(\cdot, S_t)$$

- But updates are done on alternative successor:

$$\mathrm{A}' \sim \pi(\cdot, S_t)$$

- And update *$Q(S_t,A_t)$* towards value of alternate action:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\big(R_{t+1} + \gamma Q(S_{t+1}, A') - Q(S_t, A_t)\big)$$

Q-Learning

- Both policies the behaviour policy μ and the target policy π are improved.
- The target policy π is greedy w.r.t. Q(s,a):

$$\pi(S_{t+1}) = \underset{a\prime \in A}{\operatorname{argmax}}\, Q(S_{t+1}, a')$$

- The behaviour policy μ is e.g. ε-greedy w.r.t. Q(s,a)
- The Q-learning target then simplifies to:

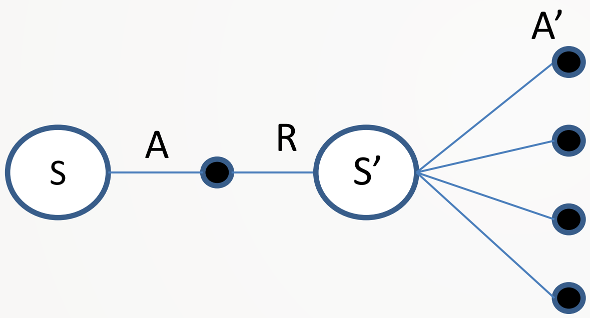
$$R_{t+1} + \gamma Q(S_{t+1}, A')$$

$$= R_{t+1} + \gamma Q\left(S_{t+1}, \underset{a\prime \in A}{\operatorname{argmax}}\, Q(S_{t+1}, a')\right)$$

$$= R_{t+1} + \max_{a\prime \in A} \gamma Q(S_{t+1}, a')$$

Q-Learning

Theorem: Q-learning control converges to the optimal action-value function, Q(S,A)→q*(s,a).



$$Q(S,A) \leftarrow Q(S,A) + \alpha\left(R + \gamma \max_{a' \in A} Q(S',a') - Q(S,A)\right)$$

Q-Learning Algorithm

```
init Q(s,a) ∀s ∈ S,a ∈ A
for n episodes:
  init s
  repeat until episode is finished:
    choose a from A(s) with π_b
    s',r = query_Env(s,a)
    Q(s,a)← Q(s,a)+α(r+γmax_a Q(S',a)-Q(s,a))
    s ← s'
  until s is terminated
//terminal state or finite horizon is reached
```

Summary

- Policy Evaluation:
  - Monte-Carlo (MC) Learning: evaluate on complete episodes:
    high variance, no bias, slow convergence
  - Temporal Difference (TD) Learning: evaluate single steps and approximate future via Bootstrapping: lower variance, faster convergence, adds bias
  - TD(λ): Links between MC and TD
- Control:
  - on-policy learning: “learn on the job”
    - GLIE Monte-Carlo Control
    - SARSA and SARSA(λ)
  - off-policy learning: learn by watching other policies
    - MC and TD with importance sampling
    - Q-learning

Literature

- Lecture notes D. Silver: Introduction to Reinforcement Learning (https://www.davidsilver.uk/teaching/)

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- R. S. Sutton, A. G. Barto: **Reinforcement Learning: An Introduction (Adaptive Computation and Machine Learning),** The MIT Press; Auflage: 2., 2018