15. 11. 23

[illegible] M: Ketmaketlik va uning limiti

$\varepsilon > 0 \quad \exists\, n_0(\varepsilon) \quad n \geq n_0 \quad |x_n - a| < \varepsilon$

$\lim\limits_{n \to \infty} x_n = a$

$$\lim_{n \to \infty} \frac{2n+1}{3n-2} = \frac{2 \cdot \infty + 1}{3 \cdot \infty - 2} = \frac{\infty}{\infty} =$$

$$= \lim_{n \to \infty} \frac{\frac{2n}{n} + \frac{1}{n}}{\frac{3n}{n} - \frac{2}{n}} = \lim_{n \to \infty} \frac{2 + \overbrace{\frac{1}{n}}^{=0}}{3 - \underbrace{\frac{2}{n}}_{=0}} = \frac{2}{3}$$

$\varepsilon = 0{,}01$

$$\left| \frac{2n+1}{3n-2} - \frac{2}{3} \right| < \frac{1}{100}$$

$$\left| \frac{6n + 3 - 6n + 4}{3(3n-2)} \right| < \frac{1}{100}$$

$$\left| \frac{7}{3(3n-2)} \right| < \frac{1}{100}$$

$$\frac{7}{3(3n-2)} < \frac{1}{100}$$

$700 < 9n - 6$

$9n > 706$

$n > 78{,}6$

$n_0 = 79$

aniqmaslik

$\infty + \infty = \infty$
$\infty \cdot \infty = \infty$
$\infty^{\infty} = \infty$

$\infty - \infty, \frac{\infty}{\infty}, \frac{0}{0}, 0^0, 1^{\infty}, 0^{\infty}, \infty^0$

1) $\lim\limits_{x \to 1} \frac{2x^2-1}{4x^2+5x+2} = \frac{2\cdot 1^2 - 1}{4\cdot 1^2 + 5\cdot 1 + 2} = \frac{1}{11}$

2) $\lim\limits_{x \to \frac{1}{2}} \frac{8x^3-1}{6x^2-5x+1} = \frac{0}{0}$

$a^3 - b^3 = \frac{(2x-1)(4x^2+2x+1)}{6(x-\frac{1}{2})(x-\frac{1}{3})} =$

$= \lim\limits_{x \to \frac{1}{2}} \frac{\cancel{(2x-1)}(4x^2+2x+1)}{\cancel{(2x-1)}(3x-1)} = \frac{4\cdot\frac{1}{4} + 2\cdot\frac{1}{2} + 1}{3\cdot\frac{1}{2} - 1} =$

$= \frac{3}{\frac{1}{2}} = 6$

3) $\lim\limits_{x \to -2} \frac{\sqrt{7-x} - \sqrt{13+2x}}{3x^2+x-10} = \frac{0}{0}$

suratni irr. dan qutqaramiz

$\frac{(\sqrt{7-x} - \sqrt{13-2x})(\sqrt{7-x} + \sqrt{13+2x})}{(3x-5)(x+2)\cdot(\sqrt{7-x} + \sqrt{13+2x})} =$

$= \lim\limits_{\to -2} \frac{7-x-13-2x}{(3x-5)(x+2)(\sqrt{7-x}+\sqrt{13+2x})} =$

$= \lim\limits_{x \to -2} \frac{-3(x+2)}{(3x-5)(x+2)(\sqrt{7-x}+\sqrt{13+2x})} =$

$= \frac{-3}{(3\cdot(-2)-5)(-2+2}$

$\lim_{x\to\infty} x(\sqrt{x^2+4}-x)$

$\infty(\sqrt{\infty^2+4}-\infty)$

$\frac{2}{6}$ $\frac{3}{6}$

$=\frac{1}{3}$ $\frac{1}{2}$

16.11.23.

Funksiya hosilasi. Hosila $\lim_{x\to 0}\frac{\sin x}{x}=1$ ^{2} $\frac{5}{4}$

tarifi. Funksiya differensiyasi

yuqori tartibli hosila

1) $y=f(x)$ egri chiziqqa $M_0(x_0 y_0)$ nuqtadan ōtkazilgan urunma tenglamalar

$$y-y_0=f'(x)(x-x_0)$$

2) $y-y_0=-\frac{1}{f'(x)}\cdot x(-x)$ $f'(x)=0$

normalning formulasi

3) $y=(f_1(x)$ $y=f_2(x)$ egri chiziqlar $M_0(x_0 y_0)$ nuqtadan kesishsin ular orasidagi burchak

$$tg\,\varphi=\frac{f_1'(x_0)-f_2(x_0)}{1+f_0'(x_0\cdot f_0'(x_0}$$ ga teng

4)

Xosila jadvali

1) $(u)' = \alpha \cdot u \;\; u'$

2) $\sqrt{u} = \frac{1}{2} \cdot \frac{1}{\sqrt{u}} \cdot u'$

3) $\left(\frac{1}{u}\right)' = -\frac{1}{u^2} \cdot u'$

4) $(a^u) = a^u \cdot \ln a \cdot u'$

5) $(e^u) = e^u \cdot u'$

6) $(u^v)' = v u^{v-1} \cdot u' + u^v \ln u \cdot v'$

7) $\left(\frac{c}{v}\right) = -\frac{c}{v^2}$

$(y') = -\frac{1}{x^{-2}} = \frac{2}{x^3}$

17.11.23

19-n

Boshlangich funksiya va aniqmas integral

Agar ☒ a ; b ☒ oraliqda aniqlangan ☒ f(x) funksiya uchun F ☒(x) ☒ f(x) tenglik ōrinli bōlsa, F(x) funksiya (uchun) f(x) funksiyaning boshlangich funksiyasi deyiladi.

f(x) funksiya rta boshlangich funksiyasi bir-biridan faqat ōzgarmas songa farq qiladi.

Boshlangich funksiyalar tōplami F(x) ☒ C, bu yerda C ōzgarmas son, f(x) funksiyadan ☒ a ; b ☒ oraliq bōyicha olingan aniqmas integral deyiladi va quyidagicha yoziladi

☒ f(x) dx ☒ F(x) ☒ C

Quyida biz integrallashning asosiy qoidalari bilan tanishamiz:

1) □ ∫ □(x) dx □□ d f □ x □ □

f □ x □ □ C

d □ f(x) dx □ d □ F □ x □ □ C □ □ f □ x □ dx

2) □□ f(x) □ g(x) □ dx □ □ f □ x □ dx □ □ g □ x □ dx

3) □ k f(x) dx □ k □ f □ x □ dx k - ōzgarmas son

4) agar □ f(x) dx □ F(x) □ C bōlsa, u holda

□ f(ax □ b) dx □ $\frac{1}{a}$ F(ax □ b) □ C bu yerda

a va b - ōzgarmas sonlar, a □ 0

5) agar □ f(x) dx □ F(x) □ C va u □□(x)

bōlsa, u holda □ f(u) dx □ F(u) □ C

22.11.23.

Mavzu: Aniq integral va uning tadbiqlari. I va II xosmas integrallar

$$S = \int_a^b f(x)\,dx = F(x)\Big|_a^b dx = F(x)\Big|_a^b = F(b) - F(a)$$

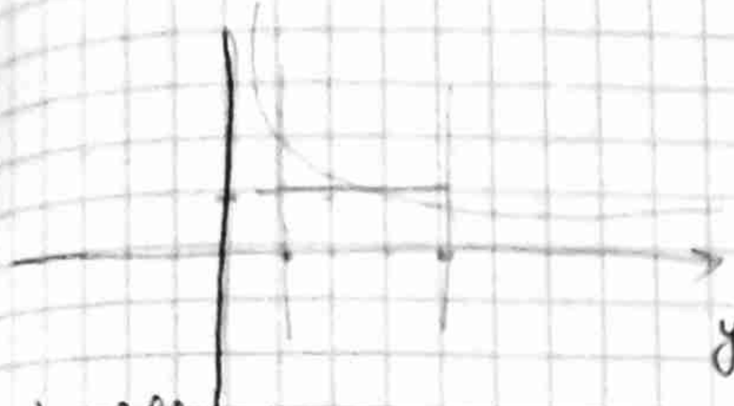
Nyuton Leybnis formulasi

1 - m

$y=\frac{1}{x}$ $y=1$, $x=4$ tog'ri chiziqlar bilan chegaralangan shakl yuzini hisoblang.

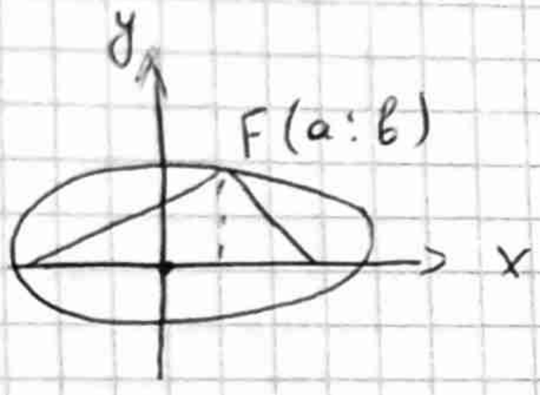
$$S=\int_1^4\left(1-\frac{1}{x}\right)dx=\left(x+\frac{1}{x^2}\right)\Big|_1^4=\left(4+\frac{1}{6}-1-1\right)=$$

$$=\frac{16}{2}+\frac{1}{16}=\frac{33}{16}$$



1) Ellips

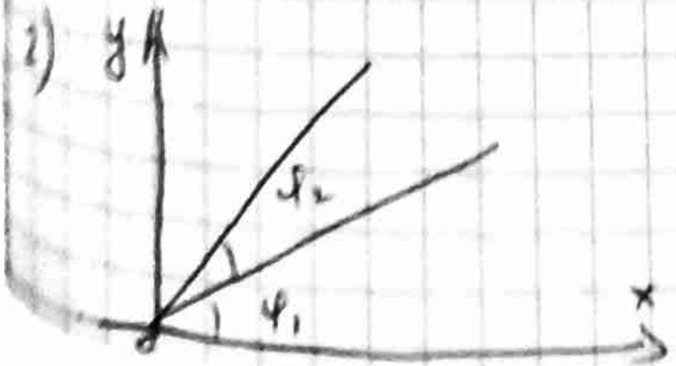
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



$$S=\int_b^a y\,dx$$

$$\begin{cases} x=a\cos t \\ y=b\cdot\sin t \end{cases}$$

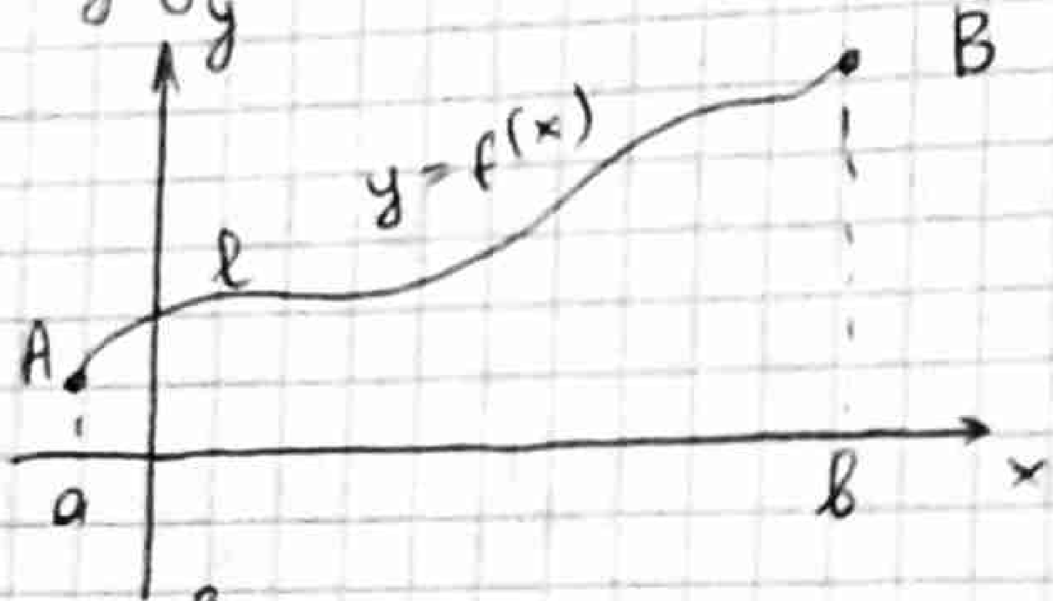
2)



$$S=\frac{1}{2}\int_{\varphi_1}^{\varphi_2} t(\varphi)\,d\varphi$$

yoy uzunliklari

$$l = \int_a^b \sqrt{1 + (f(x))^2}\, dx$$



$$l = \int_\alpha^\beta (x(t))^2 + y$$

24.11.23.

1) Sonli qatorlar, darajali qatorlar

Darajali qatorlar yaqinlashish sohasi

$$\sum^{\infty} a_n = a_1 + a_2 + a_3 + \dots + a_n$$

$$\sum_{n=1}^{\infty} \frac{1}{4n^2+4n+3} = \sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n+3)} = \frac{1}{1\cdot 5} + \frac{1}{3\cdot 7} + \frac{1}{5\cdot 9}$$

$$\frac{1}{(2n-1)(2n+3)} = \frac{\overset{2n+3}{A}}{2n-1} + \frac{\overset{2n-1}{B}}{2n+3}$$

$$1 = 2An + 3A + 2Bn + B \quad \begin{cases} 2A + 2B = 0 \\ 3A - B = 1 \end{cases}$$

$$A = \frac{1}{4} \quad B = -\frac{1}{4}$$

$$\frac{\frac{1}{4}}{(2n-1)} + \frac{\frac{1}{4}}{(2n+3)} = \frac{1}{4}\left(\frac{1}{2n-1} - \frac{1}{2n+3}\right)$$

$\sum_{n=}^{\infty} \frac{3^n + 5^n}{15^n}$ $S = \frac{b_1}{1-q}$

Garmonik qator

$\sum_{n=1}^{\infty} \frac{1}{n^\alpha}$ $\alpha > 1$ yaqin - chekli

$0 < \alpha \leq 1$ uzoq - cheksiz

geometrik progressiya

$\sum_{n=1}^{\infty} aq^{n-1} = q + aq + aq^2 + \dots + aq^{n-1}$

$|q| < 1$ yaqin

$|q| \geq 1$ uzoq

taqqoslash alomati

$\sum_{n=1}^{\infty} a_n > \sum_{n=1}^{\infty} b_n$ uzoq

$\sum_{n=1}^{\infty} a_n < \sum_{n=1}^{\infty} b_n$ - yaqin

yaqin

$\sum_{n=1}^{\infty} \frac{1}{n^2+10} < \sum_{n=1}^{\infty} \frac{1}{n^2}$ yaqin

$\sum_{n=1}^{\infty} \frac{1}{\ln n} > \sum_{n=1}^{\infty} \frac{1}{n}$ uzoq

Dalomber alomati

$\sum_{n=1}^{\infty} a_n$ $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \ell$

Koshi alomati

$\lim\limits_{n\to\infty} \sqrt[n]{a_n} = l$

$l < 1$ yaq

$l > 1$ uzoq

$l = 1$ boshqa usul

$\sum\limits_{n=1}^{\infty} \frac{1}{2^n n}$

$\lim\limits_{n\to\infty} = \dfrac{\frac{1}{2^{n+1}\cdot(n+1)!}}{\frac{1}{2^n\cdot n!}} = \lim\limits_{n\to\infty} \dfrac{2^n\cdot n!}{2^n\cdot 2\cdot n!\cdot(n+1)}$

$\lim\limits_{n\to\infty} \frac{1}{2(n+1)} = 0 < 1$ yaqin

1) $\sum\limits_{n=1}^{\infty} \frac{n}{5^n}$ $\quad \lim \dfrac{\frac{n+1}{5^{n+1}}}{\frac{n}{5^n}} = \dfrac{5^n(n+1)}{n\cdot 5^{n+1}}$

$\lim\limits_{n\to\infty} \frac{n+1}{5n} = \dfrac{1+\frac{1}{n}}{5} = \frac{1}{5} < 1$ yaqinlashuvchi

2) $\sum\limits_{n=1}^{\infty} \frac{1}{(n+1)\cdot 7^n}$

$\sum\limits_{n=1}^{\infty}$

$\lim\limits_{n\to\infty} \dfrac{\frac{1}{(n+2)\cdot 7^{n+1}}}{\frac{1}{(n+1)\cdot 7^n}} = \dfrac{7^n(n+1)}{7^n\cdot 7(n+2)} = \dfrac{n+1}{7n+14} =$

$\lim\limits_{n\to\infty} \dfrac{1+\frac{1}{n}\to 0}{7+\frac{14}{n}\to 0} = \frac{1}{7} < 1$ yaqinlashuvchi

$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{n \cdot 3^n}$$

$$\lim_{n\to\infty} \left| \frac{\frac{(x-5)^{n+1}}{(n+1)\cdot 3^{n+1}}}{\frac{(x-5)^n}{n\cdot 3^n}} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(x-5)^n}\cdot(x-5)\cdot n\cdot \cancel{3^n}}{(n+1)\cdot \cancel{3^n}\cdot 3\cdot \cancel{(x-3)^n}} \right| < 1$$

$$|x-5| \lim_{n\to\infty} \frac{n}{3n+3} < 1$$

$$|x-5| \; \frac{1}{3} < 1 \qquad |x-5| < 3 \qquad \lim_{n\to\infty} \frac{1}{3+\frac{3}{n}} = 0$$

$$-3 < x-5 < 3$$

$$2 < x < 8$$

$$R = (2 ; 8)$$

$$x = 2$$

2) Ikki karrali integral

1) $\int_2^5 6x\,dx \int_0^2 y^2\,dy = \int_2^5 6x\,dx \; \frac{y^3}{\cancel{3}} \Big|_2^0 =$

$$= \int_2^5 16x\,dx = 16\frac{x^2}{2}\Big|_2^5 = 8(25-4) =$$

$$= 168$$

2) $$\int_0^2 dx \int_0^1 (x^2+3y)\,dy = \int_0^2 dx \left(x^2 y + \frac{3y^2}{2} \Big|_0^1 \right) =$$

$$= \int_0^2 dx \left(x^2 + \frac{3}{2}\right) = \frac{x^3}{3} + \frac{3}{2}x \Big|_0^2 = \frac{8}{3} + \frac{3}{1} =$$

$$= \frac{17}{3}$$

1.19.

$z_1 = 17 - 12i \quad z_2 = 3 - 4i \quad z = -5\sqrt{3} - 5i$

$n = 6 \quad k = 4$

а) $z_1 + z_2 = 17 - 12i + 3 - 4i = 20 - 16i$

$z_1 - z_2 = 17 - 12i - 3 + 4i = 14 - 8i$

$z_1 \cdot z_2 = (17 - 12i)(3 - 4i) = 51 - 68i - 36i - 48 =$

$= 3 - 104i$

$\frac{z_1}{z_2} = \frac{(17 - 12i)(3 + 4i)}{(3 - 4i)(3 + 4i)} = \frac{51 - 68i - 36i + 48}{9 + 16}$

$= \frac{99 + 32i}{25}$

б) $z = -5\sqrt{3} - 5i$

$z = \sqrt{(-5\sqrt{3})^2 + (5i)^2} = \sqrt{75 + 25} = 10$

$\cos\varphi = \frac{-5\sqrt{3}}{10} = \frac{-\sqrt{3}}{2} \qquad \sin\varphi = \frac{-5}{10} = -\frac{1}{2}$

$\varphi = \frac{7\pi}{6}$

c) $z^6 = (-5\sqrt{3} - 5i)^6 = 5^6((\sqrt{3} + i)^2)^3 =$

$= (3 + 2\sqrt{3}i - 1)^2 = 5^6 \cdot 2^3 (\sqrt{3}i + 1)^3 =$

$= 50^3 (-3\sqrt{3}i - 9 + 3\sqrt{3}i + 1) = -8 \cdot 50^3 =$

$= -2^3 \cdot 50^3 = -100^3 = -10^6$

$\sqrt[4]{z} = \sqrt[4]{-5\sqrt{3} - 5i} = \sqrt[4]{10} \cdot \sqrt[4]{-\frac{\sqrt{3}}{2} - \frac{i}{2}} =$

$= \sqrt[4]{10} \cdot \sqrt[4]{\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}}$

2.19

a) $\lim\limits_{n \to \infty} n(\sqrt{4n^2+3} - 2n) =$

$= \lim\limits_{n \to \infty} \frac{n(\sqrt{4n^2+3} - 2n)(\sqrt{4n^2+3} + 2n)}{\sqrt{4n^2+3} + 2n} =$

$= \lim\limits_{n \to \infty} \frac{n(4n^2+3-4n^2)}{\sqrt{4n^2+3} + 2n} = \lim\limits_{n \to \infty} \frac{3n}{\sqrt{4n^2+3}+2n} =$

$= \lim\limits_{n \to \infty} \frac{3}{\sqrt{4 + \frac{3}{n^2}} + 2} = \frac{3}{\sqrt{4}+2} = \frac{3}{2+2} = \frac{3}{4} =$

$= 0.75$

b) $\lim\limits_{x \to 1} \frac{(x^3 - 3x - 2)}{(x^2 - x - 2)^2} = \lim\limits_{x \to 1} \frac{(x-2)(x+1)^2}{(x-2)^2(x+1)^2} =$

$= \lim\limits_{x \to 1} \frac{1}{x-2} = \frac{1}{-3} = -\frac{1}{3}$

d) $\lim\limits_{x \to 3} \left(\frac{9-2x}{3} \right)^{tg \frac{\pi x}{6}}$

$t = x - 3$

$x = t + 3$

$\lim\limits_{t \to 0} \left(\frac{9-2t-6}{3} \right)^{tg\left(\frac{\pi(t+3)}{6}\right)} =$

$= \lim\limits_{t \to 0} \left(1 + \left(-\frac{2t}{3}\right) \right)^{\frac{1}{\frac{3}{-2t}} \cdot \frac{3}{-2t} \cdot tg\left(\frac{\pi(t+3)}{6}\right)} =$

$= \lim\limits_{t \to 0} e^{-\frac{3}{2t} \cdot ctg \frac{\pi t}{6}} = e^{-\frac{3}{0}} = e^{-\infty} = \frac{1}{e^{\infty}} = 0$

3.19 a) $f(x)$ $\begin{cases} 3x+2, & x < -1 \\ -2x^2+1, & -1 \le x \le 1 \\ x-2, & x > 1 \end{cases}$

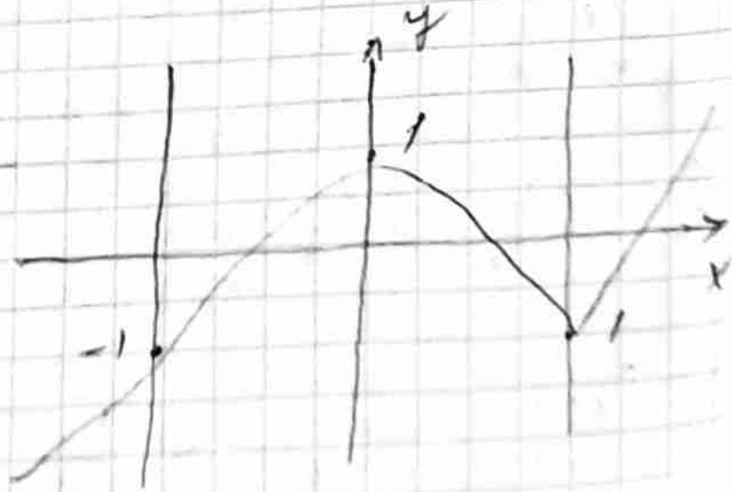
$\lim\limits_{x \to -1-0} f(x) = -1$

$\lim\limits_{x \to -1+0} f(x) = -1$

$\lim\limits_{x \to -1-0} f(x) = -1$

$\lim\limits_{x \to 1+0} f(x) = -1$

$x = -1$ $x = 1$ nuqta 1 tur uzulish nuqtasi bor taraf etish



4.19 $y = 5^{-\frac{1}{x}} \cdot \arcsin(2x+1)^3$

$y' = (5^{-\frac{1}{x}})' \arcsin(2x+1)^3 + 5^{-\frac{1}{x}} \cdot \frac{6(2x+1)^2}{\sqrt{1-(2x+1)^6}}$

$y' = 5^{-\frac{1}{x}} \left| \frac{\ln 5 \cdot \arcsin(2x+1)^3}{x^2} + \frac{6(2x+1)^2}{\sqrt{1-(2x+1)^6}} \right.$

5.19 $y^2 = \frac{x-y}{x+y}$ $\quad y^2 = \frac{x+y-2y}{x+y}$

$y^2 = 1 - \frac{2y}{x+y}$ $\quad y^2 - 1 = \frac{2y}{x+y}$

$x+y = \frac{2y}{y^2-1}$ $\quad x = \frac{2y}{y^2-1} - y$

$xy^2 + y^3 = x - y$ $\quad y'(2y+3y^2+1) = 1$

$2yy' + 3y^2y' = 1 - y'$ $\quad y' = \frac{1}{3y^2+2y+1}$

6.19. $\begin{cases} x = \sin t \\ y = \ln \cos t \end{cases}$

$y'_x = \frac{y'_t}{x'_t} = \frac{-\frac{\sin t}{\cos t}}{\cos t} = \frac{-\sin t}{\cos^2 t}$

$y''_{xx} = \frac{y'_t}{x'_t} = \frac{-\frac{\cos^2 t + 2\cos t \cdot \sin^2 t}{\cos^4 t}}{\cos t} =$

$$= \frac{-\cos^3 t + 2\cos t \sin^2 t}{\cos^4 t} = \frac{-\cos^2 t + 2\sin^2 t}{\cos^4 t} =$$

7.19

a) $\sqrt[3]{150}$ b) $\operatorname{artg}\sqrt{2.9}$

$x = 125$ $\Delta x = 25$

$f(x) = \sqrt[3]{x}$ $f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$f(x + \Delta x) = f(x) + f'(x) \cdot \Delta x$

$f(x + \Delta x) = \sqrt[3]{125} + \frac{1}{3 \cdot \sqrt[3]{125^2}} \cdot 25 =$

$5 + \frac{1}{3 \cdot 25} \cdot 25 = 5 + \frac{1}{3} = 5.33$

b) $x = 3$ $\Delta x = -0.1$

$f(x) = \operatorname{artg}\sqrt{x}$ $f(x) = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}$

$\operatorname{artg}\sqrt{2.9} = \operatorname{artg}\sqrt{3} + \frac{1}{(1+3)} \cdot \frac{1}{2\sqrt{3}} \cdot (-0.1) =$

$= \frac{\pi}{3} + \frac{1}{4} \cdot \frac{-0.1}{2\sqrt{3}} = \frac{\pi}{3} - \frac{1}{80\sqrt{3}}$

8.19

а) $\lim_{x\to 0} \frac{e^{3x}-e^{2x}}{\sin 5x} = \lim_{x\to 0} \frac{e^{3x}-e^{2x}}{5x \frac{\sin 5x}{5x}} =$

$= \lim_{x\to 0} \left(\frac{e^{3x}-e^{2x}}{5x}\right) = \lim_{x\to 0} \frac{3e^{3x}-2e^{2x}}{5} = \frac{3-2}{5} = \frac{1}{5}$

б) $\lim_{x\to 0} (\operatorname{ctg} 2x)^{\frac{1}{\ln x}} = \lim_{x\to 0} (1+\operatorname{ctg} 2x-1)^{\frac{1}{\operatorname{ctg}2x-1}\cdot\frac{\operatorname{ctg}2x-1}{\ln x}} =$

$= \lim_{x\to 0} e^{\frac{\operatorname{ctg}2x-1}{\ln x}} = \lim_{x\to 0} \frac{\frac{-2}{\sin^2 2x}}{\frac{1}{x}} = \lim_{x\to 0} e^{\frac{2x}{\sin^2 2x}} =$

$= \lim_{x\to 0} e^{\frac{1}{2x\cdot\frac{\sin^2 2x}{4x^2}}} = \lim_{x\to 0} e^{-\frac{1}{2x}} = e^{-\infty} = \frac{1}{e^{\infty}} = 0$

9.19 1) $\int \frac{3x-\arccos 2x}{\sqrt{1-4x^2}} = \int \frac{3x}{\sqrt{1-4x^2}} -$

$- \int \frac{-\arccos 2x}{\sqrt{1-4x^2}}\,dx = -\frac{3}{8}\int \frac{d(1-4x^2)}{\sqrt{1-4x^2}} - \frac{1}{2}\int$

$\int \arccos 2x\, d(\arccos 2x) = -\frac{3}{8}\cdot 2\cdot\sqrt{1-4x^2} -$

$-\frac{1}{2}\,\frac{\arccos^2 2x}{2} + C = -\frac{3}{4}\sqrt{1-4x^2} - \frac{1}{4}\arccos^2 2x + C$

2) $\int \operatorname{arcctg}\sqrt{2x+1}\,dx = \frac{1}{2}\,\frac{\operatorname{arctg}^2\sqrt{2x+1}}{2} + C$

3) $\int \frac{2x^2+6x^2+5x}{(x+2)(x+1)^3}\,dx = \int\left(\frac{2}{x+2} - \frac{1}{(x+1)^3}\right)dx =$

$$= \int \frac{d(x+2)}{x+2} - \int \frac{d(x+1)}{(x+1)^3} = \ln|x+2| + \frac{1}{2}\cdot\frac{1}{(x+1)^2}$$

4) $$\int \frac{(5x-3)\,dx}{\sqrt{2x^2+4x-5}} = \frac{5}{4}\int \frac{4(x+1)\,dx}{\sqrt{2x^2+4x-5}} - \frac{1}{4}\int \frac{8\,dx}{\sqrt{2x^2+4x-5}} = \frac{5}{4}\int \frac{d(2x^2+4x-5)}{\sqrt{2x^2+4x-5}} -$$

$$- \frac{1}{4\sqrt{8}} \int \frac{2\,dx}{\sqrt{x^2+2x+1-\frac{7}{2}}} =$$

$$= \frac{5}{2}\sqrt{2x^2+4x-5} - \sqrt{2}\cdot \ln\left|\frac{\sqrt{x+1}-\sqrt{2.5}}{\sqrt{x+1}+\sqrt{5.5}}\right| + C$$

10.19.

$z = \cos\varphi - \sin\varphi$ $\qquad -1 \le \sin\varphi \le 1$

$-2 \le z \ge 2$ $\qquad z = z \quad - \le \cos\varphi \le 1$

$-\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$

$$\int_{\varphi_1}^{\varphi_2} z^2\, d\varphi$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos\varphi - \sin\varphi)^2\, d\varphi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos^2\varphi - 2\sin\varphi\cos\varphi$$

$$+ \sin^2\varphi) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - \sin 2\varphi)\, d\varphi =$$

$\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\varphi + \frac{1}{2}\cos 2\varphi\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$

$= \frac{\pi}{2} + \frac{\pi}{2} + \frac{1}{2}(-1+1) = \pi \qquad S = \pi$

11.19.

$x = 3.5(2\cos t - \cos 2t) \qquad 0 \le t \le \frac{\pi}{2}$

$y = 3.5(2\sin t - \sin 2t)$

$\ell = \int \sqrt{f'^2(x) + g'^2(x)}\, dx =$

$= \int_0^{\frac{\pi}{2}} \sqrt{3.5^2(-2\sin t + 2\sin 2t)^2 + 3.5^2(2\cos t - 2\cos t)^2}\, dt$

$= 7\int_0^{\frac{\pi}{2}} \sqrt{(\sin^2 t - 2\sin t \sin 2t + \sin^2 2t + \cos^2 t + \cos^2 2t}$

$\overline{- 2\cos t \cos 2t}\, dt = \int \sqrt{2 - 2(\sin t \sin 2t + \cos t \cos t + }\, dt$

$=$

$= 7\int_0^{\frac{\pi}{2}} \sqrt{2(1-\cos\varphi}\, dt = 7\int_0^{\pi} 2\sin\frac{t}{2}\, dt =$

$= 14 \cdot 2 \cdot \cos\frac{t}{2}\Big|_0^{\frac{\pi}{2}} = 14$

$\ell = 14$

12.19 a) $\int_{-2}^{-1} \frac{5dx}{(x^2-4x)\ln 3} = \frac{5}{\ln 3}\int_{-2}^{-1} \frac{dx}{(x^2+4x-4)}$

$= \frac{5}{\ln 3}\int_{-2}^{-1} \frac{dx}{(x+2)^2-2^2} = \frac{5}{\ln 3} \cdot \ln\left|\frac{x+4}{4}\right| + C \Big|$

uzoglashuwchi

b) $\int \frac{^3 x\,dx}{\sqrt[3]{x^2-1}} = \frac{1}{2}\int \frac{d(x^2-1)}{(x^2-1)^{\frac{1}{3}}} = \frac{1}{2} \cdot \frac{3}{7} \cdot (x^2+1)^{\frac{7}{3}}$

$\frac{3}{14}(x^2+1)\Big|_1^3 = \frac{3}{4}(9-1) = \frac{3 \cdot 8}{4} = 6$

$6 > 1$

uzoglashuwchi

13.19 $\sum_{n=1}^{\infty} \frac{7^n+2^n}{10^n} = \lim_{n\to\infty}\left(\left(\frac{7}{10}\right)^n + \left(\frac{1}{5}\right)^n\right) =$

$\frac{\frac{7}{10}}{1-\frac{7}{10}} + \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{\frac{7}{10}}{\frac{3}{10}} + \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{7}{3} + \frac{1}{4} =$

$= \frac{28+3}{20} = \frac{31}{20}$

14.19 $\sum_{n=1}^{\infty} \frac{4n+1}{\sqrt{n \cdot 5^n}}$ $\quad \lim \frac{a_n}{a_{n+1}}$

$\lim_{n\to\infty} \frac{4n+1}{\sqrt{n \cdot 5^n}} \cdot \frac{\sqrt{(n+1)5^{n+1}}}{4n+5} =$

$= \frac{n+1}{n} \sqrt{5} = \sqrt{5}$ $\quad \sqrt{5} > 1$

qator uzoqlashuvchi

15.19.

$$\sum_{n=1}^{\infty} \frac{n^n}{3n} \qquad \lim_{n \to \infty} \frac{a^n}{a_{n+1}}$$

$$\lim_{n \to \infty} \frac{n^n}{3^n} \cdot \frac{3^{n+1}}{(n+1)^{n+1}} = \lim_{n \to \infty} \frac{3}{n+1} = 0$$

qator yaqinlashuvchi

16.19.

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3^n}{(2n+1)^n} \qquad \lim_{n \to \infty} \sqrt[n]{a_n}$$

$$\lim_{n \to \infty} \sqrt{\frac{3n}{(2n+1)^n}} = \lim_{n \to \infty} \frac{3}{(2n+1)} = 0$$

qator yaqinlashuvchi

17.19.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n} \cdot x^n}{-n} \qquad \lim_{n \to \infty} \frac{a_{n+1}}{a_n}$$

$$\lim_{n \to \infty} \frac{\sqrt{n+1} \cdot x^{n+1}}{(n+1)!} \cdot \frac{n!}{\sqrt{n} \cdot x^n} = \lim_{n \to \infty} \frac{x}{n+1}$$

$x \in (-\infty; \infty)$

18.19.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n} \cdot x^n}{n!} \qquad \lim_{n\to\infty} a$$

18.19. $z = arctg\,(x^2+y^2)$

$$\frac{dz}{dx} = \frac{2x\,dx}{1+(x^2+y^2)^2}$$

$$\frac{dz}{dy} = \frac{2y}{1+(x^2+y^2)^2}$$

$$dz = \frac{1}{1+(x^2+y^2)^2}\left(2x\,dy + 2y\,dy\right.$$

19.19. $2x^2+2y^2+y^2+8xz+z+6=0 \quad M_0(-2;1;1)$

$F'_x = 4x+8z \qquad F'_x(-2;1;1) = -8+8=0$

$F'_y = 4y \qquad F'_y(-2;1;1) = 4$

$F'_z = 2z+8x \qquad F_y = (-2;1;1) = 2-16 = -14$

$0(x+2)+4(y-1)-14(z-1) = 4(y-1)-14(z-1)=0$

urinma tenglamasi

$4(y-1)-14(y-1)=0$

normal tenglamasi

$$\frac{y-1}{4} = \frac{z-1}{-14}$$

20.20.

$$\iint (2+y)y\,dxdy \qquad D: y=-x \quad y=2x$$

$$x=-1$$

$$\int_{-x}^{2x} (2xy+y^2)dy = \left(xy^2+\frac{y^3}{3}\right)\Big|_{-x}^{2x} =$$

$$= x((4x^2-x^2)+\frac{1}{3}(8x^3+x^3) =$$

$$= 3x^3+3x^3 = 6x^3$$

$$\int_{-1}^{0} 6x^3dx = \frac{6x^4}{4}\Big|_{-1}^{0} = \frac{6}{4}$$

$$S = \frac{6}{4}$$

c) $$\iint_D \frac{x}{\sqrt{x^2+y^2}}\,dxdy \qquad D: x^2+y^2+4y=0$$

$$x \leq 0$$

$$-2 \leq x \leq 0 \qquad x^2+(y+2)^2=4$$

$$S = \frac{\pi r^2}{2} = \frac{\pi \cdot 4}{2} = 2\pi$$

$$S = 2\pi$$

№19 $2x - y = 0$ $x + y = 6$ $x = 2$

$x = 0$ $z = 0$ $z = x^2$ $y = 4$

$$\int_0^2 dz \int_0^{2\pi} d\varphi \int_0^{9 - z^2\cos^2\varphi} z\, dz =$$

$$= \int_0^2 dz \int_0^{2\pi} d\varphi \quad z(9 - r^2\cos^2\varphi)$$