

$f_n$ : FbNC数.  $f_n = f_{n-1} + f_{n-2}$

DP

非齐次通解 = 齐次通解 + 非齐特解

① 互异:  $a_n = A_1 r_1^n + A_2 r_2^n + \dots$

$A_1, A_2$  为常数,  $r$  为根

② 重根:  $a_n = (A_1 + A_2 n + \dots + A_m n^{m-1}) r_1^n$   
 $m$  为  $r_1$  的重数

③  $r = \alpha \pm \beta i$  (复根) 部分:

$\rho = \sqrt{\alpha^2 + \beta^2}, \theta = \arctan(\frac{\beta}{\alpha})$

$r = \rho(\cos\theta + i\sin\theta)$

$\bar{r} = \rho(\cos\theta - i\sin\theta)$

$a_n = A_1 \rho^n \cos n\theta + B \rho^n \sin n\theta$

$a_n = \dots + f(n) \rightarrow f(n) = P(n) \cdot S^n$   
常  $\left\{ \begin{array}{l} S \neq r, (d \neq d) S^n \\ S = r, m \text{ 重 } r \\ n^m (d \neq d) S^n \end{array} \right.$

Generating Function:

对  $a_0, \dots, a_k,$

$$\text{有: } G(x) = \sum_{k=0}^{\infty} a_k x^k = a_0 + a_1 x + a_2 x^2 + \dots$$

$$\textcircled{1} f(x) + g(x) = \sum_{k=0}^{\infty} (a_k + b_k) x^k$$

$$\textcircled{2} \text{卷积 convolution: } f(x) \cdot g(x) = \sum_{k=0}^{\infty} \left( \sum_{i=0}^k a_i b_{k-i} \right) x^k$$

应用 ★ 见作业 009