11.1 Groups and Coding

D: $\oplus$: 加和模2

D: $e: B^m \to B^n$ (字空间 → 码字空间) encoding Function, m、n为长度

D: weight 权值 → $|x|$ 中1的数

D: Hamming distance: $|x_1 \oplus x_2|$ $\delta(x,y)$

↓ the weight of $x_1 \oplus x_2$ 加和模2.

I Theorem: ① $\delta(x,y)=\delta(y,x)$ ② $\delta(x,y)\geq 0$ ③ $\delta(x,y)=0 \Leftrightarrow x=y$

④ ?走神了.

D: Minimum distance of an encoding function e:

$\min\{\partial(e(x_1), e(y_1)) \mid x,y \in B^m\}$.

II Theorem: $e(\varphi)=$ ……

↓ 字空间 ↓ 码字空间

最近距离 $\geq k+1$

↓

error

III Theorem:

$e: B^m \to B^n$ be a group code,

the min distance of e is the min weight of a nonzero code.

D: group code $e(B^m)=\{e(b) \mid e(b) \in B^n\}=Ran(e)$

$e: B^m \to B^n$ is a subgroup of B^n.

(group 0/1: ⊕ $B^3 \to B^6$) 对⊕封闭、有e、有x'、可结?!?

D: definition I.II…: Theorem.

D: Boolean ~ of matrix

$A \odot B = \cdots \bigvee a \wedge b$

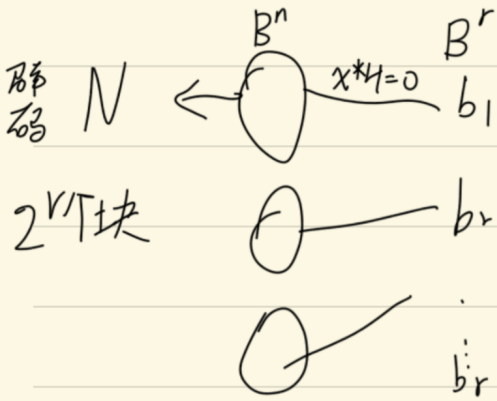
∵ Theorem:

奇偶校验矩阵

$m<n$, nonnegative, $r=n-m$, H be an $m \times n$ Boolean matrix

$f_H: B^n \to B^r$: $f_H(x) = x * H$, $x \in B^n$

is a homomorphism from Group B^n to B^r



$x_1, x_2 \in B^n$, 则 $f_H(x_1 \oplus x_2) = f_H(x_1) \oplus f_H(x_2)$

Corollary:

$N = \{x \in B^n \mid x*H=0\}$ is a normal subgroup of B^n.

$x_1, \cdots x_2 \in N$ 非空

证: ① 封闭 (Group)

② 单位元 0

③ 逆元 x^{-1}

④ $aH = Ha$

↓

$H \leq G$ 是 normal group, $\forall g \in G, h \in H, ghg^{-1} \in H$ / $gH = Hg$

$e_H: e^m \to e^n$

$b = b_1 b_2 \cdots b_m,\quad x = e_H(b) = b_1 b_2 \cdots b_m x_1 x_2 \cdots x_r$

$$x_i = b_1 h_{1i} + b_2 h_{2i} + \cdots + b_m h_{mi}$$

第i列

$\vdots$

x_r

① m的子空间 ② n=5，补3位.

$\|$
2

00
01
10
11

$b_1\ b_2$

11.2. Decoding and Error Correction.

decoding F 去校验 ???

I. e : (m,n) encoding F, d is a maximum likelihood decoding F,
then (e,d) can correct k or fewer errors

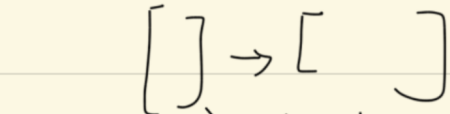
the minimum distance of e is 2k+1

$e^3 \to e^8$ $3 \geq 2k+1$

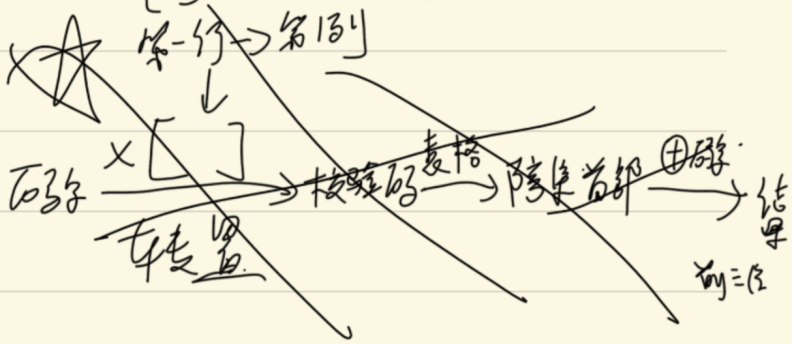
II. ?

coset header: coset中 weight最小的元素.

Decoding table



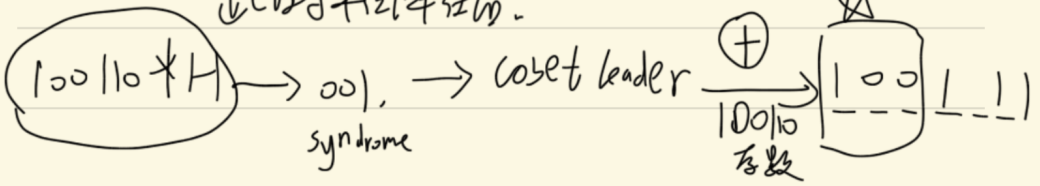
f_H is onto.



Decode:

$e(001) = m$.

此时单位阵在后.



(a).

$e(00) = 00000$

$e(01) = 01011$

$e(10) = 10011$

$e(11) = 11000$

(b).