

f_n : Fibonaci 数. $f_n = f_{n-1} + f_{n-2}$

DP

$$\text{非齐次通解} = \begin{matrix} \text{齐次通解} \\ \downarrow \\ \downarrow \end{matrix} + \begin{matrix} \text{非齐次特解} \\ \downarrow \end{matrix}$$

① 线性: $a_n = A_1 r_1^n + A_2 r_2^n + \dots$

A_1, A_2 为常数, r 为根

② 复根: $a_n = (A_{11} + A_{12} n + A_{1m} n^{m-1}) r_1^n$

$\nwarrow \swarrow$
 m 为 r_1 的重数

③ $r = \alpha \pm \beta i$ (复根部分)

$$p = \sqrt{\alpha^2 + \beta^2}, \theta = \arctan\left(\frac{\beta}{\alpha}\right),$$

$$\begin{cases} r = p(\cos \theta + i \sin \theta) \\ \bar{r} = p(\cos \theta - i \sin \theta) \end{cases}$$

$$a_n = A_1 \cdot p^n \cos n\theta + B \cdot p^n \sin n\theta$$

\nwarrow

$$a_n = \text{常} + f(n) \xrightarrow{?} f(n) = P(n) \cdot S^n$$

\uparrow 齐次通解
 S 不是 r , $(1/r)^n \times S^n$
 S 是 r , m 重
 $n^m (1/r)^n S^n$

Generating Function:

$\bar{x}^k a_0, \dots, a_k,$

有: $C(x) = \sum_{k=0}^{\infty} a_k x^k = a_0 + a_1 x + a_2 x^2 + \dots$

① $f(x) + g(x) = \sum_{k=0}^{\infty} (a_k + b_k) x^k$

② ^{卷积} convolution: $f(x) \cdot g(x) = \sum_{k=0}^{\infty} \left(\sum_{i=0}^k a_i b_{k-i} \right) x^k$

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