Homework 12

<u>due date: 5/3/12 8pm</u>

Send the notebook file of solutions to itsuko@binghamton.edu

as an e-mail attachment with subject "Phys468/568 YOUR NAME HW-12".

NAME: (Tufan Gebecelioglu)

1. Show that the Fourier series expansion of f(x) = x^2 ($-\pi < x < \pi$) is given as

   $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^n \frac{\cos n\pi}{n^2}$.

   Plot the original function f(x) and Fourier Series up to n=1,5,10 in a same graph.

   Note that the Fourier series expansion is defined as follows:

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{r=1}^{\infty}\left[a_r \cos\left(\frac{2\pi rx}{L}\right) + b_r \sin\left(\frac{2\pi rx}{L}\right)\right], \tag{4.4}$$

$$a_r = \frac{2}{L}\int_{x_0}^{x_0+L} f(x)\cos\left(\frac{2\pi rx}{L}\right)dx, \tag{4.7}$$

$$b_r = \frac{2}{L}\int_{x_0}^{x_0+L} f(x)\sin\left(\frac{2\pi rx}{L}\right)dx, \tag{4.8}$$

$$\sin n\pi = 0, \quad \sin\left(n+\tfrac{1}{2}\right)\pi = (-1)^n, \tag{4.5}$$

$$\cos n\pi = (-1)^n, \quad \cos\left(n+\tfrac{1}{2}\right)\pi = 0. \tag{4.6}$$

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Clear["Global`*"]
```

```
x0 = -π; L = 2π;
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```
f[x_] := x^2
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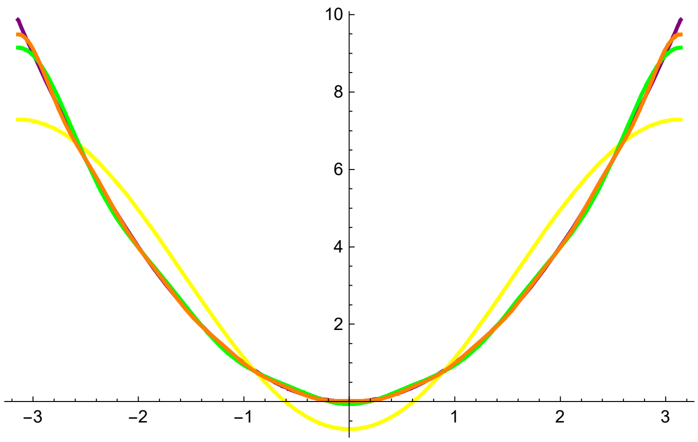
$$a[r\_] := \frac{2}{L}\int_{x0}^{x0+L} f[x]\,\mathrm{Cos}\left[\frac{2\,\pi\,r\,x}{L}\right] dx$$

$$b[r\_] := \frac{2}{L}\int_{x0}^{x0+L} f[x]\,\mathrm{Sin}\left[\frac{2\,\pi\,r\,x}{L}\right] dx$$

```
ar = a[r];
br = b[r];
```

$$F[n\_] := \frac{a[0]}{2} + \sum_{r=1}^{n}\left(ar\,\mathrm{Cos}\left[\frac{2\,\pi\,r\,x}{L}\right] + br\,\mathrm{Sin}\left[\frac{2\,\pi\,r\,x}{L}\right]\right)$$

```
F1 = F[1];
F5 = F[5];
F10 = F[10];
Plot[{f[x], F1, F5, F10}, {x, π, -π},
 PlotStyle → {{Purple, Thick}, {Yellow, Thick}, {Green, Thick}, {Orange, Thick}}]
```



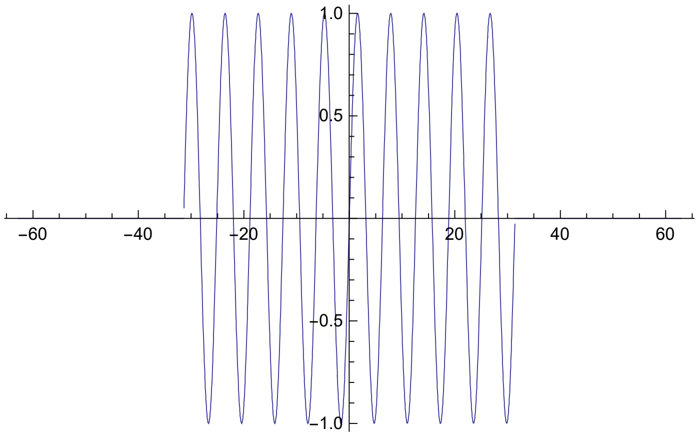
2. Calculate the Fourier Transform of the following function:

   $g[t] = \mathrm{Sin}[\omega 0\ t]$ for $|t| < \frac{N\pi}{\omega 0}$ and $g[t] = 0$ for $|t| > \frac{N\pi}{\omega 0}$

   And make plots of g[t] as a function of t ($-20\pi < t < 20\pi$) and Im G[ω] as a function of ω ($0 < \omega < 2$). Choose ω0=1 and N=10.

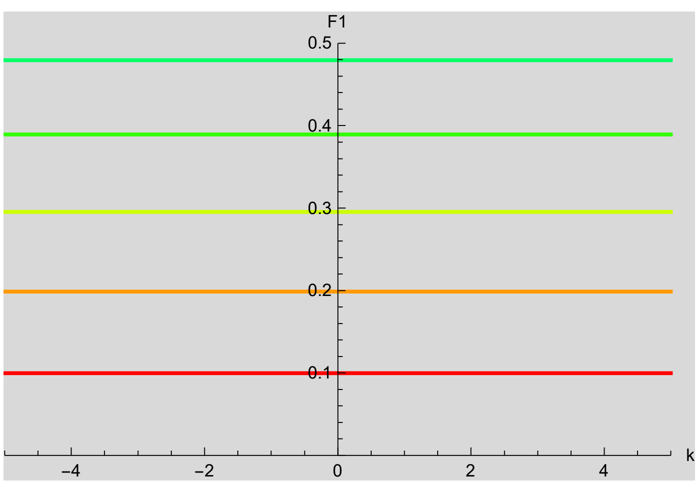
```
Clear["Global`*"]
```

```
ω0 = 1; n = 10;
x = nπ/ω0;
g[t_] = Sin[ω0 t] * (UnitStep[t + x] - UnitStep[t - x]);
Plot[g[t], {t, -20 π, 20 π}]
G1 = FourierTransform[g[t], x, k, FourierParameters → {0, -1}] // Simplify[#, t > 0] &
 Plot[Evaluate[Table[g[t], {t, 0.1, 1.0, 0.1}]], {k, -5, 5},
  PlotStyle → Table[{Hue[0.1 i], Thick}, {i, 0, 10}], PlotRange → {{-5, 5}, {0, 0.5}},
  AxesLabel → {"k", "F1"}, Background → LightGray]
```



$$\sqrt{\frac{\pi}{2}}\ \text{DiracDelta}[k]\ \text{Sin}[t] -$$

$$\sqrt{2\pi}\ \text{DiracDelta}[k] \times \text{HeavisideTheta}[-10\pi + t]\ \text{Sin}[t] + \frac{\text{Sin}[t]\ (-i\ \text{Cos}[k\,t] + \text{Sin}[k\,t])}{k\sqrt{2\pi}}$$



3. Show your own work on Animate/Manipulate!

```
Clear["Global`*"];
x0 = 0; v = 10; g = -9.8; y0 = 0;
x[t_] = x0 + v t; y[t_] = y0 + v t + 1/2 g t^2
Graphics[{Arrow[{{0, 0}, {0, 5}}], Arrow[{{0, 0}, {10, 0}}], Blue,
  Disk[{.5, 1.5}, .5], Black, Polygon[{{0, 0}, {1, 1}, {1, 3}, {0, 2}}]}]
Animate[Graphics[{Arrow[{{0, 0}, {0, 5}}],
   Arrow[{{0, 0}, {1 + x[t], 0}}], Red, Disk[{x[t] + .5, y[t] + 1.5}, .5],
   Black, Polygon[{{0, 0}, {1, 1}, {1, 3}, {0, 2}}]}], {t, 0, 2.2}]
```

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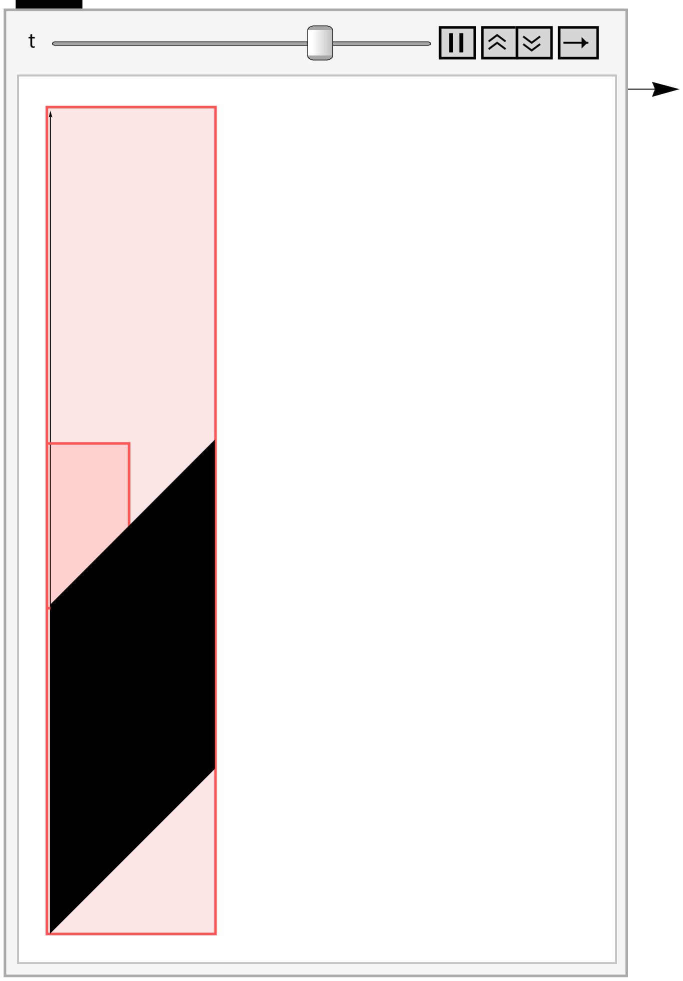
1 … 4.9 t²



```
x0 = 0; v = 10; g = -9.8; y0 = 0;
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Animate[Graphics[{Arrow[{{0, 0}, {0, 5}}],
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    Black, Polygon[{{0, 0}, {1, 1}, {1, 3}, {0, 2}}]}], {t, 0, 2.2}]
```

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$10\,t - 4.9\,t^2$