1A

→ STEP 0 : SPECIFY THE DYNAMICS

Write the difference equation $\mathbb{X}_t = A_T E_t[\mathbb{X}_{t+1}] + B_T \xi_t$ where in our case we have

DIFFERENT FROM BEFORE SINCE

AD : $\hat{y}_t = E_{t+1}[\hat{y}_{t+1}] - \sigma \hat{r}_t$

AS : $\pi_t = \kappa(\hat{y}_t - \hat{y}_t^{FLEX}) + \beta E_t[\pi_{t+1}]$

MP : $\hat{\imath}_t = \phi_\pi \pi_t + v_t$

$$\begin{vmatrix} \hat{y}_t \\ \pi_t \end{vmatrix} = A_T \begin{vmatrix} E_t[\hat{y}_{t+1}] \\ E_t[\pi_{t+1}] \end{vmatrix} + B_T \begin{vmatrix} 1 \\ \kappa \end{vmatrix} \left(\sigma \phi_\pi \kappa \underbrace{\hat{y}_t^{FLEX}}_{\text{EXOGENOUS}} - \sigma v_t \right)$$

$$\hat{y}_t^{FLEX} = \frac{1+\varphi}{\varphi+\gamma} \hat{a}_t$$

And specify dynamic of exogenous shocks: $v_t = 0$, $\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_t^a$ since we are only interested in TFP. Shock to TFP is in $\hat{y}_t^{FLEX}$ since $\hat{y}_t^{FLEX} = \frac{1+\varphi}{\varphi+\gamma} \hat{a}_t$

This is a conceptual step, irrelevant in terms of coding.

→ STEP 1 : GUESS

Conjecture about the STATIONARY SOLUTION of the dynamic system. In our case, we guess the policy function of being linear in the shock

$\hat{y}_t = \psi_{ya} \hat{a}_t$

$\pi_t = \psi_{\pi a} \hat{a}_t$

$E_t[\hat{y}_{t+1}] = \psi_{ya} E_t[\hat{a}_{t+1}] = \psi_{ya} \rho_a \hat{a}_t$

$E_t[\pi_{t+1}] = \psi_{\pi a} E_t[\hat{a}_{t+1}] = \psi_{\pi a} \rho_a \hat{a}_t$

→ STEP 2 : INSERT INTO THE SYSTEM

$-\kappa \hat{y}_t^{FLEX}$ with $\Omega \equiv (1+\varphi)(\varphi+\gamma)^{-1}$

AS : $\psi_{\pi a} \hat{a}_t = \kappa \psi_{ya} \hat{a}_t - \kappa \Omega \hat{a}_t + \beta \psi_{\pi a} \rho_a \hat{a}_t$

AD : $\psi_{ya} \hat{a}_t = \psi_{ya} \rho_a \hat{a}_t - \sigma (\phi_\pi \psi_{\pi a} \hat{a}_t - \psi_{\pi a} \rho_a \hat{a}_t)$

in red we have the monetary policy rule imposing $v_t = 0$

→ STEP 3 : COLLECT

Collect all coefficients related to each state / shock variable considering coefficient only (you can think of a unit shock $\hat{a}_t = \rho_a \hat{a}_{t-1} + 1$ with $\hat{a}_{t-1} = 0$)

AS : $(1 - \beta \rho_a) \psi_{\pi a} = \kappa (\psi_{ya} - \Omega)$

$$\psi_{\pi a} = \frac{\kappa(\psi_{ya} - \Omega)}{(1 - \beta \rho_a)}$$

AD : $\psi_{ya} = \psi_{ya} \rho_a - \sigma \left(\phi_\pi \frac{\kappa(\psi_{ya} - \Omega)}{(1-\beta\rho_a)} - \rho_a \frac{\kappa(\psi_{ya} - \Omega)}{(1-\beta\rho_a)} \right)$

$$\left(1 + \frac{\sigma \kappa}{1-\beta\rho_a}(\phi_\pi - \rho_a) - \rho_a\right)\psi_{ya} = \frac{\Omega\sigma\kappa}{1-\beta\rho_a}(\phi_\pi - \rho_a)$$

$$\psi_{ya} = \left(\frac{1-\beta\rho_a}{(1-\rho_a)(1-\beta\rho_a) + \sigma\kappa(\phi_\pi - \rho_a)}\right)\frac{\Omega\sigma\kappa}{1-\beta\rho_a}(\phi_\pi - \rho_a)$$

$$\psi_{ya} = \left(\frac{1}{(1-\rho_a)(1-\beta\rho_a) + \sigma\kappa(\phi_\pi - \rho_a)}\right)\Omega\sigma\kappa(\phi_\pi - \rho_a) = \Lambda_a\Omega\sigma\kappa(\phi_\pi - \rho_a)$$

SOLUTION :
$$\begin{cases} \psi_{ya} = \Lambda_a \Omega\sigma\kappa(\phi_\pi - \rho_a) \\ \psi_{\pi a} = \dfrac{\kappa^2}{1-\beta\rho_a}\Lambda_a\Omega\sigma(\phi_\pi - \rho_a) - \dfrac{\kappa}{1-\beta\rho_a}\Omega \end{cases}$$

→ STEP 4 : CALIBRATION

$\{\phi_\pi, \phi_y, \gamma, \varphi, \kappa, \beta, \rho_a, \rho_v\}$ using previous studies or matching moments of the data :

for example $i^{ss} = -\log\beta$, $\hat{\beta} = \text{mean}(e^{-i_t^{LR}})$ for example

→ STEP 5 : IMPULSE RESPONSE FUNCTIONS

Given the log-linear approximation around the steady state, we can express the engogenous variable $\hat{x}_t \equiv (x_t - x_t^{ss})$ as linear function of the axogenous shock.

We are computing IRF assuming all other shocks are zero, and system in steady state at time 0 : $\hat{y}_t = \hat{y}_t^{FLEX} = \tilde{y}_t = \pi_t = \hat{i}_t = E_t[\hat{\pi}_{t+1}] = \hat{m}_t = 0$

$$\begin{aligned} \hat{x}_0 &= \psi_{xa}\hat{a}_0 = \psi_{xa}\varepsilon_0^a &&= \Theta_0 \\ \hat{x}_1 &= \psi_{xa}\hat{a}_1 = \psi_{xa}\rho_a\varepsilon_0^a &&= \Theta_1 \\ &\vdots && \vdots \\ \hat{x}_T &= \psi_{xa}\hat{a}_T = \psi_{xa}\rho_a^T\varepsilon_0^a &&= \Theta_T \end{aligned}$$

SEQUENCE OF IRF

$$\Longrightarrow \begin{vmatrix} \hat{y}_t \\ \hat{y}_t^{FLEX} \\ \tilde{y}_t \\ \pi_t \\ \hat{i}_t \\ E_t[\hat{\pi}_{t+1}] \\ \hat{m}_t \\ \hat{a}_t \end{vmatrix} = \begin{vmatrix} \psi_{ya} \\ (1+\varphi)(\varphi+\gamma)^{-1} \\ \psi_{ya} - (1+\varphi)(\varphi+\gamma)^{-1} \\ \psi_{\pi a} \\ \phi_\pi\psi_{\pi a} \\ \phi_\pi\psi_{\pi a} - \psi_{\pi a}\rho_a \\ \psi_{ya} - 1 \\ 1 \end{vmatrix} (\rho_a^t \cdot \varepsilon_0^a)$$

1b

CHECK GITHUB, plots are there

1c Given the unit shock to TFP, we observe an increase in output $\hat{y}_t$ and output under flexible price $\hat{y}_t^{flex}$.
however, given

$$\hat{y}_t^{flex} = \frac{1+\varphi}{\gamma+\varphi} a_t$$

and the values of the calibration, the TFP shock transmit 1-to-1 to $\hat{y}_t^{flex}$, but $\hat{y}_t < \hat{y}_t^{flex}$ then we have a reduction in output gap.

Higher productivity reduces real marginal cost $\hat{mc}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t$ putting downward pressure on prices. In other terms, given $\hat{mc}_t = -\hat{\mu}_t$, the markup is above the desired level and firms adjust ensures producing deflation

The CB react lowering the nominal rate following the given reaction function. The decrease in nominal rate is greater than the reduction in the Expected inflation, then according to FISHER EQUATION we also have a lower real rate.

Lastly, given the production functions, we have $\hat{n}_t = \hat{y}_t - \hat{a}_t$ but since productivity shock is greater than change in output, employment decrease,

The shock is transitory, so the system converge back to steady state.

1d YES, THEY ARE THE SAME

2a it is not recursive because on the RHS we don't have P^*_{t+1}, so we can not write it in recursive form numerically. The reason why we don't have P^*_{t+1} is trivial: P^*_t is the optimal reset price for the firm as if it'll not be able to reoptimize in the future. This is the economic intuition for which we don't have a recursive formula

2b

$$F_{2t} = \sum_{k=0} \theta^k \Lambda_{t,t+k}\left(\frac{P_{t+k}}{P_t}\right)^{\varepsilon-1} Y_{t+k}$$

$$= \underset{1}{\underset{\shortparallel}{\theta^0}}\, \underset{1}{\underset{\shortparallel}{\Lambda_{t,t}}} \underset{1}{\underset{\shortparallel}{\left(\frac{P_t}{P_t}\right)}}^{\varepsilon-1} Y_t + \sum_{k=1}^{\infty} \theta^k \Lambda_{t,t+k}\left(\frac{P_{t+k}}{P_t}\right)^{\varepsilon-1} Y_{t+k}$$

$$= Y_t + \sum_{k=0}^{\infty} \theta^{k+1} \underbrace{\Lambda_{t,t+k+1}}\left(\frac{P_{t+k+1}}{P_t}\right)^{\varepsilon-1} Y_{t+k+1}$$

$\Lambda_{t,t+k+1} = \Lambda_{t,t+1} \cdot \Lambda_{t+1,t+k+1}$

$$F_{2t} = Y_t + \theta \Lambda_{t,t+1} \sum_{k=0}^{\infty} \theta^k \Lambda_{t+1,t+k+1} \left(\frac{P_{t+1}}{P_{t+1}}\right)^{\varepsilon-1}\left(\frac{1}{P_t}\right)^{\varepsilon-1}(P_{t+k+1})^{\varepsilon-1} Y_{t+k+1}$$

$$= Y_t + \theta \Lambda_{t,t+1}\left(\frac{P_{t+1}}{P_t}\right)^{\varepsilon-1} \sum_{k=0}^{\infty} \theta^k \Lambda_{t+1,t+k+1}\left(\frac{P_{t+k+1}}{P_{t+1}}\right)^{\varepsilon-1} Y_{t+k+1}$$

$$= Y_t + \theta \Lambda_{t,t+1}\, \Pi_{t+1}^{\varepsilon-1} F_{2t+1}$$

2c The procedure is identical

$$F_{1t} = (1+\mu) \sum_{k=0} \theta^k \Lambda_{t,t+k}\left(\frac{P_{t+k}}{P_t}\right)^{\varepsilon-1} Y_{t+k}\, MC_{t+k|t}$$

$$= (1+\mu)\, \underset{1}{\underset{\shortparallel}{\theta^0}}\, \underset{1}{\underset{\shortparallel}{\Lambda_{t,t}}} \underset{1}{\underset{\shortparallel}{\left(\frac{P_t}{P_t}\right)}}^{\varepsilon-1} Y_t\, MC_t + (1+\mu)\sum_{k=1}^{\infty} \theta^k \Lambda_{t,t+k}\left(\frac{P_{t+k}}{P_t}\right)^{\varepsilon-1} Y_{t+k}\, MC_{t+k|t}$$

$$= (1+\mu) MC_t Y_t + (1+\mu)\sum_{k=0}^{\infty} \theta^{k+1} \underbrace{\Lambda_{t,t+k+1}}\left(\frac{P_{t+k+1}}{P_t}\right)^{\varepsilon-1} Y_{t+k+1}\, MC_{t+k+1|t}$$

$\Lambda_{t,t+k+1} = \Lambda_{t,t+1} \cdot \Lambda_{t+1,t+k+1}$

$$= (1+\mu) Y_t MC_t + (1+\mu)\theta \Lambda_{t,t+1} \sum_{k=0}^{\infty} \theta^k \Lambda_{t+1,t+k+1}\left(\frac{P_{t+1}}{P_{t+1}}\right)^{\varepsilon-1}\left(\frac{1}{P_t}\right)^{\varepsilon-1}(P_{t+k+1})^{\varepsilon-1} Y_{t+k+1}\, MC_{t+k+1|t}$$

$$= (1+\mu) Y_t MC_t + (1+\mu)\theta \Lambda_{t,t+1}\left(\frac{P_{t+1}}{P_t}\right)^{\varepsilon-1} \sum_{k=0}^{\infty} \theta^k \Lambda_{t+1,t+k+1}\left(\frac{P_{t+k+1}}{P_{t+1}}\right)^{\varepsilon-1} Y_{t+k+1}\, MC_{t+k+1|t}$$

$$MC_{t+k+1|t} = \frac{W_{t+k+1}}{A_{t+k+1} \cdot P_t}$$

we want $MC_{t+k+1|t+1}$

$$= (1+\mu) Y_t MC_t + (1+\mu)\theta \Lambda_{t,t+1} \left(\frac{P_{t+1}}{P_t}\right)^{\varepsilon\cancel{-1}} \sum_{k=0}^{\infty} \theta^k \Lambda_{t+1,t+k+1} \left(\frac{P_{t+k+1}}{P_{t+1}}\right)^{\varepsilon-1} Y_{t+k+1} \underbrace{\frac{W_{t+k+1}}{A_{t+k+1}} \cdot \frac{P_{t+1}}{P_{t+1}}}_{= MC_{t+k+1|t+1}} \cdot \frac{1}{P_t}$$

bring outside and cancel -1 at exponent.

$$F_{1t} = (1+\mu) Y_t MC_t + (1+\mu)\theta \Lambda_{t,t+1} \Pi_{t+1}^{\varepsilon} F_{1t+1}$$

2d

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} = \left[\theta \int_0^{S(t)} P_{t-1}(i)^{1-\varepsilon} di + (1-\theta) \int_{S(t)}^{1} P_t^{*\,1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \qquad S(t) \in [0,1]$$

$$= \left[\theta \left(\int_0^{S(t)} P_{t-1}^{1-\varepsilon}(i) di\right)^{\frac{1-\varepsilon}{1-\varepsilon}} + (1-\theta)\left(\int_{S(t)}^{1} P_t^{1-\varepsilon} di\right)^{\frac{1-\varepsilon}{1-\varepsilon}}\right]^{\frac{1}{1-\varepsilon}}$$

$$= \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*\,1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$\frac{P_t}{P_{t-1}} = \left[\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

$$1 = \theta \Pi_t^{\varepsilon-1} + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon}$$

$$1 = \theta \Pi_t^{\varepsilon-1} + (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{1-\varepsilon}$$

$$1 = \theta \Pi^{\varepsilon-1} + (1-\theta) p_t^{*\,1-\varepsilon} \qquad p_t^* \equiv \text{REAL OPTIMAL RESET PRICE}$$

2e

INFLATION : $\Pi_t = \frac{P_t}{P_{t-1}} = \left[\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$ if $P_t^* > P_{t-1}$ Then $\Pi_t > 1$ $P_t^* > P_t > P_{t-1}$

2f - g

LOOK GITHUB.

(2h) Clearly $\theta = 0.0001$ is RBC world, same result as flexible price, Then output gap is zero and $\hat{m} = \hat{y}_t - \hat{a}_t = \hat{a}_t - a_t = 0$

Increase in output lead to decrease in price
=> THAN TAYLOR RULE => $\hat{R} \downarrow$ since expected inflation decrease less than inflation

With $\theta = 0.9999$ $\hat{y}$ do not increase, inflation do not move because firm cannot adjust, Then nominal rate and real rate do not move neither.

output gap is Then equal to the ONE TO ONE MOVEMENT $\hat{y}_t^{flex}$ and the same for employment given $\hat{m}_t = \hat{y}_t - \hat{a}_t = \hat{a}_t$

We can see how the other case (for different θ) stay in between

(2i) => I would Expected To observe The same IRF Than The case $\theta = 0.0001$, ie aST The flexible price case.

EQUIVALENT TO THE RBC ECONOMY WITH PERFECTLY COMPETITIVE FIRMS AND NOT NOMINAL RIGIDITIES.

↓

Yes, effect ono prices since $Y \uparrow$, Then $\pi \downarrow$, nominal rate respond, => Taylor rule and $r \downarrow$

↳ Similar To 0.0001 case