FINAL LAPO BINI

CODE IN JULIA FOR Q1 & Q2, ONLY IRF FOR R&R FROM STATA.

1e

$$\hat{y}_t = E_t[\hat{y}_{t+1}] - \sigma(\hat{i}_t - E_t[\hat{\pi}_{t+1}] - \hat{r}^N_t)$$
$$\hat{\pi}_t = \kappa \hat{y}_t + \beta E_t[\hat{\pi}_{t+1}]$$
$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \bar{i}_t$$
$$\hat{r}^N_t = \epsilon^N_t$$
$$\bar{i}_t = \epsilon^i_t$$

- POLICY SHOCK $\epsilon^N_t = 0$

$\hat{y}_t = a_{yp}\, \epsilon^i_t$

$\hat{\pi}_t = a_{\pi p}\, \epsilon^i_t$

Since shocks are IID : $E_t[\hat{y}_{t+1}] = 0$ $\quad E_t[\hat{\pi}_{t+1}] = 0$

DYNAMIC IS $\quad a_{yp}\, \epsilon^i_t = 0 - \sigma(\phi_\pi a_{\pi p}\, \epsilon^i_t + \epsilon^i_t - 0 - 0)$

$$a_{yp} = -\sigma \phi_\pi a_{\pi p} - \sigma$$

NK PHILLIPS CURVE $\quad a_{\pi p}\, \epsilon^i_t = -\kappa \sigma \phi_\pi a_{\pi p}\, \epsilon^i_t - \kappa \sigma\, \epsilon^i_t + \beta 0$

$$(1 + \kappa\sigma\phi_\pi) a_{\pi p} = -\kappa\sigma$$

SOLUTION
$$\begin{cases} a_{\pi p} = -\dfrac{\kappa\sigma}{1+\kappa\sigma\phi_\pi} \\[2ex] a_{yp} = -\dfrac{\phi_\pi \kappa \sigma^2}{1+\kappa\sigma\phi_\pi} - \sigma = -\sigma(\phi_\pi a_{\pi p} + 1) \end{cases}$$

$$\begin{vmatrix} \bar{i} \\ \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_t \\ \hat{r}_t \\ \hat{r}^N_t \end{vmatrix} = \begin{vmatrix} 1 \\ a_{yp} \\ a_{\pi p} \\ \phi_\pi a_{\pi p} + 1 \\ \phi_\pi a_{\pi p} + 1 \\ 0 \end{vmatrix} \cdot \epsilon^i_t$$

- NATURAL RATE SHOCK

$$\hat{y}_t = a_{yN} \, \epsilon^N_t$$

$$\hat{\pi}_t = a_{\pi N} \, \epsilon^N_t$$

$$E_t[\hat{y}_{t+1}] = E_t[\hat{\pi}_{t+1}]$$

DYNAMIC IS $\quad a_{yN}\epsilon^N_t = 0 - \sigma\left(\phi_\pi a_{\pi N}\epsilon^N_t - 0 - \epsilon^N_t\right)$

$$a_{yN} = -\sigma\phi_\pi a_{\pi N} + \sigma$$

NK PHILLIPS CURVE $\quad a_{\pi N}\epsilon^N_t = k a_{yN}\epsilon^N_t + \beta 0$

$$a_{\pi N} = -k\sigma\phi_\pi a_{\pi N} + k\sigma$$

$$(1 + k\sigma\phi_\pi)\, a_{\pi N} = k\sigma$$

SOLUTION

$$\begin{cases} a_{\pi N} = \dfrac{k\sigma}{1 + k\sigma\phi_\pi} \\[2ex] a_{yN} = -\dfrac{k\sigma^2\phi_\pi}{1 + k\sigma\phi_\pi} + \sigma \end{cases}$$

$$\begin{vmatrix} \bar{i} \\ \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_t \\ \hat{r}_t \\ \hat{r}^N_t \end{vmatrix} = \begin{vmatrix} 0 \\ a_{yN} \\ a_{\pi p} \\ \phi_\pi a_{\pi p} \\ \phi_\pi a_{\pi p} \\ 1 \end{vmatrix} \cdot \epsilon^i_t$$

THE IRF CHARTS HAVE THE NAME OF THE VARIABLES

1b The system is in equilibrium at $t=-1$, Shock at $t=0$, Then given that The shocks are IID, there is no propagation $\Rightarrow E_t\hat{y}_{t+1} = E_t\hat{\pi}_{t+1} = 0$

A UNIT MONETARY POLICY SHOCK (100 bp) decrease output $\hat{y}_t$ due to increase in The real rate $\hat{r}_t$. However, the increase $\hat{i}_t$ is less than one due To an overall reduction in the inflation rate, for the same reason, since $E_t\hat{\pi}_{t+1} = 0$, The real rate increase in The same proportion of The nominal rate

The increase in the real rate decrease output $\hat{y}_t$ in The same amount given $\sigma = 1$.

1c The system is in equilibrium at $t=-1$, Shock at $t=0$, Then given that The shocks are IID, there is no propagation $\Rightarrow E_t\hat{y}_{t+1} = E_t\hat{\pi}_{t+1} = 0$

An increase in The real rate increase output which also drive an increase in inflation through The slope.

The increase in inflation thus drives an increase in the policy rate That subsequently reduce the increase in output and inflation.

As before $\hat{r}_t$ tracks one to one the increase in the policy rate, since $\hat{r}_t = \hat{i}_t + \underbrace{\cancel{E_t\hat{\pi}_{t+1}}}_{=0}$

(1d)

$$\hat{y}_t = \cancel{E_t[\hat{y}_{t+1}]}^{=0} - \sigma(\hat{i}_t - \cancel{E_t[\hat{\pi}_{t+1}]}^{=0} - \hat{r}^N_t)$$
$$\hat{\pi}_t = \kappa \hat{y}_t + \beta \cancel{E_t[\hat{\pi}_{t+1}]}^{=0}$$
$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \bar{i}_t$$
$$\hat{r}^N_t = \epsilon^N_t$$
$$\bar{i}_t = \epsilon^i_t$$

SHOCKS IID

$$\hat{y}_t = -\sigma(\underbrace{\phi_\pi \underbrace{\kappa \hat{y}_t}_{\hat{\pi}_t} + \bar{i}_t}_{\hat{i}_t} - \hat{r}^N_t)$$

$$\hat{y}_t(1 + \sigma\phi_\pi\kappa) = -\sigma(\bar{i}_t - \hat{r}^N_t)$$

$$\begin{cases} \hat{y}_t = -\dfrac{\sigma}{1+\sigma\phi_\pi\kappa}(\bar{i}_t - \hat{r}^N_t) \\ \hat{\pi}_t = -\dfrac{\kappa\sigma}{1+\sigma\phi_\pi\kappa}(\bar{i}_t - \hat{r}^N_t) \\ \hat{i}_t = \phi_\pi\hat{\pi}_t + \bar{i}_t = \left[1 - \dfrac{\kappa\sigma\phi_\pi}{1+\kappa\sigma\phi_\pi}\right]\bar{i}_t + \dfrac{\kappa\sigma\phi_\pi}{1+\kappa\sigma\phi_\pi}\hat{r}^n_t \end{cases}$$

CHART CALLED "simulation.pdf"

(1e) SEE RESULT FOLDER, chart with report $\hat{\gamma}^{OLS}$, scatterplot and fitted $\hat{y}_t$ (CHART CALLED "OLS.pdf")

(1f) As we can see from the system of equation used for the simulation we have the SIMULTANEITY BIAS, the two shocks effect both output and inflation at the same time

$$\hat{y}_t = \gamma \hat{i}_t + \eta_t \qquad \text{but} \qquad \hat{y}^{TRUE}_t = -\sigma(i_t - \underbrace{r^N_t}_{})$$

This goes in error Term

$$i_t = \phi\pi_t + \bar{i}_t$$

$$\pi_t = -\frac{\kappa\sigma}{1+\sigma\phi_\pi\kappa}(\bar{i}_t - r^N_t) \implies \operatorname{Cov}(\pi, \eta_t) \neq 0$$

WHICH IMPLIES $\operatorname{Cov}(\hat{i}_t, \eta_t) \neq 0$ $\hat{\gamma}^{OLS}$ NOT CONSISTENT

(1g) If I had data on $\bar{i}_t$, I know that $\bar{i}_t \perp r_m^t$ by construction, and given

$$\hat{i}_t = \phi \pi_t + \bar{i}_t$$

$\bar{i}_t$ constitutes the EXOGENOUS PART OF $\hat{i}_t$ uncorrelated with the endogenous response due to the policy function.

(1h) If I had data on $\hat{r}^N_t$, I know that I can use it as a CONTROL VARIABLE to isolate only the movements due to $\bar{i}_t$ in the system of equations.
WE KNOW THAT IV and CF approach are equivalent / PROOF by FRISCH - WAUGH - LOVELL THEOREM

(1i) I WOULD NOT USE CHOLESKY because we need to impose an ordering structure in which slow moving variables do not react to fast moving variable.

In our system, all the variables react contemporaneously so cholesky is not the right tool in this case.
In the empirical applications, I would specify the following causal ordering

$$\begin{vmatrix} \bullet & 0 & 0 \\ \bullet & \bullet & 0 \\ \bullet & \bullet & \bullet \end{vmatrix} \begin{vmatrix} \hat{y}_t \\ \pi_t \\ \hat{i}_t \end{vmatrix} = B(L) Y_{t-1} + \eta_t$$

Where the dot's indicate the short run elasticities: interest rate reacts contemporaneously to shock to output and inflation, but output and inflation only reacts with a lag.

(1j) THE MOST RELIABLE would be using $\bar{i}_t$ as an instrument for the policy rate. Why is that the most reliable empirically although the IV and CF are the same?
Because we know that $\bar{i}_t$ is EXOGENOUS, and if the first

stage rules out the weak instrument problem, we can consistently estimate the causal parameter of interest.

For the control function approach, EMPIRICALLY, we need to impose a precise DGP s.t. the endogenous component is all captured by $\hat{\pi}^{N}_{t}$.

(1k) See chart in RESULT FOLDER ("OLS_IV.pdf")
As we can see, by using I_t as instrument, we can consistently estimate the causal parameter

Q2 A – H ANSWERS LOOK AT JULIA CODE + CHARTS

2.i The impulse response look different because in the first case we are computing the response of output gap and inflation to the change in the policy rate $\hat{i}_t$. In the second case, we are computing the IRF to the exogenous monetary policy shock, id est the component of i_t which is not due to the exogenous policy rule

2.j In the new Keynesian model, the output gap is computed with respect to the EFFICIENT LEVEL OF OUTPUT, id est the level of output without inefficiency due to price dispersion and 1^{st} welfare theorem holds.

In this exercise the output gap is computed with respect to the CBO estimate, which capture the level of output that would result from the full utilization of capital and labor. So it does not take into account any "efficiency" concept, but only factor utilization.