

# Low Latency Interest Rate Markets

## Theory, Pricing & Practice



Nicholas Burgess

## PART ONE: Theory

### IR Markets, Products & Models

- Introduction to IR Markets
- Interest Rate Swaps
- IR Products & CDS
- Yield Curves
- IR Risk
- Credit Models

## PART TWO: Pricing & Practice

### Case Studies

- IRS Pricing Formulae
- IRS Pricing Case Study
- Asset Swap Structuring
- Asset Swap Pricing Case Study
- Pricing Tricks & Rules of Thumb

Quant Research Papers

<https://ssrn.com/author=1728976>

Support Materials: Quant Research, C++ and Excel Examples

<https://github.com/nburgessx/SwapsBook>

## PART ONE - THEORY

IR Markets, Products & Models

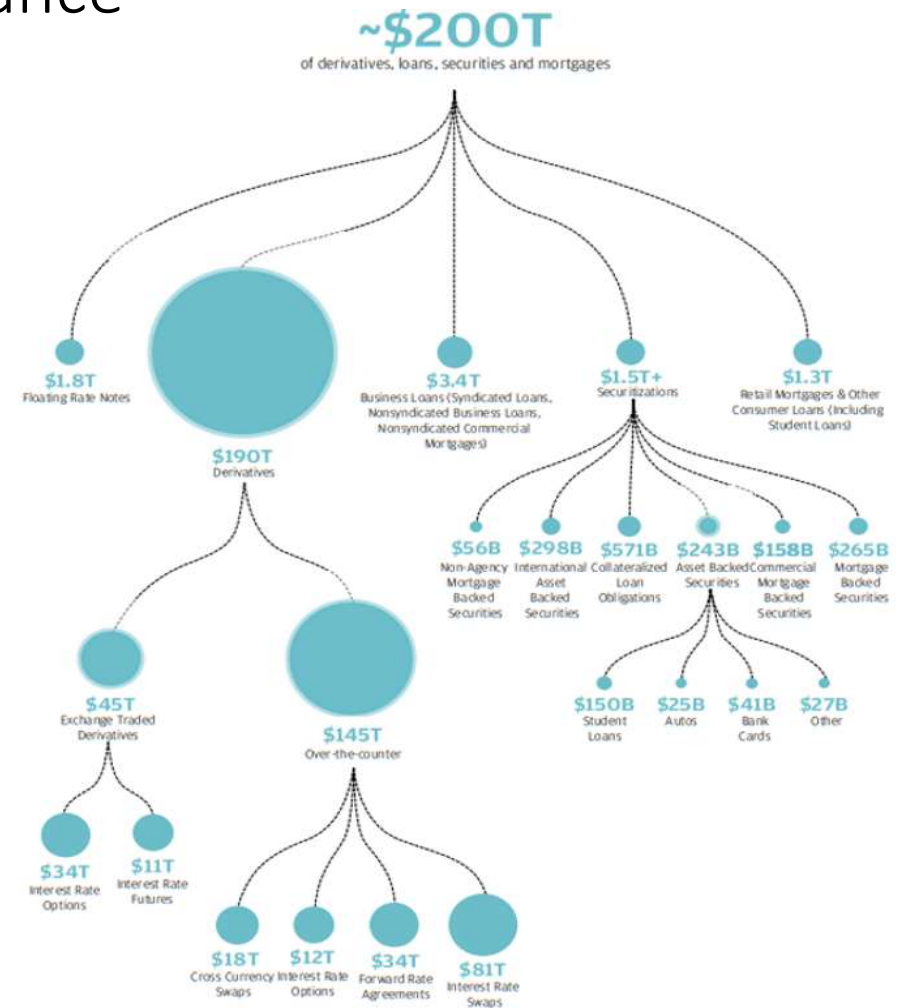
# Interest Rate Markets - Project Finance

## Purpose

- To Facilitate Government, Corporate & Project Finance
- Mortgages, Corporate Loans, Gov Projects & Infrastructure
- e.g. Hospitals, Transport (HS2), Energy & Defence Projects

## Market Size

- Market Size by Notional: \$200T (US) + \$150T (EU)
- Derivatives, Loans & Securities
- All Referencing LIBOR, until Recently



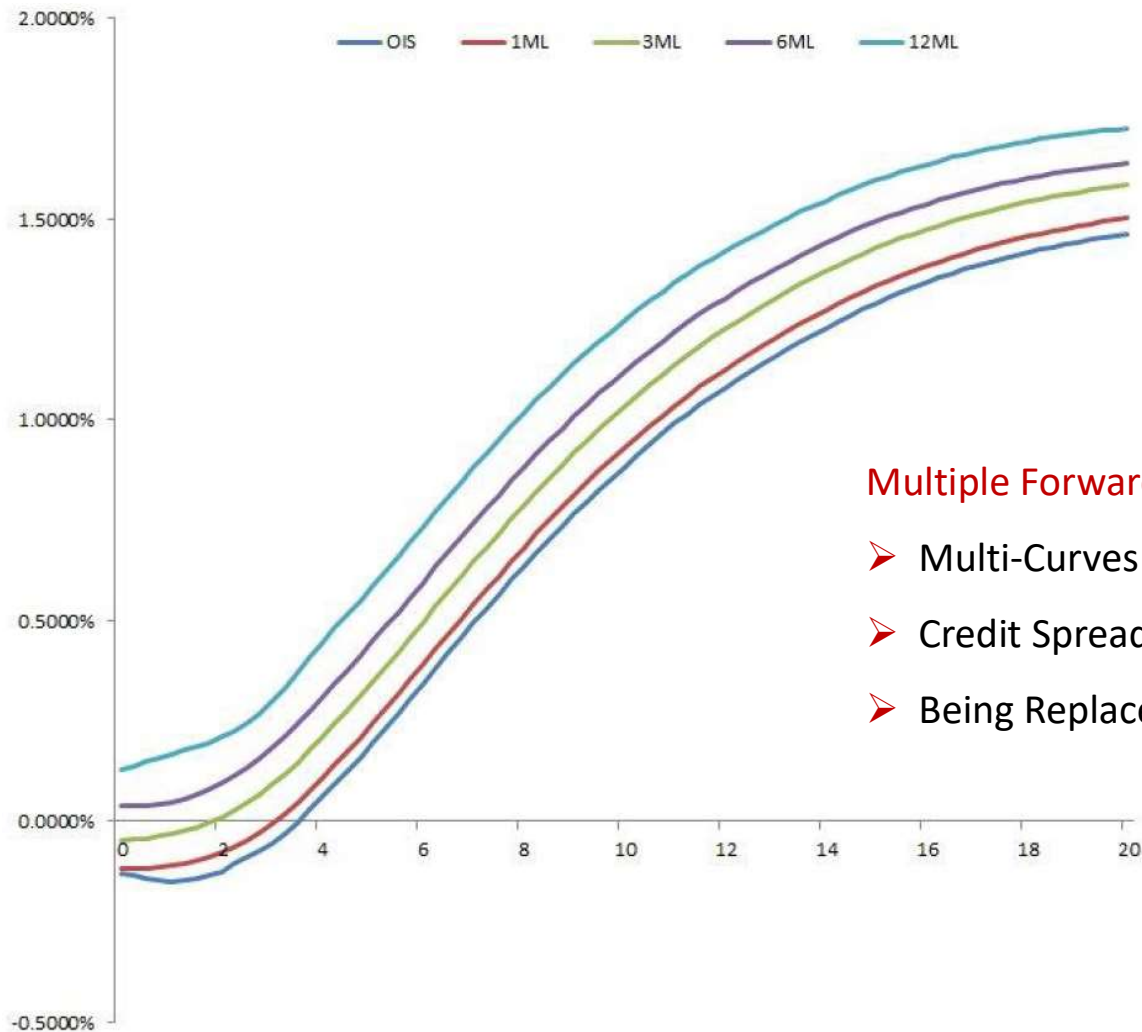


# Interest Rate Markets – Why the need for Speed?

- Cleared **Electronic Trading** & Auto-Hedging
- Real-Time, Highly Liquid & High Precision (Bid-Offer 1/10<sup>th</sup> bps i.e. USD 10 per MM)
- Trading Horizon: **High Frequency Trading** (HFT) vs Long-Term Fund Performance

USD Semi vs 3M Libor					USD Spreads vs Treasuries				
31) 1 Year	0.750 / 0.754	+0.014	≡		71) 1 Year	4.282 / 5.295	+0.687		
32) 2 Year	1.045 / 1.049	+0.017	≡		72) 2 Year	10.248 / 10.806	-0.073	≡	
33) 3 Year	1.284 / 1.287	+0.018	≡		73) 3 Year	3.337 / 3.895	-0.029	≡	
34) 4 Year	1.467 / 1.471	+0.015	≡		74) 4 Year	1.350 / 1.900	+0.161		
35) 5 Year	1.617 / 1.621	+0.014	≡		75) 5 Year	-4.020 / -3.454	+0.138	≡	
36) 6 Year	1.750 / 1.754	+0.012	≡		76) 6 Year	-8.100 / -7.550	+0.157		
37) 7 Year	1.866 / 1.870	+0.011	≡		77) 7 Year	-13.577 / -13.036	+0.382	≡	
38) 8 Year	1.966 / 1.970	+0.011	≡		78) 8 Year	-11.100 / -10.550	+0.335		
39) 9 Year	2.052 / 2.056	+0.011	≡		79) 9 Year	-9.888 / -9.088	+0.492		
40) 10 Year	2.126 / 2.129	+0.011	≡		80) 10 Year	-9.775 / -9.275	+0.537	≡	
41) 12 Year	2.250 / 2.254	+0.007	≡		81) 12 Year	2.520 / 3.320	+0.204		
42) 15 Year	2.376 / 2.380	+0.006	≡		82) 15 Year	-3.599 / -2.799	+0.110		
43) 20 Year	2.497 / 2.501	+0.002	≡		83) 20 Year	-10.100 / -9.600	+0.150		
44) 25 Year	2.558 / 2.563	+0.003	≡		84) 25 Year	-22.800 / -22.250	+0.150		
45) 30 Year	2.592 / 2.597	+0.000	≡		85) 30 Year	-38.058 / -37.491	+0.351	≡	
46) 40 Year	2.612 / 2.621	+0.003	≡						
47) 50 Year	2.598 / 2.604	+0.004	≡						

# Interest Rate Markets – Yield Curve Models



Required to Forecast Future Interest Rates

- Use **Liquid** Market Instruments
- To Imply Forward Rates & Disc. Factors

Multiple Forward Curves

- Multi-Curves Have In-Built **Credit Spread** (Tenor Homogenous)
- Credit Spread Determined by Loan Repayment Frequency
- Being Replaced by Single RFR Curves (Similar to OIS Curve)

# Interest Rate Markets – The LIBOR Problem

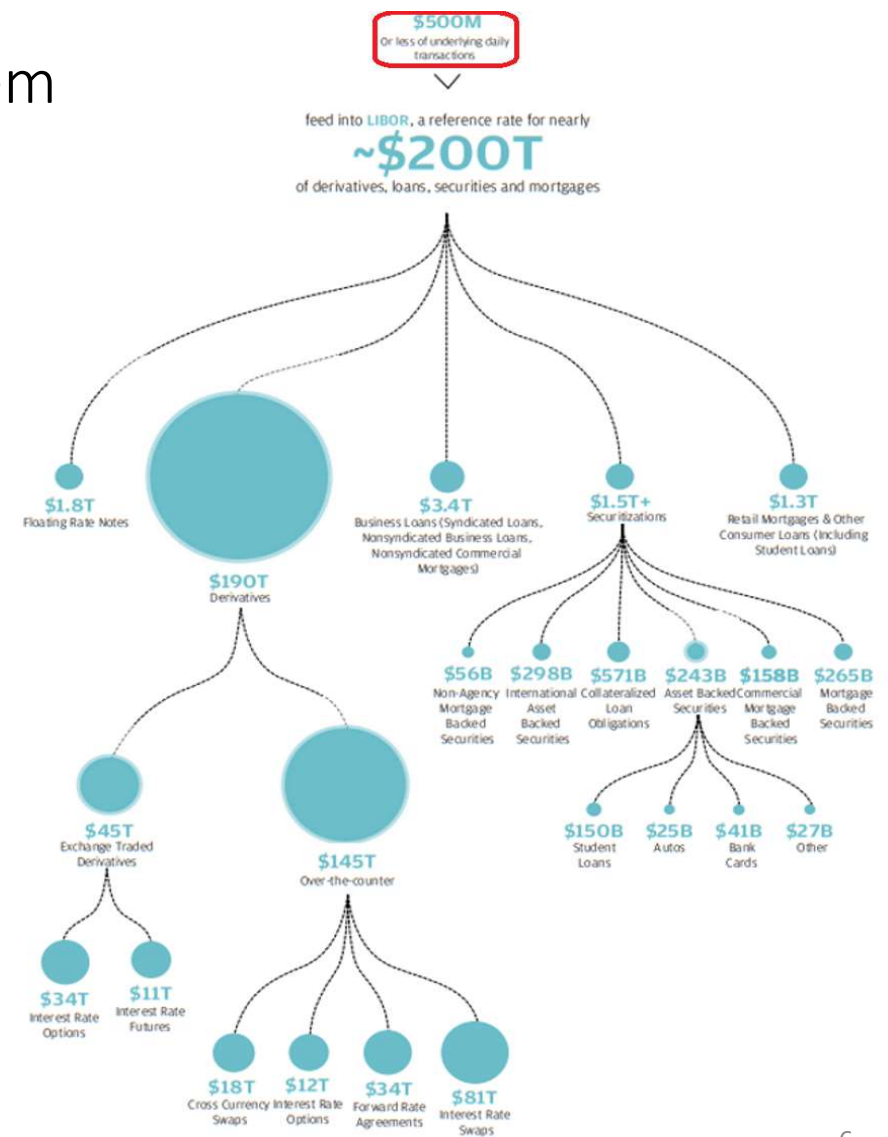
## The Problem with LIBOR

- LIBOR Market Transactions < \$500M
- Rates Do Not Reflect Actual Borrowing Levels
- LIBOR Levels Increasingly Set by Panel/Expert Judgement

## Market Size

- Market Size by Notional: \$200T (US) + \$150T (EU)

Large Market Driven by Small Number of LIBOR Transactions!!!



# Interest Rate Markets – LIBOR Benchmark Replacement

## LIBOR Rates

- Low Transaction Volume / Panel Based
- Forward Looking **Term Rate**, known **In-Advance**
- In Built Credit Risk Component

## Risk-Free Rates (RFRs)

- Transaction Based
- Backward Looking Rate, Known **In-Arrears**
- No Credit Component i.e. Risk-Free

## Market Changes

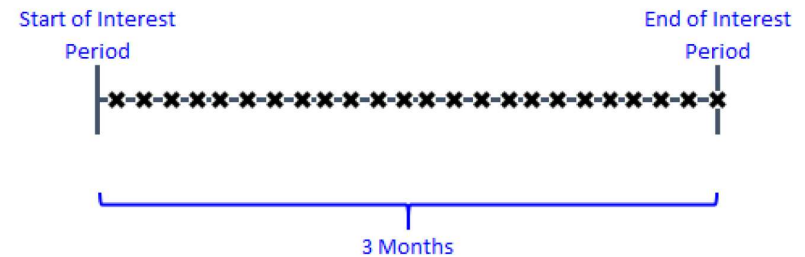
- Legacy LIBOR Contracts, Fall-Back Rates
- New RFR Products & Yield Curve Model Changes



**Rate:** Term Rate Fixed In-Advance

**Coupon:** Determined in Advance

### 3 Month Risk-Free Rate



**Rate:** Daily O/N Fixings leading to an Averaged Effective Rate

**Coupon:** Determined in Arrears



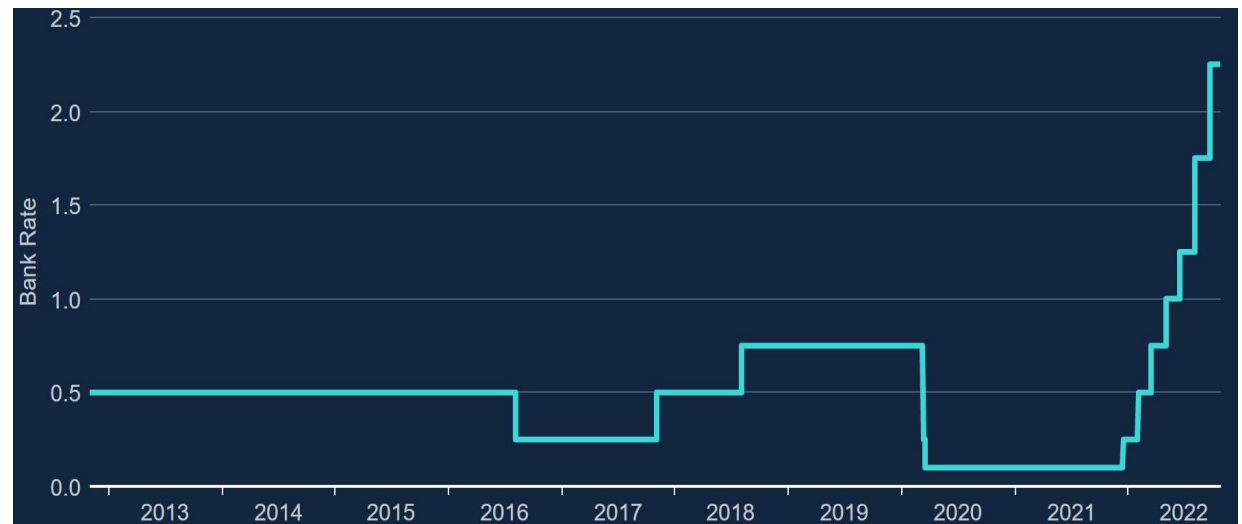
# Interest Rate Markets – Project Finance Risks & Solutions

## 1. Interest Rate Risk

- Finance linked to variable interest rates
- Use IRS to Fix Borrowing Costs

## 2. Foreign Exchange / Currency Risk

- International Finance
- Use Cross Currency Swaps to Fix FX Rates



## 3. Credit Default Risk

- Bonds, Bi-Lateral and Non-Cleared Transactions
- Risk of Counterpart Default
- Credit Default Swaps, Collateral & CSA Agreements

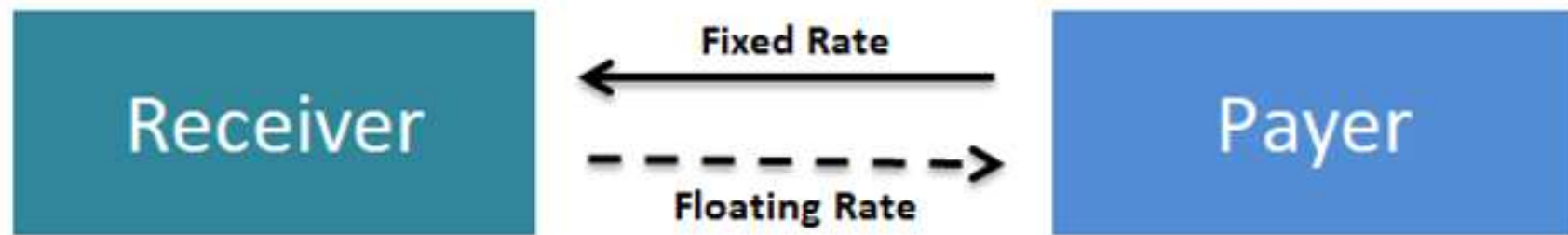
## 4. No money to invest?

- Use Asset Swaps to Borrow Funds to Invest in Bonds
- Pay LIBOR + Spread (Finance) to Receive Bond Coupons
- Floating Spread includes Funding + Credit Costs

# Interest Rate Swaps – Fixed or Variable Borrowing Costs?

## Project Finance

- Project Finance Naturally Incurs Variable Interest Costs (LIBOR + Spread)
- Exposed to Interest Rate Risk (Market may Move Against Us)



## Hedging Interest Rate Risk

- Use IRS to Exchange Floating for Fixed Interest (or Vice Versa)
- We Can Choose to Fix Borrowing Costs
- We Also Trade IRS for Speculative Purposes

# Interest Rate Swaps –Market Quotes & Pricing

USD Semi vs 3M Libor				USD Spreads vs Treasuries			
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- Standard Tenors: Spread Over US Treasury Yields
- New Swaps: Par Rate (%), since PV=0
- Existing Swaps: Present Value (USD)

# Interest Rate Swaps – Present Value

The screenshot displays a financial software interface for configuring and valuing an Interest Rate Swap. The main window is titled "Swap Manager" and contains several tabs: 3) Main, 4) Details, 5) Curves, 6) Cashflow, 7) Resets, 9) Scenario, 10) Risk, 11) CVA, 12) Matrix, 20) Properties, 21) Calculators, and 23) More Greeks. The "Main" tab is active, showing the "Fixed Float Swap" configuration.

**Deal Information:**

- Deal: Fixed Float Swap
- Counterparty: SWAP CNTRPARTY
- CCP: OTC

**Swap Legs:**

- Leg 1: Fixed**
  - Receive
  - Notional: 1MM
  - Currency: USD
  - Effective: 0D, 08/25/2015
  - Maturity: 5Y, 08/25/2020
  - Coupon: 5.000000 %
  - Pay Freq: SemiAnnual
  - Day Count: 30I/360
  - Calc Basis: Money Mkt
- Leg 2: Float**
  - Pay
  - Notional: 1MM
  - Currency: USD
  - Effective: 0D, 08/25/2015
  - Maturity: 5Y, 08/25/2020
  - Index: 3M, US0003M
  - Spread: 0.000 bp
  - Latest Index: 0.32910
  - Day Count: ACT/360
  - Reset Freq: Quarterly
  - Pay Freq: Quarterly

**Valuation Settings:**

- Curve Date: 08/21/2015
- Valuation: 08/25/2015
- OIS DC Strip: ON
- CSA Coll Ccy: USD

**Market Data:**

- Dscnt: 42, M, USD Bloomberg Curv
- Fwd: 23, M, USD Bloomberg Curv

**Valuation Results:**

Item	Value	Item	Value
Par Cpn	1.548250	Premium	16.78921
Principal	167,892.11	BP Value	1678.92112
Accrued	0.00	PV01	486.40
NPV	167,892.11	DV01	532.42
		Gamma (1bp)	0.29

**Present Value** is the Sum of **Discounted** Cash Flows

$$Swap\ PV = \underbrace{\sum_{i=1}^n N r \tau_i P(t_0, t_i)}_{\text{Fixed Cash Flows}} - \underbrace{\sum_{j=1}^m N (l_{j-1} + s) \tau_j P(t_0, t_j)}_{\text{Floating Cash Flows}}$$

## Interest Rate Swaps – Par Rate

- New Swaps Trade at Par i.e.  $PV = 0$
- Consequently such Swaps Quote as a Par Rate
- This is the fixed rate that makes both trade legs equal

$$Swap\ PV = \underbrace{r \sum_{i=1}^n N \tau_i P(t_0, t_i)}_{Fixed\ Cash\ Flows} - \underbrace{\sum_{j=1}^m N (l_{j-1} + s) \tau_j P(t_0, t_j)}_{Floating\ Cash\ Flows} = 0$$

Rearrange for the Fixed Rate  $r$  and call this the Par Rate,  $p$

$$Par\ Rate, p = \frac{PV(Float\ Leg)}{\sum_{i=1}^n N \tau_i P(t_0, t_i)} = \frac{PV(Float\ Leg)}{Annuity(Fixed\ Leg)^1}$$

<sup>1</sup> Par Rates calculated in terms of Annuity or PV01



# Interest Rate Swaps - Specification

- Majority of Swap Booking Schedule Related
- Trading **Templates**, Generators & Static Data

Swap Generator Template			
USD_SWAP_3M			
Dynamic Trade Info	LEG TYPE	LEG1:FIXED	LEG2:FLOAT
	PAY / RECEIVE	PAY	RECEIVE
Static Data + Schedule Info	NOTIONAL	1,000,000	1,000,000
	FIXED RATE (%)	1.00%	-
	FLOAT SPREAD (BPS)	-	0.00
	EFFECTIVE DATE / LAG	2D	2D
	MATURITY DATE / TENOR	2Y	2Y
	LEG CURRENCY	USD	USD
	NOTIONAL EXCHANGE	NONE	NONE
	LEVERAGE	1.00	1.00
	FRONT STUB INDEX	-	NATURAL
	BACK STUB INDEX	-	NATURAL
	VALUATION CURRENCY	USD	USD
	FORECAST INDEX	-	USD3M
	DISCOUNT INDEX	USDOIS	USDOIS
	INDEX COMPOUND METHOD	-	NONE
	SPREAD COMPOUND METHOD	-	NONE
	ROLL DAY	END	END
	STUB TYPE	SHORT START	SHORT START
	FIXING BUS DAY ADJUSTMENT	-	MODIFIED_FOLLOWING
	FIXING CALENDAR	-	NY+LDN
	FIXING LAG	-	2D
	FIXING IN-ADVANCE / IN-ARREARS	-	IN-ADVANCE
	ACCRUAL FREQUENCY	SEMI-ANNUAL	QUARTERLY
	ACCRUAL BUS DAY ADJUSTMENT	MODIFIED_FOLLOWING	MODIFIED_FOLLOWING
	ACCRUAL CALENDAR	NY	NY
	ACCRUAL DAYCOUNT	30/360	ACT/360
	PAYMENT FREQUENCY	SEMI-ANNUAL	QUARTERLY
	PAYMENT BUS DAY ADJUSTMENT	MODIFIED_FOLLOWING	MODIFIED_FOLLOWING
	PAYMENT CALENDAR	NY	NY
	PAYMENT LAG	2D	2D

TRADE ECONOMICS	TRADE PARAMETERS		LEG1	LEG2
	LegType		FLOAT	FLOAT
	Currency		EUR	USD
	Notional		8,769,622	10,000,000
	NotionalExchange		ALL	ALL
	PayReceive		PAY	RECEIVE
	EffectiveDate		Fri, 26-Oct-18	Fri, 26-Oct-18
	MaturityDateOrTenor		1Y	1Y
	FixedRate (%)		-	-
	FloatSpread (Bps)		0.00	0.00
	IndexCompoundMethod		-	NONE
	SpreadCompoundMethod		-	NONE
	Leverage		1.00	1.00
	ForecastCurve		EUR3M	USD3M
	DiscountCurve		EURDF_USDCSA	USDDF
MTM SWAPS	isMTMResetLeg		FALSE	TRUE
	ResetBaseFX		1.00000	1.14030
	ValuationCurrency		USD	USD
COUPON & STUB CONVENTIONS	CouponRollDay		NATURAL	NATURAL
	isEndOfMonth		TRUE	TRUE
	StubType		SHORT_START	SHORT_START
	FrontStubCurveIndex		NATURAL	NATURAL
	BackStubCurveIndex		NATURAL	NATURAL
	FrontStubDate		-	-
SCHEDULE INFORMATION	BackStubDate		-	-
	AccrualFrequency		QUARTERLY	QUARTERLY
	AccrualCalendar		TGT+NY+LON	TGT+NY+LON
	AccrualBusDayConv		MOD_FOLLOWING	MOD_FOLLOWING
	AccrualDaycount		ACT/360	ACT/360
	IRFixingBusDayConv		MOD_FOLLOWING	MOD_FOLLOWING
	IRFixingCalendar		TGT+NY+LON	TGT+NY+LON
	IRFixingLag		2D	2D
	IRFirstFixingLag		-	-
	PaymentFrequency		QUARTERLY	QUARTERLY
	PaymentBusDayConv		MOD_FOLLOWING	MOD_FOLLOWING
	PaymentCalendar		TGT+NY+LON	TGT+NY+LON
NON-DELIVERABLES	PaymentLag		2D	2D
	IsNonDeliverable		FALSE	FALSE
	SettlementCurrency		-	-
	FXFixingLag		-	-
	FXFixingBusDayConv		-	-
	FXFixingCalendar		-	-

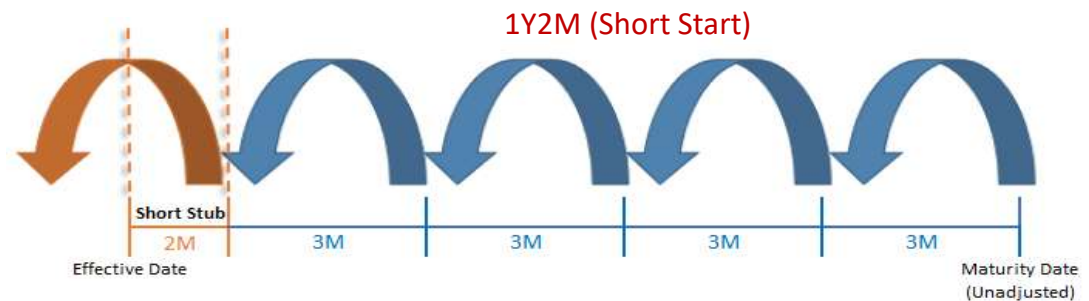
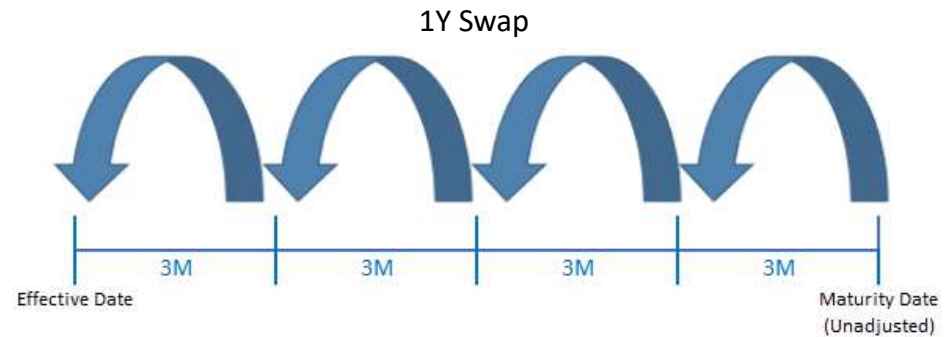
# Interest Rate Swaps - Schedules & Stubs

## Swap Schedules

- Backwards vs Forward Rolling Schedules
- Unadjusted to Preserve Roll Day
- Holiday Adjustments Ex-Ante
- Accrual Day Count Conventions

## Broken-Dated Swaps

- Stubs & Stub Rates (Linear Interp)
- Short Start/End, Long Start/End
- Market Default: **Short Start**



# IR Products – Tenor & Xccy Basis Swaps

## Tenor Basis Swaps

- Float vs Float (Same Currency)
- Exchange USD3M for USD6M say
- Match Project Cash Flow Frequency

Tenor Basis Swap Formulae (December 30, 2015).

Available at SSRN: <https://ssrn.com/abstract=2959605>

## Xccy Basis Swaps

- Float vs Float (Different Currencies)
- Exchange USD3M for EUR3M say
- Marked-to-Market / FX Notional Resets
- Reduces XVA Costs

An Illustrated Step-by-Step Guide of How to Price Cross Currency Swaps (November 11, 2018).

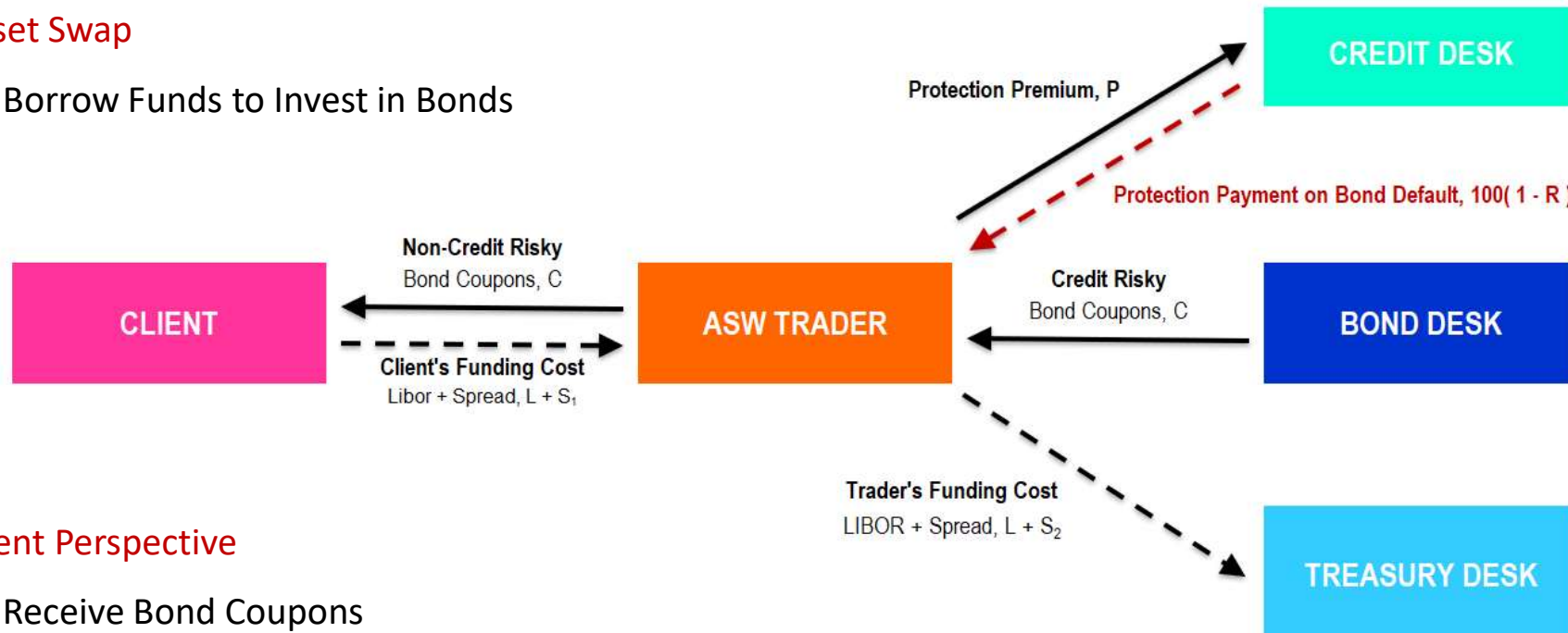
Available at SSRN: <https://ssrn.com/abstract=3278907>

91) Actions ▾		92) Products ▾		93) Views ▾		94) Info ▾		95) Settings ▾		Swap Manager	
Solver (Premium) ▾		Load		Save		Trade ▾		CCP ▾			
3) Main		4) Details		5) Curves		6) Cashflow		7) Resets		8) Scenario	
9) Risk		10) Matrix									
Deal		MTM XCCY Swap		Counterparty		SWAP CNTRPARTY ▾		Ticker / SWAP		20) Properties	
Swap		*Notional Reset b...		3 Month Euribor		Valuation Settings					
Leg 1: Float		Receive ▾		Leg 2: Float		Pay ▾		Curve Date		03/22/2019	
Notional		1MM		Notional		884,799.15		Valuation		03/26/2019	
Currency		USD		Currency		EUR		CSA Coll Ccy		USD	
Effective		0D 03/26/2019		Effective		0D 03/26/2019		Valuation Ccy		USD	
Maturity		1Y 03/26/2020		Maturity		1Y 03/26/2020		FX Rate		1.130200	
Index		3M US0003M		Index		3M EUR003M		<input checked="" type="checkbox"/> OIS DC Stripping			
Spread		0.000 bp		Spread		-12.625 bp					
Leverage		1.00000		Leverage		1.00000					
Latest Index		2.60988		Latest Index		-0.30900					
Reset Freq		Quarterly ▾		Reset Freq		Quarterly ▾					
Pay Freq		Quarterly ▾		Pay Freq		Quarterly ▾					
Day Count		ACT/360 ▾		Day Count		ACT/360 ▾					
Market											
Leg 1: NPV		1,002,566.12		Leg 2: NPV		-1,002,566.12					
Accrued		0.00		Accrued		0.00					
Premium		100.26		Premium		-100.26					
DV01		22.74		DV01		-22.74					
Valuation Results								22) Calculators ▾			
Principal		0.00		Premium		0.00000		BR01 92:EUR vs.		-102.10	
Accrued		0.00		BP Value		0.00000		DV01		0.00	
NPV		0.00						Gamma (1bp)		0.00	

# IR Products – Asset Swaps

## Asset Swap

- Borrow Funds to Invest in Bonds



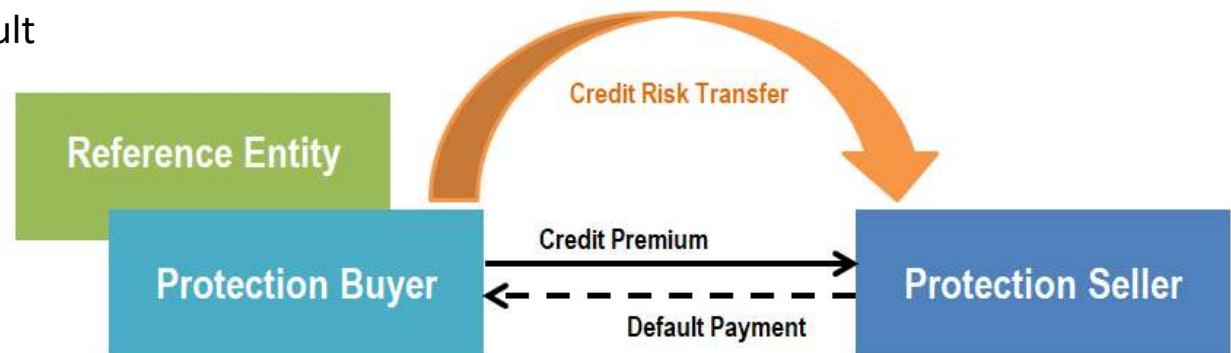
## Client Perspective

- Receive Bond Coupons
- Pay LIBOR + Spread
- Spread Includes Finance + Credit Costs

# IR Products – Credit Default Swaps (CDS)

## Insurance Against Counterparty Default

- Insuring Bond Notional Invested
- Pay Fixed Insurance Premium
- Receive Protection Payment on Default



## Credit Crisis & ISDA Big Bang (2008)

- Standardized & Cleared Contracts (IMM Dates<sup>1</sup>)
- Increased Liquidity
- Accrued Interest, Clean & Dirty Prices

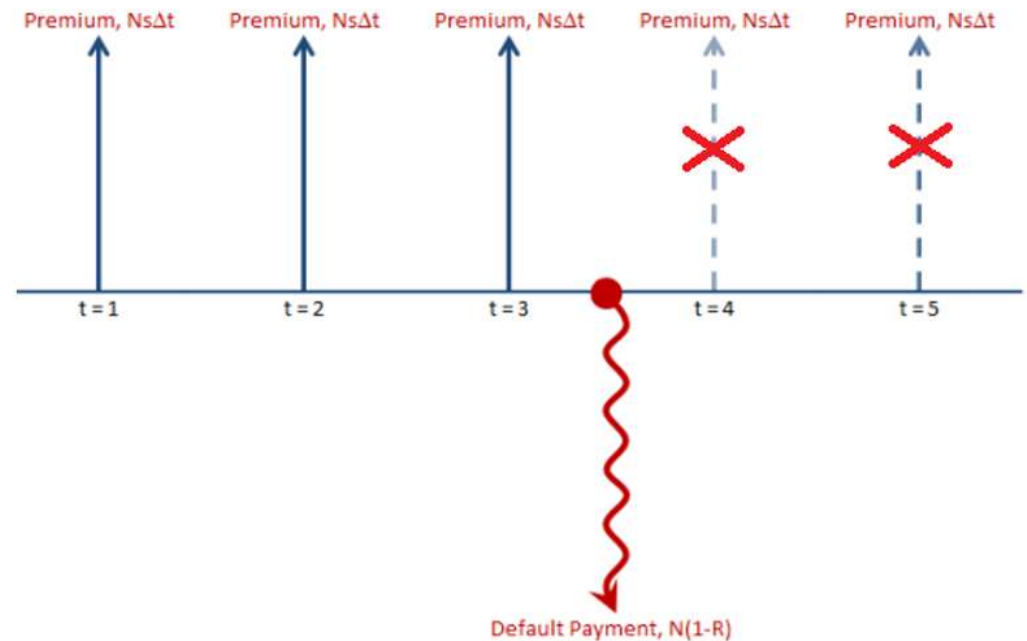
<sup>1</sup> Third Wednesday of Mar, June, Sep and Dec



# IR Products – CDS Pricing

## Pricing

- Similar to Interest Rate Swap Pricing
- With Additional Survival Probability Term,  $Q(t,T)$
- $Q(t,T) = \exp\left(-\int_t^T \lambda(t,u)du\right)$
- $\lambda$  is the 'Hazard Rate' (instantaneous prob of default)



## Buying Credit Protection

$$PV = PV(\text{Protection Leg}) - PV(\text{Premium Leg})$$

$$PV(\text{Premium Leg}) = \sum_{i=1}^n \underbrace{N s \Delta(t_{i-1}, t_i)}_{\text{Coupon}} \underbrace{Q(t_i)}_{P(\text{Survive})} \underbrace{P(t_0, t_i)}_{\text{Discount Factor}}$$

$$PV(\text{Protection Leg}) = \sum_{i=1}^n \underbrace{N(1-R)}_{\text{Loss Given Default}} \underbrace{[Q(t_{i-1}) - Q(t_i)]}_{\text{Default within Premium Period}} \underbrace{P(t_0, t_i)}_{\text{Discount Factor}}$$

# IR Risk

What are the main IR risks?

- Discount Risk (DF01)
- Forward Risk (PV01)
- Discount + Forward Risk (DV01)

Risk Calculation Methods

- Analytical
- Numerical Risk (Benchmark)
- Using Yield Curve Jacobian
- Automatic Adjoint Differentiation (AAD)

USD SOFR YIELD CURVE - CALIBRATION INSTRUMENTS

Instrument	Term	Rate
USD SOFR Swap	ON	2.37000%
USD SOFR Swap	1W	2.36510%
USD SOFR Swap	2W	2.34960%
USD SOFR Swap	3W	2.35200%
USD SOFR Swap	1M	2.34550%
USD SOFR Swap	2M	2.30320%
USD SOFR Swap	3M	2.25590%
USD SOFR Swap	4M	2.19610%
USD SOFR Swap	5M	2.14750%
USD SOFR Swap	6M	2.10350%
USD SOFR Swap	1Y	1.89350%
USD SOFR Swap	2Y	1.68360%
USD SOFR Swap	3Y	1.62600%
USD SOFR Swap	4Y	1.61700%
USD SOFR Swap	5Y	1.64200%
USD SOFR Swap	6Y	1.67900%
USD SOFR Swap	7Y	1.71600%
USD SOFR Swap	8Y	1.75700%
USD SOFR Swap	9Y	1.79800%
USD SOFR Swap	10Y	1.83200%
USD SOFR Swap	15Y	1.96800%
USD SOFR Swap	20Y	2.03300%
USD SOFR Swap	25Y	2.04100%
USD SOFR Swap	30Y	2.04900%

Bucketed DV01, USD

Instrument	Tenor	DV01
USD SOFR Swap	ON	8
USD SOFR Swap	1W	0
USD SOFR Swap	2W	0
USD SOFR Swap	3W	0
USD SOFR Swap	1M	0
USD SOFR Swap	2M	0
USD SOFR Swap	3M	0
USD SOFR Swap	4M	0
USD SOFR Swap	5M	-1
USD SOFR Swap	6M	1
USD SOFR Swap	1Y	92
USD SOFR Swap	2Y	213
USD SOFR Swap	3Y	294
USD SOFR Swap	4Y	409
USD SOFR Swap	5Y	453
USD SOFR Swap	6Y	541
USD SOFR Swap	7Y	723
USD SOFR Swap	8Y	736
USD SOFR Swap	9Y	852
USD SOFR Swap	10Y	892
USD SOFR Swap	15Y	1,320
USD SOFR Swap	20Y	1,662
USD SOFR Swap	25Y	1,979
USD SOFR Swap	30Y	2,252
<b>Total Risk</b>		<b>12,428</b>

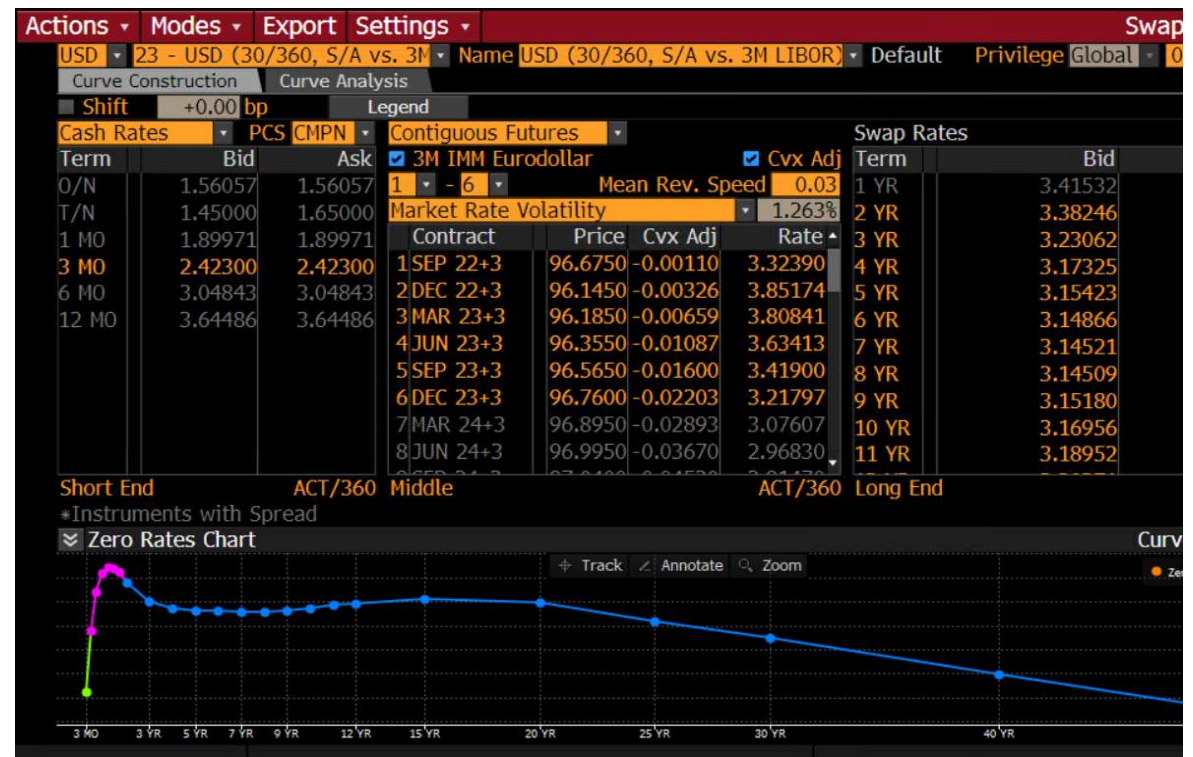
# Yield Curves - Calibration

## Model Inputs & Outputs

- Liquid Market Instrument Quotes [IN]
- Forward Rates [OUT]
- Discount Factors [OUT]

## Calibration Process

- Choose State Variable<sup>1</sup>
- Choose Interpolator (Functional Form)
- Solve and Imply Forwards & Disc Factors<sup>2</sup>



<sup>1</sup> Popular choices: forward rate, disc factor, logDF, zero rate etc.

<sup>2</sup> May need to differentiate and/or integrate state variable,  $P(t, T) = \exp\left(-\int_t^T f(t, u) du\right)$

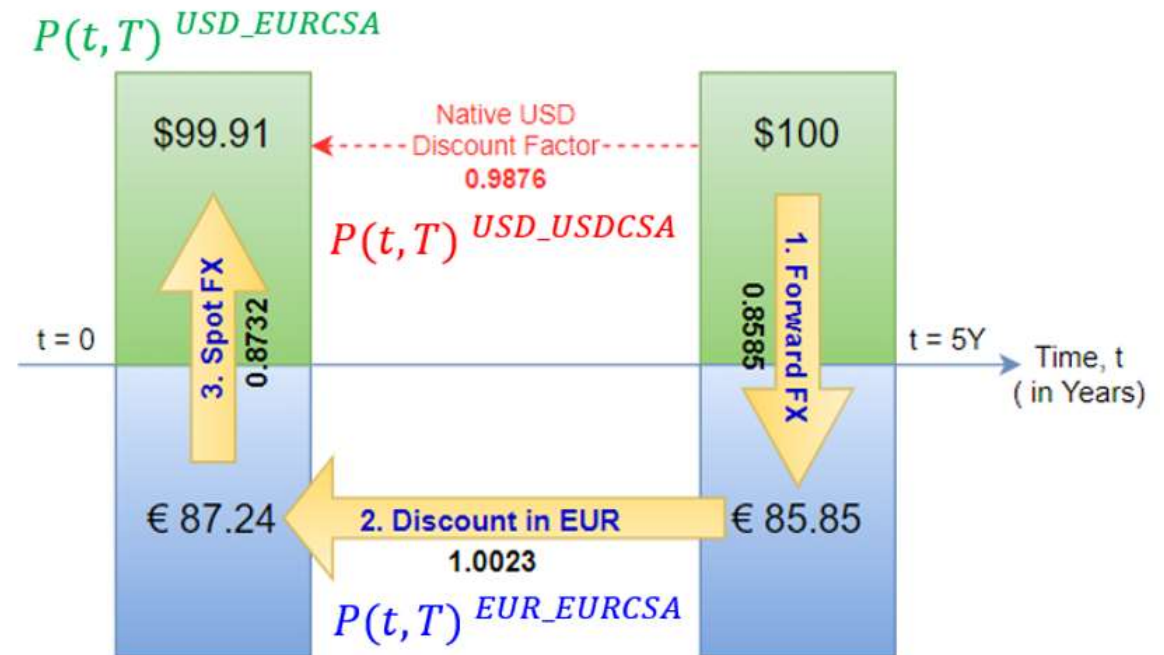
# Yield Curves – Collateral & CSA Curves

## Collateral & CSA Curves

- Calibrate to FX Forwards & Xccy Swaps
- **FX Forward Invariance** (FX Carry Trade)
- Impacts Discount Factors Only
- No Impact on Forward Rates

## Advanced CSA Topics

- Cheapest to Deliver (Multiple CSAs)
- Collateral Switch Options



$$f(t, T)^{USD/EUR} = s(t)^{USD/EUR} \underbrace{\left( \frac{P(t, T)^{EUR\_USDCSA}}{P(t, T)^{USD\_USDCSA}} \right)}_{USD \text{ CSA}} = s(t)^{USD/EUR} \underbrace{\left( \frac{P(t, T)^{EUR\_EURCSA}}{P(t, T)^{USD\_EURCSA}} \right)}_{EUR \text{ CSA}}$$

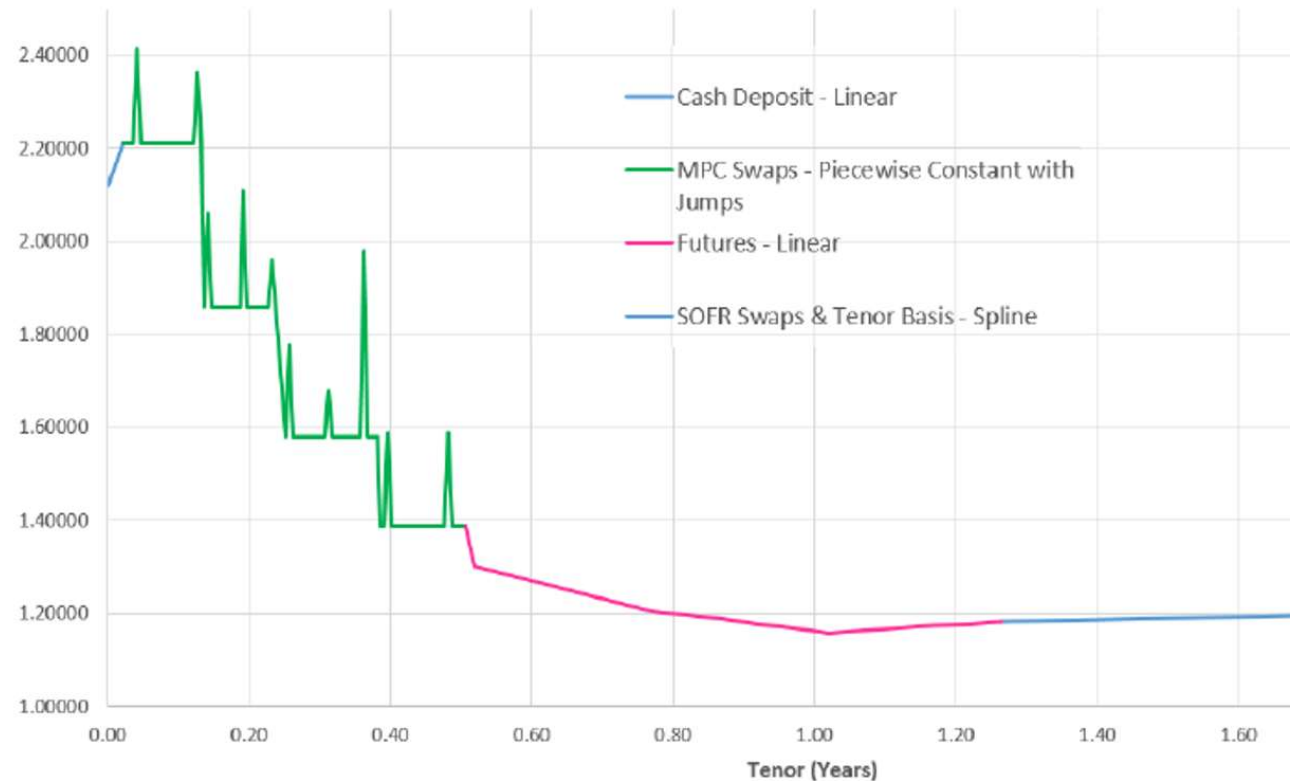
# Yield Curves - Features

## Curve Features & Considerations

- Underlying Instrument Behaviour
- Mixed Interpolation Schemes
- Turn-of-Year Effects (ToYs)

## Advanced Features for Electronic Markets

- Curve Jacobian
- Ultra-Fast Curves & Analytical Risk
- Automatic Adjoint Differentiation (AAD)





# Yield Curves – Curve Jacobian

## Electronic HFT Usage

- Ultra-Fast Rebuilds
- Real-Time Risk
- Auto-Hedging

## By-Product of Calibration Process

- Measures Changes in Market Instrument Quotes (P) on Forward Rates (L)
- First Order Derivative Matrix,  $dP/dL$  (Inverse Required)
- Controls Hedge and Risk Buckets (Same as Numerical Bumping)
- Use **Implicit Function Theorem** (IFT) to modify Risk Buckets (see Appendix)

Inverse Curve Jacobian,  $dL/dP$

Forward Pillars	Curve Calibration Instruments									
	$dP_{1Y}^{OIS}$	$dP_{2Y}^{OIS}$	$dP_{3Y}^{OIS}$	$dP_{4Y}^{OIS}$	$dP_{5Y}^{OIS}$	$dP_{1Y}^{IRS}$	$dP_{2Y}^{IRS}$	$dP_{3Y}^{IRS}$	$dP_{4Y}^{IRS}$	$dP_{5Y}^{IRS}$
$dO_{1Y}$	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$dO_{2Y}$	-1.01	2.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$dO_{3Y}$	0.00	-2.04	3.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$dO_{4Y}$	0.00	0.00	-3.08	4.08	0.00	0.00	0.00	0.00	0.00	0.00
$dO_{5Y}$	0.00	0.00	0.00	-4.13	5.13	0.00	0.00	0.00	0.00	0.00
$dL_{1Y}$	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
$dL_{2Y}$	0.00	0.00	0.00	0.00	0.00	-1.01	2.01	0.00	0.00	0.00
$dL_{3Y}$	0.00	0.00	0.00	0.00	0.00	0.00	-2.04	3.04	0.00	0.00
$dL_{4Y}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-3.08	4.08	0.00
$dL_{5Y}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-4.13	5.13

# Yield Curves – Ultra-Fast Rebuilds

New Forwards

$$L_{New} = L_{Old} + dL$$

$$= L_{Old} + (dL/dP) \cdot dP$$

New Forwards		Original Forwards		Inverse Jacobian, dL/dP											Change in Mkt Data	
L <sub>NEW</sub>		L <sub>OLD</sub>													dP	
L <sub>1Y</sub> <sup>OIS</sup>	1.44591%	L <sub>1Y</sub> <sup>OIS</sup>	1.43591%	L <sub>1Y</sub> <sup>OIS</sup>	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	L <sub>1Y</sub> <sup>OIS</sup>	0.01%
L <sub>2Y</sub> <sup>OIS</sup>	1.24323%	L <sub>2Y</sub> <sup>OIS</sup>	1.23323%	L <sub>2Y</sub> <sup>OIS</sup>	-1.01	2.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	L <sub>2Y</sub> <sup>OIS</sup>	0.01%
L <sub>3Y</sub> <sup>OIS</sup>	1.26107%	L <sub>3Y</sub> <sup>OIS</sup>	1.25107%	L <sub>3Y</sub> <sup>OIS</sup>	0.00	-2.04	3.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	L <sub>3Y</sub> <sup>OIS</sup>	0.01%
L <sub>4Y</sub> <sup>OIS</sup>	1.30130%	L <sub>4Y</sub> <sup>OIS</sup>	1.29130%	L <sub>4Y</sub> <sup>OIS</sup>	0.00	0.00	-3.08	4.08	0.00	0.00	0.00	0.00	0.00	0.00	L <sub>4Y</sub> <sup>OIS</sup>	0.01%
L <sub>5Y</sub> <sup>OIS</sup>	1.40782%	L <sub>5Y</sub> <sup>OIS</sup>	1.39782%	L <sub>5Y</sub> <sup>OIS</sup>	0.00	0.00	0.00	-4.13	5.13	0.00	0.00	0.00	0.00	0.00	L <sub>5Y</sub> <sup>OIS</sup>	0.01%
L <sub>1Y</sub> <sup>IRS</sup>	1.71896%	L <sub>1Y</sub> <sup>IRS</sup>	1.70896%	L <sub>1Y</sub> <sup>IRS</sup>	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	L <sub>1Y</sub> <sup>IRS</sup>	0.01%
L <sub>2Y</sub> <sup>IRS</sup>	1.48359%	L <sub>2Y</sub> <sup>IRS</sup>	1.47359%	L <sub>2Y</sub> <sup>IRS</sup>	0.00	0.00	0.00	0.00	0.00	-1.01	2.01	0.00	0.00	0.00	L <sub>2Y</sub> <sup>IRS</sup>	0.01%
L <sub>3Y</sub> <sup>IRS</sup>	1.50531%	L <sub>3Y</sub> <sup>IRS</sup>	1.49531%	L <sub>3Y</sub> <sup>IRS</sup>	0.00	0.00	0.00	0.00	0.00	0.00	-2.04	3.04	0.00	0.00	L <sub>3Y</sub> <sup>IRS</sup>	0.01%
L <sub>4Y</sub> <sup>IRS</sup>	1.56934%	L <sub>4Y</sub> <sup>IRS</sup>	1.55934%	L <sub>4Y</sub> <sup>IRS</sup>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-3.08	4.08	0.00	L <sub>4Y</sub> <sup>IRS</sup>	0.01%
L <sub>5Y</sub> <sup>IRS</sup>	1.63999%	L <sub>5Y</sub> <sup>IRS</sup>	1.62999%	L <sub>5Y</sub> <sup>IRS</sup>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-4.13	5.13	L <sub>5Y</sub> <sup>IRS</sup>	0.01%

## Implementation

- Slow Curve (Full-Rebuild) Ticks in Background (ca. 10ms)
- Fast Curve (Jacobian Method) Used Between Refreshes (Real-Time)

# Yield Curves – Real-Time Bucketed Risk

## Requirements

- Curve Jacobian
- Trade or Portfolio Jacobian

$$DV01(Analytical) = \underbrace{1bps \times \frac{dPV}{dL}}_{\text{Pricing Jacobian}} \times \underbrace{\frac{dL}{dP}}_{\text{Curve Jacobian}}$$

## Risk as a Matrix Operation

- Can be Parallelized / Vectorized
- Matrix Dimensions Must Agree
- Interpolation & Forward Mapping
- Barycentric Weights,  $w_j(t)$

$$p(t) = \sum_{j=0}^n w_j(t) f(t_j), \quad w_j(t) = \frac{\prod_{k=0, k \neq j}^n (t - t_k)}{\prod_{k=0, k \neq j}^n (t_j - t_k)}$$

Inverse Curve Jacobian, dL/dP

Forward Pillars	Curve Calibration Instruments									
	dP <sub>1Y</sub> <sup>OIS</sup>	dP <sub>2Y</sub> <sup>OIS</sup>	dP <sub>3Y</sub> <sup>OIS</sup>	dP <sub>4Y</sub> <sup>OIS</sup>	dP <sub>5Y</sub> <sup>OIS</sup>	dP <sub>1Y</sub> <sup>IRS</sup>	dP <sub>2Y</sub> <sup>IRS</sup>	dP <sub>3Y</sub> <sup>IRS</sup>	dP <sub>4Y</sub> <sup>IRS</sup>	dP <sub>5Y</sub> <sup>IRS</sup>
dO <sub>1Y</sub>	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
dO <sub>2Y</sub>	-1.01	2.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
dO <sub>3Y</sub>	0.00	-2.04	3.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
dO <sub>4Y</sub>	0.00	0.00	-3.08	4.08	0.00	0.00	0.00	0.00	0.00	0.00
dO <sub>5Y</sub>	0.00	0.00	0.00	-4.13	5.13	0.00	0.00	0.00	0.00	0.00
dL <sub>1Y</sub>	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
dL <sub>2Y</sub>	0.00	0.00	0.00	0.00	0.00	-1.01	2.01	0.00	0.00	0.00
dL <sub>3Y</sub>	0.00	0.00	0.00	0.00	0.00	0.00	-2.04	3.04	0.00	0.00
dL <sub>4Y</sub>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-3.08	4.08	0.00
dL <sub>5Y</sub>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-4.13	5.13

=

Trade	
Risk Bucket	IRS 3Y
OIS 1Y	0
OIS 2Y	0
OIS 3Y	0
OIS 4Y	-1
OIS 5Y	1
IRS 1Y	0
IRS 2Y	0
IRS 3Y	291
IRS 4Y	0
IRS 5Y	0

Trade Jacobian, dPV/dL  
3Y Par Swap

Trade	OIS Curve (Discount Risk)					Swap Curve (Forward Risk)				
	dO <sub>1Y</sub>	dO <sub>2Y</sub>	dO <sub>3Y</sub>	dO <sub>4Y</sub>	dO <sub>5Y</sub>	dL <sub>1Y</sub>	dL <sub>2Y</sub>	dL <sub>3Y</sub>	dL <sub>4Y</sub>	dL <sub>5Y</sub>
dS <sub>3Y</sub> <sup>IRS</sup>	0	0	0	0	0	98	97	96	0	0

Total Trade DV01

IRS 3Y
291

# Yield Curves – Automatic Adjoint Differentiation (AAD)

## Trade Jacobian

- AAD Can Compute Instrument Price & Risk Simultaneously
- Direct Differentiation of Code + Implicit Function Theorem (IFT)
- Exact & Fast (X4 Pricing Time)

## Tangent & Adjoint Modes

- Tangent Mode (dot) : **Forward** Mode - **One Risk at a Time**
- Adjoint Mode (bar) : **Backward** Mode - **All Risks Simultaneously**
- Activation Inputs Control Risk Outputs

## Implementation Methods

- By Hand (See Appendix for Swap DV01 Risk Example)
- Derivative Code by Overloading, DCO/C++
- Professional Tools: Adept, NAG

## Pricing Calculations

$$x \rightarrow f(x) \rightarrow g(f) \rightarrow h(g) \rightarrow y$$

## Chain Rule: Forwards

$$\frac{df}{dx} \cdot \frac{dg}{df} \cdot \frac{dh}{dg} \cdot \frac{dy}{dh} = \frac{dy}{dx}$$

## Chain Rule: Backwards

$$\frac{dy}{dh} \cdot \frac{dh}{dg} \cdot \frac{dg}{df} \cdot \frac{df}{dx} = \frac{dy}{dx}$$

# Yield Curves – AD Tangent Mode Example

## Tangent Mode

- Differentiate Forwards using 'Dot' Notation
- One Risk at a Time, Controlled by Dot **Input Activation Variables** 1 or 0
- For  $\frac{df}{dx_1}$  and  $\frac{df}{dx_2}$  must call tangent method twice

```

01 double function( double x1, double x2 )
02 {
03     double a = x1*x1;           // Step 1:    a = x12
04     double b = 2*a;             // Step 2:    b = 2x12
05     double c = x2;              // Step 3:    c = x2
06     double d = 3*c;             // Step 4:    d = 3x2
07     double f = b + d;           // Step 5:    f = 2x12 + 3x2
08     return f;
09 }

```

Simple Function:  $f(x_1, x_2) = 2x_1^2 + 3x_2$

Source Code: <https://onlinegdb.com/kKqaS6hJT>

```

01 tangent(2.0, 3.0, 1.0, 0.0); // Input: x1 = 2, x2 = 3, x1_d = 1, x2_d = 0   Output: 8
02 tangent(2.0, 3.0, 0.0, 1.0); // Input: x1 = 2, x2 = 3, x1_d = 0, x2_d = 1   Output: 3

```

## Function Derivatives using Tangent Mode

```

01 double tangent( double x1, double x2, double x1_dot, double x2_dot )
02 {
03     double a = x1*x1;           // Step 1:    a = x12
04     double a_dot = 2*x1*x1_dot; // Tangent:   $\dot{a} = 2x_1 \cdot \dot{x}_1$        $\dot{a} = 2x_1$ 
05     double b = 2*a;             // Step 2:    b = a
06     double b_dot = 2*a_dot;     // Tangent:   $\dot{b} = 2 \cdot \dot{a}$            $\dot{b} = 4x_1$ 
07     double c = x2;              // Step 3:    c = x2
08     double c_dot = x2_dot;      // Tangent:   $\dot{c} = \dot{x}_2$            $\dot{c} = 1$ 
09     double d = 3*c;             // Step 4:    d = 3c
10     double d_dot = 3*c_dot;     // Tangent:   $\dot{d} = 3 \cdot \dot{c}$          $\dot{d} = 3$ 
11     double f = b + d;           // Step 5:    f = 2x12 + 3x2
12     double f_dot = b_dot + d_dot; // Tangent:   $\dot{f} = \dot{b} + \dot{d}$ 
13     return f_dot;              // Result:     $\dot{f} = 4x_1 + 3$ 
14 }

```

Simple Function  $f(x_1, x_2) = 2x_1^2 + 3x_2$  with Tangent Derivatives



# Yield Curves – AD Adjoint Mode Example

## Adjoint Mode (Reverse Mode)

- Backwards Differentiation with 'Bar' Notation
- Forward Sweep then Back Propagate Risk
- Computes All Risks at Same Time
- Risk Controlled By Bar Input Activation Variable 1 or 0
- Adjoint Method Calculates Both  $\frac{df}{dx_1}$  and  $\frac{df}{dx_2}$

```

01 double function( double x1, double x2 )
02 {
03     double a = x1*x1;           // Step 1:  a = x12
04     double b = 2*a;             // Step 2:  b = 2x12
05     double c = x2;             // Step 3:  c = x2
06     double d = 3*c;            // Step 4:  d = 3x2
07     double f = b + d;          // Step 5:  f = 2x12 + 3x2
08     return f;
09 }

```

Simple Function:  $f(x_1, x_2) = 2x_1^2 + 3x_2$

```

01 adjoint(2.0, 3.0, 1.0); // Input: x1 = 3, x2 = 2, f_bar Output: df/dx1 = 8 and df/dx2 = 3

```

## Function Derivatives using Adjoint Mode

```

01 void adjoint( double x1, double x2, double f_bar )
02 {
03     // Forward Sweep
04     double a = x1*x1;           // Step 1:  a = x12
05     double b = 2*a;             // Step 2:  b = 2x12
06     double c = x2;             // Step 3:  c = x2
07     double d = 3*c;            // Step 4:  d = 3x2
10     double f = b + d;          // Step 5:  f = 2x12 + 3x2
08
09     // Back Propagation
10     double b_bar = f_bar;       // Step 5:  b_bar = 1    from input variable
11     double d_bar = f_bar;       // Step 5:  d_bar = 1    from input variable
12     double c_bar = 3*d_bar;     // Step 4:  c_bar = 3
13     double x2_bar = c_bar;      // Step 3:  x2_bar = 3    df/dx2 = 3
14     double a_bar = 2*b_bar;     // Step 2:  a_bar = 2
15     double x1_bar = 2*x1*a_bar; // Step 1:  x1_bar = 4x1  df/dx1 = 4x1
16
17     // Display Results
18     std::cout << "df/dx1: " << x1_bar << std::endl; // x̄1 = df/dx1 = 4x1
19     std::cout << "df/dx2: " << x2_bar << std::endl; // x̄2 = df/dx2 = 3
20 }

```

Simple Function  $f(x_1, x_2) = 2x_1^2 + 3x_2$  with Adjoint Derivatives

# Credit Models – Hazard Rates & Survival Probabilities

## Calibration Summary

- Yield Curve is an Input
- Calibrate to Bonds or CDS
- Imply Hazard Rates,  $\lambda$
- Used for Survival Prob,  $Q(t,T)$

## Common Assumptions

- Piecewise Constant<sup>1</sup>
- Deterministic Hazard Rates

## Rule of Thumb

$$\lambda = \frac{s}{(1 - R)}$$



<sup>1</sup> As often there is only a single calibration instrument

## PART TWO – PRICING & PRACTICE

### Case Studies Interest Rate Swaps & Asset Swaps

# Interest Rate Swap – Annuity is the Key Pricing & Risk Factor

## It's All About Annuity

- Pricing & Risk Expressed in Terms of Annuity
- Similarly Float Legs Expressed in Annuity Terms
- Can Be Used to Convert a Float Leg to Fixed Leg
- Useful for Low Latency Pricing

## Key Formulae:

- $PV = (r - p) \text{ Annuity(Fixed)}$
- $\text{Par Rate} = PV(\text{Float}) / \text{Annuity(Fixed)}$
- $PV01 = \text{Annuity(Fixed)} \times 0.01\%$
- $DV01 = PV01 + DF01 = PV01$  for Par Swaps

$$A_N = N \sum_{i=1}^n \tau_i P(t_0, t_i)$$

## Low Latency Interest Rate Swap Pricing

Electronic Rates Markets & Low Latency Interest Rate Swap Calculations (May 31, 2022).

Available at SSRN: <https://ssrn.com/abstract=4125565>

$$\text{Swap PV} = PV^{\text{Fixed Leg}} - PV^{\text{Float Leg}}$$

$$\begin{aligned} &= r \sum_{i=1}^n N_i \tau_i P(t_0, t_i) - \sum_{j=1}^m N_j l_{j-1} \tau_j P(t_0, t_j) \\ &= (r - p) A_{\text{Fixed}} \end{aligned}$$



# Interest Rate Swap – Pricing & Risk Example

Compute Annuity  $A_N$

= USD 4,863,971.74

$$A_N = N \sum_{i=1}^n \tau_i P(t_0, t_i)$$

$PV = (r - p) A_N$

= (5.00% - 1.59%)  $A_N$

= USD 167,892.11

The screenshot displays a financial software interface for swap pricing and risk management. The main window is titled 'Swap Manager' and contains several tabs: 3) Main, 4) Details, 5) Curves, 6) Cashflow, 7) Resets, 9) Scenario, 10) Risk, 11) CVA, 12) Matrix, and 20) Properties. The 'Main' tab is active, showing a 'Fixed Float Swap' with a 'Counterparty' of 'SWAP CNTRPARTY'. The swap details include: Leg 1: Fixed (Receive), Notional 1MM, Currency USD, Effective 08/25/2015, Maturity 08/25/2020, Coupon 5.000000%, Pay Freq SemiAnnual, Day Count 30I/360, Calc Basis Money Mkt. Leg 2: Float (Pay), Notional 1MM, Currency USD, Effective 08/25/2015, Maturity 08/25/2020, Index 3M US0003M, Spread 0.000 bp, Latest Index 0.32910, Day Count ACT/360, Reset Freq Quarterly, Pay Freq Quarterly. The 'Valuation Settings' panel shows Curve Date 08/21/2015, Valuation 08/25/2015, OIS DC Strip ON, and CSA Coll Ccy USD. The 'Valuation Results' panel shows Par Cpn 1.548250, Premium 16.78921, Principal 167,892.11, BP Value 1678.92112, Accrued 0.00, and NPV 167,892.11. The 'More Greeks' panel shows PV01 486.40, DV01 532.42, and Gamma (1bp) 0.29.

Par Rate =  $PV(\text{Float}) / A_N$

= 75,306 /  $A_N$

= 1.5482%

PV01

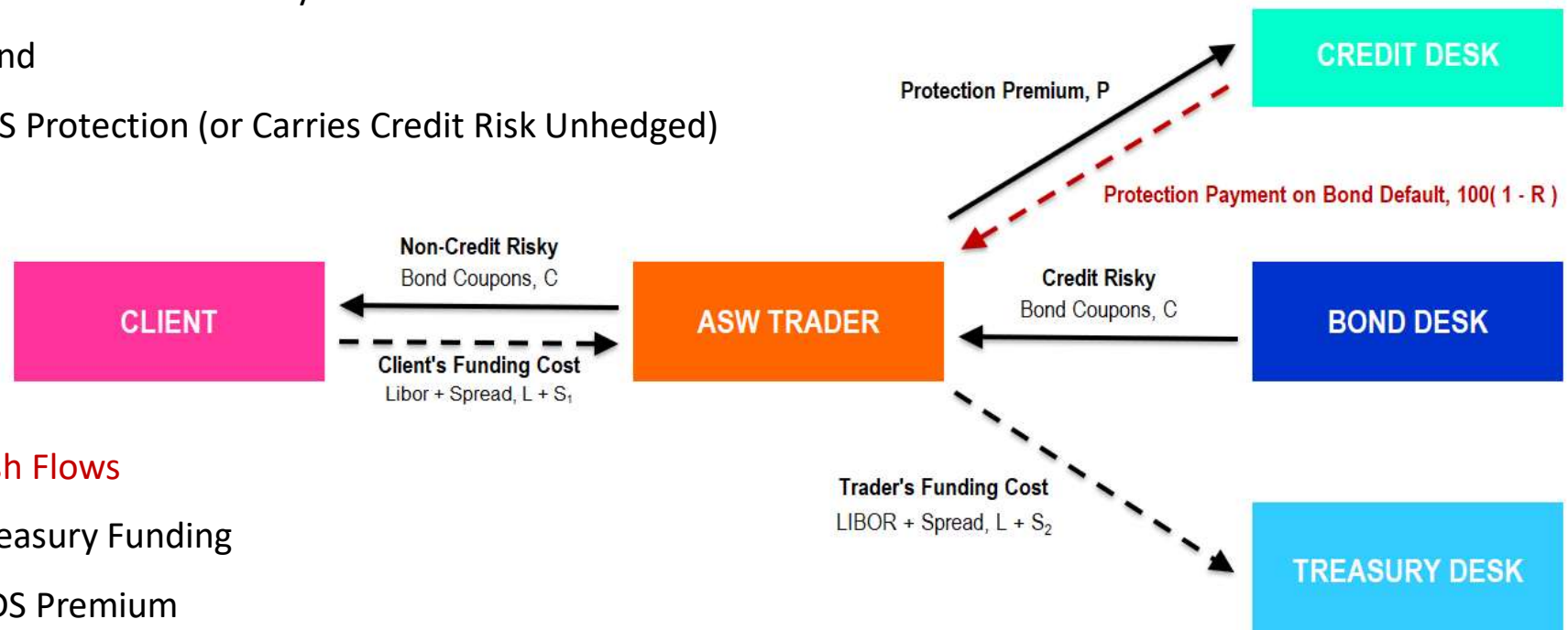
=  $A_N \times 0.01\%$

= USD 486.40

# Asset Swap – Structuring the Asset Swap Spread

## Trader Creates Synthetic Asset Swap

- Borrow Cash from Treasury to Purchase Bond
- Buy Bond
- Buy CDS Protection (or Carries Credit Risk Unhedged)



## Trader Cash Flows

- Pays Treasury Funding
- Pays CDS Premium
- Receives Bond Coupons and Passes on to Client
- Client Pays All Costs + Commission as a **Spread over LIBOR** (or RFR)



## Asset Swap – Pricing as a Spread Over LIBOR (or RFR)

DBR 0 1/2 02/15/26		5 Actions		6 Settings		Asset Swap Calculator	
1 Pricing		2 Cashflow		3 Relative Value		4 Deal Summary	
Asset Swap Analysis				Price	104.5800		
Calculate				Z-Spread	-40.9	ASW Spread	-40.6
Price -> ASW Spread				Yield(%)	0.02595	MMS Spread	-41.2
Bond JV503423		Swap		Par-Par		Matched Maturity	
Par Amount		1MM		Leg 1: Fixed		Pay	
Workout		02/15/2026		Notional		1MM	
Workout Price		100.0000		Currency		EUR	
Pay Freq		Annual		Effective Date		01/15/2016	
Day Count		ACT/ACT		Maturity Date		02/15/2026	
				Coupon		0.5	
				Pay/Reset Freq		Annual	
				Day Count		ACT/ACT	
Implied Value		100.5736		Include Accrued		Include Accrued	
Market				Discount Curve		133 Mid	
Curve Date		06/09/2016		Discount Curve		45 Mid	
Settle Date		06/13/2016		Forward Curve			
Swapped Spread Detail							
Clean Price		104.5800		Money		Spread(bp)	
Swap Price		100.0000		Cash Out		-45,800.0	
Swap Rate(%)		0.44104		Bond Cpn(%)		0.5000	
Redemption(%)		0.0000				0.0	
Funding		Spread(bp)		0.0		0.0	
Swapped Spread						-40,063.5	

- **ASW Spread** - Par-Par Spread
- **MMS Spread** - Yield-Yield Spread<sup>1</sup>

<sup>1</sup> Y/Y Spread Between Swap Rate and Benchmark Gov't Bond Yield

# Asset Swap – Pricing using Par-Par Method

## Pricing as a PV

- Valuation Method for Existing Swaps, Unwinds and Novations (trade transfers)
- Again Present Value is Simply the Sum of Incoming and Outgoing Cash Flows
- An Upfront Par-Adjustment is Made if the Underlying Bond not Trading at Par, i.e., 100

$$PV^{Asset\ Swap} = \underbrace{\phi r^{Fixed} \sum_{i=1}^n N_i \tau_i P(t_0, t_i)}_{Fixed\ Leg} - \underbrace{\phi \sum_{j=1}^m N_j (l_{j-1} + s) \tau_j P(t_0, t_j)}_{Float\ Leg} + \underbrace{\phi N_1 \left( \frac{100 - B}{100} \right)}_{Par\ Adjustment}$$

## Pricing as a Par Spread

- New Asset Swaps Price to Par i.e., zero
- Instead Quote as a Par Spread  $s$
- Rearrangement of PV formula with  $PV=0$

$$s = \left( \frac{(r^{Fixed} - p^{Market}) A^{Fixed} + \left( \frac{100 - B}{100} \right)}{A^{Float}} \right)$$

# Pricing Tricks & Rules of Thumb

## Annuity Assumption

- Need to know market par rates for standard swap maturities
- Assume Annual Coupons and Discount Factors = 1.0
- This means **Annuity = Time to Maturity**
- Used to Gain Intuition when Pricing IR Products & CDS

### Interest Rate Swap PV

➤  $PV = N ( r - p ) A(\text{Fixed})$



### Approximate PV

➤  $PV = N \Delta r T$

This gives PV as USD 100 per Million per Year per  $\Delta r$  in bps

## IRS Rule of Thumb – Multiples of a Base Case

- $PV = 100 \times \Delta N \times \Delta r \times \Delta T$
- $DV01 = PV01 = 100 \times \Delta N \times \Delta T$

# Pricing Tricks – Interest Rate Swap

## IRS – Rule of Thumb

- $PV = 100 \times \Delta N \times \Delta r \text{ in bps} \times \Delta T$
- $DV01 = PV01 = 100 \times \Delta N \times \Delta T$

## Market Par Rate

- 5Y Par Rate = 150 bps
- $\Delta r = (r - p) = (500 - 150) = 350 \text{ bps}$

## Present Value

- Here  $\Delta N = 1$ ,  $\Delta r = 350$ ,  $\Delta T = 5$
- $PV = \text{USD } 175,000$
- $DV01 = PV01 = \text{USD } 500$

PV as USD 100 per Million per Year per  $\Delta r$  in bps

The screenshot shows a financial software interface with a menu bar at the top (91) Actions, 92) Products, 93) Views, 94) Data & Settings, 95) Info, Swap Manager). Below the menu bar is a toolbar with buttons like 30) Solver (Premium), 31) Load, 32) Save, 35) Trade, 38) CCP, 43) Send to TR. The main window is divided into several panes. The left pane shows 'Deal' details for a 'Fixed Float Swap' with 'Counterparty' set to 'SWAP CNTRPARTY'. The 'Swap' section shows 'Leg 1: Fixed' with 'Receive' and 'Leg 2: Float' with 'Pay'. The 'Market' section shows 'Dscnt' as 42 M and 'Fwd' as 23 M, both using 'USD Bloomberg Curv'. The right pane shows 'Valuation Settings' with 'Curve Date' as 08/21/2015, 'Valuation' as 08/25/2015, 'OIS DC Strip' as ON, and 'CSA Coll Ccy' as USD. The bottom pane shows 'Valuation Results' with a table of values.

Valuation Results			
Par Cpn	1.548250	Premium	16.78921
Principal	167,892.11	BP Value	1678.92112
Accrued	0.00		
NPV	167,892.11		

Calculators		More Greeks	
PV01	486.40		
DV01	532.42		
Gamma (1bp)	0.29		

$$PV = 100 \times 1 \times 350 \times 5 = \text{USD } 170K$$

$$DV01 = 100 \times 1 \times 5 = \text{USD } 500$$

# Pricing Tricks – Asset Swaps

We Can Make the Same Annuity Assumption to Price Asset Swaps

## Par-Par Spread

$$S = \left( \frac{(r^{Fixed} - p^{Market})A^{Fixed} + \left(\frac{100 - B}{100}\right)}{A^{Float}} \right)$$

## IRS Rule of Thumb

$$s = (r - p) + (100 - B/100) / T$$

$$= \Delta r - (\Delta B / T)$$

where  $\Delta r = (r - p)$  in bps

and  $\Delta B = (B\% - 100\%)$  in bps

Asset Swap Calculator			
DBR 0 02/15/26	5 Actions	0 Settings	
Pricing	Cashflow	Relative Value	Deal Summary
Asset Swap Analysis	Calculate	Price	104.5800
Price -> ASW Spread	Z-Spread	-40.9	ASW Spread -40.6
	Yield(%)	0.02595	MMS Spread -41.2
Bond JV503423	Swap	Par-Par	Matched Maturity
Par Amount IMM	Leg 1: Fixed	Pay	Leg 2: Float Receive
Workout 02/15/2026	Notional	IMM	Notional IMM
Workout Price 100.0000	Currency	EUR	Currency EUR
Pay Freq Annual	Effective Date	01/15/2016	Effective Date 01/15/2016
Day Count ACT/ACT	Maturity Date	02/15/2026	Maturity Date 02/15/2026
	Coupon	0.5	Latest Index -0.112
	Pay/Reset Freq	Annual	Index EUR006M
	Day Count	ACT/ACT	Pay/Reset Freq SemiAnnual
			Day Count ACT/360
Implied Value 100.5736	Include Accrued		Include Accrued
Market	Discount Curve	133 Mid	Discount Curve 133 Mid
Curve Date 06/09/2016			Forward Curve 45 Mid
Settle Date 06/13/2016			
Swapped Spread Detail			
Clean Price	104.5800	Cash Out	4.5800
Swap Price	100.0000	Bond Cpn(%)	0.5000
Swap Rate(%)	0.44104		
Redemption(%)	0.0000		
Funding Spread(bp)	0.0		
Swapped Spread			-40.063.5
			-40.6

## Par-Par Spread

Here  $\Delta r = 0.50\% - 0.44\% = 6$  bps,  $\Delta B = 458$  bps and  $T = 10$

$$S = 6 - (458/10) \approx 6 - 46 = -40 \text{ bps}$$



# References

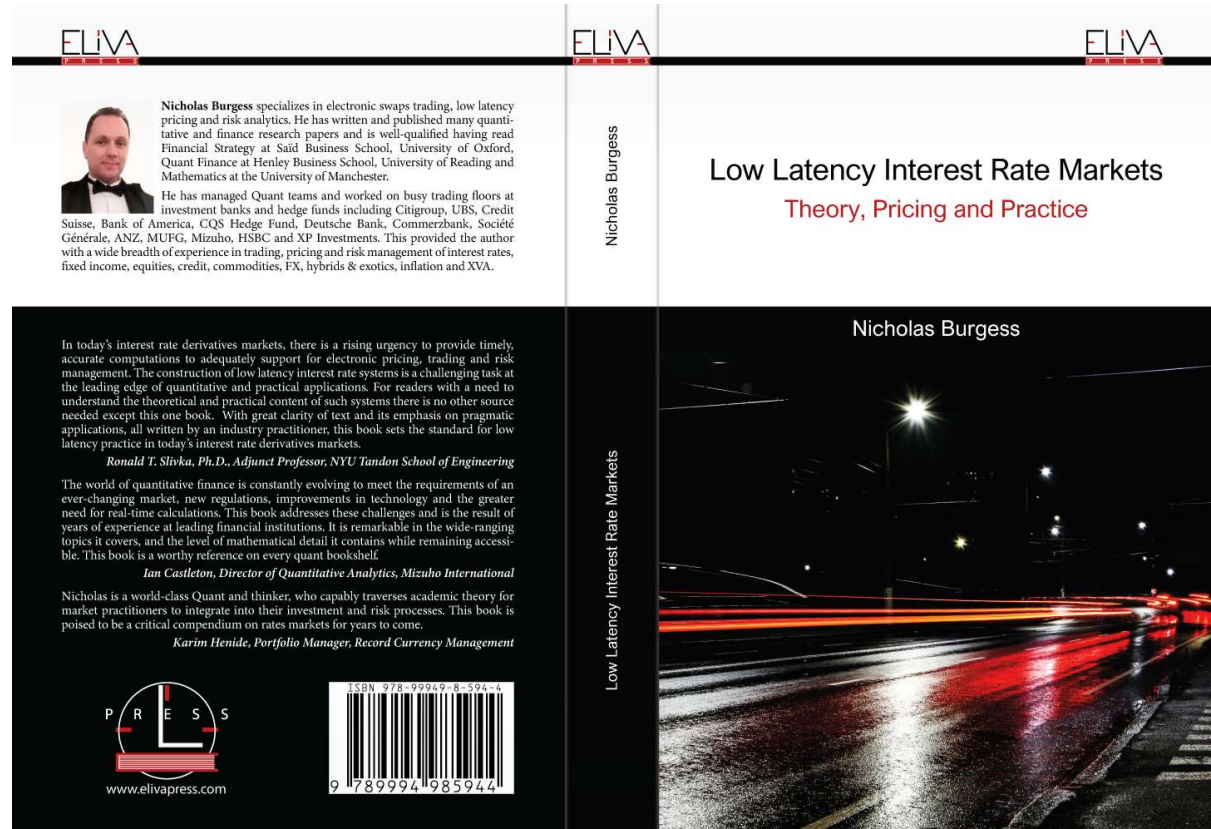


Quant Research Papers

<https://ssrn.com/author=1728976>

Support Materials, C++ & Excel Examples

<https://github.com/nburgessx/SwapsBook>



Available at Amazon: <https://amzn.eu/d/5B1bPII>

# Appendix – Implicit Function Theorem (IFT)

## IFT Theorem

To gain some intuition consider the following function  $f(x, y) = 0$  for which we have a solution  $(a, b)$ . Near the solution we can express  $y$  as function of  $x$  namely  $f(x, y(x)) = 0$ . Using this expression, we can compute the derivative in terms of  $x$  only by differentiating with respect to  $x$  as follows,

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial x} = 0$$

which gives,

$$\frac{\partial y}{\partial x} = - \left( \frac{\partial f / \partial x}{\partial f / \partial y} \right)$$

We have a solution under the condition,  $\partial f / \partial y \neq 0$ , since we cannot divide by zero.

## Yield Curve Application

In the context of a yield curve calibration, we solve for the solution of a helper target function,  $H(L, P) = 0$ , where  $L$  is the LIBOR forward rate state variable (model output) and  $P$  the yield curve par rate (model input). The helper target function computes the difference between model par rates as a function of the forward state variable  $L$  and a market instrument par rate quote,

$$H(L, P) = \text{Model Par Rate}(L) - \text{Market Par Rate}$$

## How does this Help with Sensitivity Calculations?

The IFT theorem says that having found a solution to the continuously differentiable function  $H(L, P) = 0$  in two variables we can express the solution solely in terms of the model output  $L$  namely  $H(L, P(L)) = 0$  and that the Jacobian derivative can be computed independent of model inputs i.e., the yield curve instruments and par rates as,

$$\frac{\partial P}{\partial L} = - \left( \frac{\partial H / \partial L}{\partial H / \partial P} \right)$$

Now, from the definition of the function  $H(L, P)$  we can easily determine  $dH/dP = -1$  which leads to,

$$\begin{aligned} \frac{\partial P}{\partial L} &= - \left( \frac{\partial H / \partial L}{\partial H / \partial P} \right) = \frac{\partial H}{\partial L} \\ &= \frac{d}{dL} (\text{Model Par Rate}) \end{aligned}$$

## For an Interest Rate Swap

$$\text{Par Rate}, p = \frac{PV(\text{Float Leg})}{\sum_{i=1}^n N \tau_i P(t_0, t_i)} = \frac{\sum_{j=1}^m N (l_{j-1} + s) \tau_j P(t_0, t_j)}{\text{Annuity}(\text{Fixed})}$$

- The derivative with respect to  $L$  is trivial to calculate
- We can calculate for any set of calibration instruments
- This allows us to modify and select any risk & hedge buckets

## Appendix – Swap DV01 Risk Example using AAD (Part I)

### IRS Present Value Code

- Swap Price Implementation
- Simplified for Demo Purposes
- For Full Example See

<https://bit.ly/SwapCodeAAD>

```
01 // Swap Inputs
02 // phi    Pay or Receive Fixed: Pay = 1, Receive = -1
03 // n      Swap Notional
04 // r      Fixed rate
05 // tau    Accrual year fraction
06 // t      Coupon Payment Time
07 // f      Floating Forward Rate
08 // s      Floating Spread
09 // z      Discounting Zero Rate for Discount Factor, where df = exp(-z*t)
10
11 double swap_pv(double phi, double n, double r, double tau, double t, double f, double s,
12 double z)
13 {
14     double df = exp(-z*t); // Step 1. Discount Factor using zero rate, z
15     double pv_fixed = phi*n*r*tau*df; // Step 2. Fixed PV =  $\phi N r \tau_1 P(0, t_1)$ 
16     double pv_float = -phi*n*(f+s)*tau*df; // Step 3. Float PV =  $\phi N (l_1 + s) \tau_1 P(0, t_1)$ 
17     double pv_swap = pv_fixed+pv_float; // Step 4. Swap PV = Fixed PV + Float PV
18     return pv_swap;
19 }
```

Swap Price

## Appendix – Swap DV01 Risk Example using AAD (Part II)

### Analytical DV01 Risk

- Using Adjoint Mode (AAD)
- Forward Sweep for Price
- Back Propagation for Risk
- Simultaneous Forward and Discount Risk

```

01 double adjoint(double phi, double n, double r, double tau, double t, double f, double s, double z,
02 double pv_bar)
03 {
04     // Forward Sweep
05     double df = exp(-z*t); // Step 1. Discount Factor using zero rate, z
06     double pv_fixed = phi*n*r*tau*df; // Step 2. Fixed PV =  $\phi N r \tau_1 P(0, t_1)$ 
07     double pv_float = -phi*n*(f+s)*tau*df; // Step 3. Float PV =  $\phi N (l_1 + s) \tau_1 P(0, t_1)$ 
08     double pv_swap = pv_fixed + pv_float; // Step 4. Swap PV = Fixed PV + Float PV
09     // Backward Propagation
10     double pv_fixed_bar = pv_bar; // Step 4.
11     double pv_float_bar = pv_bar; // Step 4.
12     double f_bar = -phi*n*tau*df*pv_float_bar*shift_size_f; // Step 3. *
13     double df_bar = -phi*n*f*tau*pv_float_bar*shift_size_df; // Step 3. *
14     df_bar += phi*n*r*tau*pv_fixed_bar*shift_size_df; // Step 2. *
15     double z_bar = -t*exp(-z*t)*df_bar; // Step 1.
16
17     // DV01 Result
18     return f_bar + df_bar; // Sensitivity to 1 bps change in forwards and discount factors
19 }

```

Swap DV01 using AD in Adjoint Mode

Source Code: <https://www.onlinegdb.com/edit/al8aNASJnQ>

```

01 // inputs( phi, n, r, tau, t, f, s, z, pv_bar )
02 adjoint( 1, 1000000, 0.02, 1, 1, 0.01, 0, 0.02, 1 ); // Output DV01 Risk

```

Swap DV01 Risk using Adjoint Mode

Have questions or want further info?

## Contact

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