Cross Currency Swap Trading & Pricing Formulae - A PowerPoint Overview with Excel Pricing Examples

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Executive Summary

Swap Preliminaries

- Interest Rate Swaps
- Yield Curves
- Rates Trading, Pricing & Risk

Xccy Swaps

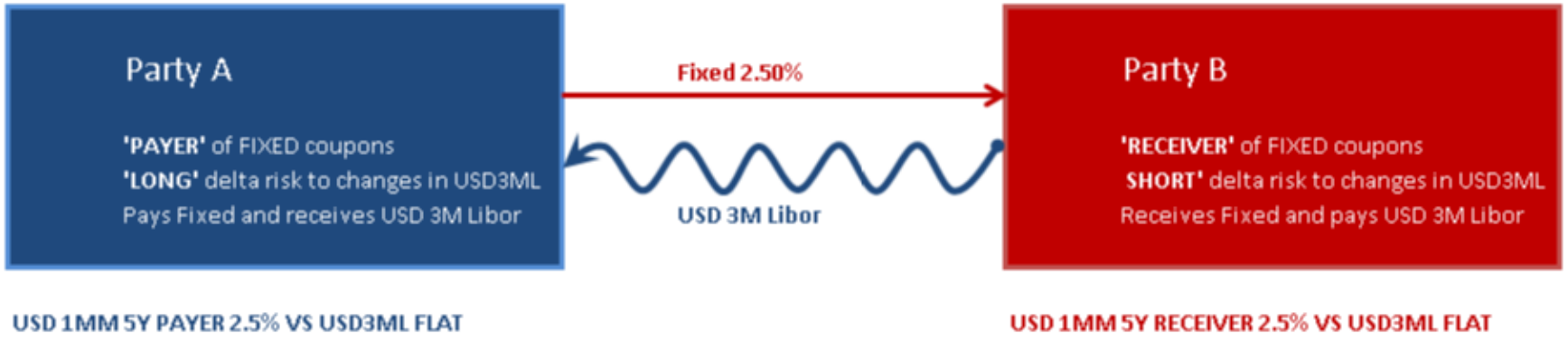
- Xccy Swap Theory - Formulae
- Xccy Swap Practice - Pricing Demo

Detailed Notes

https://ssrn.com/abstract=3278907

Interest Rate Swaps Market

- **Notional Outstanding $200tn**: Traded in large volume
- **Very Liquid** : Bid-Offer 1/10th Basis Point i.e. 0.001%
- **Swap Cashflows**: Each set of cashflows called the trade 'leg'
- **Typically Fixed-Float**: or Float-Float for Basis Swaps
- **Payer/Receiver**: of Fixed cashflows, but long/short the risk
- **Quoted at Par**: Zero upfront cost



Swap Pricing Terminology

Pricing Terminology

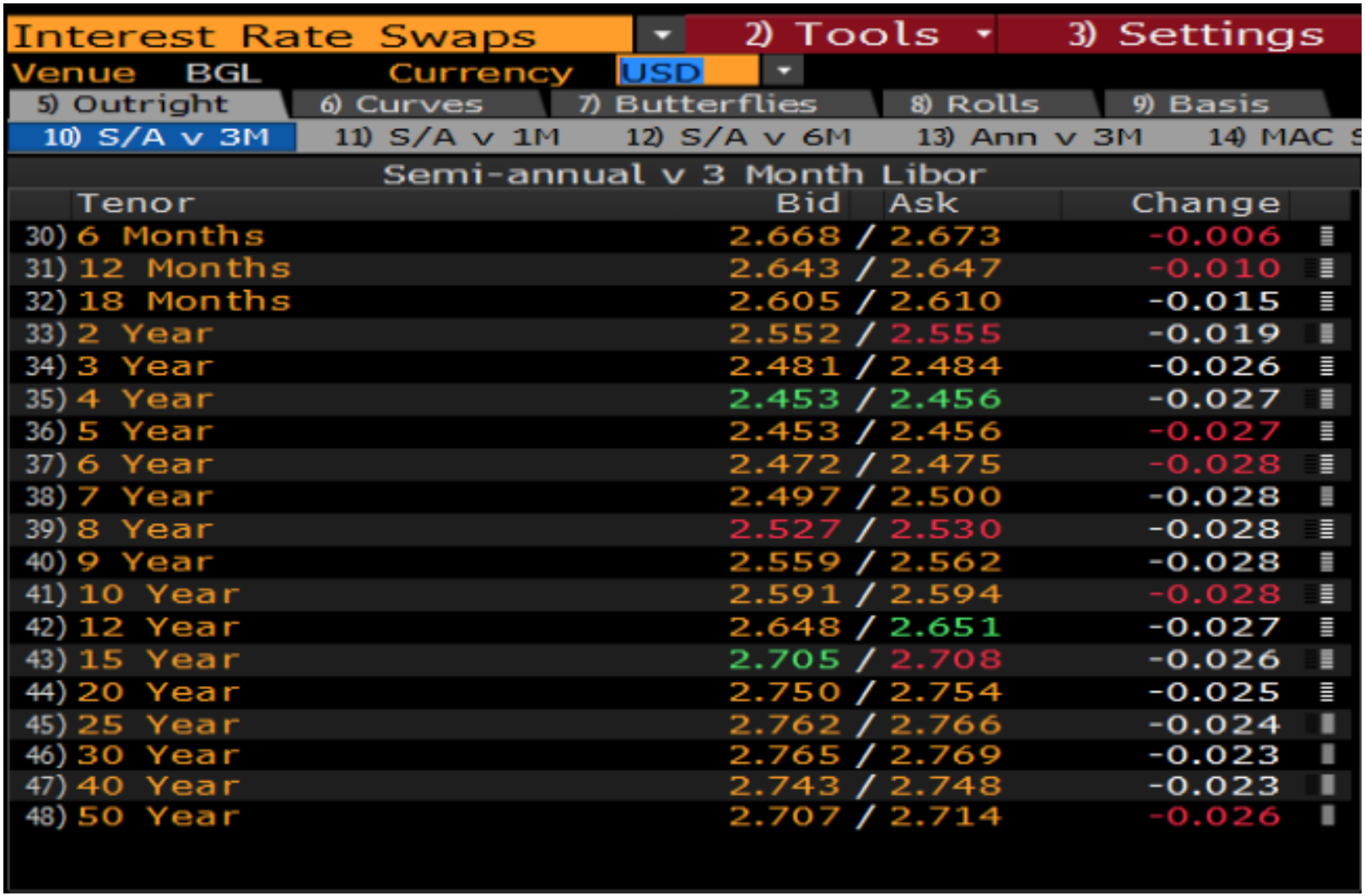
- **PV**: Present Value or Price
- **Basis Points (bps):** 1/100th of a percent i.e. 1bps = 0.01%
- **Par Rate**: The fixed rate in % to make the trade PV zero
- **Par Spread**: The float spread in bps to make trade PV zero
- **PV01**: PV sensitivity to forward rates, also called Annuity
- **DV01**: PV sensitivity to forward rates and discount factors

Quotation

- **New Deals**: Quoted as a par rate or a spread over Treasuries
- **Bespoke Deals**: Quoted as a PV, since not trading at par
- **Basis Trades**: Float-Float deals are quoted as a par spread

Bloomberg Trading Venue

Bloomberg Trading Portal, BBTI - Interest Rate Swaps



Swaps as a Spread over US Treasuries

Par Rate = US Treasury Yield + Spread (Bps)

Interest Rate Swap Pricing

Swap Specification & Pricing

To specify a swap many parameters are required to generate the swap cashflow schedules accurately. To price a swap we require Libor forecast rates, OIS discount rates and a Swap pricing formula.

$$PV^{Swap} = N \sum_{\forall i} r^{Fixed} \tau_i P(t_0, t_i) - N \sum_{\forall j} (L_j + s) \tau_j P(t_0, t_j)$$

Bloomberg Swap Manager, SWPM

Par Swap - 5Y Receiver vs USD3ML

IRS Pricing Formula

Fixed Leg

$$PV(Fixed) = N \times r^{Fixed} \underbrace{\sum_{\forall i} \tau_i P(t_0, t_i)}_{Annuity}$$

Float Leg

$$PV(Float) = N \sum_{\forall j} (L_j + s)\, \tau_j P(t_0, t_j)$$

Swap Price

$$PV(Swap) = \phi\,(PV(Fixed) - PV(Float))$$

Swap Rate

$$ParRate = \frac{PV(Float)}{N \times Annuity}$$

Other Swap Types

1. **Interest Rate Swaps** - Rate Hedging
2. **Tenor Basis Swaps** - Frequency Matching
3. **Xccy Swaps** - Funding in another Currency
4. **Asset Swaps** - Bond & Corporate Financing
5. **Credit Default Swaps** - Hedging Counterparty Risk
6. **Inflation Swaps** - Inflation Hedging

Libor Benchmark Rate Reform [New Swaps]

- Risk-Free Curves (RFRs)
- Alternative Reference Rates (ARRs)
- SOFR Swaps - Secured Overnight Funding Rate

Xccy Swap Overview

Xccy Basis Swaps

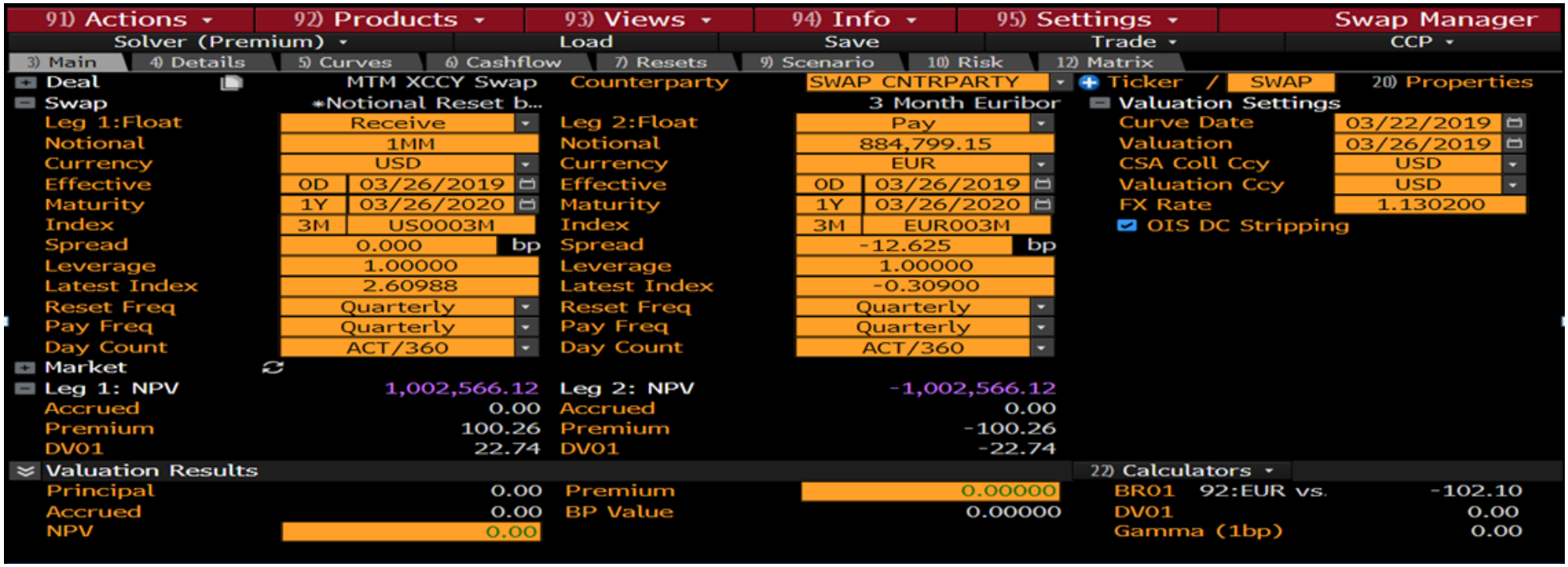
- Exchange a set of cashflows for an equivalent set in another currency
- Used to secure cheaper funding in a different currency, manage FX exposures and liquidity risk

Features

- Marked-to-Market (MTM)
- Notional Resets to Reduce Credit Exposures
- Choice of Valuation Currency
- Spot FX Required
- Multi-Currency Yield Curves Required
- OIS Discounting with Foreign CSA Collateral
- Xccy Par-Spread usually on Non-USD leg

Bloomberg MtM Xccy Swap USD/EUR

USD/EUR MtM Xccy Swap 1Y



Bloomberg Xccy Swap - USD Leg

USD/EUR MtM Xccy Swap 1Y - USD Leg

Pay Date	Accrual Start	Accrual End	Da...	FX Fixing Date	FX Rate	Notional	Principal	Reset Date
06/26/2019	03/26/2019	06/26/2019	92			1,000,000.00	-7,613.37	03/22/2019
09/26/2019	06/26/2019	09/26/2019	92	06/24/2019	0.87811	1,007,613.37	-7,662.23	06/24/2019
12/27/2019	09/26/2019	12/27/2019	92	09/24/2019	0.87149	1,015,275.60	-7,305.65	09/24/2019
03/26/2020	12/27/2019	03/26/2020	90	12/23/2019	0.86526	1,022,581.25	1,022,581.25	12/23/2019

Bloomberg Xccy Swap - EUR Leg

USD/EUR MtM Xccy Swap 1Y - EUR Leg

Yield Curves, Fwd Rates & Disc Factors

Curve Construction & Dependencies

1. First Calibrate Standard/Native CSA Curves
2. Then build USD CSA Curves using Xccy Swaps
3. Then build Non-USD CSA Curves using FX Fwd Invariance

CSA Discount Factors

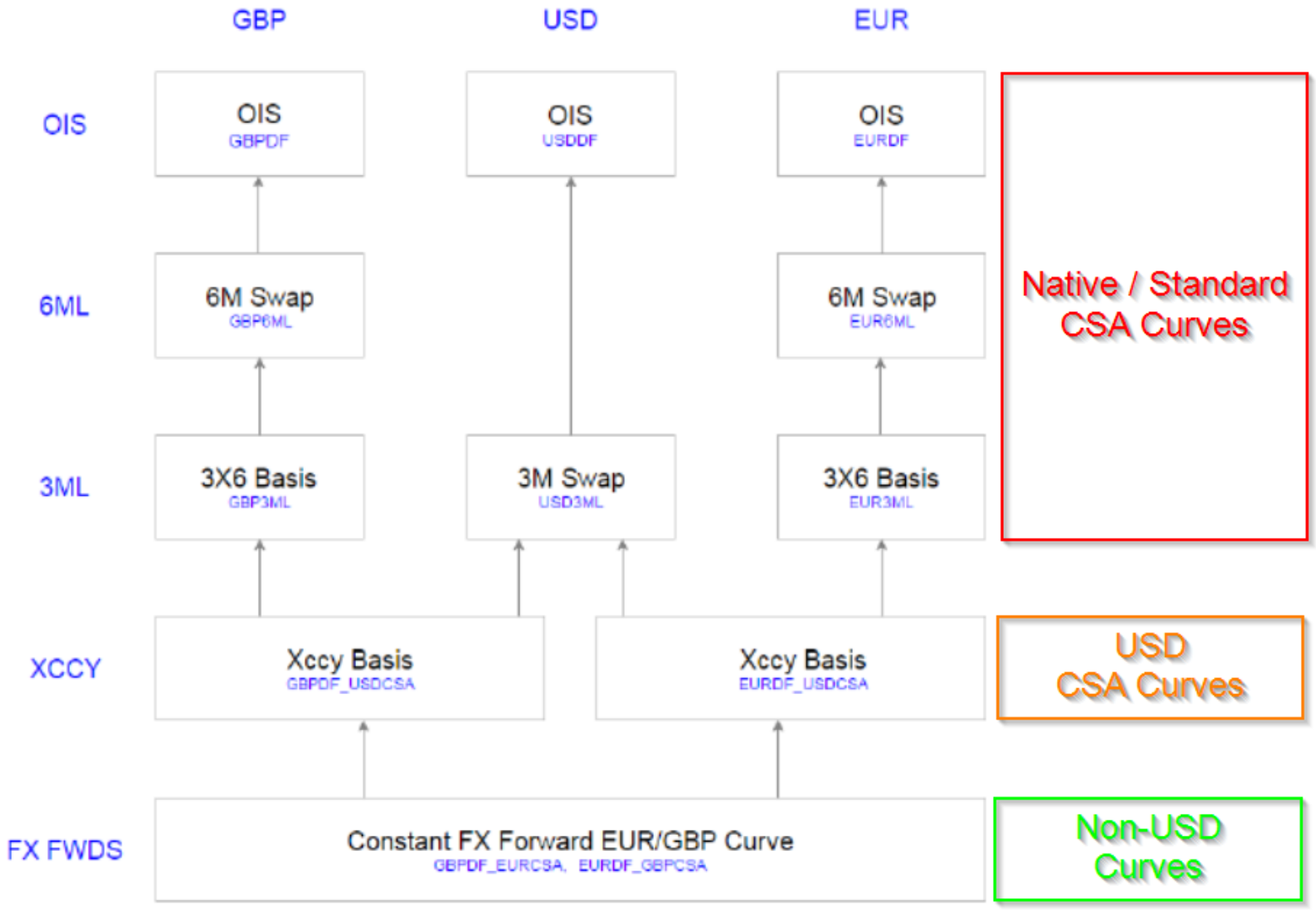
- Forward rates are independent of CSA
- Discount factors depend on CSA

Standard/Native CSA Disc Factors

- Native CSA discount factors same as No CSA
- Example: USDOIS_USDCSA = USDOIS

Curve Dependency Tree

Example: EURDF with GBP Collateral
EUROIS-GBPCSA



Xccy Curves - Discounting with Collateral

USD CSA Discount Factors

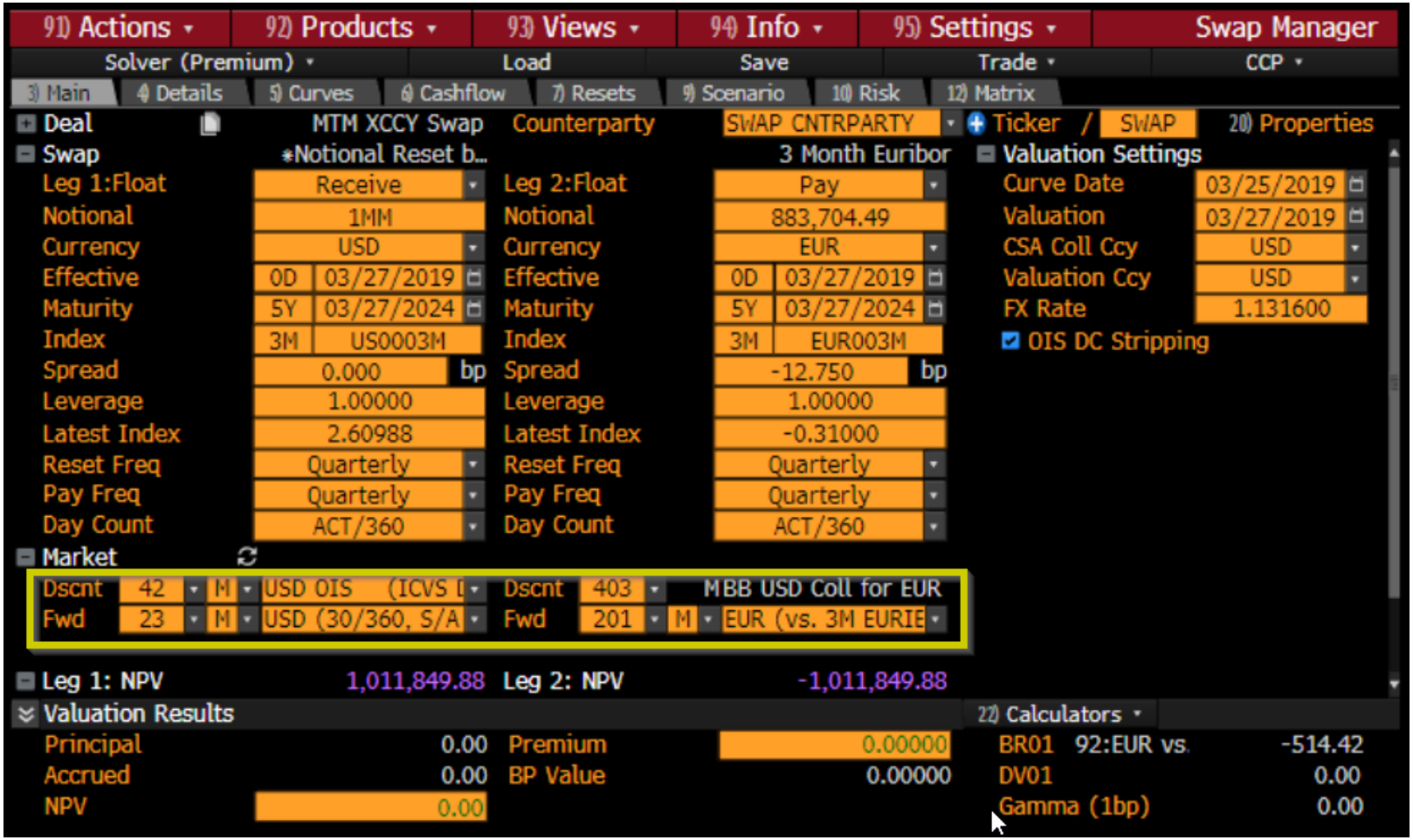
- Implied directly from market Xccy Swap Spreads
- Typically Xccy trades have a USD leg and post USD collateral

Example: USD/EUR Xccy

- We **know**: Par Spread, USDOIS, USD3ML and EURIBOR3M
- from which we **imply** EUROIS-USDCSA

Xccy Trade - Curves & DF Requirements

SWPM -FLFL -XCCY -MTM -USD 1MM 5Y



Non-USD CSA Curves

FX Forward Invariance

For Non-USD CSA calculations we assume FX forward invariance

FX Forward Invariance Example for EUR_JPYCSA

$$FwdFX(t,T)^{EUR/JPY} = \underbrace{S\frac{P(t,T)^{EUR\_JPYCSA}}{P(t,T)^{JPY\_JPYCSA}}}_{JPY\_JPYCSA \text{ from OIS Curve}} = \underbrace{S\frac{P(t,T)^{EUR\_USDCSA}}{P(t,T)^{JPY\_USDCSA}}}_{\text{From Xccy Curve}}$$

where t denotes the valuation date, S the FX spot rate, P(t,T) the discount factor at time t for tenor T with $0 \leq t \leq T$

Non-USD CSA Discount Factors

- Use LHS to imply from using FX Forwards
- Use RHS to imply from Xccy Basis quotes

Replication Logic

- We can create a synthetic EUR/JPY forward FX rate
- Borrow JPY, buy spot EUR/JPY and deposit the EUR
- The forward FX rate is constant for any given CSA

Xccy Swap Pricing Formula

FOR/DOM Xccy Price

$$PV(\Omega_{Xccy}) = \phi\Big[PV(\Omega_{FOR}) - PV(\Omega_{DOM})\Big]$$

$$\begin{aligned} PV(\Omega_{Xccy}) = \phi\Big[&PV\Big(\text{Cpn}, \Omega_{FOR}\Big) + PV\Big(\text{Exch}, \Omega_{FOR}\Big) + PV\Big(\text{Resets}, \Omega_{FOR}\Big) \\ &- PV\Big(\text{Cpn}, \Omega_{DOM}\Big) - PV\Big(\text{Exch}, \Omega_{DOM}\Big) - PV\Big(\text{Resets}, \Omega_{DOM}\Big)\Big] \end{aligned}$$

Example: USD/EUR MtM Xccy Swap

- Valuation Currency: USD
- Collateral Currency: USD
- Reset Currency: USD
- Quotation: EUR Par-Spread
- EUR Leg: 3M EURIBOR with EUROIS-USDCSA discounting
- USD Leg: 3M USD Libor with USDOIS discounting

Xccy Swap Pricing Formula [Expanded]

$$
\begin{aligned}
PV\left(\Omega_{Xccy}\right) = \phi\Big[& \underbrace{\sum_{j=1}^{m} N_{t_0}^{FOR}\Psi(t_j)^{FOR}(l_j + s_{FOR})\tau_j P(0,t_j)^{FOR-CSA}}_{\textit{Foreign Float Coupons}} \\
& + \Big(\underbrace{N_{t_0}^{FOR}\Psi(t_m)^{FOR}P(0,t_m)^{FOR-CSA}}_{\textit{Foreign Final Exchange}} - \underbrace{N_{t_0}^{FOR}\Psi(t_0)^{FOR}P(0,t_0)^{FOR-CSA}}_{\textit{Foreign Upfront Exchange}} \Big) \\
& + \mathbb{1}_{\{\Omega=MtM\}}\mathbb{1}_{\{FOR=C^{Reset}\}} \underbrace{\sum_{j=1}^{m-1} N_{t_0}^{FOR}\Big(\Psi(t_j)^{FOR} - \Psi(t_{j+1})^{FOR}\Big)P(0,t_j)^{FOR-CSA}}_{\textit{Foreign Notional Resets}} \\
& - \underbrace{\sum_{j=1}^{m} N_{t_0}^{DOM}\Psi(t_j)^{DOM}(l_j + s_{DOM})\tau_j P(0,t_j)^{DOM-CSA}}_{\textit{Domestic Float Coupons}} \\
& - \Big(\underbrace{N_{t_0}^{DOM}\Psi(t_m)^{DOM}P(0,t_m)^{DOM-CSA}}_{\textit{Domestic Final Exchange}} - \underbrace{N_{t_0}^{DOM}\Psi(t_0)^{DOM}P(0,t_0)^{DOM-CSA}}_{\textit{Domestic Upfront Exchange}} \Big) \\
& - \mathbb{1}_{\{\Omega=MtM\}}\mathbb{1}_{\{DOM=C^{Reset}\}} \underbrace{\sum_{j=1}^{m-1} N_{t_0}^{DOM}\Big(\Psi(t_j)^{DOM} - \Psi(t_{j+1})^{DOM}\Big)P(0,t_j)^{DOM-CSA}}_{\textit{Domestic Notional Resets}} \Big]
\end{aligned}
$$

Xccy Features: MtM

Marked-to-Market

- Xccy swaps present users with FX risk
- The MtM feature mitigates this
- MtM Xccy Swaps track and reset the FX rate each period
- FX losses on the Notional reimbursed on each FX fixing date

Understanding the Notional

Xccy Coupon Notional

We always scale the **initial** notional by Ψ_i

$$N_i = N_0 \Psi_i$$

Notional Reset Factor

$$\Psi_i = \underbrace{\alpha(t_0, C^{Leg})}_{\textit{Valuation Adj}} \underbrace{\beta(t_i, C^{Leg})}_{\textit{FX Reset Adj}}$$

Spot FX Valuation Adjustment, α

$$\alpha(t, C^{Leg}) = \begin{cases} 1 & , \textit{if } C^{Leg} = C^{Val} \\ s^{FOR/DOM} & , \textit{if } C^{Leg} \neq C^{Val} \textit{ and } C^{Leg} = C^{FOR} \\ s^{DOM/FOR} & , \textit{if } C^{Leg} \neq C^{Val} \textit{ and } C^{Leg} = C^{DOM} \end{cases}$$

Forward FX MtM Reset Adjustment, β

$$\beta(t, C^{Leg}) = \begin{cases} 1 & , \textit{if } C^{Leg} \neq C^{Reset} \\ \left(\frac{f(t)^{FOR/DOM}}{s^{FOR/DOM}}\right) & , \textit{if } C^{Leg} = C^{Reset} \textit{ and } C^{Leg} = C^{FOR} \\ \left(\frac{f(t)^{DOM/FOR}}{s^{DOM/FOR}}\right) & , \textit{if } C^{Leg} = C^{Reset} \textit{ and } C^{Leg} = C^{DOM} \end{cases}$$

Notional Resets & Exchanges

Initial & Final Exchanges

Borrow funds at start and return at end

$$\text{Initial Exchange} = N_0 \Psi_0 P(0, t_0)$$

$$\text{Final Exchange} = -N_0 \Psi_n P(0, t_n)$$

Notional FX Resets

FX immunized: FX losses reimbursed each FX fixing date

$$\begin{aligned} \text{FX Reset} &= N_i P(0, t_i) - N_{i+1} P(0, t_i) \\ &= N_0 \Psi_i P(0, t_i) - N_0 \Psi_{i+1} P(0, t_i) \end{aligned}$$

Coupon Pricing

Floating Cashflows

$$
\begin{aligned}
PV(Coupon) &= \sum_{i=1}^{n} N_0 \Psi_i (l_i + s) \tau_i P(0, t_i) \\
&= \sum_{i=1}^{n} N_0 \Psi_i l_i \tau_i P(0, t_i) + s \sum_{i=1}^{n} N_0 \Psi_i \tau_i P(0, t_i) \\
&= \sum_{i=1}^{n} N_0 \Psi_i l_i \tau_i P(0, t_i) + s \text{ Annuity}
\end{aligned}
$$

where

$$
Annuity = \sum_{i=1}^{n} N_0 \Psi_i \tau_i P(0, t_i)
$$

Xccy Par Spread

Par Spread, s

Applied to the Non-USD Leg

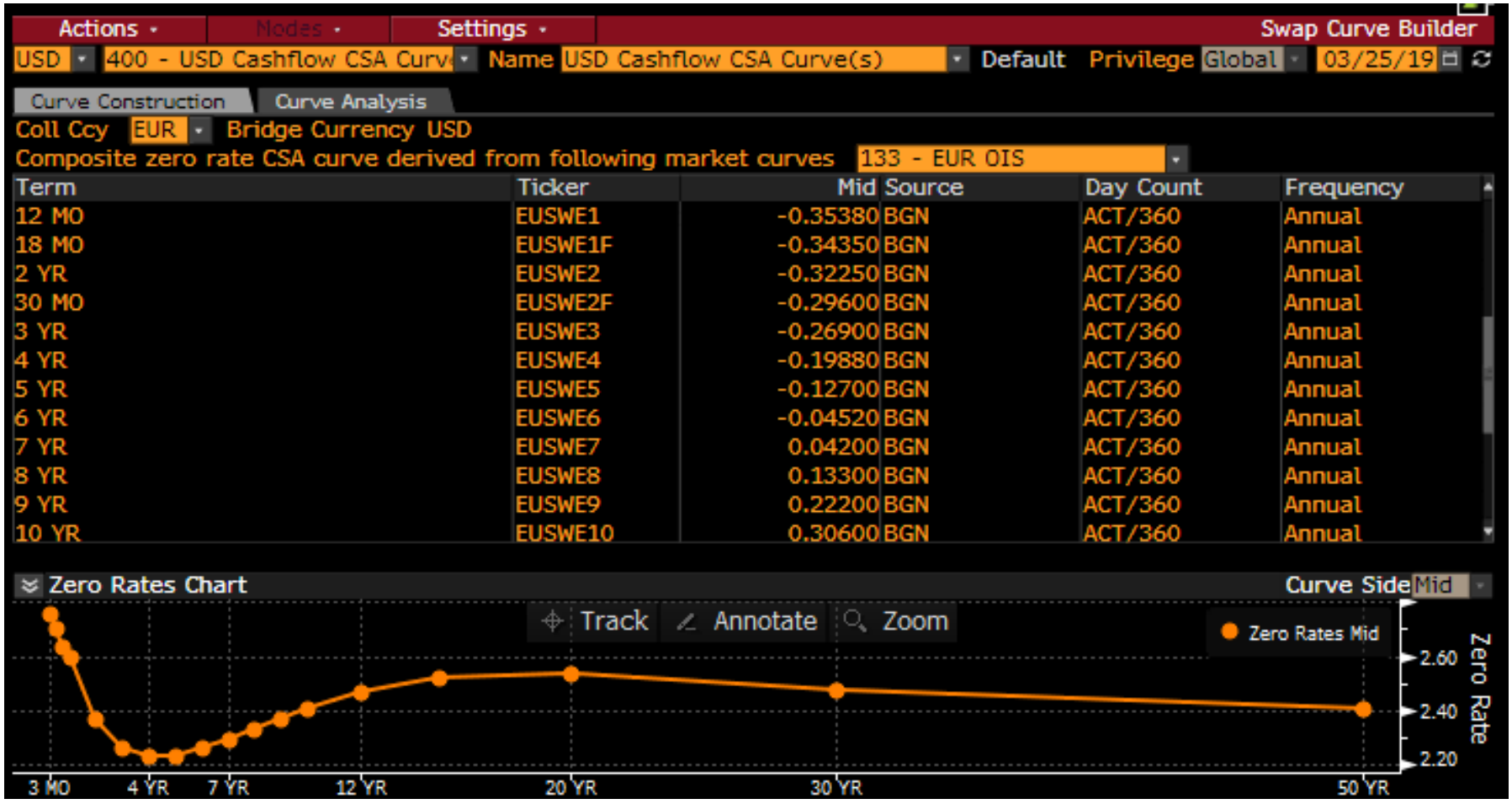
$$s = -\left(\frac{\text{PV(Trade with No Spread)}}{\text{Annuity(Non-USD Leg)}} \right)$$

where

$$Annuity = \sum_{i=1}^{n} N_0 \Psi_i \tau_i P(0, t_i)$$

Bloomberg: Xccy Quotes

Bloomberg Par Rates



Xccy Pricing Demo Workbook

Demo Workbook:

5Y USD/EUR MTM XCCY SWAP USD 1MM

Cross Currency Swap, Ω_{Xccy}		
TradeDate	Fri, 26-Oct-18	
Maturity (Years)	5Y	Wed, 25-Oct-23
Trade Notional	1,000,000	
Trade Currency	USD	
MtM	YES	
NotionalExchanges	YES	
Reset Currency	USD	USD
CSA Currency	USD	
Valuation Currency	USD	
SpotFX	1.14030	USD/EUR
LegCurrency	EUR	USD
LegNotional	876,962	1,000,000
PayOrReceive	PAY	RECEIVE
LegType	FLOATING	FLOATING
RateOrSpread (%)	0.00000%	0.00000%
FloatIndex	EUR EURIBOR 3M	USD LIBOR 3M
Frequency	QUARTERLY	QUARTERLY
LegResetsRequired	NO	YES
LegSpotFX	0.87696	1.14030
ValuationFXAdj	1.14030	1.00000
DaycountBasis	ACT/360	ACT/360
UseMarketSchedule	NO	NO

Prices / ParSpreads	LEG1: EUR	LEG2: USD
LegPV	-24,992	15,686
SwapPV	-9,306	USD

Leg1 - EUR Cashflows

	Notional	FXFixingDate	ForwardFX	NotionalExchange	Spread	FloatRate	Coupon	DiscountFactor	CouponPV	SpotFX	ValuationPV
0				876,962			876,962	1.000000	876,962	1.1403	1,000,000
1	-876,962	Fri, 26-Oct-18	1.00000	0	0.00000%	-0.31695%	703	1.002365	704	1.1403	803
2	-876,962	Fri, 25-Jan-19	1.00000	0	0.00000%	-0.31644%	701	1.004182	704	1.1403	803
3	-876,962	Fri, 26-Apr-19	1.00000	0	0.00000%	-0.28931%	641	1.005926	645	1.1403	736
4	-876,962	Fri, 26-Jul-19	1.00000	0	0.00000%	-0.22709%	503	1.007807	507	1.1403	579
5	-876,962	Sat, 26-Oct-19	1.00000	0	0.00000%	-0.13634%	302	1.009467	305	1.1403	348
6	-876,962	Sat, 25-Jan-20	1.00000	0	0.00000%	-0.05021%	111	1.010835	113	1.1403	128
7	-876,962	Sat, 25-Apr-20	1.00000	0	0.00000%	0.02216%	-49	1.011997	-50	1.1403	-57
8	-876,962	Sat, 25-Jul-20	1.00000	0	0.00000%	0.08249%	-183	1.013047	-185	1.1403	-211
9	-876,962	Sun, 25-Oct-20	1.00000	0	0.00000%	0.13501%	-299	1.013962	-303	1.1403	-346
10	-876,962	Sun, 24-Jan-21	1.00000	0	0.00000%	0.19845%	-440	1.014734	-446	1.1403	-509
11	-876,962	Sun, 25-Apr-21	1.00000	0	0.00000%	0.27912%	-619	1.015295	-628	1.1403	-716
12	-876,962	Sun, 25-Jul-21	1.00000	0	0.00000%	0.37754%	-837	1.015577	-850	1.1403	-969
13	-876,962	Mon, 25-Oct-21	1.00000	0	0.00000%	0.48748%	-1,081	1.015536	-1,097	1.1403	-1,251
14	-876,962	Mon, 24-Jan-22	1.00000	0	0.00000%	0.58832%	-1,304	1.015210	-1,324	1.1403	-1,510
15	-876,962	Mon, 25-Apr-22	1.00000	0	0.00000%	0.67584%	-1,498	1.014663	-1,520	1.1403	-1,733
16	-876,962	Mon, 25-Jul-22	1.00000	0	0.00000%	0.74980%	-1,662	1.013957	-1,685	1.1403	-1,922
17	-876,962	Tue, 25-Oct-22	1.00000	0	0.00000%	0.81171%	-1,799	1.013132	-1,823	1.1403	-2,079
18	-876,962	Tue, 24-Jan-23	1.00000	0	0.00000%	0.87156%	-1,932	1.012214	-1,956	1.1403	-2,230
19	-876,962	Tue, 25-Apr-23	1.00000	0	0.00000%	0.93160%	-2,065	1.011157	-2,088	1.1403	-2,381
20	-876,962	Tue, 25-Jul-23	1.00000	-876,962	0.00000%	0.99282%	-879,163	1.009939	-887,901	1.1403	-1,012,474

Leg2 - USD Cashflows

	Notional	FXFixingDate	ForwardFX	NotionalExchange	Spread	FloatRate	Coupon	DiscountFactor	CouponPV	SpotFX	ValuationPV
0				-1,000,000			-1,000,000	1.000000	-1,000,000	1.0000	-1,000,000

Appendix - Useful Resources

References

1. Collateralization & CSA Fundamentals
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