Humans and animals are able to make decisions when they faced a binary alternative. In this section, we wille model decision making using sequential probability ratio test (SPRT). This model can be used in a random dot motion task (see here for an example). In this paradigm, a patch of points moving either on average to the left or to the right is shown to the subject that has to determine the direction of movement. Subject's goal is to determine the direction of the dots which can be easy or though depending on the coherence between dots.

In this tutorial, we consider a simplified version of the random dot motion task. On each trial i, the subject is shown a single dot moving at velocity v_i generated by a fixed probability distribution, which we know to be either:

$$
\begin{aligned}
p_L &= \mathcal{N}\left(-1, \sigma^2\right) && (1)\\
&\text{or} && (2)\\
p_R &= \mathcal{N}\left(+1, \sigma^2\right) && (3)\\
& && (4)
\end{aligned}
$$

This means that the dot is moving leftward or rightward and that its speed is normally distributed around $|1|$. We want to determine which distribution amongst p_L and p_R is the true data generating distribution. In order to do that, we will define two alternative hypotheses, the first one H_L states that p_L is the data generating distribution while H_R states that it is p_R. The decision process is based on the time-evolution of the log likelihood ratio between these two hypotheses. At every time step, we will define the likelihood functions for both hypotheses. These functions quantify how probable it is that the data point x_i is generated from a given distribution. For a given occurence of the point x_i, the two likelihood functions will be defined by $p_L\left(x_i|z=0\right)$ and $p_R\left(x_i|z=1\right)$, which are two gaussian distributions.

Using the following gaussian observations models

$$
\begin{aligned}
p_L\left(x|z=0\right) &= \mathcal{N}\left(\mu_L, \sigma_L^2\right) && (5)\\
p_R\left(x|z=1\right) &= \mathcal{N}\left(\mu_R, \sigma_R^2\right) && (6)\\
& && (7)
\end{aligned}
$$

and the definition of the log-likelihood ratio

Compute the expression of $\log \Lambda_i$, the log-likelihood ratio at iteration i

Without loss of generality, let's further assume the true data generating distribution is p_R. In this case x_i can be expressed as $x_i = \mu_R + \sigma_R \epsilon$ where ϵ comes from a standard Gaussian. The foregoing formula can then be rewritten as

$$
\log \Lambda_i = \left(\log \frac{\sigma_L}{\sigma_R} + 0.5 \frac{\left(\mu_R - \mu_L\right)^2}{\sigma_L^2} \right) + \left(\frac{\mu_L - \mu_R}{\sigma_L^2} \sigma_R \, \epsilon - 0.5 \left[1 - \left(\frac{\sigma_R}{\sigma_L} \right)^2 \right] \epsilon^2 \right)
$$

Where the first two constant terms serve as the drifting part and the last terms are the diffusion part. If we further let $\sigma_L = \sigma_R$, we can get rid of the quadratic term and this reduces to the classical deiscrete drift-diffusion equation where we have analytical solutions for mean and expected auto-covariance:

$$\log \Lambda_i = 0.5 \frac{(\mu_R - \mu_L)^2}{\sigma_L^2} + \frac{\mu_R - \mu_L}{\sigma_L^2} \epsilon, \quad \text{where } \epsilon \sim \mathcal{N}(0, 1)$$