Path Integral Quantum Annealing (PIQA)

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Overview

- Introduction
- Encoding Optimisation Problems
- Simulated Annealing
  - Preliminary Results
- Path Integral Quantum Annealing
- Conclusions & Future Work

Introduction

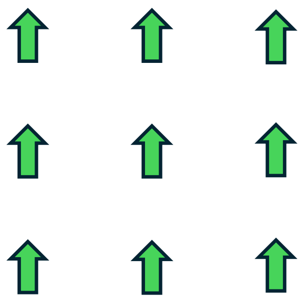
- Real-world optimisation problems are often too complex for brute-force approach
- Quantum annealing (QA) is a **heuristic search**[1] approach to solving such problems, resembling classical simulated annealing (SA)
- *Objective*: to compare speed-up of QA (specifically, PIQA) over SA

> ***Heuristic search*** – a problem-generic solution strategy for solving NP-hard optimisation problems when exact solutions are impractical

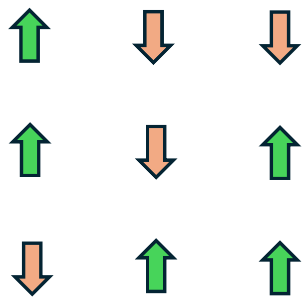
[1] *McGeoch, Adiabatic Quantum Computation and Quantum Annealing: Theory and Practice. Springer Nature, 2022.*

Encoding Optimisation Problems

- A method of encoding combinatorial optimisation (CO) problems is required prior to solving



Ferromagnet



Spin-glass

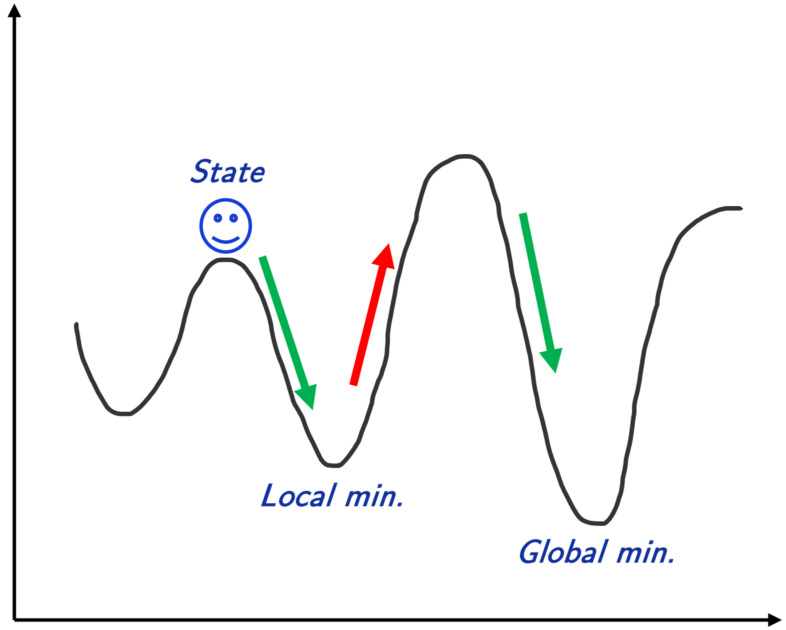
Classical Spin-glass Ising Hamiltonian[2]

$$\mathcal{H}_{Ising} = -\sum_{\langle i,j \rangle} J_{ij} s_i s_j - \sum_i h_i s_i$$

[2] *Nishimori, Statistical Physics of Spin Glasses and Information Processing. Oxford University Press, 2001.*

Simulated Annealing

- Start with random spin configuration
- Prepare new state with random spin flipped
- Accept or reject new state based on the **Metropolis criterion**
- Gradually lower temperature[3]



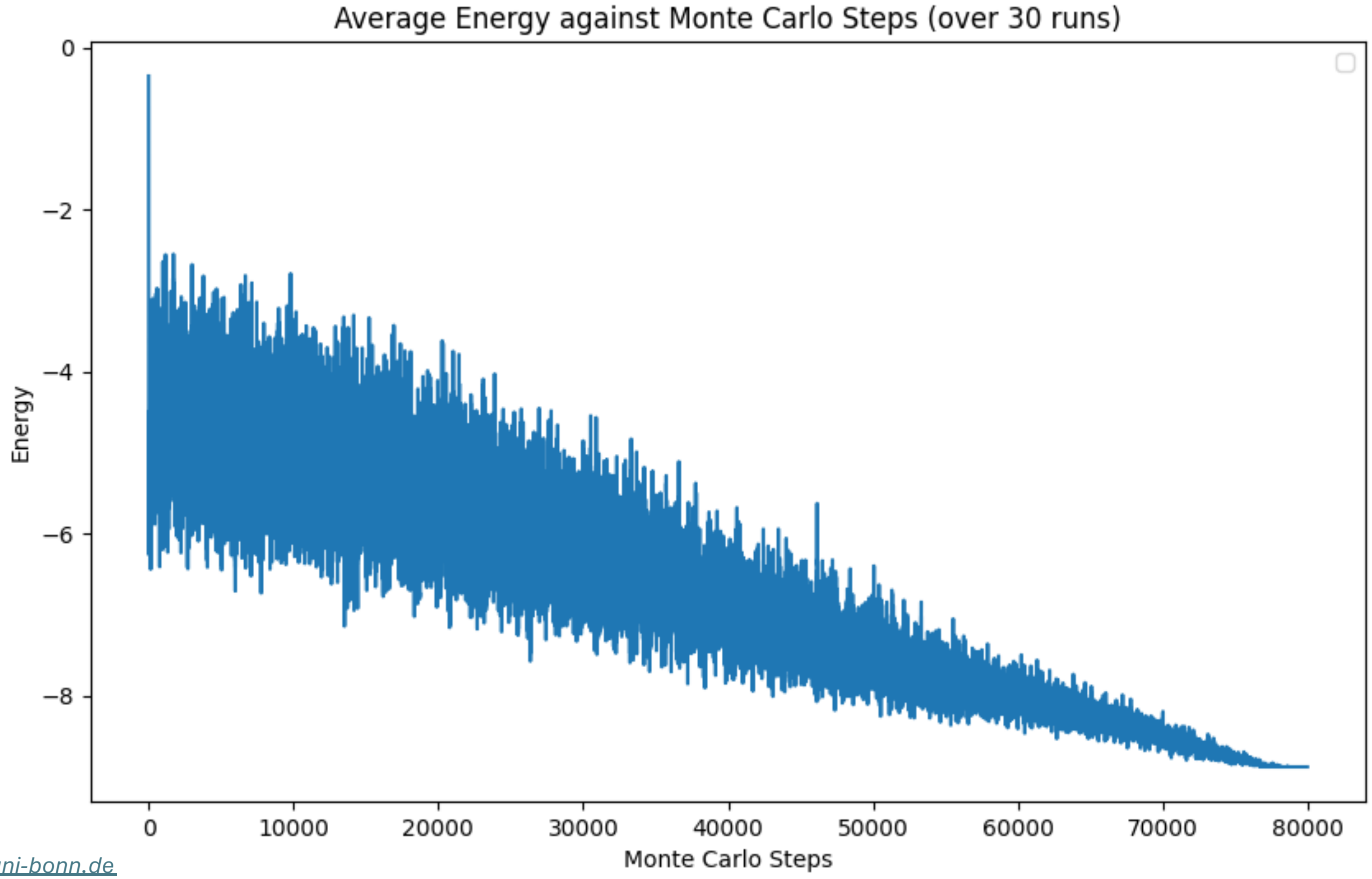
Metropolis Criterion

$$\mathcal{P}(accept) = e^{\frac{-\Delta E}{T_t}}$$

[3] *Kirkpatrick et al., Science, 1983.*

Preliminary Results

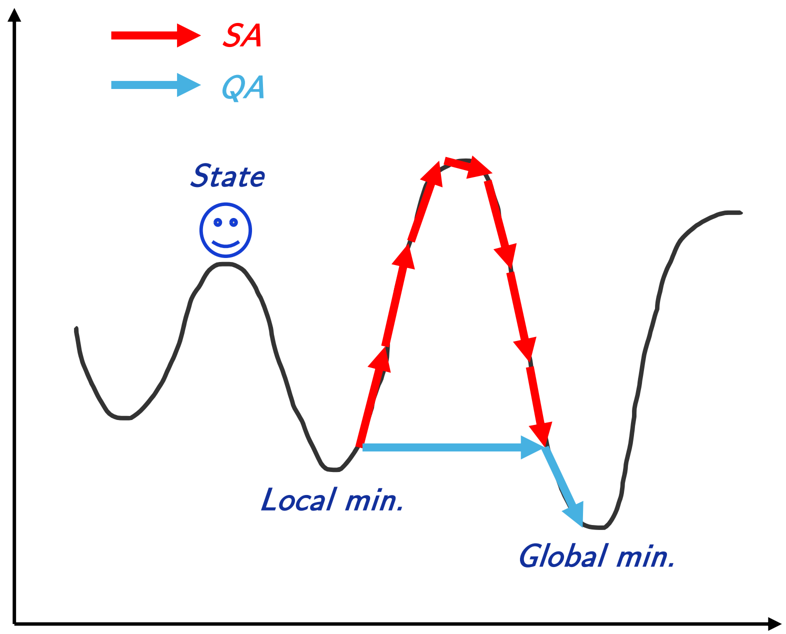
- SA performed and verified for a small 9-spin instance[4]



[4] *Spin-glass server*, http://spinglass.uni-bonn.de

Path Integral Quantum Annealing

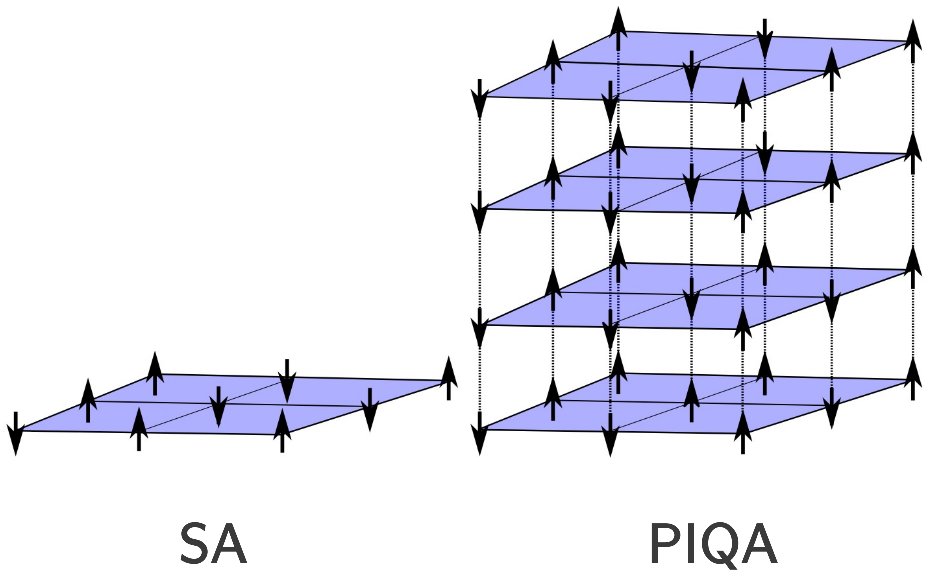
- QA leverages **quantum tunnelling** to escape local minima
- PIQA approximates QA using **Trotter decomposition**, effectively discretising the quantum system[5]



> ***Trotter decomposition*** – Approximates the *d*-dimensional quantum system onto a (*d+1*)-dimensional classical system with multiple "Trotter slices" evolving in imaginary time

[5] *Battaglia et al., Phys. Rev. E, 2005.*

Path Integral Quantum Annealing (ctd.)



Effective PIQA Hamiltonian and Coupling Term[6]

$$\mathcal{H}_{PIQA} = -\sum_{k}^{P}\left(\sum_{i,j} J_{ij} s_i^k s_j^k + J_\perp \sum_i s_i^k s_i^{k+1}\right) \qquad J_\perp = -\frac{PT}{2}\ln\tanh\left(\frac{\Gamma}{PT}\right)$$

[6] *Martoňák et al., Phys. Rev. B, 2002.*
Figure: Heim et al., Science, 2014.

Conclusions & Future Work

- SA and QA both aim to solve **optimisation problems** encoded into spin-glass Ising Hamiltonians
- SA may struggle to converge to a solution due to **local minima**
- QA can perform **quantum tunnelling**, but requires fine tuning of parameters
- PIQA implements QA numerically using a **Monte Carlo approach**
- *Future Directions:* Investigate larger-scale problems, parameter selection