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Part 1 “Resolution”

This is five sentences with the question written in natural language.

1. All cats eat dry food.
2. Anyone who has any dog will not have any bird.
3. Allergic people do not have anything that eat dry food.
4. Rima has either a dog or a cat.
5. If Rima is allergic, then Rima does not have any bird. (This is what will be proved by resolution).
6. Rima is allergic.

a- Convert the facts into First Order Logic “FOL”.

1. $\forall$ x (cat(x) $\rightarrow$ eat(x))
2. $\forall$ x $\forall$ y (have(x,y) $\wedge$ dog(y) $\rightarrow \neg \exists$ z (have(x,z) $\wedge$ bird (z)))
3. $\forall$ x (al(x) $\rightarrow \neg \exists$ y (have (x,y) $\wedge$ eat(y)))
4. $\exists$ x (have (rima,x) $\wedge$ (dog(x) $\vee$ cat(x)))
5. al(rima) $\rightarrow \neg \exists$ z (have(rima,z) $\wedge$ bird(z))
6. al(rima).

Convert the FOL into Prenex Normal Form “PNF”, conjunctive normal form “CNF”, and doing the Skolemization process “removing the existential quantifier”.

1. $\neg$ cat(x) $\vee$ eat(x)
2. $\neg$ have(x,y) $\vee \neg$ dog(y) $\vee \neg$ have(x,z) $\vee \neg$ bird(z)
3. $\neg$ al(x) $\vee \neg$ have(x,y) $\vee \neg$ eat(y)
4. have(rima,a) $\wedge$ (cat(a) $\vee$ dog(a))
5. $\neg$ al(rima) $\vee \neg$ (have(rima,z) $\wedge$ bird(z))
6. al(rima).

“5” After negation it will be

- $\neg$ ($\neg$ al(rima) $\vee \neg$ (have(rima,z) $\wedge$ bird(z)))
- al(rima) $\wedge$ have(rima,b) $\wedge$ bird(b)

b- Proving manually that the question is logically entailed from knowledge base by applying Resolution.

This is the Knowledgebase.

1) $\neg$ cat(X) $\vee$ eat(X)
2) $\neg$ have(Y,B) $\vee \neg$ dog(B) $\vee \neg$ have(Y,Z) $\vee \neg$ bird(Z)
3) $\neg$ al(Y) $\vee \neg$ have(Y,X) $\vee \neg$ eat(X)
4) have(rima,A)

5) (cat(A) ∨ dog(A))
6) al(rima)
7) have(rima,Z)
8) bird(Z)
9) al(rima)

I negate the question to prove it at the end.

[¬ have(rima,Z)]

[¬ have(Y,B) ∨ ¬ dog(B) ∨ ¬ have(Y,Z) ∨ ¬ bird(Z)]

(rima,Y) (Z,B)

[¬ have(rima,Z) ∨ ¬ dog(Z) ∨ ¬ bird(Z)]

[have(rima,A)]

(A/Z)

¬ dog(A) ∨ ¬ bird(A)

(cat(A) ∨ dog(A))

¬ bird(A) ∨ cat(A)

Bird(Z)

(Z/A)

cat(Z)

¬ cat(X) ∨ eat(X)

(Z/X)

eat(Z)

¬ al(Y) ∨ ¬ have(Y,X) ∨ ¬ eat(X)

(Z/X)

¬ al(Y) ∨ ¬ have(Y,Z)

al(rima)

(rima,Y)

¬ have(rima,Z)

have(rima,Z)

[ ]

c- Proving automatically that the question is logically entailed from knowledge base by applying Resolution.

Here I explained how I wrote the knowledge base in **SWI Prolog**.

animal(dog).

animal(cat).

animal(bird).

eat(cat,dryfood).

al(rima).

neg(have(al,cat)).

have(X,bird) :- neg(have(X,dog)).

have(rima,bird) :- neg(al(rima)).

have(rima,dog) :- al(rima).

I asked Prolog if have(rima,bird). ? and the answer is False. Of course because Rima has allergic from birds.

d- Using implementation of the Resolution for the propositional case, for the following sets of propositional clauses, written in CNF.

I. **[[¬a,b],[c,d],[¬d,b],[¬b],[¬c,b]]**

[c,d]	[¬d,b]
[c,b]	[¬b]
[c]	[¬c,b]
[b]	[¬a,b]
[¬a,b]	

II. **[[¬b,a],[¬a,b,e],[a, ¬e],[¬a],],[e]]**

[a,¬e]	[e]
[a]	[¬a,b,e]
[b,e]	[¬b,a]
[e,a]	[¬a]
[e]	

III. **[[¬a,b],[c,f],[¬c],[¬f,b],[¬c,b]]**

1. [c,f] [¬f,b]
   [c,b] [¬c,b]
   [b] [¬a,b]
   [b,¬a] [¬c]
   [b, ¬a, ¬c]

2. [¬c] [¬c,b]
   [¬c,b] [c,f]
   [b,f] [¬f,b]
   [b] [¬a,b]
   [b,¬a]

IV. **[[a,b],[¬a, ¬b]]**

[a,b] [¬a, ¬b]

[ ]

Part 2 "SAT Solver"

Problem one. [[toddler],[¬toddler,child],[¬child,¬male, boy],[¬infant, child], [¬child,¬female, girl], [female], [girl]]

Strategy no. 1 I chose "toddler" at the beginning because it appears in one of the shortest clauses in C.

C•toddler = { [child] , [¬child,¬male, boy] , [¬infant, child] , [¬child,¬female, girl] , [female], [girl] }
(C•toddler)•$\overline{child}$ = { [] , [¬infant] , [female], [girl] }
((C•toddler)•$\overline{child}$)•$\overline{infant}$ = {[female], [girl]}
(((C•toddler)•$\overline{child}$)•$\overline{infant}$)•female = { [girl] }
((((C•toddler)•$\overline{child}$)•$\overline{infant}$)•female)•girl = {}

Result: YES satisfiable if ({toddler/true ; child/false ; infant/false ; female/true ; girl/true})

Strategy no. 2 I chose "child" because it appears in the most clauses in C.

C•child = { [toddler] , [¬male, boy] , [¬female, girl], [female], [girl] }
(C•child)•female = { [toddler] , [¬male, boy] , [girl], [girl] }
((C•child)•female)•girl = { [toddler] , [¬male, boy] }
(((C•child)•female)•girl)•$\overline{male}$ = { [toddler] }
((((C•child)•female)•girl)•$\overline{male}$)•toddler = { }

Result: YES satisfiable if ({child/true ; female/true ; girl/true ; male/false ; toodler/true})

Strategy no. 3 I chose "boy" because it appears in the fewest clauses in C.

C•boy = { [toddler] ,[¬toddler, child] , [¬infant, child] , [¬child,¬female, girl] , [female] , [girl] }
(C•boy)•female = { [toddler] ,[¬toddler, child] , [¬infant, child] , [¬child, girl] , [girl] }
((C•boy)•female)•child = { [toddler] , [girl] , [girl] }
(((C•boy)•female)•child)•girl = { [toddler] }
((((C•boy)•female)•child)•girl) • $\overline{toddler}$ = { [] }

Result: NO UNSAT

In my opinion, choosing different atoms each time may affect the final result as in the example I got yes in strategy one and two but the last one I got no.

Problem two. [[toddler],[¬toddler,child],[¬child,¬male,boy],[¬infant,child], [¬child,¬female,girl], [female], [¬girl]]

Strategy no. 1 I chose "female" because it appears ones in C.

C•$\overline{female}$ = { [toddler],[¬toddler,child],[¬child,¬male,boy],[¬infant,child], [], [¬girl] }
(C• $\overline{female}$)•$\overline{toddler}$= { [] ,[¬child,¬male,boy] , [¬infant,child], [¬girl] }
((C•$\overline{female}$)•$\overline{toddler}$)•child= { [¬male,boy] , [¬girl] }
(((C•$\overline{female}$)•$\overline{toddler}$)•child)•boy= { [¬girl] }
((((C•$\overline{female}$)•$\overline{toddler}$)•child)•boy)•$\overline{girl}$= { }

Result: YES satisfiable if ({female/false ; toddler/false ; child/true ; boy/true ; girl/false})

Strategy no. 2 I chose "girl" because it is balanced atom in C.

C•girl = { [toddler] , [¬toddler,child] , [¬child,¬male, boy] , [¬infant, child] , [female], [] }
(C•girl)•$\overline{infant}$ = { [toddler] , [¬toddler,child] , [¬child,¬male, boy] , [female] }
((C•girl)•$\overline{infant}$)•boy= { [toddler] , [¬toddler,child] , [female] }
(((C•girl)•$\overline{infant}$)•boy)•female= { [toddler] , [¬toddler,child] }
((((C•girl)•$\overline{infant}$)•boy)•female)•$\overline{toddler}$= { [] }

Result: NO unsatisfiable

Problem three. [[¬a,b],[c,d],[¬d,b],[¬c,b],[¬b]]

Strategy no. 1 I chose "b" because it is the least balanced atom in C.

C•b = { [c,d] ,[] }
C•b•d = { }

Result: Yes satisfiable if ({b/true ; d/true})

Strategy no. 2 I chose "ā" because it appears in the fewest clauses in C.

C•ā= { [c,d],[¬d,b],[¬c,b],[¬b] }
(C•ā)•c = { [¬d,b],[b],[¬b] }
((C•ā)•c)•b = { [] }

Result: NO Unsatisfiable

Problem four. [[¬b,a],[¬a,b,e],[e],[a, ¬e],[¬a]]

Strategy no. 1 I chose "$\bar{b}$" because it is the most balanced atom in C.

C•$\bar{b}$ = { [¬a,e],[e],[a,¬e],[¬a] }
(C•$\bar{b}$)•$\bar{e}$ = { [¬a],[],[¬a] }
((C•$\bar{b}$)•$\bar{e}$)•a = { [] }

Result: NO Unsatisfiable

Strategy no. 2 I chose "a" because it appears in the most clauses in C.

C•a = { [b,e] , [e] , [] }
(C•a)•e = { }

Result: YES satisfiable if ({a/true ; e/true})

Problem five. [[¬a,¬e,b],[¬d,e,¬b],[¬e,f,¬b],[f,¬a,e],[e,f,¬b]]

Strategy no.1 I chose "e" because it appears in the most clauses in C.

C•e = { [¬a,b],[f,¬b] }
(C•e)•b = { [f] }
((C•e)•b)•f = { }

Result: YES satisfiable if ({e/true ; b/true ; f/true})

Strategy no.2 I chose "e" again with different choices at the end.

C•e = { [¬a,b],[f,¬b] }
(C•e)•ā = { [f,¬b] }
((C•e)•ā)•$\bar{b}$ = { }

Result: YES satisfiable if ({e/true ; a/false ; b/false})

Strategy no.3

$C \cdot \bar{b}$ = { [¬a,¬e] , [f,¬a,e] }

$(C \cdot \bar{b}) \cdot \bar{a}$ = { }

Result: YES satisfiable if ({b/false ; a/false})

Strategy no.4

$C \cdot \bar{d}$ = { [¬a,¬e,b], [¬e,f,¬b],[f,¬a,e],[e,f,¬b] }

$(C \cdot \bar{d}) \cdot e$= { [¬a,b], [f,¬b] }

$((C \cdot \bar{d}) \cdot e) \cdot b$= { [f] }

$(((C \cdot \bar{d}) \cdot e) \cdot b) \cdot \bar{f}$= { [] }

Result: No Unsatisfiable

Problem six. [[a,b],[¬a,¬b] ,[¬a,b] ,[a,¬b]]

Strategy no. 1 I chose “a” first time

$C \cdot a$ = { [¬b] ,[b] }

$(C \cdot a) \cdot \bar{b}$ = { [] }

Result: UNSAT

Strategy no.2 I chose “$\bar{b}$” second time

$C \cdot \bar{b}$ = { [a] , [¬a] }

$(C \cdot \bar{b}) \cdot \bar{a}$ = { [] }

Result: UNSAT

Here I attached a photo from my SWI Prolog, with some queries and their answers.