Differential Calculus
MTH 62-140
Laws/Theorems/Identities in Trigonometry

1. (a) $\pi = \dfrac{\text{circumference(c)}}{\text{diameter(d)}}$

   (b) $c = \pi d = 2\pi r$

2. (a) $\theta = \dfrac{s}{r}$

   (b) $\pi\,\text{rad} = 180\,\text{deg}$

3. (a) Trigonometric ratios: $\sin\theta = \dfrac{y}{r}$, $\cos\theta = \dfrac{x}{r}$, $\tan\theta = \dfrac{y}{x}$, $\csc\theta = \dfrac{r}{y}$, $\sec\theta = \dfrac{r}{x}$, $\cot\theta = \dfrac{x}{y}$.

   (b) CAST rule

4. Trigonometric Identities

   (a) Type 1: $\csc\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$, $\cot\theta = \dfrac{1}{\tan\theta}$,

   (b) Type 2: $\sin^2\theta + \cos^2\theta = 1$

      i. $\sin^2\theta + \cos^2\theta = 1$

      ii. $1 + \tan^2\theta = \sec^2\theta$

      iii. $1 + \cot^2\theta = \csc^2\theta$

   (c) Type 3:

$$\sin\left(\frac{n\pi}{2} + \theta\right) = \begin{cases} m\sin\theta, & \text{if } n \text{ is even} \\ m\cos\theta, & \text{if } n \text{ is odd} \end{cases}$$

   m is determined by the CAST rule.

   (d) Type 4: $\sin(x+y) = \sin x\cos y + \cos x\sin y$

      i. $\sin(x+y) = \sin x\cos y + \cos x\sin y$

      ii. $\sin(x-y) = \sin x\cos y - \cos x\sin y$

      iii. $\cos(x+y) = \cos x\cos y - \sin x\sin y$

      iv. $\cos(x-y) = \cos x\cos y + \sin x\sin y$

      v. $\sin 2x = 2\sin x\cos x$

vi. $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

vii. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

viii. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

ix. $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

x. $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

xi. $\tan(2x) = \dfrac{2\tan x}{1 - \tan^2 x}$

5. (a) If a function is not one-to-one then it does not have inverse function. Trigonometric functions are not one-to-one. Therefore they do not have inverse functions. By restricting the domain of trigonometric functions we can make them one-to-one and then we can define inverse functions.

(b) i. $\sin^{-1} x = y \Leftrightarrow \sin y = x$ and $-\dfrac{\pi}{2} \leq y \leq \dfrac{\pi}{2}$

ii. $\cos^{-1} x = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$

iii. $\tan^{-1} x = y \Leftrightarrow \tan y = x$ and $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

iv.
$$\sin^{-1}(\sin x) = x \text{ for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$
$$\sin(\sin^{-1} x) = x \text{ for } -1 \leq x \leq 1$$

v.
$$\cos^{-1}(\cos x) = x \text{ for } 0 \leq x \leq \pi$$
$$\cos(\cos^{-1} x) = x \text{ for } -1 \leq x \leq 1$$

vi.
$$\tan^{-1}(\tan x) = x \text{ for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$
$$\tan(\tan^{-1} x) = x \text{ for } -\infty < x < \infty$$