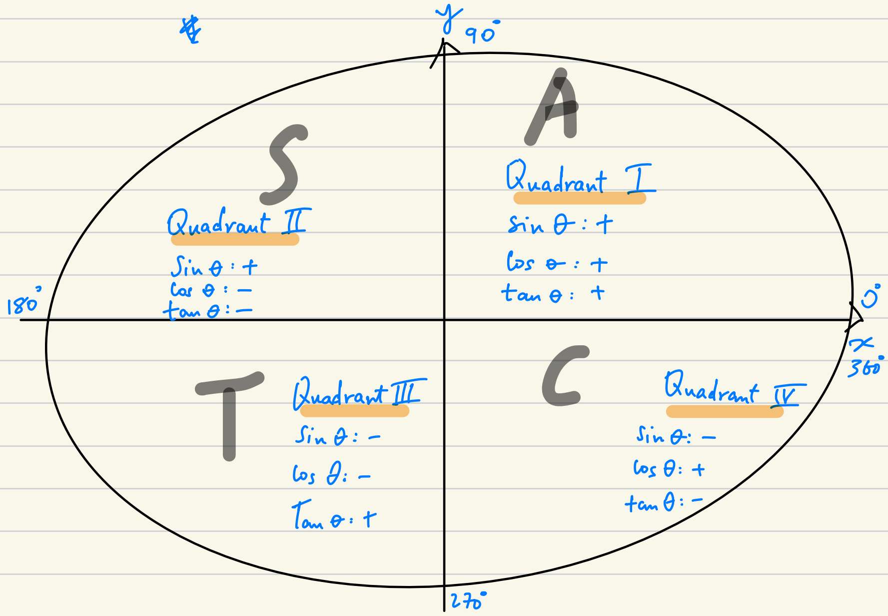
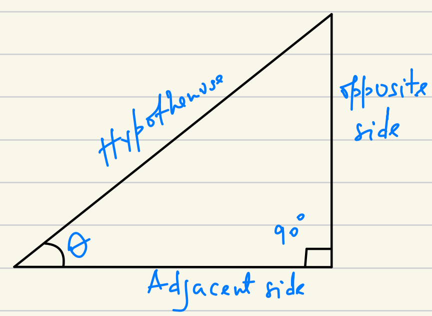
09-13-2024
Friday

θ \ $f(x)$	Sin	Cos	Tan	Cot
30 ($\pi/6$)	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$	$\sqrt{3}$
45 ($\pi/4$)	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1
60 ($\pi/3$)	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
90 ($\pi/2$)	1	0	∞	0
180 (π)	0	-1	0	∞
360 (2π)	0	1	0	∞



* Note : * Acute angles are $(0° \leq \theta \leq 90°)$

* Obtuse angle are $(90° < \theta)$



* $\sin\theta = \frac{opp}{hyp} = \frac{1}{\csc\theta}$ * $\csc = \frac{hyp}{opp} = \frac{1}{\sin\theta}$

* $\cos\theta = \frac{adj}{hyp} = \frac{1}{\sec\theta}$ * $\sec\theta = \frac{hyp}{adj} = \frac{1}{\cos\theta}$

* $\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$ * $\cot\theta = \frac{adj}{opp} = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

* If θ is an acute angle, then the following relationships are true:

1. $\sin(90°-\theta) = \cos\theta$
2. $\cos(90°-\theta) = \sin\theta$
3. $\tan(90°-\theta) = \cot\theta$
4. $\cot(90°-\theta) = \tan\theta$
5. $\sec(90°-\theta) = \csc\theta$
6. $\csc(90°-\theta) = \sec\theta$

* $\sin^2\theta + \cos^2\theta = 1$

* $1 + \tan^2\theta = \sec^2\theta$

* $1 + \cot^2\theta = \csc^2\theta$

* $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
* $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$
* $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
* $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
* $\tan(A+B) = \dfrac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$

* $\tan(A-B) = \dfrac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$

* $\sin(2A) = 2\sin(A)\cos(A)$
* $\tan(2A) = \dfrac{2\tan(A)}{1-\tan^2(A)}$
* $\cos(2A) = \cos^2(A) - \sin^2(A)$

Since

$\sin^2(A)+\cos^2(A)=1$

$\rightarrow \cos^2 A = 1-\sin^2 A$

or $\rightarrow \sin^2 A = 1-\cos^2 A$

Resulting into

$\cos(2A) = 1-\sin^2 A - \sin^2 A = \underline{1-2\sin^2 A}$

or

$= \cos^2 A - (1-\cos^2 A) = \underline{2\cos^2 A - 1}$

Product identities

* $\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$
* $\sin(A)\sin(B) = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$
* $\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$

Sum identities

* $\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

* $\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

* $\sin(A) - \sin(B) = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$

* $\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

Half-angle identities

* $\sin^2\left(\frac{A}{2}\right) = \frac{1-\cos(A)}{2}$

* $\cos^2\left(\frac{A}{2}\right) = \frac{1+\cos(A)}{2}$

* $\tan^2\left(\frac{A}{2}\right) = \frac{1-\cos(A)}{1+\cos(A)} = \frac{\sin^2(A/2)}{\cos^2(A/2)}$

Example: Find the exact ratio of $\sin(15^\circ)$

Solution

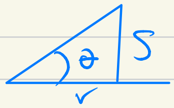
$$\sin(15^\circ) = \sin(45^\circ - 30)$$

$$= \sin 45^\circ \cos 30^\circ - \cos(45)\sin(30)$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

From radian to degree

$\theta = \frac{S}{r}$ rad 

Note:

To define the inverse function for f, the function f has to be one to one. If f(x) is no

Ex: $f(x) = x^2$ is not one-to-one because (-1) and (1) have the same range, which is 1

Inverse function steps

Step 1: verify if the function is one-to-one.

Step 2: solve for x

Step 3: switch x for y

Ex: $f(x) = 3x + 2$

Step 1: let $f(x_1) = f(x_2)$

$$3x_1 + 2 = 3x_2 + 2$$

$$3x_1 = 3x_2$$

$$x_1 = x_2$$

Step 2: $y = 3x + 2$

$$y - 2 = 3x$$

$$x = \frac{y-2}{3}$$

Step 3 $x = \frac{y-2}{3}$

$$y = \frac{x-2}{3}$$

Ex 2: $f(x) = \dfrac{2x+1}{4x+3}$

Step 1 let $f(x_1) = f(x_2)$

$$\frac{2x_1+1}{4x_1+3} = \frac{2x_2+1}{4x_2+3}$$

$$(2x_1+1)(4x_2+3) = (2x_2+1)(4x_1+3)$$

$$\cancel{8x_1x_2} + 6x_1 + 4x_2 + \cancel{3} = \cancel{8x_1x_2} + 6x_2 + 4x_1 + \cancel{3}$$

$$6x_1 + 4x_2 = 6x_2 + 4x_1$$

$$\cancel{2}(3x_1 + 2x_2) = \cancel{2}(3x_2 + 2x_1)$$

$$3x_1 - 2x_1 = 3x_2 - 2x_2$$

$$x_1 = x_2 \quad (\text{Hence } f(x) \text{ is one-to-one})$$

Step 2: $y = \dfrac{2x+1}{4x+3}$

$$y(4x+3) = 2x+1$$

$$4xy + 3y = 2x+1$$

$$4xy - 2x = 1 - 3y$$

$$2x(2y-1) = 1-3y$$

$$\frac{2x}{2} = \frac{1-3y}{2y-1} \div 2$$

$$x = \frac{2-6y}{2y-1}$$

Step 3:

$$y = \frac{2-6y}{2y-1}$$

$$4x+3 \neq 0$$

$$4x \neq -3$$

$$x \neq -3/4$$

	Domain	range
$f(x)$		
$f^{-1}(x)$		

Ex $f(x) = \sqrt{3x-10}$

Find $f^{-1}(x)$. Also find the domain and range of f and f^{-1}.

Outline