Differential Calculus
MTH 62-140

1. Suppose c is a constant and the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist. Then

   (a) $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

   (b) $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

   (c) $\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$

   (d) $\lim_{x\to a}[f(x)\cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

   (e) $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$

2. $\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$ where n is a positive integer.

3. $\lim_{x\to a} c = c$ and $\lim_{x\to a} x = a$

4. $\lim_{x\to a} x^n = a^n$ where n is a positive integer

5. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer. If n is even, we assume that $a > 0$.

6. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ where n is a positive integer. If n is even, we assume that $\lim_{x\to a} f(x) > 0$.

7. Direct Sustitution Property: If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x\to a} f(x) = f(a)$$

8. If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$, provided the limits exist.

9. The Squeeze Theorem: If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibley at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then

$$\lim_{x \to a} g(x) = L$$

10. If $r > 0$ is a rational number, then

$$\lim_{x \to \infty} \frac{1}{x^r} = 0$$

If $r > 0$ is a rational number such that x^r is defined for all x, then

$$\lim_{x \to -\infty} \frac{1}{x^r} = 0$$

11.

$$\lim_{x \to \infty} \frac{1}{r^x} = \begin{cases} \infty, & \text{if } 0 < r < 1 \\ 1, & \text{if } r = 1 \\ 0, & \text{if } r > 1 \end{cases}$$

12. (a) $\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$

(b) $\lim_{x \to 0} \left(\frac{e^x - 1}{x}\right) = 1$

13. (a) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(b) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

(c) $\lim_{x \to 0} \sin x = 0$

(d) $\lim_{x \to 0} \cos x = 1$

(e) $\lim_{x \to 0} \tan x = 0$

(f) $\lim_{x \to (\frac{\pi}{2})^-} \tan x = \infty$

(g) $\displaystyle\lim_{x\to(\frac{\pi}{2})^+} \tan x = -\infty$

(h) $\displaystyle\lim_{x\to 0^+} \ln x = -\infty$

14. (a) $\displaystyle\lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}$

(b) $\displaystyle\lim_{x\to-\infty} \tan^{-1} x = -\frac{\pi}{2}$

15. (a) If $\displaystyle\lim_{x\to a} f(x) = \infty$ and $\displaystyle\lim_{x\to a} g(x) = \infty$, then $\displaystyle\lim_{x\to a} f(x)\cdot g(x) = \infty$

(b) If $\displaystyle\lim_{x\to a} f(x) > 0$ and $\displaystyle\lim_{x\to a} g(x) = \infty$, then $\displaystyle\lim_{x\to a} f(x)\cdot g(x) = \infty$