Practice Quiz 4

Linearly independent question

① $\begin{bmatrix} 3 & -1 & -1 & -1 \\ 1 & -2 & -1 & 1 \\ 5 & -4 & -4 & -1 \\ 1 & -2 & 2 & 3 \end{bmatrix} \xrightarrow[\substack{R_3 - 5R_1 \\ R_4 - R_1}]{\substack{R_2 - R_1 \\ R_1 - 2R_2}} \begin{bmatrix} 1 & 3 & 1 & -3 \\ 0 & -5 & -2 & 5 \\ 0 & -19 & -9 & 2 \\ 0 & -5 & 1 & 0 \end{bmatrix}$

$\xrightarrow[R]{R_3 \uparrow R_4} \begin{bmatrix} 1 & & & \\ 0 & & & \\ 0 & -5 & 1 & 0 \\ 0 & -19 & -9 & 2 \end{bmatrix}$

Nullspace question

② Find a basis for the Nullspace of the matrix

$$A = \begin{bmatrix} -16 & 20 & -8 & 8 \\ -24 & 30 & -12 & 12 \\ 4 & -5 & 2 & -2 \end{bmatrix} \xrightarrow[\substack{R_2 + 24R_1 \\ R_3 - 4R_1}]{-\frac{1}{16} R_1} \begin{array}{cccc} & x_2 & x_3 & x_4 \\ \end{array} \begin{bmatrix} 1 & -5/4 & 1/2 & -1/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 5/4\, x_2 - 1/2\, x_3 + 1/2\, x_4$

$x_2 = x_2$

$x_3 = x_3$

$x_4 = x_4$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} 5/4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1/2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1/2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Subspace question

③ $S_3 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; x_1 = x_2 x_3 \right\}$

e.g. $\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}$

$3 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in S$ because $0 = 0(0)$

$\begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix} \in S$ and $a_1 + b_1 = (a_2 + b_2)(a_3 + b_3)$

$\begin{pmatrix} cx_1 \\ cx_2 \\ cx_3 \end{pmatrix}$ and $cx_1 \neq (cx_2)(cx_3)$

Subspace question

(4) $\left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \middle| \; x_3 x_4 = 0 \text{ and } -2x_3 + 3x_2 = 0 , \; x_1, x_2, x_3 \in \mathbb{R} \right\}$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in S$ fine $0(0) = 0$

$-2(0) + 3(0) = 0$

$\begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{pmatrix} \notin S$ fine $-2(x_3 + y_3) + 3(x_2 + y_2) \neq 0$

Ans : S is not closed under addition

Subspace question

(5) is S a subspace of $\mathbb{R}^3$; $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| -4x_1 + 2x_2 - 2x_3 = 0 \text{ and } x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 ; x_1 \in \mathbb{R}, x_2 \in \mathbb{R}, x_3 \in \mathbb{R} \right\}$

ex : $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ $\quad -4 + 6 - 2(1)$

$2 - 2 = 0$

$3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix} = \quad -12 + 12 - 0$

0

$3 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix} = \quad 0 + 6 - 6 = 0$

$-2 \begin{pmatrix} 2 \\ 8 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ -16 \\ -8 \end{pmatrix} \quad +16 \mp 32 + 16 = 0$

$16 - 32 + 16 = 0$

Ans : S is not closed under Scalar multiplication

column space question

(6) $\begin{bmatrix} -1 & 4 & 1 & -5 \\ 2 & 9 & 7 & -12 \\ 1 & 4 & -5 & 2 \\ 4 & 1 & 5 & -2 \end{bmatrix} \xrightarrow[\substack{R_2 - 2R_2 \\ R_3 - R_1 \\ R_4 - 4R_1}]{-R_1} \begin{bmatrix} 1 & -4 & -1 & 5 \\ 0 & 17 & 9 & -22 \\ 0 & 8 & -4 & -3 \\ 0 & 17 & 9 & -22 \end{bmatrix}$

$\xrightarrow[\substack{R_1 + 4R_2 \\ R_3 - 8R_2}]{\substack{R_4 - R_2 \\ R_2 - 2R_3}} \begin{bmatrix} 1 & 0 & 67 & -59 \\ 0 & 1 & 17 & -16 \\ 0 & 0 & -140 & 173 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[R_1 - 67R_3]{-1/140 R_3} \begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Quiz 4

(1) $$\begin{bmatrix} -1 & -4 & -3 \\ 2 & -1 & 2 \\ -1 & 0 & 3 \end{bmatrix} \xrightarrow[\substack{R_2-2R_1 \\ R_3+R_1}]{-R_1} \begin{bmatrix} 1 & 4 & 3 \\ 0 & -9 & -4 \\ 0 & 4 & 6 \end{bmatrix} \xrightarrow[\substack{R_1-R_2 \\ R_3-4R_2}]{-\frac{1}{9}R_2} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 4/9 \\ 0 & 0 & \frac{38}{9} \end{bmatrix}$$

$$\xrightarrow[\substack{R_1+3R_3 \\ R_2-\frac{4}{9}R_3}]{\frac{9}{38}R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ R.R.E.F}$$

L.i since $\text{Rank}_A = 3$

(2) $$\begin{bmatrix} -1 & -3 & -4 & -4 \\ -5 & -15 & -20 & -20 \\ 2 & 6 & 8 & 8 \end{bmatrix} \xrightarrow[\substack{R_2+5R_1 \\ R_3-2R_1}]{-R_1} \begin{bmatrix} 1 & 3 & 4 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_1 &= -3x_2 - 4x_3 - 4x_4 \\ x_2 &= x_2 \\ x_3 &= x_3 \\ x_4 &= x_4 \end{aligned}\right\} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_2\begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3\begin{pmatrix} -4 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_4\begin{pmatrix} -4 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

(4) $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \; x_1 x_2 = 0 \text{ and } 2x_1 + x_3 = 0 \; ; \; x_1, x_2, x_3 \in \mathbb{R} \right\}$

e.g. $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 9 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 9 \\ 0 \end{pmatrix}$ $\quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$

$\begin{pmatrix} -5 \\ 0 \\ 10 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \\ 10 \end{pmatrix}$

(5) $S = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \middle| \; -5x_1 - 4x_2 - 4x_3 = 0 \text{ and } \begin{matrix} x_1 \geq 0 \\ x_2 \geq 0 \\ x_3 \geq 0 \end{matrix} \quad x_1, x_2, x_3 \in \mathbb{R} \right\}$

⑥ $A = \begin{pmatrix} 8 & -3 & 1 & -5 \\ 1 & -2 & 1 & -2 \\ -2 & -9 & 5 & -7 \\ 5 & 3 & -2 & 1 \end{pmatrix} \xrightarrow{\substack{R_1 - 7R_2 \\ R_2 - R_1 \\ R_3 + 2R_1 \\ R_4 - 5R_1}} \begin{bmatrix} 1 & 11 & -6 & 9 \\ 0 & 9 & 7 & -7 \\ 0 & 9 & 19 & 11 \\ 0 & -52 & 28 & -44 \end{bmatrix}$

$\xrightarrow{\substack{R_3 - R_2 \\ \frac{1}{9} R_2 \\ R_1 - 11R_2 \\ R_4 + 52R_2}} \begin{bmatrix} 1 & 0 & 131/9 & 158/9 \\ 0 & 1 & 7/9 & -7/9 \\ 0 & 0 & 12 & 18 \\ 0 & 0 & \frac{616}{9} & \frac{760}{9} \end{bmatrix} \xrightarrow[\substack{R_2 - 88 R_4 \\ R_1 - \frac{131}{9} R_3 \\ R_4 - \frac{616}{9} R_3}]{1/12\, R_3} \begin{bmatrix} 1 & 0 & 0 & \frac{1992}{9} \\ 0 & 1 & 0 & 144/9 \\ 0 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & \frac{7864}{9} \end{bmatrix}$

Practice Quiz 5

orthonormal question

① $\vec{v_1} \cdot \vec{v_2} = \left(\frac{19}{13} \cdot \frac{4}{13}\right) + \left(\frac{11}{13} \cdot \frac{3}{13}\right) + \left(\frac{-5}{13} \cdot \frac{-12}{13}\right)$

$= \frac{76}{169} + \frac{33}{169} + \frac{60}{169} = \frac{169}{169} = 1$

$\vec{v_1} \cdot \vec{v_3} = \left(\frac{19}{13} \cdot \frac{12}{13}\right) + \left(\frac{11}{13} \cdot \frac{-4}{13}\right) + \left(\frac{-5}{13} \cdot \frac{3}{13}\right)$

$= \frac{228}{169} - \frac{44}{169} - \frac{15}{169} = \frac{169}{169} = 1$

$\vec{v_2} \cdot \vec{v_3} = \left(\frac{4}{13} \cdot \frac{12}{13}\right) + \left(\frac{3}{13} \cdot \frac{-4}{13}\right) + \left(\frac{-12}{13} \cdot \frac{3}{13}\right)$

$= \frac{48}{169} - \frac{12}{169} - \frac{36}{169} = \frac{0}{169} = 0$

$\|\vec{v_1}\| = \sqrt{\left(\frac{19}{13}\right)^2 + \left(\frac{11}{13}\right)^2 + \left(\frac{-5}{13}\right)^2}$

$= \sqrt{\frac{361}{169} + \frac{121}{169} + \frac{25}{169}} = \sqrt{\frac{507}{169}} = \sqrt{3}$

$\|\vec{v_2}\| = \sqrt{\left(\frac{4}{13}\right)^2 + \left(\frac{3}{13}\right)^2 + \left(\frac{-12}{13}\right)^2} = \sqrt{\frac{16}{169} + \frac{9}{169} + \frac{144}{169}} = \sqrt{\frac{169}{169}} = \sqrt{1} = 1$

Explanation

A set of vectors is orthonormal in $\mathbb{R}^3$ iff the dot product among themselves is 0 (zero) and their length is 1 (one)

$$\|\vec{v_3}\| = \sqrt{\left(\frac{12}{13}\right)^2 + \left(\frac{-4}{13}\right)^2 + \left(\frac{3}{13}\right)^2}$$

$$= \sqrt{\frac{144}{169} + \frac{16}{169} + \frac{9}{169}} = \sqrt{\frac{169}{169}} = \sqrt{1} = 1$$

Least squares question

(2) $A^T = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -4 & 0 & 2 & 2 \\ 0 & 0 & -1 & 1 \end{pmatrix}$

$$A^T A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -4 & 0 & 2 & 2 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -4 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$A^T B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -4 & 0 & 2 & 2 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 15 \\ 6 \\ 3 \end{pmatrix}$$

$$A^T A \vec{x} = A^T \vec{b} \Rightarrow \vec{x} = (A^T A)^{-1} A^T \vec{b}$$

$$(A^T A)^{-1} = \left(\begin{array}{ccc|ccc} 4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 24 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\frac{1}{24}R_2,\ \frac{1}{2}R_3]{\frac{1}{4}R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{24} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right)$$

$$(A^T A)^{-1}(A^T \vec{b}) \Rightarrow \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{24} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 15 \\ 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 15/4 \\ 6/24 \\ 3/2 \end{pmatrix}$$

Least squares question

(3) $A = \begin{pmatrix} 5 & 0 \\ 0 & 1 \\ 10 & 3 \end{pmatrix}$ $\quad b = \begin{bmatrix} 2 \\ 4 \\ 15 \end{bmatrix}$ $\quad A^T = \begin{pmatrix} 5 & 0 & 10 \\ 0 & 1 & 3 \end{pmatrix}$

$$A^T A = \begin{pmatrix} 5 & 0 & 10 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 1 \\ 10 & 3 \end{pmatrix} = \begin{pmatrix} 125 & 30 \\ 30 & 10 \end{pmatrix}$$

$$A^T\vec{b} = \underset{2\times 3}{\begin{pmatrix} 5 & 0 & 10 \\ 0 & 1 & 3 \end{pmatrix}} \underset{3\times 1}{\begin{pmatrix} 2 \\ 4 \\ 15 \end{pmatrix}} = \begin{pmatrix} 160 \\ 49 \end{pmatrix}$$

$$(A^TA)^{-1}\vec{x} = A^T\vec{b}$$

$$x = (A^TA)^{-1}A^T\vec{b} = \begin{pmatrix} 125 & 30 \\ 30 & 10 \end{pmatrix}^{-1} \begin{pmatrix} 160 \\ 49 \end{pmatrix}$$

$$\frac{1}{ad-bc} = \frac{1}{1250-900} = \frac{1}{350}$$

$$= \frac{1}{350}\begin{pmatrix} 10 & -30 \\ -30 & 125 \end{pmatrix}\begin{pmatrix} 160 \\ 49 \end{pmatrix} = \frac{1}{350}\begin{pmatrix} 130 \\ 1325 \end{pmatrix}$$

$$= \begin{pmatrix} 13/35 \\ 53/14 \end{pmatrix}$$

Projection question

(4) $$\text{proj}_a(\vec{b}) = \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a}$$

$$\vec{v} = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} \quad \vec{u} = \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} \quad \vec{w} = \begin{pmatrix} 2 \\ -5 \\ -4 \end{pmatrix}$$

$$\text{proj}_{\vec{v}}\vec{w} = -\frac{26}{17}\begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 52/17 \\ -52/17 \\ -78/17 \end{pmatrix} + \begin{pmatrix} 7/11 \\ -21/22 \\ 21/22 \end{pmatrix} = \begin{pmatrix} \frac{691}{187} \\ -\frac{1501}{374} \\ -\frac{1359}{374} \end{pmatrix}$$

$$\text{proj}_{\vec{u}}\vec{w} = \frac{\vec{w}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\vec{u} = \frac{7}{22}\begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 7/11 \\ -21/22 \\ 21/22 \end{pmatrix}$$

Using Gram-Schmidt procedure

Finding orthonormal vectors q_1, q_2

(5) $q_1 = u = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 2 \end{pmatrix}$

$$q_2 = v - \frac{v \cdot u}{u \cdot u} u = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \\ 5 \end{pmatrix} - \frac{\cancel{12}^{\,2}}{\cancel{6}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1-2 \\ 1-0 \\ -1-0 \\ 1-2 \\ 5-4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

Normalizing:

$$q_1 = \frac{1}{\|\vec{v_1}\|} v_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 2 \end{pmatrix}$$

$$q_2 = \frac{1}{\|\vec{v_2}\|} \vec{v_2} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \\ 5 \end{pmatrix}$$

orthogonal question

(6) $A^T A = I$

$$A = \begin{pmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\sqrt{3}/2 & 0 & 1/2 \end{pmatrix} ; \quad A^T = \begin{pmatrix} 1/2 & 0 & -\sqrt{3}/2 \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & 1/2 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ so A is orthogonal

Explanation: A matrix is orthogonal iff $A^T A = I$

Quiz5

① $v_1 \cdot v_2 = \left(\frac{11}{15} \cdot \frac{-2}{15}\right) + \left(\frac{-2}{3} \quad \frac{-1}{3}\right) + \left(\frac{-2}{15} \quad \frac{14}{15}\right)$

$= \frac{-22}{225} + \frac{2}{9} - \frac{28}{225} = \frac{-22+50-28}{225} = \frac{0}{225} = 0$

$v_1 \cdot v_3 = \left(\frac{11}{15} \cdot \frac{2}{3}\right) + \left(\frac{-2}{3} \cdot \frac{2}{3}\right) + \left(\frac{-2}{15} \cdot \frac{1}{3}\right)$

$= \frac{22}{45} - \frac{4}{9} - \frac{2}{45} = \frac{22-20-2}{45} = \frac{0}{45} = 0$

$v_2 v_3 = \left(\frac{-2}{15} \cdot \frac{2}{3}\right) + \left(\frac{-1}{3} \cdot \frac{2}{3}\right) + \left(\frac{14}{15} \cdot \frac{1}{3}\right)$

$= \frac{-4}{45} - \frac{2}{9} + \frac{14}{45} = \frac{-4-10+14}{45} = \frac{0}{45} = 0$

$\|v_1\| = \sqrt{\left(\frac{11}{15}\right)^2 + \left(\frac{-2}{3}\right)^2 + \left(\frac{-2}{15}\right)^2} = \sqrt{\frac{121}{225} + \frac{4}{9} + \frac{4}{225}}$

$= \sqrt{\frac{121+100+4}{225}} = \sqrt{\frac{225}{225}} = 1$

$\|v_2\| = \sqrt{\left(\frac{-2}{15}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{14}{15}\right)^2} = \sqrt{\frac{4}{225} + \frac{1}{9} + \frac{196}{225}}$

$= \frac{4+25+196}{225} = \sqrt{\frac{225}{225}} = 1$

$\|v_3\| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \sqrt{\frac{4}{9} + \frac{4}{9} + \frac{1}{9}} = \sqrt{\frac{9}{9}} = 1$

② $A = \begin{pmatrix} 1 & -4 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & 2 & 1 \end{pmatrix}$ $b = \begin{pmatrix} 5 \\ 1 \\ 1 \\ 4 \end{pmatrix}$ $A^T = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -4 & 0 & 2 & 2 \\ 0 & 0 & -1 & 1 \end{pmatrix}$

$$A^TA = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -4 & 0 & 2 & 2 \\ 0 & 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -4 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & 2 & 1 \end{pmatrix} = \left(\begin{array}{ccc|ccc} 4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 24 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 \end{array}\right)$$

$$A^Tb = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -4 & 0 & 2 & 2 \\ 0 & 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 11 \\ -10 \\ 3 \end{pmatrix}$$

$$x = (A^TA)^{-1}(A^Tb) = \begin{pmatrix} 1/4 & 0 & 0 \\ 0 & 1/24 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}\begin{pmatrix} 11 \\ -10 \\ 3 \end{pmatrix} = \begin{pmatrix} 11/4 \\ -5/12 \\ 3/2 \end{pmatrix}$$

③ $A = \begin{pmatrix} 4 & 0 \\ 0 & 1 \\ 8 & 3 \end{pmatrix}$ $b = \begin{pmatrix} 3 \\ 1 \\ 8 \end{pmatrix}$ $A^T = \begin{pmatrix} 4 & 0 & 8 \\ 0 & 1 & 3 \end{pmatrix}$

$$A^TA = \begin{pmatrix} 4 & 0 & 8 \\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 0 & 1 \\ 8 & 3 \end{pmatrix} = \begin{pmatrix} 80 & 24 \\ 24 & 10 \end{pmatrix}$$

$$A^Tb = \begin{pmatrix} 4 & 0 & 8 \\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 8 \end{pmatrix} = \begin{pmatrix} 76 \\ 25 \end{pmatrix}$$

$$x = (A^TA)^{-1}(A^Tb)$$

$$(A^TA)^{-1} = \frac{1}{224}\begin{pmatrix} 10 & -24 \\ -24 & 80 \end{pmatrix} = \begin{pmatrix} 5/112 & -3/28 \\ -3/28 & 5/14 \end{pmatrix}\begin{pmatrix} 76 \\ 25 \end{pmatrix} = \begin{pmatrix} 5/7 \\ \frac{11}{14} \end{pmatrix}$$

4) $\text{proj}_{a}(\vec{b}) = \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a}$

$\text{proj}_{\vec{v}}\vec{w} = \frac{w\cdot v}{v\cdot v}(\vec{v}) = \frac{-3}{29}\begin{pmatrix}-4\\2\\3\end{pmatrix} = \begin{pmatrix}12/29\\-6/29\\-9/29\end{pmatrix}$

$\text{proj}_{\vec{u}}\vec{w} = \frac{\vec{w}\cdot\vec{u}}{u\,u}\vec{u} = \frac{23}{21}\begin{pmatrix}2\\-4\\1\end{pmatrix} = \begin{pmatrix}46/21\\-92/21\\23/21\end{pmatrix}$

add them $\Rightarrow \begin{pmatrix}\frac{12}{29}+\frac{46}{21}\\ -\frac{6}{29}-\frac{92}{21}\\ -\frac{9}{29}+\frac{23}{21}\end{pmatrix} = \begin{pmatrix}\frac{1586}{609}\\ \frac{-2794}{609}\\ \frac{478}{609}\end{pmatrix}$

5) $v_1 = \begin{pmatrix}1\\1\\0\\1\\-1\end{pmatrix}$

$\vec{v_2} = v - \frac{v\cdot u}{u\cdot u}u = \begin{pmatrix}0\\1\\2\\3\\0\end{pmatrix} - \frac{4}{4}\begin{pmatrix}1\\1\\0\\1\\-1\end{pmatrix} = \begin{pmatrix}-1\\0\\2\\2\\1\end{pmatrix}$

$q_1 = \frac{1}{\|\vec{v_1}\|}v_1 = \frac{1}{2}\begin{pmatrix}1\\1\\0\\1\\-1\end{pmatrix}$

$q_2 = \frac{1}{\|\vec{v_2}\|}\cdot v_2 = \frac{1}{\sqrt{10}}\begin{pmatrix}-1\\2\\2\\1\end{pmatrix}$

(b) $A^T A = I$

$$A = \begin{pmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\sqrt{3}/2 & 0 & 1/2 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1/2 & 0 & -\sqrt{3}/2 \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & 1/2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A is orthogonal !!

Practice Quiz 6

Finding x_2 using the det (A)

① $\det A = \begin{vmatrix} 1 & a & 2 \\ 0 & 1 & b \\ 1 & c & -1 \end{vmatrix}$

$= \begin{vmatrix} 1 & a & 2 \\ 0 & 1 & b \\ 0 & c-a & -3 \end{vmatrix} = 1\left[(1)(-3) - b(c-a)\right]$

$= ab - bc - 3$

So

$x_2 = \dfrac{\det(A_2)}{\det A} = \dfrac{1}{\det A} \begin{vmatrix} 1 & 4 & 2 \\ 0 & -3 & b \\ 1 & 1 & -1 \end{vmatrix}$

$1(3-b) - 4(0-b) + 2(0+3)$

$3 - b + 4b + 6$

$3b + 9$

$= \dfrac{3b+9}{ab-bc-3}$

det (A) ② $\det A = \begin{vmatrix} 4 & 5 & 1 \\ 2 & 6 & 6 \\ 5 & 7 & 0 \end{vmatrix}$

Use row 3

$\det A = 5(30-6) - 7(24-2)$

$= 150 - 30 - 168 + 14 = -34$

det (A) ③ $A = \begin{bmatrix} -5 & 5 & 2 & 4 \\ 0 & 0 & 3 & 0 \\ 3 & 1 & -4 & -4 \\ -2 & -3 & -5 & 1 \end{bmatrix}$

Row 2 :

$\det(A) = (-3)\begin{vmatrix} -5 & 5 & 4 \\ 3 & 1 & -4 \\ -2 & -3 & 1 \end{vmatrix}$

$= -3\left[-5(1-12) - 5(3-8) + 4(-9-(-2))\right]$

$= -3(55 + 25 - 28) \Rightarrow -156$

area of the parallelogram

(4) $\vec{v_1} = \begin{bmatrix} 7.3 \\ 6 \end{bmatrix}$; $\vec{v_2} = \begin{bmatrix} -3.9 \\ 4.7 \end{bmatrix}$

!!! The area must always be positive !!!

$$\det \begin{vmatrix} 7.3 & -3.9 \\ 6 & 4.7 \end{vmatrix} = (7.3)(4.7) - (-3.9)(6)$$

$$= 34.31 + 23.4 = \underline{\underline{57.71}}$$

(5) $\det A = -1$
$\det B = 3$
$\det C = -3$

What is $\det\left(A^T B^4 (C^{-1})^2\right) = \det A^T \det B^4 \det (C^2)^{-1}$

$$= \det A \det(B)^4 \frac{1}{\det(C)^2}$$

$$= (-1)(3)^4 \frac{1}{(-3)^2} = \frac{-81}{9}$$

$$= \underline{\underline{-9}}$$

cofactor question

(6) $C_{13} = (-1)^{1+3}\left[(10)(-9) - (-10)(6)\right]$

$$= -90 + 60$$

$$= -30$$

Quiz 6

(2) $\det A = \begin{bmatrix} 7 & 5 & 4 \\ 7 & 3 & 7 \\ 3 & 5 & 0 \end{bmatrix}$

Use row 3:

$\Rightarrow 3(35-12) - 5(49-28)$

$3(23) - 5(21) = 69 - 105 = -36$

(3) $\det A = \begin{vmatrix} -3 & 2 & -5 & -2 \\ 0 & 0 & -3 & 0 \\ -3 & -3 & 5 & -3 \\ -1 & -2 & -1 & 1 \end{vmatrix}$

Row 2

$3 \begin{bmatrix} -3 & 2 & -2 \\ -3 & -3 & -3 \\ -1 & -2 & 1 \end{bmatrix} = 3 \left[-1 \left(-6-6\right) + 2\left(9-6\right) + 1\left(9+6\right) \right]$ row 3

$\Rightarrow 3 \left[12 + 6 + 15 \right] = 99$

(4) $|\det A| = \begin{vmatrix} -2.3 & -3 \\ -5.5 & 5.1 \end{vmatrix} = (-2.3)(5.1) - (-3)(-5.5)$

$= -11.73 - 16.5$

$= \underline{\underline{-28.23}} = 28.23$

does not contain (−) sign because it is the area of the parallelogram

(5) $\det(A) = -4$; $\det(B) = 1$; $\det(C) = 1$

$\det\left(A^T B^4 (C^{-1})^2\right) = \det A^T \cdot \det B^4 \cdot \det (C^{-1})^2$

$= \det A \, \det B^4 \, \dfrac{1}{\det(C)^2}$

$= (-4)(1)^4 \left(\dfrac{1}{1^2} \right)$

(6) C_{33} of $\begin{vmatrix} 6 & -8 & -6 \\ -4 & 7 & 0 \\ -2 & 7 & -6 \end{vmatrix} = (-1)^{3+3}\left(42 - 32\right)$

$\Rightarrow \underline{\underline{10}}$

① $\det A = \begin{vmatrix} 1 & a & 1 \\ 0 & 4 & b \\ 1 & c & -2 \end{vmatrix} = \begin{vmatrix} 1 & a & 1 \\ 0 & 4 & b \\ 0 & c-a & -3 \end{vmatrix} \Rightarrow 1\big(4(-3)-(c-a)(b)\big)$

$-12 - bc + ab$

$\underline{\underline{ab - bc - 12}}$

$x_2 = \dfrac{\det A_2}{\det A} = \dfrac{1}{ab-bc-12} \begin{vmatrix} 1 & 1 & 1 \\ 0 & -3 & b \\ 1 & 3 & -2 \end{vmatrix}$

$\det(A_2) \Rightarrow (6-3b) - (0-b) + (0-(-3))$

$6 - 3b + b + 3$

$-2b + 9$

Test 2

① $A = \underset{3\times 2}{\begin{pmatrix} 5 & 0 \\ 0 & 1 \\ 10 & 3 \end{pmatrix}}$; $A^T = \underset{2\times 3}{\begin{pmatrix} 5 & 0 & 10 \\ 0 & 1 & 3 \end{pmatrix}}$; $b = \underset{3\times 1}{\begin{pmatrix} 2 \\ 3 \\ 12 \end{pmatrix}}$

$A^T A = \begin{pmatrix} 125 & 30 \\ 30 & 10 \end{pmatrix}$

$A^T b = \begin{pmatrix} 130 \\ 39 \end{pmatrix}$

$x = (A^T A)^{-1} A^T b$

$(A^T A)^{-1} = \dfrac{1}{350} \begin{pmatrix} 10 & -30 \\ -30 & 125 \end{pmatrix} \begin{pmatrix} 130 \\ 39 \end{pmatrix} = \dfrac{1}{350} \begin{pmatrix} 130 \\ 975 \end{pmatrix}$

$= \begin{pmatrix} \cancel{\frac{130}{350}}^{26}_{7} \\ \cancel{\frac{975}{350}}^{39}_{14} \end{pmatrix} = \begin{pmatrix} \frac{26}{7} \\ \frac{39}{14} \end{pmatrix}$

② $\text{proj}_a(b) = \frac{a \cdot b}{a \cdot a} a$

$\text{proj}_{\vec{v}} \vec{w} = \frac{v \cdot w}{v \cdot v} \vec{v} = \frac{-10}{17} \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$

$\text{proj}_{\vec{u}} \vec{w} = \frac{\vec{u} \cdot \vec{w}}{\vec{u} \cdot \vec{u}} \vec{u} = \frac{7}{29} \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$

add them

$$\Rightarrow \begin{pmatrix} -\frac{20}{17} + \frac{14}{29} \\ \frac{-30}{17} - \frac{28}{29} \\ \frac{-20}{17} + \frac{21}{29} \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{-580 + 238}{493} \\ \frac{-870 - 476}{493} \\ \frac{-580 + 357}{493} \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{-342}{493} \\ \frac{-1346}{493} \\ \frac{-223}{493} \end{pmatrix}$$

③ $A = \begin{bmatrix} 1 & a & 4 \\ 0 & 4 & b \\ 1 & c & -2 \end{bmatrix}$; $\vec{b} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$

$$\det A = \begin{vmatrix} 1 & a & 4 \\ 0 & 4 & b \\ 0 & c-a & -6 \end{vmatrix} = 1(-24 - b(c-a))$$

$$= -24 - bc + ba$$

$$= ab - bc - 24$$

$$\det(A_2) = \begin{vmatrix} 1 & 4 & 4 \\ 0 & -1 & b \\ 1 & 1 & -2 \end{vmatrix} = 1(2-b) + 4b + 4$$

$$3b + 6$$

$$x_i = \frac{\det(A_2)}{\det A} = \frac{3b+6}{ab-bc-24}.$$

$x_1 = \frac{5}{3} x_2 + \frac{5}{3} x_3 + \frac{5}{3} x_4$

$x_2 = x_2$

$x_3 = x_3$

$x_4 = x_4$

④ $A = \begin{bmatrix} -3 & 5 & 5 & 5 \\ -15 & 25 & 25 & 25 \\ -6 & 10 & 10 & 10 \end{bmatrix} \xrightarrow[\substack{R_3 - 2R_1 \\ -\frac{1}{3}R_1}]{R_2 - 5R_1} \begin{array}{cccc} & ② & ③ & ④ \\ \left[\begin{matrix} 1 \\ 0 \\ 0 \end{matrix}\right. & \begin{matrix} -\frac{5}{3} \\ 0 \\ 0 \end{matrix} & \begin{matrix} -\frac{5}{3} \\ 0 \\ 0 \end{matrix} & \left.\begin{matrix} -\frac{5}{3} \\ 0 \\ 0 \end{matrix}\right] \end{array} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} \frac{5}{3} \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} \frac{5}{3} \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} \frac{5}{3} \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(7) $\vec{v}_1 = \vec{u} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$

$$\vec{v}_2 = \vec{v} - \frac{v \cdot u}{u \cdot u} u = \begin{bmatrix} 3-1 \\ -2-(-1) \\ 0-(-1) \\ 0-1 \\ 0-(-1) \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

Normalisiy

$$q_1 = \frac{1}{\|\vec{v}_1\|} \vec{v}_1 = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$q_2 = \frac{1}{\|\vec{v}_2\|} \vec{v}_2 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 2 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = 1/\sqrt{2}$$

(8) $A = \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\sqrt{3}/2 & 0 & 1/2 \end{bmatrix}$; $A^T = \begin{bmatrix} 1/2 & 0 & -\sqrt{3}/2 \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & 1/2 \end{bmatrix}$

$$A^T A = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ A is orthogonal

(9) $\det A = \begin{vmatrix} 5 & -2 & -1 & -1 \\ 0 & 0 & -4 & 0 \\ 5 & 2 & 1 & -1 \\ -3 & 3 & 1 & 5 \end{vmatrix} = 4 \begin{vmatrix} 5 & -2 & -1 \\ 5 & 2 & -1 \\ -3 & 3 & -5 \end{vmatrix} = 4(-3(4) - 3(0) - 5(20))$

$= 4(-12 - 100) = -448$

(10) $\det A = 4$; $\det(B) = -4$; $\det(C) = 5$

$\det(A^T B^4 (C^{-1})^4) = \det A^T \det B^4 \det(C^{-1})^4 = \det A \det B^4 \frac{1}{\det C^4} = 4(-4)^4 \frac{1}{5^4} = 4(256)\frac{1}{625}$

$= \frac{1024}{625} = 1.6384$

$$a_n = 2a_{n-1} + 3, \quad a_0 = 1$$

$a_0 = 1$

$a_1 = 2a_0 + 3$

$a_2 = 2a_1 + 3 = 2(2a_0 + 3) + 3 = 2(2a) + 6 + 3 = 2(2a) + 9$

$a_3 = 2a_2 + 3 = 2(2(2a_0 + 3) + 3) + 3 = 2(2(2a) + 9) + 3 = 2(2(2a)) + 21$

$a_4 = 2a_3 + 3 = 2(2(2(2a_0 + 3) + 3) + 3) + 3$

$$\sum_{j=1}^{n} = j^2 = \underline{n(n+1)(2n+1)}$$

$$\sum_{i=1}^{k}$$

Math 1250 Final

* Orthogonal if $v_i \cdot v_j = 0$ $\{\text{if } i \neq j\}$
* Orthogonal if $v_i \cdot v_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

$\Rightarrow 0 + 0 + \frac{1}{3} + 0 = \frac{1}{3}$

$\Rightarrow (-1, -1, 1, 0) \cdot (1, 1, 0, 1)$

$-1 - 1 + 0 + 0 = \frac{-2}{3}$

<u>Closest to the vector $\vec{v}$</u>

$\text{proj}_u(\vec{v}) = (\vec{v} \cdot \vec{w_1})\,\vec{w_1} + (\vec{v} \cdot \vec{w_2})\,\vec{w_2}$

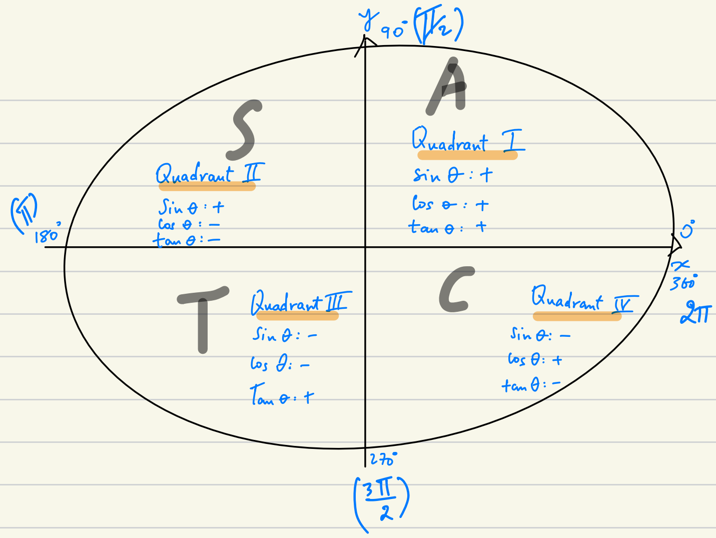
$123 \% 10 = 3$

$0 + 3 = 3$

$123 / 10 = 12$

$12 \% 10 = 2$

$f(x)$ / θ	Sin	Cos	Tan	Cot
30 ($\pi/6$)	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$	$\sqrt{3}$
45 ($\pi/4$)	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1
60 ($\pi/3$)	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
90 ($\pi/2$)	1	0	∞	0
180 (π)	0	-1	0	∞
360 (2π)	0	1	0	∞



Practice Lab 2

① $3 - \cot^2 x = 0 \iff \cot^2 x = 3 \iff \frac{1}{\tan^2 x} = 3$

$\tan^2 x = \frac{1}{3} \iff \tan x = \pm\sqrt{\frac{1}{3}}$

$\Rightarrow \tan(x) = \pm\frac{\sqrt{3}}{3}$

(a) In the first quadrant, $\tan(x)$ is positive

So, $\tan x = \frac{\sqrt{3}}{3} \iff x = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

(b) In the second quadrant, $\tan(x)$ is negative

So, $\tan(x) = -\frac{\sqrt{3}}{3} \iff x = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{5\pi}{6}$

(c) In third quadrant, $\tan(x)$ is positive so,

$\tan(x) = \frac{\sqrt{3}}{3} \Leftrightarrow x = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6} + \frac{6\pi}{6} = \frac{7\pi}{6}$

(d) In fourth quadrant, $\tan(x)$ is negative, so $\tan x = -\frac{\sqrt{3}}{3} = \frac{5\pi}{6} + \pi = \frac{5\pi + 6\pi}{6}$

$= \frac{11\pi}{6}$

2 $6\sin^2 x = 97\cos(x) - 44.$

since $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$6(1 - \cos^2 x) = 97\cos(x) - 44$

$6 - 6\cos^2 x = 97\cos x - 44$

$97\cos x - 44 - 6 + 6\cos^2 x = 0$

$6\cos^2 x + 97\cos x - 50 = 0$

Let $\cos x$ be α, then

$\underset{a}{6}\alpha^2 + \underset{b}{97}\alpha \underset{c}{-50} = 0$

$6\alpha^2 + 100\alpha - 3\alpha - 50 = 0$

$(2\alpha - 1)(3\alpha + 50) = 0$

$\Downarrow \qquad \Downarrow$

$2\alpha - 1 = 0 \qquad 3\alpha + 50 = 0$

$2\alpha = 1 \qquad \alpha = -\frac{50}{3}$ not possible since $-1 \le \cos x \le 1$

$\alpha = 1/2$

since α is $\cos x$, then

$\cos x = \frac{1}{2}$

$x = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\frac{5\pi}{3} - \frac{\pi}{3} = \frac{4\pi}{3}$

$y = \cos(14x+13)$; $y = \cos(x)$

$f(x) = \ln(x+7)$

$g(x) = e^{39x+19}$

$(g \circ f)(x) = e^{39[\ln(x+7)]+19}$

$f(x) = \frac{5x}{x+6}$

$g(x) = \cot(8x)$

$(g \circ f)(x) = \cot\left[8\left(\frac{5x}{x+6}\right)\right]$

$3 - \cot^2 x = 0$

$\cot^2 x = 3$

$\tan^2 x = \frac{1}{3}$

$f(x) = \sqrt{5 - 40x}$

$f(-1) = \sqrt{5 - 40(-1)}$

$= \sqrt{5 + 40}$

$\sqrt{45} = \underline{\underline{6.70}}$

$5 - 40(\cancel{1})$

$\sqrt{5 - 40}$

$\sqrt{-35}$

$f^{-1}(x) = \dfrac{5 - x^2}{40}$

$5 - x^2 \geq 0$

$5 \geq x^2$

$x^2 \leq 5$

$x \leq \sqrt{5}$

$x \leq 2.236$

$f^{-1}(x) = \frac{5}{40} - \frac{x^2}{40}$

$\frac{1}{8} - \frac{x^2}{40}$

$f(x) = \frac{67-9x}{2-x}$

$2-x \neq 0 \qquad 67-9x \geq 0$

$2 \neq x \qquad 67 \geq 9x$

$x \neq 2 \qquad x \geq \frac{67}{9}$

$y = \frac{67-9x}{2-x}$

$x = \frac{67-9y}{2-y}$

$x(2-y) = 67-9y$

$2x-2y = 67-9y$

$2x-67 = -9y+2y$

$2x-67 = -7y$

$y = \frac{2x-67}{-7}$

$y = \frac{67-2x}{7}$

$y = \frac{67-9x}{2-x}$

$y(2-x) = 67-9x$

$2y-yx = 67-9x$

$2y-67 = yx-9x$

$2y-67 = x(y-9)$

$x = \frac{2y-67}{y-9}$

$y = \frac{-67+2x}{-9+x}$

(3) $f(x) = e^{11-20x}$

$\ln y = \ln e^{11-20x}$

$\ln y = (11-20x)\ln e$

$\ln y = 11-20x$

$\ln y - 11 = -20x$

$x = \frac{11-\ln y}{20}$

$y = \frac{11-\ln x}{20}$

$$f(x) = e^{11-20x}$$

$$f^{-1}(x) = \frac{11 - \ln x}{20}$$

$$e^{7-6x} \geq 14$$

$$\ln e^{7-6x} \geq \ln(14)$$

$$7 - 6x \geq \ln(14)$$

$$-6x \geq \ln(14) - 7$$

$$x \leq \frac{7 - \ln(14)}{6}$$

$$\log_{104} x + \log_{104}(-5 + 84x) = 1$$

$$\log_{104} x(-5 + 84x) = 1$$

$$84x^2 - 5x = 104^1$$

$$84x^2 - 5x - 104 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(84)(-104)}}{2(84)}$$

$$= \frac{5 \pm 187}{168}$$

$$\frac{192}{168} = \frac{8}{7}$$

$$-\frac{182}{168} = -\frac{91}{84} \rightarrow \text{not valid}$$

$$\cos^{-1}\left(\cos\left(\frac{\pi}{27}\right)\right)$$

$$y = \sqrt{5 - 40x}$$

$$x = \sqrt{5 - 40y}$$

$$x^2 = 5 - 40y$$

$$x^2 - 5 = -40y$$

$$y = \frac{5 - x^2}{40}$$

5

0002

$P1 \Rightarrow x$

$P2 \rightarrow y$

0001

8

ℓ

output

5

0001

$P1 \rightarrow x$

$P2 \rightarrow y$

0002

8

BEFORE

* Values = $x = 5.5$; $y = 8.1$

* Value address = $x =$ 0x16d7634c8 ; $y =$ 0x16d76
34c4

* Pointer address = $x =$ 0x16d7634c4 ; $y =$ 0x16d7634c4

0x16d7634c8 0x16d76
34c4

0x16d7634c4 0x16d7634c

$g = 42$

$x = \& g$

$k = 0 \rightarrow 9$

① `*(arr + 0) = 11 - *(arr + 0);`

`a[a[0]]`

`a[4]`

`++a[1];`

`a[1] = a[1] + 1;`

$8 + 1$

`a[1] = 9;`

Midterm schedule (after reading week)

* Comp 1410 (Midterm 1) => Saturday, October 26th 2024 (5:00 pm - 7:00 pm)
Erie 3123

* Fren 1410 => Thursday, October 24th 2024 (13:00 -> 14:00) Dillon Hall 255

* Comp 2650 => October 21st (4 pm - 5:15 pm) odette building 104

* Math 1720 => November 2nd 10:00 am on Mobius

Priority

1. Comp 2650
2. French 1410
3. Comp 1410
4. Math 1720

array [3][2]

$$\begin{array}{cc} 00 & 01 \\ 1 & 2 \\ 10 & 11 \\ 3 & 4 \\ 20 & 21 \\ 5 & 6 \end{array} \rightarrow \begin{array}{ccc} 00 & 10 & 20 \\ 1 & 3 & 5 \\ 01 & 11 & 21 \\ 2 & 4 & 6 \end{array}$$

arr(i)(j) $\rightarrow$ arr 2 (j)(i)

arr[10] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

arr2[2][5] = {{1, 2, 3, 4, 5}, {6, 7, 8, 9, 10}}

10/2

rows = 40

cols = 5

k = 3

j = 2

i * cols + j

3 × 5 + 2

15 + 2

17

4 × 5 + 5

/ 20 20

① (110110)

$UBI = (110110)_2 = (32+16+4+2) \Rightarrow \begin{array}{r} 32 \\ 22 \\ \hline 54 \end{array} \Rightarrow (54)_{10}$

(digit positions under 110110: 5 4 3 2 1 0)

$1CF = (110110)_2 \Rightarrow -(001001)_2 \Rightarrow -(8+1) = (-9)_{10}$

(digit positions under 001001: 5 4 3 2 1 0)

$2CF = (110110) = -(001001 + 000001) \Rightarrow -(001010) = -(8+2) = (-6)_{10}$

(digit positions under 001010: 5 4 3 2 1 0)

SMF (Standard Modified Form)

$$\begin{array}{r} 0101 \\ 0011 \\ \hline 1000 \end{array}$$

$$\begin{array}{l}
57 \,\lfloor 2 \\
\underline{56} \quad 28 \,\lfloor 2 \\
1 \quad \underline{28} \quad 14 \,\lfloor 2 \\
\quad 0 \quad \underline{14} \quad 7 \,\lfloor 2 \\
\quad\quad 0 \quad \underline{6} \quad 3 \,\lfloor 2 \\
\quad\quad\quad 1 \quad \underline{2} \quad 1 \,\lfloor 2 \\
\quad\quad\quad\quad 1 \quad 0 \quad 0 \\
\quad\quad\quad\quad\quad \overline{1}
\end{array}$$

$(111001)_2 = (000110)_2 = (000111)_2$

$$\begin{array}{r} 1\ \cancel{1}0110\cancel{1}\cancel{1}0 \\ -\ 11010011 \\ \hline 01100001 \end{array}$$

because $x+(-y)$
$\Rightarrow x-y$.

$$\begin{array}{r} 1\ \cancel{1}0110\cancel{1}\cancel{1}0 \\ -\ 01010011 \\ \hline 11100001 \end{array}$$

$\underline{x+1000}-1000\ \underline{-y}$

$-1000+ans$

$-(1000-ans)$

$(00110100)_2 = (32+16+4)_{10} = (52)_{10}$

(-83)

$64+16+2+1$

$(11010011)_2 = (128+64+16+2+1) = -(211)$

16
4 x
64
2
128

$-(1)0101010) \Rightarrow -(32+8+2) = -(42)_{10}$

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = \frac{\ln 1+0}{0}$$

$Z'Y'X' =$ mo

$Z'Y'X =$

$Z'YX =$

$ZY'X =$

$ZYX' =$

$2^4 = 16 \quad 2$

8

$01 - 2's(01)$

10
1
$\overline{11}$

$1\cancel{1}01$

11

$-3 = 11\,(0)$

$3 = (11) = (00) = (01)$

$\cancel{1}00$ $01 = 10 \quad 11$

-01
$\overline{11}$

$1\cancel{1}00$ 1101
11 10
$\overline{101}$ $\overline{1111}$

$+01$
$0,0$
11
100

$\cancel{10}$
11
$\overline{(1)11}$

11 $\cancel{1}00$ $\cancel{1}00$
$10\;(01)$ -11
$\overline{110\;10}$ $\overline{101}$

11
$\overline{00}$

1101
11
$\overline{110}$

$\cancel{10}$
11
$\overline{111}$

1

$$Y = \frac{18x^{74} + 67x^{73} - 13}{31x^{74} + 49x^{72} + 60.}$$

$$x^{74}\left(\frac{49x^{72}}{x^{74}}\right)$$

$$49x^{72-74}$$

$$49x^{-2}$$

$$49(x^2)^{-1}$$

$$\frac{49}{x^2}$$

$$y = \frac{87e^{83x} + 76e^{-18x} + 20}{10e^{83x} - 29e^{-18x} + 48}$$

$$e^{83x}\left(\frac{29e^{-18x}}{e^{83x}}\right)$$

$$29e^{-18x-83x}$$

$$29e^{-101}$$

$$\frac{29}{e^{101x}}$$

$$\frac{87e^{83x}}{e^{-18x}} + 12$$

$$87e^{101x}$$

0 1 2 3

$$
\begin{array}{r}
0\,1\,0\,1 \\
+\;1\,0\,1\,0 \\
\hline
1\,1\,1\,1 \\
+\;0\,1\,0\,1 \\
\hline
0\,1\,0\,2
\end{array}
$$

$$(\sec x)' = \left(\frac{1}{\cos x}\right)' = (\cos^{-1} x)'$$

$$(\sec x)'$$

$$\lim_{x \to 0} \frac{\sec(x+h) - \sec x}{h}$$

$$\frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h}$$

$$\frac{\cos x - \cos(x+h)}{h \cos x \cos(x+h)}$$

$$\frac{\cos x - (\cos x \cos h + \sin x \sin h)}{h \cos x (\cos x \cos h + \sin x \sin h)}$$

$\cos x$

$$(\ln x)' =$$

$(6/2)/3$ $\qquad$ $6/(2/3)$

3 $\qquad$ $6 : \frac{2}{3}$

$1 \times$ $\qquad$ $6 \times \frac{3}{2}$

$\frac{18}{2} \to 9$

$9c + d = 5$

$c + 18d = 5$

$d = 5 - 9c$

$c + 18(5 - 9c) = 5$

$c + 90 - 162 = 5$

$c = 5 - 90 + 162$

$c = 77$

$d = 5 - 9(c)$

$= 5 - 9(77)$

$5 - 693$

$d = -688$

⑧ $f(x) = \frac{3}{x^2}$

$f(x+h) = \frac{3}{(x+h)^2}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\frac{3}{(x+h)^2} - \frac{3}{x^2}}{h}$$

$$\lim_{h \to 0} \frac{3x^2 - 3(x+h)^2}{h x^2 (x+h)^2}$$

$$\lim_{h \to 0} \frac{3\left[x^2 - (x+h)^2\right]}{h x^2 (x+h)^2}$$

$$\lim_{h \to 0} \frac{3(\cancel{x^2} - \cancel{x^2} - 2xh - h^2)}{h x^2 (x+h)^2}$$

$$\lim_{h \to 0} \frac{3(-2xh - h^2)}{h x^2 (x^2 + 2xh + h)}$$

$$\lim_{h \to 0} \frac{-3\cancel{h}(2x + h)}{\cancel{h} x^2 (x+h)^2}$$

$$\lim_{h \to 0} \frac{-3(2x+h)}{x^2 (x+h)^2} \Rightarrow \frac{-3(2x+0)}{x^2 (x+0)^2} \Rightarrow \frac{-3(2\cancel{x})}{\cancel{x^4} x^3} = \frac{-6}{x^3}$$

9)

$$f(x) = \sqrt{x+23}$$

$$f(x+h) = \sqrt{(x+h)+23}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\sqrt{(x+h)+23} - \sqrt{x+23}}{h}$$

$$\lim_{h \to 0} \frac{\left(\sqrt{x+h+23} - \sqrt{x+23}\right) \cdot \left(\sqrt{x+h+23} + \sqrt{x+23}\right)}{h\left(\sqrt{x+h+23} + \sqrt{x+23}\right)}$$

$$(a-b)(a+b)$$

$$a^2 - \cancel{ab} + \cancel{ab} - b^2$$

$$a^2 - b^2$$

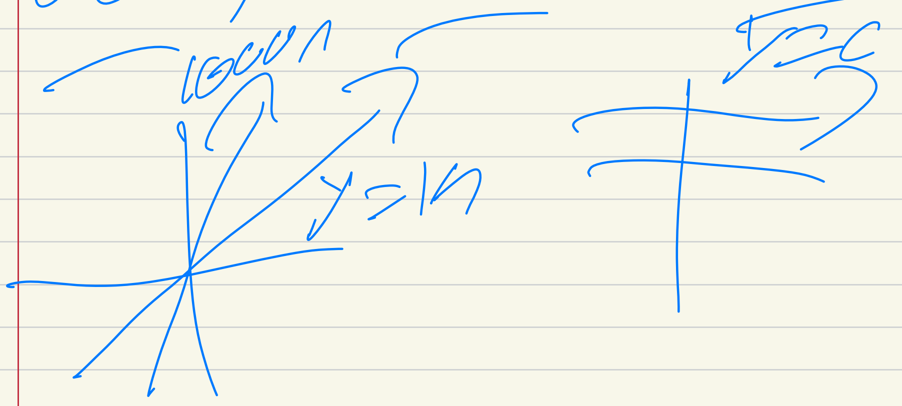
array [6] [5]

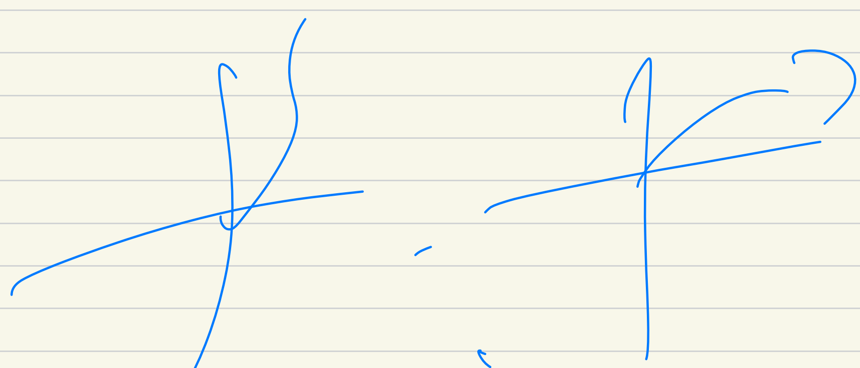
오) ↓

5 - 1

(a) $\sum_{j=0}^{n} \sum_{k=0}^{n} O(1)$

$O(1)$, $O(n)$, $O(n^2)$





$$\sum_{j=0}^{1000n} \sum_{k=0}^{n} O(1)$$

$$\sum_{j=0}^{c000}\sum_{k=0}^{N} O(1)$$

$$= \sum_{j=0}^{1000} N \cdot O(1)$$

$$= \cancel{1000 \cdot O(N)} \; O(1)$$

$$= O(N)$$

$$\sum_{k=0}^{N} 1 = N \cdot c_i$$

$$\sum_{k=j}^{N} 1 = (N - j + 1)$$

$(j+1)(j+1)(j+2)$

$\Sigma(j,\ j+1,\ j+2,\ j+3)$

$N=4$

$\Sigma = (N - j + 1) 1$

$(4 - 2 + 1)$

$\Sigma = (2+1) 1$
3

$\Sigma = 3$

$j = j/2$

$0/2$

$j = 0$

Loops

for (i = 0 ; i < 2, i ++) n, 1000

→ print ("ronard");

{ → for 0 → 3
  → for 0 → 3

$$\rightarrow \sum_{j=0}^{12} \sum_{k=0}^{12} O(1)$$

low
log → $\log_a b^n = n \log_a b$

$$\sum_{j=0}^{12} 12 \; O(1)$$

$12 \times 12 \; O(1)$

$144 \; O(1) = O(1)$

$$\sum_{d=0}^{1{,}000} \sum_{k=0}^{N} O(1)$$

$$\sum_{j=0}^{1000} N \; O(1)$$

$1000 \cdot N \; O(1)$

example

[5, 4, 3, 2, 1]

```
{
  i = 0
  pos = 0
  {
    d = 1
    a[1] < a[0]
    {
      pos = 1
    }
  }
  swap(& a[0], & a[1])
}
```

5 4 5 4

sub max = max(N, 4)

$$\log_a \frac{a}{x} = t$$

$$\log_a x \cdot \log_x a = \log t$$

$$\log_x a \cdot \log a = \log t$$

$\log_b a = c$

$\boxed{a = b^c}$

$\frac{1}{\cos x} = y$

$1 = y \cos x$

$a = b^{\log_b a}$

$1 =$

$\bar{a} + \overline{(c\, d\, e)}$

$5^{\sec x} = y$

$\ln 5^{\sec x} = \ln y$

$(\ln x)' = 1/x$

$(\sec x \ln 5)' = (\ln y)'$

$y' \cdot \frac{1}{y}$

$\sec x \tan x \ln 5 + 0 \cdot \sec x = y' \cdot \frac{1}{y}$

0

$$\sec x \tan x \ln 5 = y' \cdot \frac{1}{y}$$

$$y \cdot \sec x \tan x \ln 5 = y'$$

$$5^{\sec x} \cdot \sec x \tan x \ln 5 = y'$$

$$[\sin^{-1} x] = y$$

$$\sin^{-1} y = x$$

$$\frac{d}{dy}(y)' = \frac{d}{dy}(\sin x)$$

$$1 = x' \cos x$$

$$1 = y' \cos y$$

$$y' = \frac{1}{\cos(\sin^{-1} x)} = \frac{1}{\sqrt{1 - x^2}}$$

(*) $\cos^2 y = 1 - \sin^2 y$

$$\cos y = \sqrt{1 - \sin^2 y}$$

$$= \sqrt{1 - \sin^2(\sin^{-1} x)}$$

$$\sqrt{1 - x^2}$$

$$\tan^{-1} x = y$$

$$(x)' = (\tan y)'$$

$$1 = y' \sec^2 y$$

$$y' = \frac{1}{\sec^2(\tan^{-1} x)}$$

$$y' = \frac{1}{1 + [\tan(\tan^{-1} x)]^2}$$

$$y' = \frac{1}{1 + x^2}$$

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin(A+B) + \sin(A-B)$$

$$\sin A \cos B + \cancel{\sin B \cos A} + \sin A \cos B - \cancel{\sin B \cos A}$$

$$\frac{\cancel{2} \sin A \cos A}{\cancel{2}} = \frac{1}{\cancel{2}}[\sin(A+B) + \sin(A-B)]$$

$x^2 - 10x + 1$

$\text{delta} = b^2 - 4ac$

$= (-10)^2 - 4(1)(1)$

$= 100 - 4 = 96$

$x_1 = \dfrac{10 + \sqrt{96}}{2} = \dfrac{\cancel{10}^{5} + \cancel{4}^{2}\sqrt{6}}{\cancel{2}} = 5 + 2\sqrt{6}$

$= 5 - 2\sqrt{6}$

$z'y'x' + z'yx' + zy'x' + zyx'$

$z'x'(y' + y) + zx'(y' + y)$

$z'x'(1) + zx'(1)$

$x'(z' + z)$

$x'(1)$

$= x'$

$$y'x + yx'$$

$$z'y'x' + z'yx' + zy'x' + zyx'$$

$$C'D + CD'$$

C	D	C'	D'	C'D	+ CD'	f	
0	0	1	1	0	0	0	m_0
0	1	1	0	1	0	1	m_1
1	0	0	1	0	1	1	m_2
1	1	0	0	0	0	0	m_3

$$A'B'CD + A'BCD + ABC'D + ABCD + ABCD' + A'BCD$$

$$A'CD(B'+B) + ABD(C'+C) + ABC(D'+D)$$

$$A'CD + ABD + ABC$$

if $\overline{\overline{X}Y} = X\overline{Y}$, then $\overline{X}Y + X\overline{Y} = 1$

X	Y	$\overline{X}$	$\overline{Y}$	$\overline{X}Y$	$\overline{\overline{X}Y}$ $\neq$	$X\overline{Y}$
0	0	1	1	0	1	0
0	1	1	0	1	0	0
1	0	0	1	0	1	1
1	1	0	0	0	1	0

$$y - b = m(x - a)$$

$$y = -\frac{9}{5}x - \frac{2}{5}$$

$$-\frac{9}{5}x$$

$$y' = \left(-\frac{9}{5}x\right)'$$

$$0 + -\frac{9}{5}$$

$$0 - \frac{9}{5} = \frac{-9}{5} = -\frac{9}{5}$$

$$5xe^{y}-14yx^{3}+8y^{7}=10$$

$$5e^{y}+5xy'e^{y}-14y'x^{3}-42yx^{2}+56y'y^{6}=0$$

$$y'[5xe^{y}-14x^{3}+56y^{6}]=-5e^{y}+42yx^{2}$$

② $\ln(23y)\cos x = y^{6}+20xy$

$$\frac{\cancel{23}y'}{\cancel{23}y}\cos x-\ln 23y\sin x = 6y^{5}y'+20y+20xy'$$

$$y'=\frac{20y^{2}+y\ln(23y)\sin x}{\cos x-6y^{6}-20xy}$$

$$y=(x^{4}+1)^{13x+10}$$

$$y = (x^4+1)^{13x+10}$$

$$\ln y = (13x+10)\ln(x^4+1)$$

$$\frac{y'}{y} = 13\ln(x^4+1) + (13x+10)\frac{4x^3}{x^4+1}$$

$$y' = (x^4+1)^{13x+10}$$

$$y - 1 = 10\ln 9(x-0)$$

$$y = 10\ln 9\, x + 1$$

$$y = 9^{\tan 10x}$$

$$\ln y = \tan 10x \ln 9$$

$$\frac{y'}{y} = 10\sec^2 10x \ln 9.$$

$$y' = 9^{\tan 10x} \cdot 10\sec^2 10x \cdot \ln 9$$

$$10 \cdot 9^0 \cdot 1 \cdot \ln 9$$

$$10 \cdot \ln 9$$

first last

0 1 2 3 4

4 5 9 3 6

6 5 9 3 4

9 5 6 3 4

4 5 6 3 9

$\sum_{0}^{12} \sum_{k=0}^{12} = 12 \times 12 \; O(1) = 144 \; O(1) = O(1)$

(b) $\sum_{j=0}^{1000} \sum_{k=0}^{N} O(1)$

(d) $\sum_{j=0}^{N} \sum_{k=0}^{N}$

$(i-1)\; O(1) = O(i)$

$\sum_{j=0}^{i} \sum_{i=0}^{n-1} O(1)$

$\sum_{0}^{n-1} O(1) + O(n-1)$

$(i-0+1)$ n $O(1)$

$(i+1)$

$O(n-1)$

$O(n^2)$

$O(1) + O(n)$

$O(1)(O(n))$

5, 4, 3, 2, 1

n=5

4

$O(1) + T(n-1)$

0 1

1

2^2

{5, 4, 3, 2, 1}

i = 0

j = 0-1 = -1

arr[-1] > temp

z/yx	00	01	10	11
0	m_0 1	m_1 0	m_2 0	m_3 0
1	m_4 1	m_5 1	m_6 0	m_7 0

$\Pi = (z'+y'+x)(z'+y+x')(z'+y+x)(z+y+x')(z+y+x)$

flip

[illegible] → [illegible]

$8 \to 7$

$yz + zy'$

$z(y+y')$

$0 \quad 1 = 1$

$1 \quad 0 = 1$

$z(1) + z = z$

$0 \quad 0 = 0$

$1+1=1$

$xy' + yx$

$(x, y, z) = \sum(1, 2, 4, 5)$

$3 = 180 \rightarrow 9 \rightarrow$

20 min/Q

DAYS / CLASS	Monday	Tuesday	Wednesday	Thursday	Friday
COMP 2560	(LAB) COMP-2560 11:30 AM → 12:50 AM	(LEC) COMP-2560 8:50 am → 9:50 am		(LEC) COMP-2560 8:50 am → 9:50 am	
MATH 1020	(LEC) Math-1020 8:30 am → 9:20 am		(LEC) Math-1020 8:30 am → 9:20 am (LAB) MATH-1020 5:00 pm → 6:50 pm		
MATH 1730		Math 1730 (LEC) 10:00 → 11:20 am		Math 1730 (LEC) 10:00 → 11:20 am	Math-1730 (LAB) 10:30 am → 11:20 am ??
COMP 2540	(LEC) COMP-2540 1:00 PM – 2:20 PM (LAB) COMP-2540 4:00 PM → 5:20 pm		(LEC) COMP-2540 1:00 PM – 2:20 PM		
STAT 2910	STAT-2910 (LEC) 10:00 AM → 11:20 AM		STAT-2910 (LEC) 10:00 AM → 11:20 AM		STAT-2910 (LAB) ?? 10:30 AM → 11:20 AM

?? => means time conflict. So, you are gonna have to choose one.