* ngabira@uwindsor.ca
* Student ID = 110182078
* Names: Trésor Dan Minor NGABIRANO

Lec_05

① (a) Commutative : $x \oplus y = y \oplus x$

Exclusive-OR (XOR) $\oplus$

XOR ; * $x \oplus y = x\bar{y} + \bar{x}y$

* $y \oplus x = y\bar{x} + \bar{y}x = \bar{x}y + x\bar{y}$

Truth table

x	y	$\bar{x}$	$\bar{y}$	$x\bar{y}$	$\bar{x}y$	$x\oplus y$	$y \oplus x$
0	0	1	1	0	0	0	0
0	1	1	0	0	1	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	0	0	0

Therefore, XOR is Commutative.

(b) Associative = $x \oplus (y \oplus z) = (x \oplus y) \oplus z = x \oplus y \oplus z$

XOR * $x \oplus (y \oplus z) = \bar{x}(y\oplus z) + x\overline{(y \oplus z)} = \bar{x}(\bar{y}z + y\bar{z}) + x(\overline{\bar{y}z + y\bar{z}})$

1st Truth table

x	y	z	$\bar{x}$	$\bar{y}$	$\bar{z}$	$\bar{y}z+y\bar{z}$	$\overline{\bar{y}z+y\bar{z}}$	$\bar{x}(\bar{y}z+y\bar{z})$	$x(\overline{\bar{y}z+y\bar{z}})$	$x\oplus(y\oplus z)$ or $\bar{x}(\bar{y}z+y\bar{z})+x(\overline{\bar{y}z+y\bar{z}})$
0	0	0	1	1	1	0	1	0	0	0
0	0	1	1	1	0	1	0	1	0	1
0	1	0	1	0	1	1	0	1	0	1
0	1	1	1	0	0	0	1	0	0	0
1	0	0	0	1	1	0	1	0	1	1
1	0	1	0	1	0	1	0	0	0	0
1	1	0	0	0	1	1	0	0	0	0
1	1	1	0	0	0	0	1	0	1	1

2nd truth table $\Rightarrow (x\oplus y)\oplus z = \overline{(x\oplus y)}z + (x \oplus y)\bar{z} \Rightarrow (\overline{\bar{x}y + x\bar{y}})z + (\bar{x}y + x\bar{y})\bar{z}$

$\bar{x}y$	$x\bar{y}$	$\bar{x}y+x\bar{y}$	$(\overline{\bar{x}y+x\bar{y}})$	$(\overline{\bar{x}y+x\bar{y}})z$	$(\bar{x}y+x\bar{y})\bar{z}$	$(x\oplus y)\oplus z$
0	0	0	1	0	0	0
0	0	0	1	1	0	1
1	0	1	0	0	1	1
1	0	1	0	0	0	0
0	1	1	0	0	1	1
0	1	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	1	1	0	1

Conclusion

Therefore, since

$\bar{x}(\bar{y}z+y\bar{z}) + x(\overline{\bar{y}z+y\bar{z}}) = (\overline{\bar{x}y+x\bar{y}})z + (\bar{x}y+x\bar{y})\bar{z}$

OR $x\oplus(y\oplus z) = (x\oplus y)\oplus z$

We can conclude that XOR is Associative

② Prove that XOR is not distributive over AND;

i.e $x \oplus (yz) \neq (x \oplus y)(x \oplus z)$

$\Rightarrow x(\overline{yz}) + \bar{x}(yz) \neq (x\bar{y} + \bar{x}y)(x\bar{z} + \bar{x}z)$

Truth table

x	y	z	$\bar{x}$	$\bar{y}$	$\bar{z}$	$\overline{yz}$	$x(\overline{yz})$	$\bar{x}(yz)$	$x \oplus (yz)$	$x\bar{y}$	$\bar{x}y$	$x\bar{z}$	$\bar{x}z$	$(x\bar{y}+\bar{x}y)(x\bar{z}+\bar{x}z)$ or $(x \oplus y)(x \oplus z)$
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
0	0	1	1	1	0	1	0	0	0	0	0	0	1	0
0	1	0	1	0	1	1	0	0	0	0	1	0	0	0
0	1	1	1	0	0	0	0	1	1	0	1	0	1	1
1	0	0	0	1	1	1	1	0	1	1	0	1	0	1
1	0	1	0	1	0	1	1	0	1	1	0	0	0	0
1	1	0	0	0	1	1	1	0	1	0	0	1	0	0
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0

Therefor, $x \oplus (yz) \neq (x \oplus y)(x \oplus z)$ as shown above

(not equal or different)