LAB 5

① $A = \{2, 4, 7\}$

$B = \{2, 7\}$

$C = \{4, 7\}$

$D = \{4, 6, 9\}$

(a) Let $X = A\ \{2, 4, 7\}$

$Y = B\ \{2, 7\}$

Then $y \subseteq A$

(b) Let $x = B\ \{2, 7\}$

$Y = C\ \{4, 7\}$ Then $Y \nsubseteq A$

OR

We can also use a table like following row to column:

	A	B	C	D
A		No	No	No
B	Yes		No	No
C	Yes	No		No
D	No	No	No	

There are two ways for inclusion

$y \subset x$: Y is included in x

$y \subseteq x$: y is properly included to x (or y might be equal to x)

③ What is the cardinality of each of these sets. (we put "| |")

(a) $|\phi| = 0$ where ϕ denotes an empty set.

(b) $|\{0\}| = 1$

(c) $|\{\phi, \{\phi\}\}| = 2$

(d) $|\{\underbrace{\phi}_{1}, \underbrace{\{\phi\}}_{2}, \underbrace{\{\phi, \{\phi\}\}}_{3}\}| = 3$

④ Find the truth set of each of these predicates when the domain of x is the set of integers, $\mathbb{Z}$.

(a) $P(x) : x^3 \geq 1$

$\{x \in \mathbb{Z} \mid P(x)\}$; $\{x \in \mathbb{Z} \mid x^3 \geq 1\}$; $\{1, 2, 3, 4, \cdots\}$

(b) $Q(x) : x^2 = 2$

$\{x \in \mathbb{Z} \mid Q(x)\}$; $\{x \in \mathbb{Z} \mid x^2 = 2\}$; ϕ

(c) $R(x) : x \leq x^2$

$\{x \in \mathbb{Z} \mid R(x)\}$; $\{x \in \mathbb{Z} \mid x \leq x^2\}$; Truth set $\{\cdots, -2, -1, 0, 1, 2, 3, \cdots\} = \mathbb{Z}$

(d) $R(x) = 2^x < x^2$

$\{x \in \mathbb{Z} \mid R(x)\}$; $\{x \in \mathbb{Z} \mid 2^x < x^2\}$; Truth set $= \{\cdots, -3, -2, -1, 3\}$

⑥ Using set builder notation, describe the set of integers last digit is 7. Hint what is x mod 10

Set of +ve integer $\{7, 17, 27, 37, 47, \cdots\} = \{x \in \mathbb{Z}^+ \mid x \bmod 10 = 7\}$

$\{x \mid x \in Z^- \wedge x \bmod 10 = 3\}$ (For negative numbers)

Note When dividing a with b

$a = qb + r$ where $0<$

⑦ $A = \{1, 2, 3\}$. Find the elements of the power set of A. What's the cardinality of P(A).

$P(A) = 2^{|A|} = 2^3 = 8 = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$|P(A)| = 8$ (Cardinality)

⑧ Show the set of even integers is a subset of the set Z of all integers.

Solution

Let $E = \{x \mid x = 2k \wedge x \in Z\}$.

$I = \{x \mid x \in Z\}$

$E \subseteq I$

$E = \{x \mid x = 2k\}$

Through simplification

$$\frac{p \wedge a}{\therefore p}$$

③ $A = \{e, g, f\}$; $B = \{x, y\}$; $C = \{2, 3\}$

$A \times B = \{(e, x, 2), (e, x, 3), (e, y, 2), (e, y, 3),$
$(g, x, 2), (g, x, 3), (g, y, 2), (g, y, 3)$
$(f, x, 2), (f, x, 3)\ (f, y, 2), (g, y, 3\}$

(b) $C \times B \times C = \{(2,x,2), (2,x,3), (2,y,2), (2,y,3), (3,x,2), (3,x,3), (3,y,2), (3,y,3)\}$

(5) (a) $A \cup B = \{a,b,c,d,e,f,g,h\}$

(b) $A \cap B = \{a,b,c,d,e\}$

(c) $A - B = \phi$

(d) $B - A = \{f,g,h\}$

(9) (a) $A \cap B \subseteq A$

1. $x \in A \cap B$
2. $x \in A \wedge x \in B$ (def. of intersection)
3. $x \in A$ (Simplification)

(b) $A \subseteq (A \cup B)$

1. $\{x / x \in A\}$ (Assumption)
2. $\{x \mid x \in A \vee x \in B\}$ (Addition)
3. $\{x / x \in A \cup B\}$ (Def. of union)

(11) Prove $A - B = A \cap \bar{B}$

(*) $\{x / x \in (A-B)\}$ (Assumption)

$\{x \mid x \in A \wedge x \notin B\}$ (Def. of substraction)

$\{x / x \in A \cap \bar{B}\}$ (Def. of complement)

$A - B = A \cap \overline{B}$

$A - B \subseteq A \cap \overline{B}$ and $A \cap \overline{B} = A - B$

$\{x \mid x \in A - B\}$

$\{x \mid x \in A \wedge x \notin B\}$ (Def. of difference)

$\{x \mid x \in A \wedge x \in \overline{B}\}$ (Def. of complements)

$\{x \mid x \in A \cap \overline{B}\}$ (Def. of intersection)

$A \cap \overline{B}$

A	B	$\overline{B}$	$A - B$	$A \cap \overline{B}$
1	1	0	0	0
1	0	1	1	1
0	1	0	0	0
0	0	1	0	0

Extra: $A \cup (A \cap B) = A$

$A \cup (A \cap B) = A$

$\{x \mid x \in A \cup (A \cap B)\}$

$\{x \mid x \in A \vee x \in (A \cap B)\}$ (Def. of union)

$\{x \mid x \in A \vee (x \in A \wedge x \in B)\}$ (Def. of intersection)

$\{x \mid (x \in A \vee x \in A) \wedge (x \in A \vee x \in B)\}$ (distribution)

$\{x \mid x \in A \wedge (x \in A \vee x \in B)\}$

$\{x \mid x \in A\}$. Simplification

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(1) a) $f(n) = n-1$ (one-to-one)

b) $f(n) = n^2+1$ (none)

eg $K = n^2+1$

$-1+5 = n^2+\cancel{1}-1$

$n^2 = 4$

$n = \pm 2$

(c) $f(n) = n^3$ (one-to-one)

(d) $f(n) = \lceil n/2 \rceil$ (onto)

eg $f(1) = \lceil 1/2 \rceil = \lceil 0.5 \rceil = 1$

$f(2) = \lceil 2/2 \rceil = \lceil 1 \rceil = 1$

(3) $f(x) = x^2+2$; $g(x) = x+3$

$f(x)+g(x) = x^2+2+x+3$

$= x^2+x+5$

$fg = f(x)\,g(x) = (x^2+2)(x+3) = x^3+3x^2+2x+6$

② * $f \circ g = f(g(x)) = (x+3)^2 + 2 = x^2 + 6x + 9 + 2 = x^2 + 6x + 11$

* $g \circ f = g(f(x)) = (x^2 + 2) + 3 = x^2 + 5$

④ $f(x) = x^3 + 2$

$y = x^3 + 2$

$x^3 = y - 2$

$x = \sqrt[3]{y-2}$

$f^{-1}(x) = \sqrt[3]{x-2}$

⑤ The image of $S = \{b, c, d\}$

$S = \{1, 4, 1\}$

⑥ $A = \{a, b, c\}$; $B = \{1, 2, 3\}$

$f(a) = 2$; $f(b) = 3$; $f(c) = 1$

$f^{-1}(2) = a$; $f^{-1}(3) = b$; $f^{-1}(1) = c$

⑦ (a) $a_n = -3a_{n-1}$; $a_0 = -2$

$a_1 = -3a_0 = -3(-2) = 6$

$a_2 = -3a_1 = -3(6) = -18$

$a_3 = -3a_2 = -3(-18) = 54$

$a_4 = -3a_3 = -3(54) = -162$

$a_5 = -3a_4 = -3(-162) = 486$

(b) $a_n = 2a_{n-1}^2 + 3$; $a_0 = 1$

$a_1 = 2a_0^2 + 3 = 2(1)^2 + 3 = 5$

$a_2 = 2a_1^2 + 3 = 2(5)^2 + 3 = 53$

$a_3 = 2a_2^2 + 3 = 2(53)^2 + 3 = 5621$

$a_4 = 2a_3^2 + 3 = 2(31{,}595{,}641)^2 + 3$

$a_5 = 2a_4^2 + 3 = 2[2(31{,}595{,}641)^2 + 3] + 3$

(c) $a_n = 2a_{n-1} - a_{n-2} + 2a_{n-3}$; $a_0 = 1$; $a_1 = 1$; $a_2 = 2$

$a_1 = 1$
$a_2 = 2$
$a_3 = 2(2) - 1 + 2(1) = 5$
$a_4 = 2(5) - 2 + 2(1) = 10$

$a_5 = 2(10) - 5 + 2(2) = 19$

8) Find the solution to each of these recurrence relations

(a) $a_n = -a_{n-1}$; $a_0 = 5$

$a_1 = -5$; $a_2 = 5$; $a_3 = -5$; $\{a_n = (-1)^n 5\}$;

(b) $a_n = a_{n-1} + 3$; $a_0 = 1$

$a_0 = 1$

$a_1 = a_0 + 3$

$a_2 = a_0 + 3 + 3$

$a_3 = a_0 + 3 + 3 + 3$

$\vdots$

$a_n = a_0 + 3n$

$\{a_n = 1 + 3n$

9 (a) $\sum_{j=1}^{6} j^2 \Leftrightarrow 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 \Rightarrow 1 + 4 + 9 + 16 + 25 + 36$

$= 91$

(c) $\sum_{i=0}^{2} \sum_{j=1}^{3} ij \Leftrightarrow (0)(1) + (1)(2) + (2)(3)$

$= 0 + 2 + 6 = 8$

(b) $\sum_{k=50}^{100} k^2 \Leftrightarrow \underline{3825}$

10 (a) $\sum_{j \in S} j \Rightarrow 1 + 3 + 7 = 11$

(b) $\sum_{j \in S} j^2 \Rightarrow 1^2 + 3^2 + 7^2 = 1 + 9 + 49 = 59$

(c) $\sum_{j \in S} 1 \Rightarrow 1$

LAB 7

1, 3, 4, 5, 6, 8, 9, 11

(1)

i	n : 8	x : 9	action
1	$1 \leq 8 \equiv T$	$1 \neq 9 \equiv T$	$i = 1+1 \Rightarrow 2$
2	$2 \leq 8 \equiv T$	$3 \neq 9 \equiv T$	$i = 2+1 \Rightarrow 3$
3	$3 \leq 8 \equiv T$	$4 \neq 9 \equiv T$	$i = 3+1 \to 4$
4	$4 \leq 8 \equiv T$	$5 \neq 9 \equiv T$	$i = 5$
5	$5 \leq 8 \equiv T$	$6 \neq 9 \equiv T$	$i = 6$
6	$6 \leq 8 \equiv T$	$8 \neq 9 \equiv T$	$i = 7$
7	$7 \leq 8 \equiv T$	$9 \neq 9 \equiv F$	location = 7

return location

(b)

i	j	i<j	$m = \lfloor (i+j)/2 \rfloor$	9 > am	Action
1	8	1<8	$1+8 = 9/2 = 4$	$9 > 5 \equiv T$	$i = 5$
5	8	5<8	$5+8 = 13/2 = 6$	$9 > 8 \equiv T$	$i = 6+1 = 7$
7	8	7<8	$7+8 = 15/2 = 7$	$9 > 9 \equiv F$	$j = 7$
7	7	7<7			location = 7 return location

(2)

i	5, 1, 4, 2, 8
1	1, 4, 2, 5, 8
2	1, 2, 4, 5, 8
3	1, 2, 4, 5, 8
4	1, 2, 4, 5, 8
5	1, 2, 4, 5, 8

Finals Revision

(9) (a) $a_n = a_{n-1} + n$; $a_0 = 1$

$a_1 = 1 + 1 = 2 \quad -1 = 1$
$a_2 = 2 + 2 = 4 \quad -1 = 3$
$a_3 = 4 + 3 = 7 \quad -1 = 6$
$a_4 = 7 + 4 = 11 \quad -1 = 10$
$a_5 = 11 + 5 = 16 \quad -1 = 15$

Solution to recurrence

$$1 + \frac{n(n+1)}{2}$$