

Given any set  $U$  with  $U \neq \emptyset$ , the following are true.

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#### The Distributive Lattice Axioms

##### Idempotence

A1: For all  $X \subseteq U$ ,  $X \cap X = X$ .

O1: For all  $X \subseteq U$ ,  $X \cup X = X$ .

##### Commutativity

A2: For all  $X, Y \subseteq U$ ,  $X \cap Y = Y \cap X$ .

O2: For all  $X, Y \subseteq U$ ,  $X \cup Y = Y \cup X$ .

##### Associativity

A3: For all  $X, Y, Z \subseteq U$ ,  $(X \cap Y) \cap Z = X \cap (Y \cap Z)$ .

O3: For all  $X, Y, Z \subseteq U$ ,  $(X \cup Y) \cup Z = X \cup (Y \cup Z)$ .

##### Absorption

A4: For all  $X, Y \subseteq U$ ,  $X \cup (X \cap Y) = X$ .

O4: For all  $X, Y \subseteq U$ ,  $X \cap (X \cup Y) = X$ .

##### Distributivity

D1: For all  $X, Y, Z \subseteq U$ ,  $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$  and  $(Y \cup Z) \cap X = (Y \cap X) \cup (Z \cap X)$ .

D2: For all  $X, Y, Z \subseteq U$ ,  $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$  and  $(Y \cap Z) \cup X = (Y \cup X) \cap (Z \cup X)$ .

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#### The Boundary Axioms

##### Universal Complement

U1:  $U^c = \emptyset$ .

U2:  $\emptyset^c = U$ .

##### Identity

A5: For all  $X \subseteq U$ ,  $X \cap U = X$  and  $U \cap X = X$ .

O5: For all  $X \subseteq U$ ,  $X \cup \emptyset = X$  and  $\emptyset \cup X = X$ .

##### Complementation

A6: For all  $X \subseteq U$ ,  $X \cap (X^c) = \emptyset$  and  $X^c \cap X = \emptyset$ .

O6: For all  $X \subseteq U$ ,  $X \cup (X^c) = U$  and  $X^c \cup X = U$ .

##### Annihilator

A7: For all  $X \subseteq U$ ,  $X \cap \emptyset = \emptyset$  and  $\emptyset \cap X = \emptyset$ .

O7: For all  $X \subseteq U$ ,  $X \cup U = U$  and  $U \cup X = U$ .

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#### DeMorgan's Laws

DM1: For all  $X, Y \subseteq U$ ,  $(X \cap Y)^c = X^c \cup Y^c$ .

DM2: For all  $X, Y \subseteq U$ ,  $(X \cup Y)^c = X^c \cap Y^c$ .

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#### The Order Axioms

##### Reflexivity

I1: For all  $X \subseteq U$ ,  $X \subseteq X$ .

##### Antisymmetry

I2: For all  $X, Y \subseteq U$ , if  $X \subseteq Y$  and  $Y \subseteq X$ , then  $X = Y$ .

##### Transitivity

I3: For all  $X, Y, Z \subseteq U$ , if  $X \subseteq Y$  and  $Y \subseteq Z$ , then  $X \subseteq Z$ .

##### Consistency

I4: For all  $X, Y, Z \subseteq U$ ,  $X \subseteq X \cup Y$  and  $X \cap Y \subseteq X$ .

##### Order Preservation

I5: For all  $X, Y, Z \subseteq U$ , if  $X \subseteq Y$ , then  $X \cap Z \subseteq Y \cap Z$  and  $X \cup Z \subseteq Y \cup Z$ .