Differential Calculus
MTH 62-140
Laws/Theorems/Definitions About Continuity

1. (a) A function f is continuous at a number a if

$$\lim_{x \to a} f(x) = f(a)$$

   (b) A function f is continuous on an interval if it is continuous at every number in the interval.

2. (a) If f and g are continuous at a and c is a constant then the following functions are also continuous at a: $f + g$, $f - g$, $f \cdot g$, cf, $\frac{f}{g}$ if $g(a) \neq 0$.

   (b)
   i. If f is continuous at b and $\lim_{x \to a} g(x) = b$ then $\lim_{x \to a} f(g(x)) = f(b)$. In other words,

   $$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$$

   ii. If g is continuous at a and f is continuous at $g(a)$ then $f \circ g$ is continuous at a, i.e. $f(g(x))$ is continuous at a.

3. The following functions are continuous at every number in their domains: polynomials, rational functions, root functions, trigonometric functions, inverse trigonometric functions, exponential functions, logarithmic functions.

4. Intermediate Value Theorem: Suppose that f is continuous on the closed interval $[a, b]$ and N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.