1. Calcular $\int_0^1 \int_0^{1-x} \int_{2y}^{1+y^2} x \, dzdydx.$ Rpta. 1/10

$$\int_0^1 \int_0^{1-x} xz \Big|_{2y}^{1+y^2} dy\,dx$$

$$\int_0^1 \int_0^{1-x} x(1+y^2) - x2y \, dydx$$

$$\int_0^1 \left(xy + \frac{y^3}{3}x - y^2x\right)\Big|_0^{1-x} dx$$

$$\int_0^1 \frac{xy(3+y^2-3y)}{3}\Big|_0^{1-x} dx$$

$$\int_0^1 x(1-x)\frac{(3+1-2x+x^2-3+3x)}{3} dx$$

$$(x-x^2)(x^2+x+1)$$

$$\frac{1}{3}\int_0^1 x-x^4 \, dx$$

$$\frac{1}{3}\left(\frac{x^2}{2} - \frac{x^5}{5}\Big|_0^1\right)$$

$$\frac{1}{3}\left(\frac{1}{2} - \frac{1}{5}\right)$$

$$\frac{1}{3}\left(\frac{3}{10}\right) = \frac{1}{10}$$

2. Calcular $\int_0^1 \int_0^x \int_0^{x+y} (x+y+z)\, dzdydx.$ Rpta. 7/8

$$\int_0^1 \int_0^x \left(xz + yz + \frac{z^2}{2} \Big|_0^{x+y} \right) dy\, dx$$

$$= \int_0^1 \int_0^x \left(x^2 + yx + yx + y^2 + \frac{x^2 + 2xy + y^2}{2} \right) dy\, dx$$

$$= \int_0^1 x^2 y + \frac{y^2 x}{2} + \frac{y^2 x}{2} + \frac{y^3}{3} + \frac{x^2 y}{2} + \frac{x y^2}{2} + \frac{y^3}{6} \Big|_0^x \, dx$$

$$= \int_0^1 \left(x^3 + \frac{x^3}{2} + \frac{x^3}{2} + \frac{x^3}{3} + \frac{x^3}{2} + \frac{x^3}{2} + \frac{x^3}{6} \right) dx$$

$$= \frac{x^4}{4} + \frac{x^4}{8} + \frac{x^4}{8} + \frac{x^4}{12} + \frac{x^4}{8} + \frac{x^4}{8} + \frac{x^4}{24} \Big|_0^1$$

$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{12} + \frac{1}{8} + \frac{1}{8} + \frac{1}{24} = \frac{7}{8}$$

3. Calcular $\int_1^2 \int_y^{y^2} \int_0^{ln\ x} y\ e^z\ dzdxdy.$ Rpta. 47/24

$$\int_1^2 \int_y^{y^2} \left(y e^z \Big|_0^{\ln x} \right) dx\,dy$$

$$\int_1^2 y \int_y^{y^2} (x - 1) dx\,dy$$

$$\int_1^2 y \left(\frac{x^2}{2} - x \Big|_y^{y^2} \right) dy$$

$$\int_1^2 y \left(\frac{y^4}{2} - y^2 - \left(\frac{y^2}{2} - y \right) \right) dy$$

$$\int_1^2 y \left(\frac{y^4}{2} - \frac{3y^2}{2} + y \right) dy$$

$$\int_1^2 \left(\frac{y^5}{2} - \frac{3y^3}{2} + y^2 \right) dy$$

$$\frac{y^6}{12} - \frac{3y^2}{8} + \frac{y^3}{3} \Big|_1^2$$

$$\frac{128}{24} - \frac{144}{24} + \frac{64}{24} - \frac{2}{24} + \frac{9}{24} - \frac{8}{24} = \boxed{\frac{47}{24}}$$

4. Calcular $\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} xy\ \sin(yz)\ dzdxdy.$ Rpta. $\frac{\pi^3-\pi\sin(\pi^2)}{2}$

$$\int_0^{\pi}\int_0^{\pi} xy\,\frac{-\cos(yz)}{y}\Big|_0^{\pi} dxdy$$

$$= \int_0^{\pi}\int_0^{\pi} x - \cos(y\pi) - \cancel{xy\,\frac{-\cos(0)}{y}}$$

$$= \int_0^{\pi}\int_0^{\pi} -x\cos(y\pi) + x\,dxdy$$

$$= \int_0^{\pi} -\frac{x^2}{2}\cos y\pi + \frac{x^2}{2}\Big|_0^{\pi} dy$$

$$= \int_0^{\pi} -\frac{\pi^2}{2}\cos(y\pi) + \frac{\pi^2}{2}\,dy$$

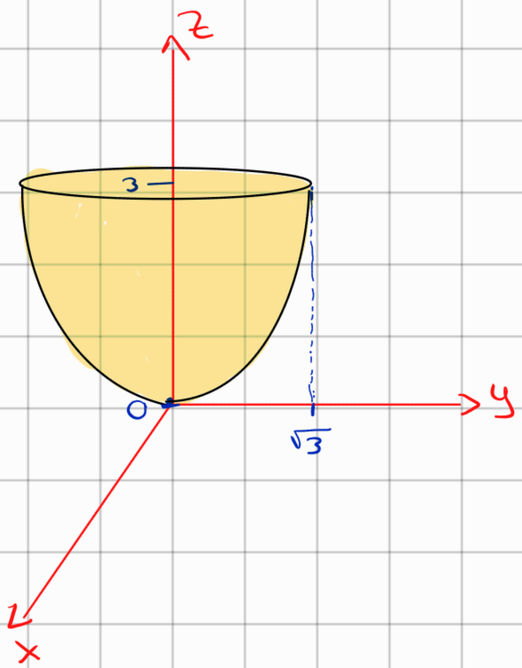
$$= -\frac{\pi^2}{2}\,\frac{\operatorname{Sen}(\pi y)}{\pi} + \frac{y\pi^2}{2}\Big|_0^{\pi}$$

$$= \frac{-\pi}{2}\operatorname{Sen}(\pi^2) + \frac{\pi^3}{2}$$

$$= \boxed{\frac{\pi^3 - \pi\operatorname{Sen}(\pi^2)}{2}}$$

7. Calcular $\iiint_\Omega x \, dV$, donde Ω es el recinto de todos los puntos que cumplen $0 \leq z \leq 3$, $x^2 + y^2 \leq z$.

Rpta. 0



$$x^2 + y^2 = 3$$
$$r^2 = 3$$
$$r = \sqrt{3}$$

$$0 \leq z \leq 3 \; ; \; 0 \leq r \leq \sqrt{3} \; ; \; 0 \leq \theta \leq 2\pi$$

$$\int_0^{2\pi} \int_0^{\sqrt{3}} \int_0^{3} r^2 \operatorname{Cos}(\theta) \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\sqrt{3}} r^2 \operatorname{Cos}(\theta)(3) \, dr \, d\theta$$

$$= \int_0^{2\pi} 3 \operatorname{Cos}(\theta) \left(\frac{r^3}{3} \Big|_0^{\sqrt{3}} \right) d\theta$$

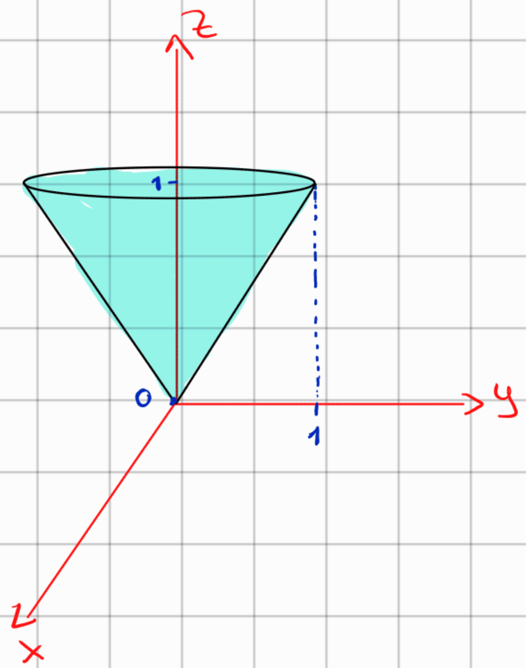
$$= \int_0^{2\pi} 3 \operatorname{Cos}(\theta) \left(\frac{3^{2/3}}{3} \right) d\theta$$

$$= 3^{2/3} \left(\operatorname{Sen}(\theta) \Big|_0^{2\pi} \right)$$

$$= 3^{2/3} \left(\operatorname{Sen}(2\pi) - \operatorname{Sen}(0) \right)$$

$$= 3^{2/3}(0 - 0) = 3^{2/3}(0) = \underline{0}$$

8. Calcular $\iiint_\Omega \sqrt{x^2+y^2}\, dV$, donde Ω es el sólido limitado por $z=\sqrt{x^2+y^2}$, $z=1$. Rpta. $\pi/6$



$r \leq z \leq 1$; $0 \leq r \leq 1$; $0 \leq \theta \leq 2\pi$

$$\int_0^{2\pi}\int_0^1\int_r^1 \sqrt{r} \cdot r\, dz\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^1\int_r^1 r^2\, dz\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^1 \left(r^2\left(z\Big|_r^1\right)\right) dr\, d\theta$$

$$\int_0^{2\pi}\int_0^1 \left(r^2(1-r)\right) dr\, d\theta$$

$$\int_0^{2\pi}\int_0^1 r^2 - r^3\, dr\, d\theta$$

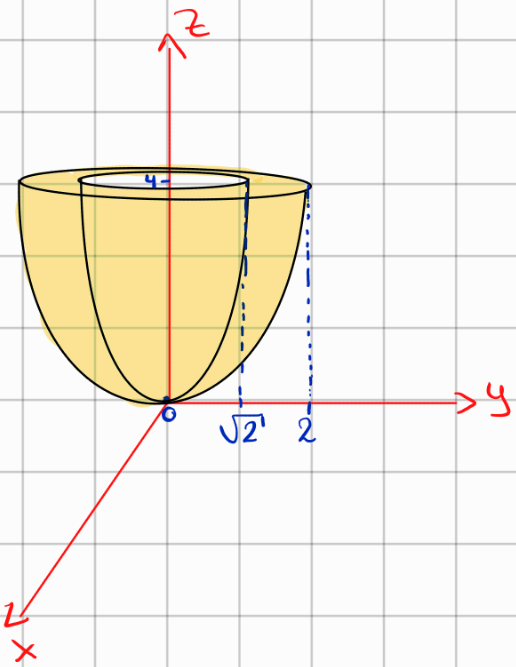
$$\int_0^{2\pi} \left(\frac{r^3}{3} - \frac{r^4}{4}\Big|_0^1\right) d\theta$$

$$\int_0^{2\pi} \left(\frac{1}{3} - \frac{1}{4}\right) d\theta$$

$$\int_0^{2\pi} \frac{1}{12}\, d\theta$$

$$\frac{1}{12}\left(\theta\Big|_0^{2\pi}\right) = \frac{2\pi}{12} = \boxed{\frac{\pi}{6}}$$

9. Calcular $\iiint_\Omega \cos(x^2+y^2+z)\,dV$, donde Ω es el sólido acotado por las superficies $x^2+y^2=2,\ \ x^2+y^2=4,\ \ z=0,\ \ z=4.$ Rpta. $\pi(\cos 4-\cos 8+\cos 6-\cos 2)$



$x^2+y^2=2\ ;\ x^2+y^2=4$

$r^2=2\ ;\ r^2=4$

$0\le z\le 4\ ;\ \sqrt{2}\le r\le 2\ ;\ \le 0\le\theta\le 2\pi$

$$=\int_0^4\int_0^{2\pi}\int_{\sqrt{2}}^{2}\cos(r^2+z)\,r\,dr\,d\theta\,dz$$

$u=r^2+z$

$du=2r\,dr$

$$=\int_0^4\int_0^{2\pi}\frac{\mathrm{Sen}(r^2+z)}{2}\bigg|_{\sqrt{2}}^{2}d\theta\,dz$$

$$=\int_0^4\int_0^{2\pi}\frac{1}{2}\left(\left(\mathrm{Sen}(4+z)\right)-\left(\mathrm{Sen}(2+z)\right)\right)d\theta\,dz$$

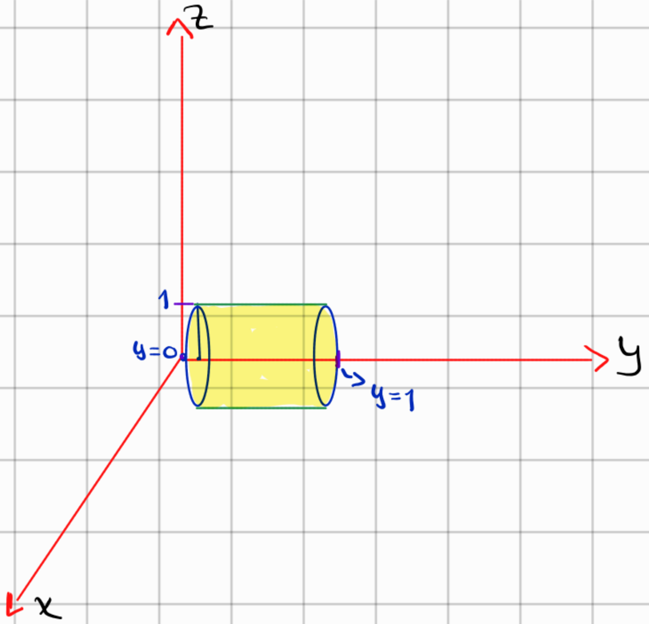
$$=\int_0^4\pi\left(\mathrm{Sen}(4+z)\right)-\mathrm{Sen}(2+z)\big)\,dz$$

$$=-\pi\left(\mathrm{Cos}(4+z)\Big|_0^4\right)-\pi\left(\mathrm{Cos}(2+z)\Big|_0^4\right)$$

$$=-\pi\left(\mathrm{Cos}(8)-\mathrm{Cos}(4)\right)-\left(-\pi\left(\mathrm{Cos}(6)+\mathrm{Cos}(2)\right)\right)$$

$$=\pi\left(\mathrm{Cos}(4)-\mathrm{Cos}(8)+\mathrm{Cos}(6)-\mathrm{Cos}(2)\right)$$

11. Calcular $\iiint_\Omega (x^2+y^2+z^2)\, dV$, si el dominio Ω está limitado por el cilindro $x^2+z^2=1$ y los planos $y=0$, $y=1$. Rpta. $5\pi/6$



$0 \leq y \leq 1$; $0 \leq r \leq 1$; $0 \leq \theta \leq 2\pi$

$$\iiint \left(r^2 + r^2 \operatorname{Sen}^2(\theta)\right) r \, dy\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1\int_0^1 r^2 + y^2 \; r\, dy\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left(r^3 y + r\frac{y^3}{3} \Big|_0^1 \right) dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left(r^3 + \frac{r}{3} \right) dr\, d\theta$$

$$= \int_0^{2\pi} \left(\frac{r^4}{4} + \frac{r^2}{6} \Big|_0^1 \right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{4} + \frac{1}{6} \right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{10}{24} \right) d\theta$$

$$= \frac{10r}{24} \Big|_0^{2\pi}$$

$$\frac{20\pi}{24} = \boxed{\frac{5\pi}{6}}$$