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Ecuaciones diferenciales de variable separable

1. $e^x dx + (1+e^x)Tgydy = 0$

$e^x dx = -(1+e^x)\,Tg\,y\,dy$

$\frac{e^x}{1+e^x}dx = -Tg(y)dy$

$\int \frac{1}{u}du = -\int Tg(y)dy$

$\ln|e^x+1| = -\ln|Sec(y)| + \ln c$

$\ln c = \ln|e^x+1| + \ln|Sec y|$

Reorganizamos

$c = (e^x+1)\,Sec y \Rightarrow \boxed{c = e^x Sec y + Sec y}$

2. $x\sqrt{1+y^2} + y\sqrt{1+x^2}\,y' = 0$

$x(1+y^2)^{1/2} + y(1+x^2)^{1/2}\frac{dy}{dx} = 0$

$x(1+y^2)^{1/2}dx + y(1+x^2)^{1/2}dy = 0$

$x(1+y^2)^{1/2}dx = -y(1+x^2)^{1/2}dy$

$\frac{x}{(1+x^2)^{1/2}}dx = -\frac{y}{\sqrt{1+y^2}}dy$

Integramos

$\int \frac{x}{(1+x^2)^{1/2}}dx = -\int \frac{y}{(1+y^2)^{1/2}}dy$

$u = 1+x^2$ $\quad$ $v = 1+y^2$

$du = 2x\,dx$ $\quad$ $dv = 2y\,dy$

Realizamos el cambio

$\frac{1}{2}\int \frac{du}{u^{1/2}} = -\frac{1}{2}\int \frac{dv}{v^{1/2}}$

$\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\int v^{-1/2}dv$

$\frac{1}{2}\left(\frac{u^{1/2}}{\frac{1}{2}}\right) = -\frac{1}{2}\left(\frac{v^{1/2}}{\frac{1}{2}}\right) + C$

Reemplazamos u, v

$\sqrt{1+x^2} = -\sqrt{1+y^2} + C$

Reorganizamos

$\boxed{C = \sqrt{1+x^2} + \sqrt{1+y^2}}$

3. $(xy^2 - y^2 + x - 1)dx + (x^2y - 2xy + x^2 + 2y - 2x + 2)dy = 0$

$$(y^2(x-1)+x-1)dx + (y(x^2-2x+2)+x^2-2x+2)dy = 0$$

Reorganizamos

$$(x-1)(y^2+1)\,dx + (x^2-2x+2)(y+1)\,dy = 0$$

$$(x-1)(y^2+1)\,dx = -(x^2-2x+2)(y+1)dy$$

$$\frac{(x-1)}{(x^2-2x+2)}\,dx = -\frac{(y+1)}{(y^2+1)}\,dy$$

Integramos

$$\int \frac{x-1}{x^2-2x+2}\,dx = -\int \frac{y}{1+y^2}dy - \int \frac{1}{1+y^2}dy$$

$u = x^2-2x+2$ $\qquad$ $v = 1+y^2$

$du = 2x-2\ dx$ $\qquad$ $dv = 2y\,dy$

$$\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2}\int \frac{dv}{v} - \int \frac{1}{1+y^2}dy$$

$$\frac{1}{2}+\ln|u| = -\frac{1}{2}\ln|v| - \arctan y + c$$

$$\frac{1}{2}\ln|x^2-2x+2| = -\frac{1}{2}\ln|1+y^2| - \arctan y + c$$

Reordenamos

$$c = \frac{1}{2}\ln|x^2-2x+2| + \frac{1}{2}\ln|1+y^2| + \arctan y$$

$$c = \frac{1}{2}\ln|(x^2-2x+2)(1+y^2)| + \arctan y$$

4. $(y^2 + xy^2)\frac{dy}{dx} + x^2 - x^2y = 0$

Multiplicamos por dx

$$(y^2+xy^2)dy+(x^2-x^2y)dx=0$$

$$y^2(x+1)dy + x^2(1-y)dx=0$$

Reorganizamos

$$x^2(1-y)dx=-y^2(x+1)dy$$

$$\frac{x^2}{x+1}dx = +\frac{y^2}{y-1}dy$$

Realizamos la división $\frac{x^2}{x+1}$ y $\frac{y^2}{1-y}$

$$\frac{x^2}{x+1}=(x-1)+\frac{1}{x+1} \quad \wedge \quad \frac{y^2}{y-1}=(y+1)+\frac{1}{y-1}$$

Reemplazamos e integramos

$$\int\left((x-1)+\frac{1}{x+1}\right)dx=\int\left((y+1)+\frac{1}{y-1}\right)dy$$

$$\int(x-1)dx+\int\frac{1}{x+1}dx= \int(y+1)dy+\int\frac{1}{y-1}dy$$

$$\frac{x^2}{2}-x+\ln|x+1|=\frac{y^2}{2}+y+\ln|y-1|+c$$

Reordenamos

$$c=\frac{x^2}{2}-x+\ln|x+1|-\frac{y^2}{2}-y-\ln|y-1|$$

5. $(1 + y^2)dx = (y - \sqrt{1+y^2})(1 + x^2)dy$

$$\frac{1}{1+x^2}dx = \frac{(y-(1+y^2)^{1/2})}{1+y^2}dy$$

Integramos

$$\int\frac{1}{1+x^2}dx=-\int\frac{(1+y^2)^{1/2}}{1+y^2}dy+\int\frac{y}{1+y^2}dy \Rightarrow \begin{array}{l} v=1+y^2 \\ dv=2ydy \end{array}$$

$$\int\frac{1}{1+x^2}dx=-\int\frac{1}{\sqrt{1+y^2}}dy+\frac{1}{2}\int\frac{dv}{v}$$

$$\arctan x=-\arctan y+\frac{1}{2}\ln|1+y^2|+c$$

Reordenamos

$$c=\arctan(x)+\arctan(y)-\frac{\ln|1+y^2|}{2}$$

Ecuaciones diferenciales reducibles a variable separable

1. $\frac{dy}{dx} = Cos(x+y)$

$$\frac{dy}{dx} = Cos\,x\,Cos\,y - Sen\,x\,Sen\,y$$

$$dy = (Cos\,x\,cos\,y - Sen\,x\,Sen\,y)dx$$

Integramos

$$\int dy = \int cos\,y\,cos\,x\,dx - \int sen\,y\,sen\,x\,dx$$

$$y = cos\,y\,sen\,x + sen\,y\,Cos\,x + c$$

$$y = sen(y-x) + c$$

$$\boxed{c = y - sen(y-x)}$$

2. $y' = Sen^2(x-y+1)$

$$dy = sen^2(x-y+1)\,dx$$

$$z = x-y+1$$

$$dz = dx - dy$$

$$dy = dx - dz$$

$$dx - dz = sen^2 z\,dx$$

$$-dz = sen^2 z\,dx - dx$$

$$-dz = (Sen^2 z - 1)\,dx$$

$$-\frac{1}{Sen^2(z)-1}\,dz = dx$$

$$+\int \frac{1}{1-Sen^2(z)}\,dz = \int dx$$

$$1 - Sen^2 z = 2Sen^2\left(\tfrac{z}{2}\right)$$

$$\frac{1}{2}\int \frac{1}{Sen^2\left(\frac{z}{2}\right)}\,dz = \int dx$$

$$\frac{1}{2}\int Csc^2\left(\tfrac{z}{2}\right)dz = \int dx \qquad u = \frac{z}{2} \qquad du = dz$$

$$\frac{1}{2}\int Csc^2(u)\,du = \int dx$$

$$-Ctg\left(\tfrac{z}{2}\right) = x + c \Rightarrow \boxed{c = x + ctg\left(\frac{x+y+1}{2}\right)}$$

3. $y' = \dfrac{x+y}{x+y+2}$

$$\frac{dy}{dx} = \frac{u}{u+2} \qquad u = x+y$$
$$du = dx + dy$$

$$\frac{du - dx}{dx} = \frac{u}{u+2}$$

$$du = \left(\frac{u}{u+2}\right)dx + dx$$

$$du = \frac{2u+2}{u+2}\,dx$$

$$\frac{u+2}{2u+2}\,du = dx$$

$$\int \frac{u+2}{2u+2}\,du = \int dx$$

Por fracciones parciales

$$\int 2 + \frac{1}{2}\left(\frac{1}{u+1}\right)du = dx$$

$$2\int du + \frac{1}{2}\int \frac{1}{u+1}\,du = \int dx$$

$$z = u+1 \Rightarrow dz = du$$

$$2\int du + \frac{1}{2}\int \frac{1}{z}\,dz = \int dx$$

$$2u + \frac{1}{2}\ln(z) = x + C$$

$$2u + \frac{1}{2}\ln|u+1| = x + C$$

$$2(x+y) + \ln^{1/2}|x+y+1| = x + C$$

$$\boxed{C = x + 2y + \ln^{1/2}|x+y+1|}$$

4. $(x+y)^2\dfrac{dy}{dx} = b^2$, b *constante*

$$(x+y)^2 dy = b^2 dx$$

$$z = x+y \Rightarrow dz = dx+dy \Rightarrow dy = dz - dx$$

$$(z^2)(dz-dx) = b^2 dx$$

$$z^2 dz - z^2 dx = b^2 dx$$

$$z^2 dz = (b^2+z^2)dx$$

$$\frac{z^2}{b^2+z^2}dz = dx$$

Integramos

$$\int \frac{z^2}{b^2+z^2}dz = \int dx$$

$$\int dz - b^2\int\frac{dz}{b^2+z^2} = \int dx$$

$$z - (b^2(\tfrac{1}{b})\operatorname{arct}\tfrac{z}{b} = x + C$$

$$-x+x+y - b\operatorname{arctg}\left(\frac{x+y}{b}\right) = C$$

$$y - b(\operatorname{arctg}\left(\frac{x+y}{b}\right) = C$$

$$\boxed{C = y - b(\operatorname{arctg}\left(\frac{x+y}{b}\right)}$$

5. $xy^2(xy' + y) = c^2$, c *cte.*

$$x^2y^2\frac{dy}{dx} + xy^3 = c^2$$

$$x^2y^2dy + xy^3dx - c^2dx = 0$$

$$x^2y^2dy + (xy^3 - c^2)dx = 0$$

$$x^2y^2dy = -(xy^3-c^2)dx$$

$$x^2y^2dy = (c^2 - xy^3)dx$$

$$z = c^2 - xy^3$$

$$dz = -\left(x'y^3 + x(y^3)'\right)$$

$$dz = -y^3dx - 3xy^2dy$$

$$\frac{dz + y^3dx}{3} = -xy^2dy$$

$$\frac{xdz}{3} + \frac{xy^3dx}{3} = -xy^2dy$$

$$-\frac{xdz}{3} - \frac{xy^3dx}{3} = z\,dx$$

$$-\frac{xdz}{3} - \frac{x}{3}\left(\frac{c^2-z}{x}\right)dx = z\,dx$$

$$\frac{-xdz}{3} - \frac{c^2-z}{3}dx = z\,dx$$

$$-xdz = (c^2 - z + 3z)\,dx$$

$$-xdz = (c^2+2z)\,dx$$

$$-\int\frac{1}{c^2+2z}dz = \int\frac{1}{x}dx$$

$$u = c^2 + 2z$$

$$du = 2dz$$

$$-\frac{1}{2}\int\frac{du}{u} = -\frac{1}{2}\ln|u| \Rightarrow$$

$$-\frac{1}{2}\ln|u| = \ln x + \ln c$$

$$\ln c = \frac{1}{2}\ln|2z - c^2| + \ln x$$

$$\boxed{c = (2z - c^2)^{1/2} + x}$$

Ecuaciones diferenciales homogéneas

1. $\underbrace{(y^2 - xy)}_{M}dx + \underbrace{x^2}_{N}dy = 0$

Sustituimos $y = ux$

$y = ux$

$dy = udx + xdu$

$((ux)^2 - x(ux))dx + x^2(udx + xdu) = 0$

$u^2x^2dx - \cancel{ux^2dx} + \cancel{x^2udx} + x^3du = 0$

$u^2x^2dx + x^3du = 0$

Reordenamos

$\frac{x^2}{x^3}dx = -\frac{1}{u^2}du$

$\int \frac{1}{x}dx = -\int u^{-2}du$

$\frac{1}{x} = -\left(\frac{u^{-1}}{-1}\right) + c$

$\ln x = \frac{1}{u} + c$

$\ln x = \frac{x}{y} + c$

$c = \ln x - \frac{x}{y}$

2. $\underbrace{\left(y - xCtg\frac{x}{y}\right)}_{M}dy + \underbrace{yCtg\frac{x}{y}}_{N}dx = 0$

$x = vy$

$dx = vdy + ydv$

$(y - vyctg\,v)dy + (yCtg\,v)(vdy + ydv) = 0$

$ydy - \cancel{vyctgvdy} + \cancel{vyctgvdy} + yctgv\,ydv = 0$

$ydy + yctgvydv = 0$

$ydy = -y^2 ctgv\,dv$

$\frac{1}{y}dy = -\,Ctg\,v\,dv$

$\int \frac{1}{y}dy = -\int ctgv\,dv$

$\ln y = -\ln|sen(\frac{x}{y})| + \ln c$

$\ln y = -\ln|Sen\frac{x}{y}| + \ln c$

$\ln c = \ln|y| + \ln|Sen\frac{x}{y}| \Rightarrow c = y\,Sen\frac{x}{y}$

3. $\left(ySen\frac{y}{x} + xCos\frac{y}{x}\right)dx - xSen\frac{y}{x}dy = 0$

$y = ux$

$dy = udx + xdu$

$(uxsen\,u + x\cos u)dx - x\,sen\,u(udx + xdu) = 0$

$\cancel{uxsen\,u\,dx} + x\cos u\,dx - \cancel{xusen\,u\,dx} - x^2 sen\,u\,du = 0$

$x\cos u\,dx = +x^2 sen\,u\,du$

$\frac{1}{x}dx = \frac{sen\,u}{\cos u}du$

Integramos

$\int \frac{1}{x}dx = \int \tan u\,du$

$\ln|x| = \ln|\sec u| + c$

$\ln c = \ln|\sec u| - \ln|x|$

$\ln c = \ln\left|\frac{\sec\frac{y}{x}}{x}\right| \Rightarrow c = \frac{\sec\frac{y}{x}}{x}$

4. $y' = \frac{x^3}{4x^3 - 3x^2y}$

$\frac{dy}{dx} = \frac{x^3}{4x^3 - 3x^2y}$

$4\cancel{x^2} - 3\cancel{x^2}y\,dy = \cancel{x^2}dx$

$4x - 3y\,dy = xdx$

$xdx - (4x - 3y)dy = 0$

$x = vy \Rightarrow dx = vdy - ydv$

$vy(vdy - ydv) - (4(vy) - 3y)dy = 0$

$v^2y\,dy - vy^2dv - 4vydy + 3ydy = 0$

$v^2y\,dy - 4vy\,dy + 3y\,dy - vy^2dv = 0$

$y(v^2 - 4v + 3)dy - vy^2dv = 0$

$y(v^2 - 4v + 3)dy = vy^2dv$

$\frac{y}{y^2}dy = \frac{v}{v^2 - 4v + 3}dv$

Integramos

$\int \frac{1}{y}dy = \int \frac{v}{(v-3)(v-1)}dv$

$\int \frac{1}{y}dy = \frac{3}{2}\int \frac{dv}{v-3} - \frac{1}{2}\int \frac{dv}{v-1}$

$\ln|y| = \frac{3}{2}\ln|v-3| - \frac{1}{2}\ln|v-1| + \ln c$

$\ln c = \ln|y| - \frac{3}{2}\ln|v-3| + \frac{1}{2}\ln|v-1|$

$\ln c = \ln|y| - \ln\left|\frac{x}{y} - 3\right|^{3/2} + \ln\left|\frac{x}{y} - 1\right|^{1/2}$

$\ln c = \ln\left|\frac{y\left(\frac{x}{y} - 1\right)^{1/2}}{\left(\frac{x}{y} - 3\right)^{3/2}}\right|$

$c = \frac{y\left(\frac{x}{y} - 1\right)^{1/2}}{\left(\frac{x}{y} - 3\right)^{3/2}}$

5. $(y+\sqrt{x^2-y^2})dx = xdy$

$$(y+\sqrt{x^2-y^2})dx - xdy = 0$$

$$y = \mu x \Rightarrow dy = \mu dx + x d\mu$$

$$(\mu x+\sqrt{x^2-\mu^2x^2})dx - x(\mu dx + x d\mu) = 0$$

$$\cancel{\mu x dx} + \sqrt{x^2(1-\mu^2)}dx - \cancel{x\mu dx} - x^2 d\mu = 0$$

$$x(1-\mu^2)^{1/2}dx = x^2 d\mu$$

$$\frac{1}{x}dx = \frac{1}{\sqrt{1-\mu^2}}d\mu$$

Integramos

$$\ln|x| = \operatorname{arcsen}\mu + C$$

$$\ln|x| = \operatorname{arcsen}\frac{y}{x} + C$$

$$\boxed{C = \ln|x| - \operatorname{arcsen}\frac{y}{x}}$$

6. $2xydx + (y^2-x^2)dy = 0$

$$x = vy \Rightarrow dx = vdy + ydv$$

$$2(vy)y(vdy+ydv) + (y^2-(vy)^2)dy = 0$$

$$2v^2y^2dy + 2vy^3dv + y^2dy - v^2y^2dy = 0$$

$$v^2y^2dy + y^2dy + 2vy^3dv = 0$$

$$y^2(v^2+1)dy = -2vy^3dv$$

$$\frac{y^2}{y^3}dy = -\frac{2v}{v^2+1}dv$$

Integramos

$$\int\frac{1}{y}dy = -\int\frac{2v}{v^2+1}dv \Rightarrow m = v^2+1 \Rightarrow dm = 2v\,dv$$

$$\int\frac{1}{y}dy = -\int\frac{1}{m}dm$$

$$\ln|y| = -\ln|m| + c$$

$$\ln|y| = -\ln|v^2+1| + c$$

$$\ln|y| = -\ln\left|\frac{x^2}{y^2}+1\right| + c$$

$$\ln C = \ln\left|(y)\left(\frac{x^2}{y^2}+1\right)\right|$$

$$\boxed{C = (y)\left(\frac{x^2}{y^2}+1\right)}$$

7. $3ydx + (x + 2y)dy = 0$

$x = vy \Rightarrow dx = vdy + ydv$

Reemplazamos

$3y(vdy + ydv) + (vy + 2y)dy = 0$

$3yvdy + 3y^2dv + vydy + 2ydy = 0$

$y(3v + v + 2)dy + 3y^2dv = 0$

$y(4v + 2)dy = -3y^2dv$

$\frac{2y}{y^2}dy = -\left(\frac{3}{2v+1}\right)dv$

Integramos

$\int \frac{2}{3y}dy = -\int \frac{3}{2v+1}dv \Rightarrow r = 2v+1 \Rightarrow dr = 2dv$

$2\ln|y| = -\frac{3}{2}\int \frac{1}{r}dr$

$2\ln|y| = -\frac{3}{2}\ln|r| + c$

Reemplazamos (r)

$2\ln|y| = -\frac{3}{2}\ln|2v+1| + \ln c$

Reemplazamos (v)

$2\ln|y| = -\frac{3}{2}\ln|2\left(\frac{x}{y}\right)+1) + c$

$C = 2\ln|y| + \frac{3}{2}\ln|\frac{2x}{y} + 1|$

$\ln C = \ln\left|(y^2)\left(\frac{2x+y}{y}\right)^{3/2}\right.$

$$\boxed{C = (y^2)\left(\frac{2x+y}{y}\right)^{3/2}}$$

Ecuaciones diferenciales reducibles a homogéneas

1. $(x-4y-9)dx + (4x+y-2)dy = 0$

$L_1 = x-4y-9$

$x-4y-9=0$

$4y = x-9$

$y = \frac{x}{4} - \frac{9}{4} \quad m_1 = \frac{1}{4}$

$L_2 = 4x+y-2$

$4x+y-2=0$

$y=-4x+2$

$m_2 = -4$

Entonces $L_1 \not\parallel L_2$

$L_1: x-4y-9=0$

$L_2: 4x+y-2=0$

$x-4y-9=0$

$16x+4y-8=0$

$x=1 \wedge y=-2$

$\exists(x_0,y_0)=(1,-2)$

$(x = x_0+u \Rightarrow dx = du) \Rightarrow x = 1+u$

$(y = y_0+v \Rightarrow dy = dv) \Rightarrow y = -2+v$

Reemplazamos la ecuación principal

$(1+u-4(-2+v)-9)du + (4(1+u)+(-2+v-2))dv = 0$

$(1+u+8-4v-9)du + (4+4u-2+v-2)dv = 0$

$(u-4v)du + (4u+v)dv = 0$

Es homogenea

$\underbrace{(u-4v)}_{M}du + \underbrace{(4u+v)}_{N}dv = 0 \qquad \frac{u}{v}$

$u = av \Rightarrow du = adv + vda$

Reemplazamos

$(av-4v)(adv+vda) + (4av+v)dv = 0$

$a^2vdv + av^2da - \cancel{4vadv} - 4v^2da + \cancel{4avdv} + vdv = 0$

$a^2vdv + vdv + av^2da - 4v^2da = 0$

$v(a^2+1)dv + v^2(a-4)da = 0$

$(a^2+1)dv = -(a-4)da$

$\frac{1}{v}dv = \frac{-(a-4)da}{a^2+1}$

Integramos

$\int \frac{1}{v}dv = -\int \frac{a-4}{a^2+1}da$

$\ln|v| = -\left(\frac{1}{2}\ln|a^2+1| - 4\arctan a\right) + C$

Reemplazamos "a" y "v"

$\ln|y+2| = \ln^{-1/2}\left|\left(\frac{x-1}{y+2}\right)^2+1\right| + 4\arctan\frac{x-1}{y+2} + C$

$C = \ln^{1/2}\left|\left(\frac{x-1}{y+2}\right)^2+1\right| - \ln|y+2| + 4\arctan\left(\frac{x-1}{y+2}\right)$

$$2.\quad \frac{dy}{dx} = \frac{x+3y-5}{x-y-1}$$

$L_1 = x+3y-5$

$x+3y-5=0$

$3y=-x+5$

$y=-\frac{x}{3}+\frac{5}{3}$

$m_1=-\frac{1}{3}$

$L_2 = x-y-1$

$x-y-1=0$

$y=x-1$

$m_2=1$

Entonces $L_1 \not\parallel L_2$

$L_1 \cap L_2$

$L_1: x+3y-5=0$

$L_2: x-y-1=0$

$$\begin{array}{r} x+3y-5 \\ -x+y+1 \\ \hline 0+4y-4=0 \\ 4y=4 \\ y=1 \end{array} \qquad x=2$$

$y=1 \wedge x=2$

$x=x_0+u \Rightarrow dx=du$

$y=y_0+v \Rightarrow dy=dv$

$x=2+u$

$y=1+v$

Reemplazamos la ecuación principal

$$\frac{dv}{du} = \frac{2+u+3+3v-5}{2+u-1-v-1} \Rightarrow \frac{u+3v}{u-v}$$

Reordenamos

$(u+3v)du-(u-v)dv=0$

$u=av \Rightarrow du=adv+vda$

$(av+3v)(adv+vda)-(av-v)dv=0$

$a^2vdv+av^2da+3vadv+3v^2da-avdv+vdv=0$

$a^2vdv+2vadv+vdv+av^2da+3v^2da=0$

$v(a^2+2a+1)dv+v^2(a+3)da=0$

$(a^2+2a+1)dv=-v(a+3)da$

$\frac{1}{v}dv=-\frac{a+3}{a^2+2a+1}da$

Integramos

$\int \frac{1}{v}dv=-\int \frac{a+3}{a^2+2a+1}da$

$\int \frac{1}{v}dv=-\int \frac{a+3}{(a+1)^2}da \qquad u=a+1$

$\int \frac{1}{v}dv=-\int \frac{u+2}{u^2}du$

$\int \frac{1}{v}dv=-(\int \frac{u}{u^2}du+\int 2u^{-2}du$

$\ln|v|=-(\ln u+-2u^{-1})+C$

$\ln|v|=\ln^{-1}(a+1)+\frac{2}{a+1}+C$

$C=\ln|v|+\ln(a+1)-\frac{2}{a+1}$

$$\boxed{C=\frac{\ln^{-1}|(y-1)}{\frac{x-2}{y-1}+1} \quad \Big| -\frac{2}{\frac{x-2}{y-1}+1}}$$

3. $(x-2y+5)dx+(2x-y+4)dy = 0$

$x=-1$

$L_1 = x-2y+5$ $\quad$ $L_2 = 2x-y+4$

$x-2y+5=0$ $\quad$ $2x-y+4=0$

$2y=x+5$ $\quad$ $y=2x+4$

$y=\frac{x}{2}+\frac{5}{2}$ $\quad$ $m_2=2$

$m_1=\frac{1}{2}$

Entonces $L_1 \not\parallel L_2$

$L_1: \; x-2y+5=0$

$L_2: \; 2x-y+4=0$

$$\begin{array}{r} -2x+4y-10 \\ 2x-y+4 \\ \hline 3y-6=0 \end{array}$$

$y=2$ $\quad$ $2x+2=0$, $x=-1$

$$\left.\begin{array}{l} x=x_0+u \Rightarrow dx=du \\ y=y_0+v \Rightarrow dy=dv \end{array}\right\} \begin{array}{l} x=-1+u \\ y=2+v \end{array}$$

Reemplazamos en la ecuación principal

$(-1+u-4-2v+5)du+(-2+2u-2-v+4)dv=0$

$(u-2v)du+(2u-v)dv=0$

$u=av \Rightarrow du=adv+vda$

$(av-2v)(adv+vda)+(2(av)-v)dv=0$

$a^2vdv+av^2da-\cancel{2vadv}-2v^2da+\cancel{2avdv}-vdv=0$

$a^2vdv-vdv+av^2da-2v^2da=0$

$v(a^2-1)dv+v^2(a-2)da=0$

$(a^2-1)dv=-v(a-2)da$

$\frac{1}{v}dv=-\frac{a-2}{a^2-1}da$

Integramos

$\int\frac{1}{v}dv=-\int\frac{a-2}{a^2-1}da$

$\int\frac{1}{v}dv=-\int\left(\frac{-1/2}{x-1}+\frac{3/2}{x+1}\right)dx$

$\ln(v)=-\frac{1}{2}\ln|x-1|+\frac{3}{2}\ln|x+1|+\ln c$

$\ln c=\ln|y-2|+\ln|x-1|^{1/2}-\ln|x+1|^{3/2} \Rightarrow \boxed{C=\frac{(y-2)(x-1)^{1/2}}{(x+1)^{3/2}}}$

4. $\frac{dy}{dx} = \frac{x + y + 4}{x - y - 6}$

$L_1 = x+y+4$

$x+y+4=0$

$y=-x-4$

$m_1=-1$

$L_2 = x-y-6$

$x-y-6=0$

$y=x-6$

$m_2=1$

Entonces $L_1 \nparallel L_2$

$L_1: x+y+4=0$

$L_2: x-y-6=0$

$2x-2=0$

$x=1$

$x+y+4=0$

$1+y+4=0$

$y=-5$

$x = x_0 + u \Rightarrow x = 1 + u \Rightarrow dx = du$

$y = y_0 + v \Rightarrow y = -5 + v \Rightarrow dy = dv$

Reemplazamos la ecuación principal

$\frac{dv}{du} = \frac{1+u-5+v+4}{1+u+5-v-6}$

$(u+v)du = (u-v)dv$

$(u+v)du - (u-v)dv = 0$

$u = av \Rightarrow du = a\,dv + v\,da$

$(av+v)(a\,dv + v\,da) - (av-v)dv = 0$

$a^2v\,dv + av^2da + \cancel{va\,dv} + v^2da - \cancel{av\,dv} + v\,dv = 0$

$a^2v\,du + v\,dv + av^2da + v^2da = 0$

$v(a^2+1)dv + v^2(a+1)da = 0$

$v(a^2+1)dv = -v^2(a+1)da$

$\frac{1}{v}dv = -\frac{a+1}{a^2+1}da$

Integramos

$\int \frac{1}{v}dv = -\int \frac{a+1}{a^2+1}da$

$\int \frac{1}{v}dv = -\int \frac{a}{a^2+1}da - \int \frac{1}{1+a^2}da$

$\int \frac{1}{v}dv = -\int \frac{a}{a^2+1}da - \int \frac{1}{1+a^2}da$

$\int \frac{1}{v}dv = -\frac{1}{2}\int \frac{1}{u}du - \int \frac{1}{1+a^2}da$

$\ln|v| = -\frac{1}{2}\ln|u| - \arctan x + C$

$C = \ln|v| + \frac{1}{2}\ln|u| + \arctan x$

Reemplazamos "v" y "u"

$C = \ln|y+5| + \frac{1}{2}\ln|x-1| + \arctan x$

$C = \ln|(y+5)(x-1)^{1/2}| + \arctan x$

C) Ecuaciones diferenciales exactas

1. $\underbrace{2xy}_{M}dx + \underbrace{(x^2-1)}_{N}dy = 0$

$\frac{\partial M}{\partial y} = 2x \quad \frac{\partial N}{\partial x} = 2x$

Entonces $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ la E.D. es exacta

$\frac{\partial F}{\partial x} = M$

Derivamos respecto x

$\frac{\partial F}{\partial x} = 2xy$

$\partial F = 2yx\,\partial y$

$\int \partial F = \int 2yx\,\partial x$

$F(x,y) = yx^2 + g(y)$

Derivamos respecto a y

$\frac{\partial F(x,y)}{\partial y} = x^2 + g'(y)$

$\frac{\partial F(x,y)}{\partial y} = N$

$x^2 + g'(y) = x^2 - 1$

$g'(y) = -1$

Integramos

$\int g'(y) = -\int dy$

$g(y) = -y + c$

Reemplazamos g(y) en f(x,y)

$F(x,y) = yx^2 - y + c$

$c = y(x^2-1)$

2. $\underbrace{(2x+4y-5)}_{M}dx + \underbrace{(6y+4x-1)}_{N}dy = 0$

$\frac{\partial M}{\partial y} = 4 \quad \frac{\partial N}{\partial x} = 4$

Entonces $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ la E.D. es exacta

$\frac{\partial F}{\partial x} = M$

Derivamos respecto x

$\frac{\partial F}{\partial x} = 2x + 4y - 5$

$\partial F = (2x+4y-5)\partial x$

Integramos

$\int \partial F = \int (2x+4y-5)\partial x$

$F(x,y) = x^2 + 4yx - 5x + g(y)$

Derivamos respecto a y

$\frac{\partial F(x,y)}{\partial y} = 4x + g'(y)$

$\frac{\partial F(x,y)}{\partial y} = N$

$4x + g'(y) = 6y + 4x - 1\ \partial y$

$\int g'(y) = \int (6y + -1)\partial y$

$g(y) = 3y^2 - y + c$

Reemplazamos g(y) en F(x,y)

$F(x,y) = x^2 + 4yx - 5x + 3y^2 - y + c$

$0 = x^2 + 4yx - 5x + 3y^2 - y$

$c = x^2 + 4yx - 5x + 3y^2 - y$

3. $\left(x^2y^3 - \frac{1}{1+9x^2}\right)\frac{dx}{dy} + x^3y^2 = 0$

$$\underbrace{\left(x^2y^3 - \frac{1}{1+9x^2}\right)}_{M}dx + \underbrace{x^3y^2}_{N}dy = 0$$

$$\frac{\partial M}{\partial y} = 3x^2y^2 \qquad \frac{\partial N}{\partial x} = 3x^2y^2$$

Entonces $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ la E.D. es exacta

$$\frac{\partial F}{\partial y} = x^3y^2$$

$$\partial F = x^3y^2\,\partial y$$

$$\int \partial F = x^3 \int y^2 dy$$

$$F(x,y) = \frac{x^3y^3}{3} + g(x)$$

$$\frac{\partial F(x,y)}{\partial x} = x^2y^3 + g'(x)$$

$$\frac{\partial F(x,y)}{\partial x} = M$$

$$x^2y^3 + g'(x) = x^2y^3 - \frac{1}{1+9x^2}$$

$$\int g'(x) = -\int \frac{1}{1+(3x)^2}\partial x \qquad u = 3x \qquad du = 3dx$$

$$g(x) = -\frac{1}{3}\int \frac{1}{1+u^2}du$$

$$g(x) = -\frac{1}{3}(\arctan u) + C$$

$$g(x) = -\frac{\arctan(3x)}{3} + C$$

Reemplazamos $g(x)$ en $F(x,y)$

$$F(x,y) = \frac{x^3y^3}{3} - \frac{\arctan(3x)}{3} + C$$

$$\boxed{C = \frac{x^3y^3 - \arctan(3x)}{3}}$$

4. $\underbrace{(y^2Cosx - 3x^2y - 2x)}_{M}dx + \underbrace{(2ySenx - x^3 + Lny)}_{N}dy = 0$

$$\frac{\partial M}{\partial y} = 2yCosx - 3x^2 \qquad \frac{\partial N}{\partial x} = 2yCosx - 3x^2$$

Entonces $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$; la E.D. es exacta

$$\frac{\partial F}{\partial x} = M$$

$$\frac{\partial F}{\partial x} = y^2 Cosx - 3x^2y - 2x$$

$$\partial F = y^2 cosx - 3x^2y - 2x \, \partial x$$

Integramos

$$\int \partial F = \int (y^2 cosx - 3x^2y - 2x)\partial x$$

$$F(x,y) = y^2 senx - x^3y - x^2 + g(y)$$

Derivamos respecto a y

$$\frac{\partial F(x,y)}{\partial y} = 2ysenx - x^3 + g'(y)$$

$$\frac{\partial F}{\partial y} = N$$

$$2ysenx - x^3 + g'(y) = 2ysenx - x^3y + lny$$

$$g'(y) = ln(y)$$

$$\int g'(y) = ln(y)\,\partial y$$

$$g(y) = y\,ln(y) - y + C$$

Reemplazamos g(y) en F(x,y)

$$F(x,y) = y^2 Senx - x^3y - x^2 + y\,ln(y) - y + C$$

$$\boxed{C = y^2 Senx - x^3y - x^2 + y\,ln(y) - y + C}$$

5. $\underbrace{\left(Seny + ySenx + \frac{1}{x}\right)}_{M} dx + \underbrace{\left(xCosy - Cosx + \frac{1}{y}\right)}_{N} dy = 0$

$$\frac{\partial M}{\partial y} = Cos\,y + Senx \qquad \frac{\partial N}{\partial x} = Cosy + Senx$$

Entonces $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$; la E.D. es exacta

$$\frac{\partial F}{\partial x} = M$$

$$\frac{\partial F}{\partial x} = Seny + ysenx + \frac{1}{x}$$

$$\partial F = \left(Seny + ysenx + \frac{1}{x}\right)\partial x$$

$$\int \partial F = \int Seny + ysenx + \frac{1}{x}\,\partial x$$

$$F(x,y) = xSeny - ycosx + \ln x + g(y)$$

Derivamos respecto a y

$$\frac{\partial F(x,y)}{\partial y} = xCosy - Cosx + g'(y)$$

$$\frac{\partial F}{\partial y} = N$$

$$xcosy - cos\,x + g'(y) = xCosy - Cosx + \frac{1}{y}$$

$$g'(y) = \frac{1}{y}$$

$$\int g'(y) = \int \frac{1}{y} dy$$

$$g(y) = \ln y$$

Reemplazamos g(y) en F(x,y)

$$F(x,y) = xseny - ycos\,x + \ln x + \ln y$$

$$\boxed{C = xseny - y\,Cosx + \ln((x)(y))}$$

6. $\underbrace{(y^3 - y^2Senx - x)}_{M}dx + \underbrace{(3xy^2 + 2yCosx)}_{N}dy = 0$

$$\frac{\partial M}{\partial y} = 3y^2 - 2ySenx \qquad \frac{\partial N}{\partial x} = 3y^2 - 2ySenx$$

Entonces $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$; la E.D. es exacta

$$\frac{\partial F}{\partial x} = M$$

$$\frac{\partial F}{\partial x} = y^3 - y^2 Senx - x$$

$$\partial F = (y^3 - y^2 Senx - x)\partial x$$

$$\int \partial F = \int (y^3 - y^2 Senx - x)\partial x$$

$$F(x,y) = xy^3 + y^2 cosx - \frac{x^2}{2} + g(y)$$

Derivamos respecto a y

$$\frac{\partial F(x,y)}{\partial y} = 3xy^2 + 2y Cosx + g'(y)$$

$$\frac{\partial F}{\partial y} = N$$

$$3xy^2 + 2y cosx + g'(y) = 3xy^2 + 2y Cosx$$

$$g'(y) = 0$$

$$\int g'(y) = \int (0) dy$$

$$g(y) = 0$$

Reemplazamos g(y) en F(x,y)

$$F(x,y) = xy^3 + y^2 Cosx - \frac{x^2}{2} + C$$

$$\boxed{C = xy^3 + y^2 cosx - \frac{x^2}{2}}$$

D) Ecuaciones diferenciales lineales

1. $\frac{dy}{dx} - 3y = 0$, con $y(0) = -3$

$y' - \underbrace{3y}_{P(x)} = \underbrace{0}_{Q(x)}$

$\mu = e^{-\int (3\ )dx}$

$\mu = e^{-3x}$

$e^{-3x} y' - e^{-3x} 3y = 0$

$\frac{d}{dx}(y e^{-3x}) = 0$

$\int d(y e^{-3x}) = \int 0 dx$

$y e^{-3x} = c$

$y = \frac{c}{e^{-3x}} \Rightarrow -3 = C \Rightarrow C = -3 \Rightarrow y = \frac{-3}{e^{-3x}}$

2. $x\frac{dy}{dx} + y = 2x$, con $y(1) = 0$

$xy' + y = 2x$

$y' + \frac{y}{x} = 2$

Factor integral

$\mu(x) = e^{\int \frac{1}{x} dx}$

$\mu(x) = e^{\ln x}$

$\mu(x) = x$

$x\left(y' + \frac{y}{x} = 2\right)$

$xy' + y = 2x$

$\frac{\partial}{\partial x}(xy) = 2x$

$\partial(xy) = 2x\ \partial x$

$\int \partial(xy) = \int 2x\, \partial x$

$xy = x^2 + C$

$y = \frac{x^2 + C}{x} \Rightarrow \frac{1+C}{1} = 0 \Rightarrow C = -1 \Rightarrow y = \frac{x^2 - 1}{x}$

3. $x\frac{dy}{dx} - 4ydy = x^6 e^x$

$$y' - \frac{4}{x}y = x^5 e^x$$

$$\mu(x) = e^{-4\int \frac{1}{x}dx} \Rightarrow \mu(x) = e^{-4\ln x} \Rightarrow \mu(x) = x^{-4}$$

$$\frac{d(x^{-4}y)}{dx} = x^5 e^x$$

$$\int d(x^{-4}y) = \int \underbrace{x^5}_{u} \underbrace{e^x dx}_{dv} \Rightarrow \begin{array}{l} u = x^5 \\ du = 5x^4 dx \end{array} \quad \begin{array}{l} \int dv = \int e^x dx \\ v = e^x \end{array}$$

$$x^{-4}y = x^5 e^x - \int e^x 5x^4 dx$$

$$x^{-4}y = x^5 e^x - 5(e^x(x^4 - 4x^3 + 12x^2 - 24x + 24 + C \Rightarrow \boxed{C = x^5 e^x - 5e^x x^4 + 20e^x x^3 - 60e^x x^2 + 120e^x x + 120e^x - x^{-4}y}$$

4. $\frac{dy}{dx} + 2y = x^2 + 2x$

$$y' + \underbrace{2}_{P(x)}y = \underbrace{x^2 + 2x}_{Q(x)}$$

Factor integral

$$\mu(x) = e^{\int 2dx}$$

$$\mu(x) = e^{2x}$$

$$e^{2x}(y' + 2y) = (x^2 + 2x)e^{2x}$$

$$\frac{d}{dx}(e^{2x}y) = (x^2 e^{2x} + e^{2x}2x)$$

$$d(e^{2x}y) = (x^2 e^{2x} + e^{2x}2x)dx$$

$$\int d(e^{2x}y) = \int (x^2 e^{2x} + e^{2x}2x)dx$$

$$\int \underbrace{x^2}_{u} \underbrace{e^{2x}dx}_{dv} + \int \underbrace{e^{2x}}_{dv} \underbrace{2x}_{u} \underbrace{dx}_{dv}$$

$u = x^2$	$u = 2x$
$du = 2x\,dx$	$du = 2dx$
$dv = e^{2x}dx$	$dv = e^{2x}dx$
$\int dv = \int e^{2x}dx$	$\int dv = \int e^{2x}dx$
$v = e^{2x}$	$v = e^{2x}$
$uv - \int v\,du$	$uv - \int v\,du$
$x^2 e^{2x} - \int e^{2x} 2x\,dx$	$2xe^{2x} - 2\int e^{2x}dx$
$x^2 e^{2x} - 2xe^{2x} + 2e^{2x}$	$2xe^{2x} - 2e^{2x}$

$$e^{2x}y = x^2 e^{2x} - \underline{2xe^{2x}} + 2e^{2x} + \underline{2xe^{2x}} - 2e^{2x} + c \Rightarrow e^{2y}y = x^2 e^{2x} - 4e^{2x} + C \Rightarrow \boxed{y = \frac{x^2 e^{2x} - 4e^{2x} + C}{e^{2y}}}$$

5. $(x^2 + 9)\frac{dy}{dx} + xy = 0$

$$y' + \underbrace{\frac{x}{x^2+9}}_{P(x)} y = \underbrace{0}_{Q}$$

Factor integral

$u = x^2+9$

$du = 2x\,dx$

$$\mu(x) = e^{\int\left(\frac{x}{x^2+9}\right)dx}$$

$$\mu(x) = e^{\frac{1}{2}\int\frac{1}{u}\,du}$$

$$\mu(x) = e^{\ln u^{1/2}}$$

$$\mu(x) = (x^2+9)^{1/2}$$

$$\frac{d((x^2+9)^{1/2}y)}{dx} = 0$$

$$\int d(x^2+9)^{1/2}y = \int 0\,dx$$

$$(x^2+9)^{1/2}y = C$$

$$\boxed{y = \frac{C}{(x^2+9)^{1/2}}}$$

6. $xLnx\frac{dy}{dx} - y = x^3(3Lnx - 1)$

$$y' - \frac{1}{x\ln x}y = \frac{x^3(3\ln x-1)}{x(\ln x)}$$

Factor integral

$$\mu(x) = e^{-\int(\frac{1}{x\ln x})dx}$$

$$u = \ln x$$
$$du = \frac{1}{x}dx$$

$$\mu(x) = e^{\int \frac{1}{u}du}$$

$$\mu(x) = e^{-\ln(u)}$$

$$\mu(x) = e^{-\ln(\ln x)}$$

$$\mu(x) = \ln(x)^{-1}$$

$$\ln(x)^{-1}\left(y' - \frac{1}{x\ln x}y\right) = \ln x^{-1}\left(\frac{x^3(3\ln x-1)}{x\ln(x)}\right)$$

$$\frac{d(-\ln(x)y)}{dx} = \frac{x^2(3\ln x-1)}{\ln x^2}$$

$$\int d(-\ln(x)y = \int \frac{x^2(3\ln x-1)dx}{\ln x^2}$$

$$\frac{y}{\ln x} = 3\int x^2 \ln x^{-1}dx - \int \frac{x^2}{\ln(x)^2}dx$$

$$\frac{y}{\ln x} = 3\left(\ln^{-1}x\left(\frac{x^3}{3}\right) - \frac{1}{3}\int -\frac{x^2}{x\ln x^2}dx - \int \frac{x^2}{\ln x^2}dx\right.$$

$$\frac{y}{\ln x} = x^3\ln x^{-1} + \int \frac{x^2}{\ln x^2}dx - \int \frac{x^2}{\ln x^2}dx$$

$$\ln x^{-1}y = x^3\ln x^{-1}$$

$$\boxed{y = x^3}$$