DATA STRUCTURES AND ALGORITHMS
LECTURE 5

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In Lecture 4...

- Containers
    - ADT Priority Queue
    - ADT Deque
    - ADT List

    - Singly linked list

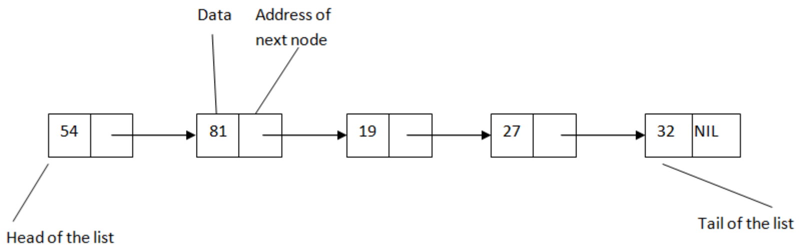
Today

- Singly linked list iterator

- Doubly linked list
- Sorted list
- Circular list

Singly Linked Lists - Recap

- Example of a singly linked list with 5 nodes:



Singly Linked Lists - Representation

- For the representation of a SLL we need two structures: one structure for the node and one for the list itself.

```
SLLNode:
    info: TElem //the actual information
    next: ↑ SLLNode //address of the next node
```

Singly Linked Lists - Representation

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```
SLLNode:
    info: TElem //the actual information
    next: ↑ SLLNode //address of the next node
```

```
SLL:
    head: ↑ SLLNode //address of the first node
```

- Usually, for a SLL, we only memorize the address of the head. However, there might be situations when we memorize the address of the tail as well (if it helps us implement the operations).

SLL - Iterator

- How can we define an iterator for a SLL?
- Remember, an iterator needs a reference to a *current element* from the data structure it iterates over. How can we denote a *current element* for a SLL?

SLL - Iterator

- How can we define an iterator for a SLL?
- Remember, an iterator needs a reference to a *current element* from the data structure it iterates over. How can we denote a *current element* for a SLL?
- Remember, for the dynamic array the current element was the index of the element. Can we do the same here?

SLL - Iterator

- In case of a SLL, the current element from the iterator is actually a node of the list.

```
SLLIterator:
    list: SLL
    currentElement: ↑ SLLNode
```

SLL - Iterator - init operation

- What should the *init* operation do?

SLL - Iterator - init operation

- What should the *init* operation do?

```
**subalgorithm** init(it, sll) **is:**
//pre: sll is a SLL
//post: it is a SLLIterator over sll
    it.sll ← sll
    it.currentElement ← sll.head
**end-subalgorithm**
```

- Complexity:

SLL - Iterator - init operation

- What should the *init* operation do?

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```

- Complexity: $\Theta(1)$

SLL - Iterator - getCurrent operation

- What should the *getCurrent* operation do?

SLL - Iterator - getCurrent operation

- What should the *getCurrent* operation do?

```
function getCurrent(it) is:
//pre: it is a SLLIterator, it is valid
//post: getCurrent ← e, e is TElem, the current element from it
//throws: exception if it is not valid
   if it.currentElement = NIL then
      @throw an exception
   end-if
   e ← [it.currentElement].info
   getCurrent ← e
end-function
```

- Complexity:

SLL - Iterator - getCurrent operation

- What should the *getCurrent* operation do?

```
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```

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SLL - Iterator - next operation

- What should the *next* operation do?

SLL - Iterator - next operation

- What should the *next* operation do?

```
**subalgorithm** next(it) **is:**
//pre: it is a SLLIterator, it is valid
//post: it' is a SLLIterator, the current element from it' refers to
the next element
//throws: exception if it is not valid
  **if** it.currentElement = NIL **then**
    @throw an exception
  **end-if**
  it.currentElement ← [it.currentElement].next
**end-subalgorithm**
```

- Complexity:

SLL - Iterator - next operation

- What should the *next* operation do?

```
subalgorithm next(it) is:
//pre: it is a SLLIterator, it is valid
//post: it' is a SLLIterator, the current element from it' refers to
the next element
//throws: exception if it is not valid
   if it.currentElement = NIL then
      @throw an exception
   end-if
   it.currentElement ← [it.currentElement].next
end-subalgorithm
```

- Complexity: $\Theta(1)$

SLL - Iterator - valid operation

- What should the *valid* operation do?

SLL - Iterator - valid operation

- What should the *valid* operation do?

```
function valid(it) is:
//pre: it is a SLLIterator
//post: true if it is valid, false otherwise
  if it.currentElement $\neq$ NIL then
    valid $\leftarrow$ True
  else
    valid $\leftarrow$ False
  end-if
end-subalgorithm
```

- Complexity:

SLL - Iterator - valid operation

- What should the *valid* operation do?

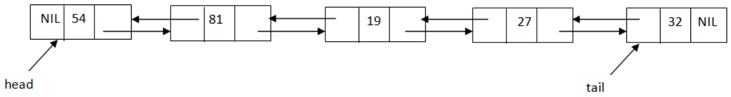
```
function valid(it) is:
//pre: it is a SLLIterator
//post: true if it is valid, false otherwise
  if it.currentElement $\neq$ NIL then
    valid $\leftarrow$ True
  else
    valid $\leftarrow$ False
  end-if
end-subalgorithm
```

- Complexity: $\Theta(1)$

Doubly Linked Lists - DLL

- A doubly linked list is similar to a singly linked list, but the nodes have references to the address of the previous node as well (besides the *next* link, we have a *prev* link as well).
- If we have a node from a DLL, we can go to the next node or to the previous one: we can walk through the elements of the list in both directions.
- The *prev* link of the first element is set to *NIL* (just like the *next* link of the last element).

Example of a Doubly Linked List



- Example of a doubly linked list with 5 nodes.

Doubly Linked List - Representation

- For the representation of a DLL we need two structures: one struture for the node and one for the list itself.

```
DLLNode:
    info: TElem
    next: ↑ DLLNode
    prev: ↑ DLLNode
```

Doubly Linked List - Representation

- For the representation of a DLL we need two structures: one struture for the node and one for the list itself.

```
DLLNode:
    info: TElem
    next: ↑ DLLNode
    prev: ↑ DLLNode
```

```
DLL:
    head: ↑ DLLNode
    tail: ↑ DLLNode
```

DLL - Operations

- We can have the same operations on a DLL that we had on a SLL:
    - search for an element with a given value
    - add an element (to the beginning, to the end, to a given position, etc.)
    - delete an element (from the beginning, from the end, from a given positions, etc.)
    - get an element from a position
- Some of the operations have the exact same implementation as for SLL (e.g. search, get element), others have similar implementations. In general, if the structure of the list needs to be modified, we need to modify more links and have to pay attention to the *tail* node.

DLL - Insert at the end

- Inserting a new element at the end of a DLL is simple, because we have the *tail* of the list, we do not have to walk through all the elements (like we have to do in case of a SLL).

```
subalgorithm insertLast(dll, elem) is:
//pre: dll is a DLL, elem is TElem
//post: elem is added to the end of dll
    newNode ← allocate() //allocate a new DLLNode
    [newNode].info ← elem
    [newNode].next ← NIL
    [newNode].prev ← dll.tail
    if dll.head = NIL then //the list is empty
        dll.head ← newNode
        dll.tail ← newNode
    else
        [dll.tail].next ← newNode
        dll.tail ← newNode
    end-if
end-subalgorithm
```

- Complexity:

```
subalgorithm insertLast(dll, elem) is:
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    [newNode].next ← NIL
    [newNode].prev ← dll.tail
    if dll.head = NIL then //the list is empty
        dll.head ← newNode
        dll.tail ← newNode
    else
        [dll.tail].next ← newNode
        dll.tail ← newNode
    end-if
end-subalgorithm
```

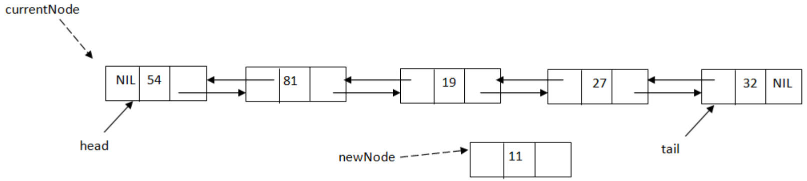
- Complexity: $\Theta(1)$

DLL - Insert on position

- The basic principle of inserting a new element at a given position is the same as in case of a SLL.
- The main difference is that we need to set more links (we have the *prev* links as well) and we have to check whether we modify the tail of the list.
- In case of a SLL we *had to* stop at the node after which we wanted to insert an element, in case of a DLL we can stop before or after the node (but we have to decide in advance, because this decision influences the special cases we need to test).

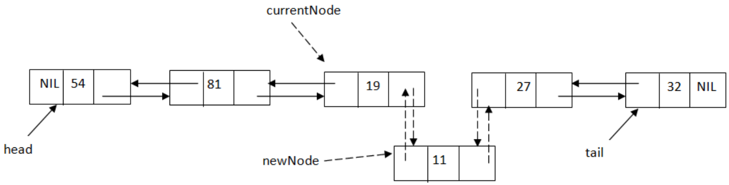
DLL - Insert on position

- Let's insert value 46 at the 4th position in the following list:



DLL - Insert on position

- We move with the *currentNode* to position 3, and set the 4 links.



DLL - Insert at a position

```
subalgorithm insertPosition(dll, pos, elem) is:
//pre: dll is a DLL; pos is an integer number; elem is a TElem
//post: elem will be inserted on position pos in dll
    if pos $<$ 1 then
        @ error, invalid position
    else if pos $=$ 1 then
        insertFirst(dll, elem)
    else
        currentNode $\leftarrow$ dll.head
        currentPos $\leftarrow$ 1
        while currentNode $\neq$ NIL and currentPos $<$ pos - 1 execute
            currentNode $\leftarrow$ [currentNode].next
            currentPos $\leftarrow$ currentPos $+$ 1
        end-while
//continued on the next slide...
```

DLL - Insert at position

```
        if currentNode = NIL then
            @error, invalid position
        else if currentNode = dll.tail then
            insertLast(dll, elem)
        else
            newNode ← alocate()
            [newNode].info ← elem
            [newNode].next ← [currentNode].next
            [newNode].prev ← currentNode
            [[currentNode].next].prev ← newNode
            [currentNode].next ← newNode
        end-if
    end-if
end-subalgorithm
```

- Complexitate: $O(n)$

DLL - Insert at a position

- Observations regarding the *insertPosition* subalgorithm:
  - We did not implement the *insertFirst* subalgorithm, but we assume it exists.
  - The order in which we set the links is important: reversing the setting of the last two links will lead to a problem with the list.
  - It is possible to use two *currentNodes*: after we found the node after which we insert a new element, we can do the following:

```
nodeAfter ← currentNode
nodeBefore ← [currentNode].next
//now we insert between nodeAfter and nodeBefore
[newNode].next ← nodeBefore
[newNode].prev ← nodeAfter
[nodeBefore].prev ← newNode
[nodeAfter].next ← newNode
```

DLL - Delete a given element

- If we want to delete a node with a given element, we first have to find the node:
    - we can use the *search* function (discussed at SLL, but it is the same here as well)
    - we can walk through the elements of the list until we find the node with the element (this is implemented below)

DLL - Delete a given element

```
function deleteElement(dll, elem) is:
//pre: dll is a DLL, elem is a TElem
//post: the node with element elem will be removed and returned
    currentNode ← dll.head
    while currentNode ≠ NIL and [currentNode].info ≠ elem execute
        currentNode ← [currentNode].next
    end-while
    deletedNode ← currentNode
    if currentNode ≠ NIL then
        if currentNode = dll.head then
            deleteElement ← deleteFirst(dll)
        else if currentNode = dll.tail then
            deleteElement ← deleteLast(dll)
        else
//continued on the next slide...
```

DLL - Delete a given element

```
            [[currentNode].next].prev ← [currentNode].prev
            [[currentNode].prev].next ← [currentNode].next
            @set links of deletedNode to NIL
        **end-if**
    **end-if**
    deleteElement ← deletedNode
**end-function**
```

- Complexity: $O(n)$
- If we used the *search* algorithm to find the node to delete, the complexity would still be $O(n)$ - *deleteElement* would be $\Theta(1)$, but searching is $O(n)$

DLL - Iterator

- The iterator for a DLL is identical to the iterator for the SLL (but *currentNode* is *DLLNode* not *SLLNode*).

Algorithmic problems using Linked Lists

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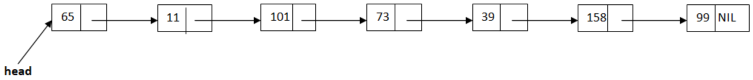
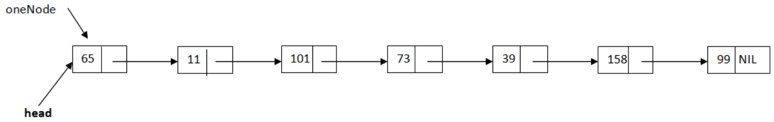
- Find the n^{th} node from the end of a SLL.
- Simple approach: go through all elements to count the length of the list. When we know the length, we know at which position the n^{th} node from the end is. Start again from the beginning and go to that position.
- Can we do it in one single pass over the list?

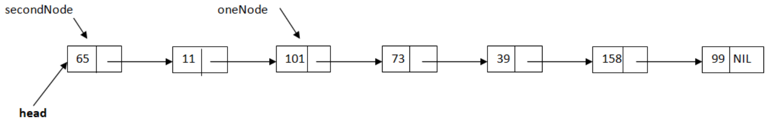
Algorithmic problems using Linked Lists

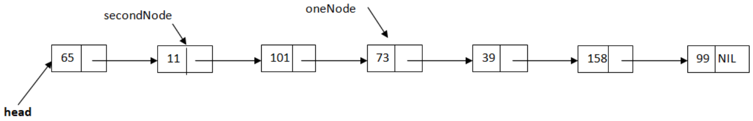
- Find the n^{th} node from the end of a SLL.
- Simple approach: go through all elements to count the length of the list. When we know the length, we know at which position the n^{th} node from the end is. Start again from the beginning and go to that position.
- Can we do it in one single pass over the list?
- We need to use two auxiliary variables, two nodes, both set to the first node of the list. At the beginning of the algorithm we will go forward $n - 1$ times with one of the nodes. Once the first node is at the n^{th} position, we move with both nodes in parallel. When the first node gets to the end of the list, the second one is at the n^{th} element from the end of the list.

- We want to find the 3rd node from the end (the one with information 39)




oneNode
65
11
101
73
39
158
99
NIL
head


secondNode
oneNode
65
11
101
73
39
158
99
NIL
head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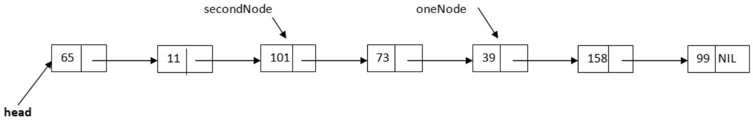
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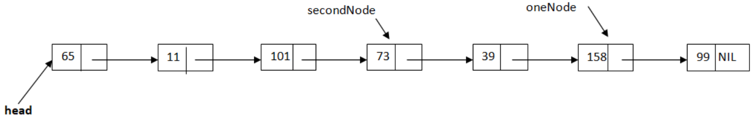
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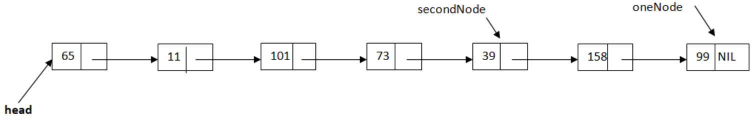
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oneNode
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158
99
NIL
head

N-th node from the end of the list

```
function findNthFromEnd (sll, n) is:
//pre: sll is a SLL, n is an integer number
//post: the n-th node from the end of the list or NIL
    oneNode ← sll.head
    secondNode ← sll.head
    position ← 1
    while position < n and oneNode ≠ NIL execute
        oneNode ← [oneNode].next
        position ← position + 1
    end-while
    if oneNode = NIL then
        findNthFromEnd ← NIL
    else
    //continued on the next slide...
```

N-th node from the end of the list

```
        while [oneNode].next $\neq$ NIL execute
            oneNode $\leftarrow$ [oneNode].next
            secondNode $\leftarrow$ [secondNode].next
        end-while
        findNthFromEnd $\leftarrow$ secondNode
    end-if
end-function
```

- Is this approach really better than the simple one (does it make fewer steps)?

- Write a subalgorithm which rotates a singly linked list (moves the first element to become the last one).

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    - We have to do two things: remove the first node and then attach it after the last one.
    - Special cases:

- Write a subalgorithm which rotates a singly linked list (moves the first element to become the last one).
    - We have to do two things: remove the first node and then attach it after the last one.
    - Special cases:
        - an empty list
        - list with a single node

```
subalgorithm rotate(sll) is:
  if NOT (sll.head = NIL OR [sll.head].next = NIL) then
    first ← sll.head //save the first node
    sll.head ← [sll.head].next remove the first node
    current ← sll.head
    while [current].next ≠ NIL execute
      current ← [current].next
    end-while
    [current].next ← first
    [first].next ← NIL
    //make sure it does not point back to the new head node
  end-if
end-subalgorithm
```

- Complexity:

```
subalgorithm rotate(sll) is:
  if NOT (sll.head = NIL OR [sll.head].next = NIL) then
    first ← sll.head //save the first node
    sll.head ← [sll.head].next remove the first node
    current ← sll.head
    while [current].next ≠ NIL execute
      current ← [current].next
    end-while
    [current].next ← first
    [first].next ← NIL
    //make sure it does not point back to the new head node
  end-if
end-subalgorithm
```

- Complexity: $\Theta(n)$

Think about it

- Given the first node of a SLL, determine whether the list ends with a node that has NIL as *next* or whether it ends with a cycle (the *last* node contains the address of a previous node as *next*).
- If the list from the previous problems contains a cycle, find the length of the cycle.
- Find if a SLL has an even or an odd number of elements, without counting the number of nodes in any way.
- Reverse a SLL non-recursively in linear time using $\Theta(1)$ extra storage.

Sorted Linked Lists

- A *sorted linked list* (or ordered list) is a linked list in which the elements from the nodes are in a specific order, given by a *relation*.
- This *relation* can be $<$, $\leq$, $>$ or $\geq$, but we can also work with an abstract relation.
- Using an abstract relation will give us more flexibility: we can easily change the relation (without changing the code written for the sorted list) and we can have, in the same application, lists with elements ordered by different relations.

The relation - recap

- You can imagine the *relation* as a function with two parameters (two *TComp* elems):

$$relation(c_1, c_2) = \begin{cases} true, & "c_1 \leq c_2" \\ false, & otherwise \end{cases}$$

- $"c_1 \leq c_2"$ means that c_1 should be in front of c_2 when ordering the elements.

Sorted List - representation

- When we have a sorted list (or any sorted structure or container) we will keep the relation used for ordering the elements as part of the structure. We will have a field that represents this relation.

- In the following we will talk about a *sorted singly linked list* (representation and code for a *sorted doubly linked list* is really similar).

Sorted List - representation

- We need two structures: *Node - SSLLNode* and *Sorted Singly Linked List - SSLL*

```
SSLLNode:
    info: TComp
    next: ↑ SSLLNode
```

```
SSLL:
    head: ↑ SSLLNode
    rel: ↑ Relation
```

SSLL - Initialization

- The relation is passed as a parameter to the *init* function, the function which initializes a new SSLL.
- In this way, we can create multiple SSLLs with different relations.

```
subalgorithm init (ssll, rel) is:
//pre: rel is a relation
//post: ssll is an empty SSLL
    ssll.head ← NIL
    ssll.rel ← rel
end-subalgorithm
```

- Complexity: $\Theta(1)$

SSLL - Insert

- Since we have a singly-linked list we need to find the node *after* which we insert the new element (otherwise we cannot set the links correctly).
- The node we want to insert after is the first node whose successor is *greater than* the element we want to insert (where *greater than* is represented by the value *false* returned by the relation).
- We have two special cases:
  - an empty SSLL list
  - when we insert before the first node

SSLL - insert

```
subalgorithm insert (ssll, elem) is:
//pre: ssll is a SSLL; elem is a TComp
//post: the element elem was inserted into ssll to where it belongs
   newNode ← allocate()
   [newNode].info ← elem
   [newNode].next ← NIL
   if ssll.head = NIL then
   //the list is empty
      ssll.head ← newNode
   else if ssll.rel(elem, [ssll.head].info) then
   //elem is "less than" the info from the head
      [newNode].next ← ssll.head
      ssll.head ← newNode
   else
//continued on the next slide...
```

SSLL - insert

```
    cn ← ssll.head //cn - current node
    while [cn].next $\neq$ NIL and ssll.rel(elem, [[cn].next].info) = false execute
      cn ← [cn].next
    end-while
    //now insert after cn
    [newNode].next ← [cn].next
    [cn].next ← newNode
  end-if
end-subalgorithm
```

- Complexity:

SSLL - insert

```
        cn ← ssll.head //cn - current node
        while [cn].next $\neq$ NIL and ssll.rel(elem, [[cn].next].info) = false execute
            cn ← [cn].next
        end-while
        //now insert after cn
        [newNode].next ← [cn].next
        [cn].next ← newNode
    end-if
end-subalgorithm
```

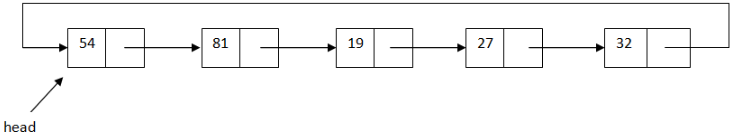
- Complexity: $O(n)$

SSLL - Other operations

- The search operation is identical to the search operation for a SLL (except that we can stop looking for the element when we get to the first element that is "greater than" the one we are looking for).
- The delete operations are identical to the same operations for a SLL.
- The return an element from a position operation is identical to the same operation for a SLL.
- The iterator for a SSLL is identical to the iterator to a SLL.

Circular Lists

- For a SLL or a DLL the last node has as *next* the value *NIL*. In a *circular list* no node has *NIL* as next, since the last node contains the address of the first node in its next field.



Circular Lists

- We can have singly linked and doubly linked circular lists, in the following we will discuss the singly linked version.
- In a circular list each node has a successor, and we can say that the list does not have an end.
- We have to be careful when we iterate through a circular list, because we might end up with an infinite loop (if we set as stopping criterion the case when *currentNode* or *[currentNode].next* is *NIL*.
- There are problems where using a circular list makes the solution simpler (for example: Josephus circle problem, rotation of a list)

Circular Lists

- Operations for a circular list have to consider the following two important aspects:
    - The *last* node of the list is the one whose *next* field is the *head* of the list.
    - Inserting before the head, or removing the head of the list, is no longer a simple $\Theta(1)$ complexity operation, because we have to change the *next* field of the last node as well (and for this we have to find the last node).
    - However, retaining the tail node as well, even in case of singly linked list, will help with these operations.

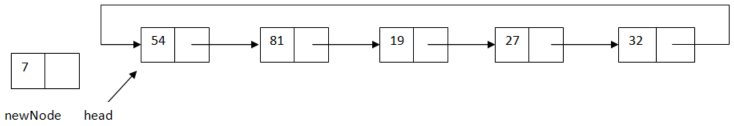
Circular Lists - Representation

- The representation of a circular list is exactly the same as the representation of a simple SLL. We have a structure for a *Node* and a structure for the *Circular Singly Linked Lists - CSLL*.

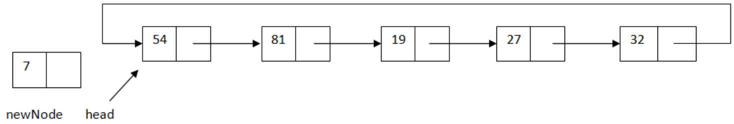
```
CSLLNode:
    info: TElem
    next: ↑ CSLLNode
```

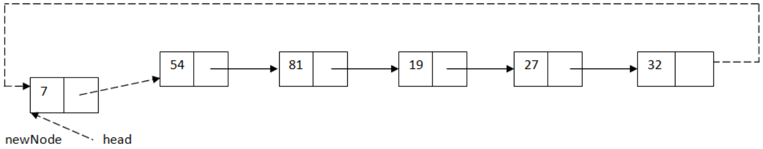
```
CSLL:
    head: ↑ CSLLNode
```

CSLL - InsertFirst



CSLL - InsertFirst





CSLL - InsertFirst

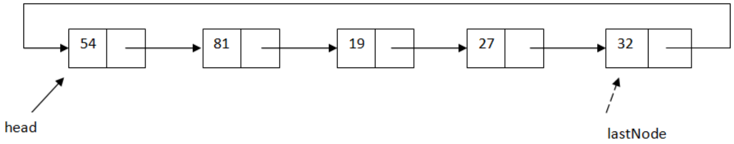
```
subalgorithm insertFirst (csll, elem) is:
//pre: csll is a CSLL, elem is a TElem
//post: the element elem is inserted at the beginning of csll
   newNode ← allocate()
   [newNode].info ← elem
   [newNode].next ← newNode
   if csll.head = NIL then
      csll.head ← newNode
   else
      lastNode ← csll.head
      while [lastNode].next ≠ csll.head execute
         lastNode ← [lastNode].next
      end-while
//continued on the next slide...
```

CSLL - InsertFirst

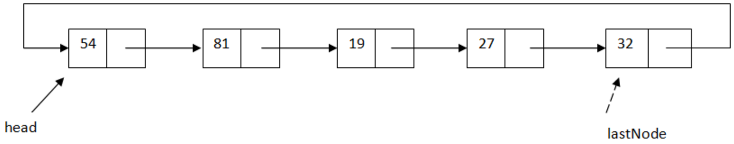
```
        [newNode].next $\leftarrow$ csll.head
        [lastNode].next $\leftarrow$ newNode
        csll.head $\leftarrow$ newNode
    **end-if**
**end-subalgorithm**
```

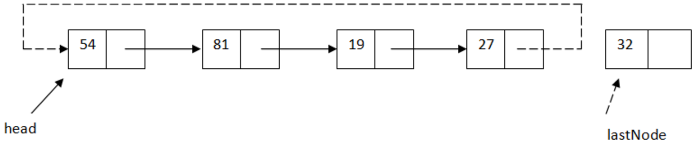
- Complexity: $\Theta(n)$
- Note: inserting a new element at the end of a circular list looks exactly the same, but we do not modify the value of *csll.head* (so the last instruction is not needed).

CSLL - DeleteLast



CSLL - DeleteLast





CSLL - DeleteLast

```
function deleteLast(csll) is:
//pre: csll is a CSLL
//post: the last element from csll is removed and the node
//containing it is returned
   deletedNode ← NIL
   if csll.head ≠ NIL then
      if [csll.head].next = csll.head then
         deletedNode ← csll.head
         csll.head ← NIL
      else
         prevNode ← csll.head
         while [[prevNode].next].next ≠ csll.head execute
            prevNode ← [prevNode].next
         end-while
//continued on the next slide...
```

```
            deletedNode ← [prev].next
            [prev].next ← csll.head
        end-if
    end-if
    [deletedNode].next ← NIL
    deleteLast ← deletedNode
end-function
```

- Complexity: $\Theta(n)$

Summary

- Linked list variants:
  - Doubly linked list
  - Sorted list
  - Circular list
- Extra reading - A think about problem for which the solution will be in next week's extra reading.