

# Warm-up - Seminar 4

u.1. Determine a cartesian equation for the line  $\ell$  in the following cases:

a)  $\ell$  contains  $A(-2, 3)$ ,  $\gamma 60^\circ$  w/  $Ox$

A line's direction vector is given by:

$$v(\cos \alpha, \sin \alpha) \rightsquigarrow \begin{array}{c} \text{Diagram of a unit circle with radius } r \\ \text{The angle } \alpha \text{ is measured from the positive } Ox \text{ axis.} \\ \text{The cosine value is labeled as } \cos \text{ along the horizontal axis.} \\ \text{The sine value is labeled as } \sin \text{ along the vertical axis.} \end{array}$$

We have the unit vector  
 $v(\cos 60^\circ, \sin 60^\circ) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$   
 in the opposite dir:

$$v\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

Writing the parametric equations for  $A(-2, 3)$ :

$$\left\{ \begin{array}{l} x = -2 + \frac{1}{2}t \\ y = 3 + \frac{\sqrt{3}}{2}t \end{array} \right. \quad \left| \quad \left\{ \begin{array}{l} x = -2 + \frac{1}{2}t \\ y = 3 - \frac{\sqrt{3}}{2}t \end{array} \right. \right.$$

$$\left. \begin{array}{l} t = \frac{x+2}{\frac{1}{2}} = 2(x+2) \\ t = \frac{y-3}{\frac{\sqrt{3}}{2}} = \frac{2(y-3)}{\sqrt{3}} \end{array} \right\} \Rightarrow \left( \begin{array}{l} \frac{x+2}{t} = 2(x+2) \\ \frac{y-3}{t} = \frac{2(y-3)}{\sqrt{3}} \end{array} \right)$$

b)  $B(1,7)$  and orthogonal to  $m(4,3)$

If a line is perpendicular to a vector  $m(A,B) \Rightarrow$

$$l: A(x - x_0) + B(y - y_0) = 0$$

$$\begin{matrix} B(1,4) \\ m(4,3) \end{matrix} \Rightarrow l: 4(x-1) + 3(y-4) = 0$$

$$\begin{aligned} 4x - 4 + 3y - 12 &= 0 \\ l: 4x + 3y - 16 &= 0 \end{aligned}$$

4.7. Consider the line  $l$

a) if  $v(v_1, v_2)$  is a dir. vector for  $l$ , then  
 $m(v_2, -v_1)$  is a normal vector

In dim 2 + vector orthogonal to a direction is a  
normal vector  $\hookrightarrow \perp$

$$\langle v, m \rangle = v_1 v_2 + v_2 (-v_1) = 0 \Rightarrow v \perp m \Rightarrow v \text{-normal}$$

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$J \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -v_2 \\ v_1 \end{bmatrix}$$

! Any vector + to a normal vector is a direction

!  $\text{J}$ -operator is defined as follows,

$\forall v \in V^2$ :  $\exists J(v)$ :

a)  $J(iv) \perp v$

b)  $|J(v)| = |v|$

c)  $(v, J(v))$  is right-oriented