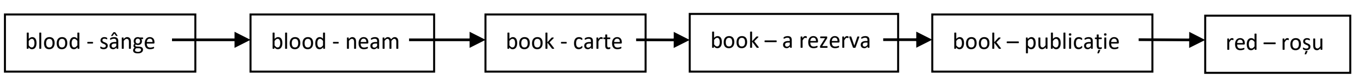
DSA – Seminar 3
Sorted MultiMap (SMM)

- Map – contains key-value pairs. Keys are unique, each key has a single associated value.
- MultiMap – a key can have multiple associated values (can be considered a list of values).
- Sorted MultiMap – there is a relation R defined on the keys and they are ordered based on the keys. There is no particular order of the values belonging to a key (we do not order based on the values)

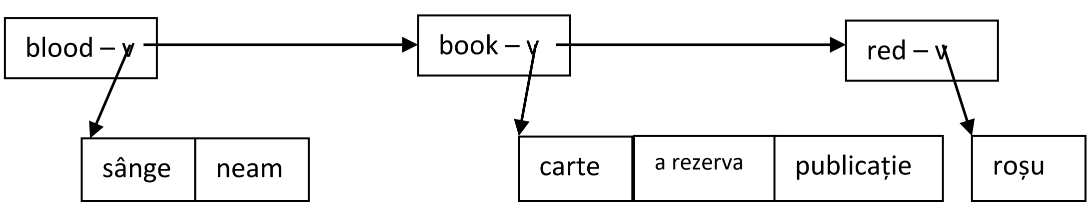
Problem: Implement the SortedMultiMap ADT – use a singly linked representation with dynamic allocation

Ex. a multimap with the translation of different English words into Romanian
- book – carte, a rezerva, publicație
- red – roșu
- blood – sânge, neam

Representation 1: Singly linked list of <key, value> pairs. There might be multiple nodes with the same key, they will be placed one after the other (since the nodes are sorted based on the keys). Relative order of nodes with the same key is not important.



Representation 2: Singly linked list of <key, list of values> pairs. The keys are unique and sorted.



No matter which representation we choose, the content of the SMM is the same: we have 6 key-value pairs.

How could we represent the *list of values* from the second representation?

- Data structure level:
  - Dynamic array, SLL, DLL
- ADT level:
  - List, Bag

We will consider that the *list of values* is actually an ADT List, already implemented (together with the ListIterator).

Representation:

TElem:
k: TKey
vl: List

Node:
info: TElem
next: ↑Node

SMM:
head: ↑Node
R: Relation

$$R(k_1, k_2) = \begin{cases} true, if \text{"}k_1 \leq k_2\text{"} \ (k_1 comes\ before\ k_2) \\ false, otherwise \end{cases}$$

Iterator:

We need to keep in the iterator:

- the SMM
- a reference to the current node from the SMM
- an iterator for the list of values associated to the current node

Obs 1: In a SMM we have key-value pairs, so current element from the iterator has to be a key-value pair. Even if we chose representation 2, we cannot say that our current element is a key and a list of values.

Obs 2: Instead of an iterator over the list of values associated to the current node we could have used the index/position of the current element from the list of values (since it is a list and it has positions). But working with an iterator over the value list is more efficient.

IteratorSMM:
smm: SMM
current: ↑Node
itL: IteratorList

Iterator operations: init, valid, next, getCurrent (returns a <key, value> pair).

Printing the elements of a SMM using the iterator:

```
Subalgorithm print(smm) is:
	iterator(smm, it)
	while valid(it) execute:
		getCurrent(it, <k,v>)
		@print k and v
		next(it)
	end-while
end-subalgorithm
```

The print subalgorithm looks in the same way independently of the representation of the iterator and the representation of the map!

Operations for the iterator

```
subalgorithm init (it, smm) is:
```

```
        it.smm ← smm
        it.current ← smm.head
        if it.current ≠ NIL then:
                iterator([it.smm.head].info.vl, it.itL)
        end-if
end-subalgorithm
Complexity: θ(1)


subalgorithm getCurrent(it) is: // result will be a <k, v> pair
        if it.current = NIL then
                @throw exception
        end-if
        k ← [it.current].info.k
        v ← getCurrent(it.itL)
        getCurrent ← <k,v>
end-subalgorithm
Complexity: θ(1)


function valid(it):
        if it.current ≠ NIL then
                valid ← true
        else
                valid ← false
end-function
Complexity: θ(1)



subalgorithm next(it) is:
        if it.current = NIL then
                @throw exception
        end-if
        next(it.itL)
        if not valid(it.itL) then
                it.current ← [it.current].next
                if it.current ≠ NIL then
                        iterator ([it.current].info.vl, it.itL)
                end-if
        end-if
end-subalgorithm
Complexity: θ(1)

subalgorithm first(it) is:
        it.current ← it.smm.head
        if it.current ≠ NIL then:
                iterator([it.smm.head].info.vl, it.itL)
        end-if
end-subalgorithm
Complexity: θ(1)
```

Operations for the sorted multi map

Notations for the complexities:

n – number of distinct keys

smm – total number of elements

```
subalgorithm init(smm, R) is:
	smm.R ← R
	smm.head ← NIL
end-subalgorithm
Complexity: θ(1)


subalgorithm destroy(smm) is:
	while smm.head ≠ NIL execute:
		aux ← smm.head
		smm.head ← [smm.head].next
		destroy([aux].info.vl)
		free(aux)
	end-while
end-subalgorithm
Complexity:
If destroy for list is θ(1) => θ(n)
If destroy for list is θ(length of list) => θ(smm)

//auxiliary function that will help us with the other operations (private function,
it is not part of the interface).
//pre: smm is  SMM, k is a Tkey
//post: kNode is a ↑Node, prevNode is a ↑Node. If there is a node with k as key,
kNode will be that node and prevNode will be the previous node. If there is no node
with k as key, kNode will be NIL and prevNode will be the node after which the key k
should be.

For the previous example (the one with the words and translations):
searchNode for „book” -> kNode the node with „book”, prevNode the node with „blood”
searchNode for „blood” -> kNode the node with „blood”, prevNode will be NIL
searchNode for „day” -> kNode will be NIL, prevNode the node with „book”
searchNode for „air” -> kNode will be NIL, prevNode will be NIL

subalgorithm searchNode(smm, k, kNode, prevNode) is:
	aux ← smm.head
	prev ← NIL
	found ← false
	while aux ≠ NIL and smm.R([aux].info.k, k) and  not found execute
		if [aux].info.k = k then
			found ← true
		else
			prev ← aux
			aux ← [aux].next
		end-if
	end-while
	if found then
		kNode ← aux
		prevNode ← prev
	else
		kNode ← NIL
```

```
            prevNode ← prev
        end-if
end-subalgorithm
Complexity: O(n)

subalgorithm search(smm, k, list) is:
        searchNode (smm, k, kNode, prevNode)
        if kNode = NIL then
                init(list) // return an empty list
        else
                list ← [aux].info.vl
        end-if
end-subalgorithm
Complexity: O(n)


subalgorithm add(smm, k, v) is:
        searchNode(smm, k, kNode, prevNode)
        if kNode = NIL then
                addANewKey (smm, k, v, prevNode)
        else
                addEnd([kNode].info.vl, v) //an operation from the interface of the list
        end-if
end-subalgorithm
Complexity:
//searchNode is O(n)
//addANewKey is Θ(1) operation (we will use the prevNode)
//instead of addEnd another add function can be used (so it can have Θ(1) complexity)
If addEnd (or whatever function is used for values) is Θ(1) => O(n)
If addEnd (or whatever function is used for values) is Θ(length of the list) =>
O(smm)

//auxiliary operation (not part of interface)
//pre: smm is a SMM, k is a TKey, v is a TElem/ TValue,  prevNode is a ↑Node (the
node after which the new node should be added)
//post: a new node with key k and value v is added to the smm. The order of the keys
will respect the relation.
subalgorithm addANewKey (smm, k, v, prevNode) is:
        allocate(newNode)
        [newNode].info.k ← k
        init ([newNode].info.vl)
        addEnd([newNode].info.vl, v)
        if prevNode = NIL then
                [newNode].next ← smm.head
                smm.head ← newNode
        else
                [newNode].next ← [prevNode].next
                [prevNode].next ← newNode
        end-if
end-subalgorithm
Complexity: Θ (1) //supposing addToEnd it Θ(1) – which is true since in this
situation we will always add an element into an empty list


function remove(smm, k, v) is:
```

```
        searchNode(smm, k, kNode, prevNode)
        if kNode ≠ NIL then
                pos ← indexOf([kNode].info.vl, v)
                if pos ≠ -1 then
                        remove([kNode].info.vl, pos, e)
                end-if
                if isEmpty([kNode].info.vl) then
                        removeKey(smm, k, prevNode)
                end-if
                remove ← true
        end-if
        remove ← false
end-subalgorithm
Complexity: O(smm)

//auxiliary operation (not part of the interface)
//pre: smm is a SMM, k is a TKey, prevNode is a ↑Node, smm contains a node with key k
after the node prevNode (if prevNode is NIL, then the first node of smm contains the
key k). The value list of the node with key k is empty.
//post: the node containing key k is removed from smm
subalgorithm removeKey(smm, k, prevNode) is:
        if prevNode = NIL then
                deleted ← smm.head
                smm.head ← [smm.head].next
                destroy([deleted].info.vl)
                free(deleted)
        else
                deleted ← [prevNode].next
                [prevNode].next ← [[prevNode].next].next
                destroy([deleted].info.vl)
                free(deleted)
        end-if
end-subalgorithm
Complexity: Θ(1)
Destroy will destroy an empty list => Θ(1)
```