3.1. Let $a \in \mathbb{R}$ be a parameter. Consider the lines $\ell_1 : 2x + y = 1$ and $\ell_2 : x + ay = -1$. Discuss the relative position of the two lines in terms of the parameter a. Furthermore, determine the values a for which the origin and the point $P(-2,-2)$ lie on the same side of ℓ_2.

3.2. Consider the lines $\ell_1 : x - 1 = \frac{y-1}{2} = z - 3$ and $\ell_2 : \frac{x-3}{3} = \frac{y}{6} = \frac{z+2}{3}$. Are the two lines parallel? If they are, give an equation for the plane containing them. Moreover, check if this plane separates the point $P(1,1,1)$ from the origin.

3.3. What is the relative position of the lines $x = -3t, y = 2+3t, z = 1, t \in \mathbb{R}$ and $x = 1+5s, y = 1+13s, z = 1 + 10s, s \in \mathbb{R}$?

3.4. Determine the values a and d for which the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-3}{-2}$ is contained in the plane $ax + y - 2z + d = 0$.

3.5. Determine the relative positions of the line $\ell : \frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-1}{-4}$ and the plane $\pi : 2x - y + z - 1 = 0$. If the line punctures the plane, determine the intersection point.

3.6. Determine Cartesian equations for the line ℓ passing through $Q = (1,1,2)$ and coplanar to the lines $\ell' : 3x - 5y + z = -1, 2x - 3z = -9$ and $\ell'' : x + 5y = 3, 2x + 2y - 7z = -7$. Furthermore, establish whether ℓ meets or is parallel to ℓ' and ℓ''.

3.7. Determine the value k for which the lines $\ell : x = k + t, y = 1 + 2t, z = -1 - kt, t \in \mathbb{R}$ and $\ell' : x = 2 - 2t, y = 3 + 3t, z = 1 + t, t \in \mathbb{R}$ are coplanar. Moreover, determine a Cartesian equation for the plane that contains them and the intersection point $\ell \cap \ell'$ if it exists.

3.8. Determine the relative positions of the planes in the following cases

a) $\pi_1 : x + 2y + 3z - 1 = 0$, $\pi_2 : x + 2y - 3z - 1 = 0$.

b) $\pi_1 : 3x + y + z - 1 = 0$, $\pi_2 : 2x + y + 3z + 2 = 0$, $\pi_3 : -x + 2y + z + 4 = 0$.

c) $\pi_1 : 3x + y + z - 5 = 0$, $\pi_2 : 2x + y + 3z + 2 = 0$, $\pi_3 : 5x + 2y + 4z + 1 = 0$.

3.9. Consider two lines given as intersections of planes:

$$\ell_1 : \begin{cases} a_1x + a_2y + a_3z + a_4 = 0 \\ b_1x + b_2y + b_3z + b_4 = 0 \end{cases} \qquad \ell_2 : \begin{cases} c_1x + c_2y + c_3z + c_4 = 0 \\ d_1x + d_2y + d_3z + d_4 = 0 \end{cases}.$$

Show that ℓ_1 and ℓ_2 are coplanar if and only if

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix} = 0.$$