Dynamical Systems – Seminar Test

May 13, 2024

Name_____

Group_____

1. (3 p)

Find the general solution of the following linear differential equation:
(a) $x' - 3x = 0$; (b) $x'' + 3x = 0$; (c) $x' - 3tx = 0$.

2. (4 p)

Specify the type and stability of the following planar linear differential system $\dot{x} = 3y$, $\dot{y} = -2x$. Find a global first integral. Represent its phase portrait.

3. (3 p)

Represent the phase portrait of the following scalar nonlinear dynamical system $\dot{x} = 6 + x - x^2$. What can you say about its solutions $\varphi(t, 2)$ and, respectively, $\varphi(t, 3)$?

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1. (3 p)

Find the general solution of the following linear differential equation:
(a) $x' + 5x = 0$; (b) $x'' - 5x = 0$; (c) $x' + \frac{5}{t}x = 0$.

2. (4 p)

Specify the type and stability of the following planar linear differential system $\dot{x} = 3x$, $\dot{y} = -2y$. Find a global first integral. Represent its phase portrait.

3. (3 p)

Represent the phase portrait of the following scalar nonlinear dynamical system $\dot{x} = 6 - x - x^2$. What can you say about its solutions $\varphi(t, -3)$ and, respectively, $\varphi(t, 0)$?

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May 20, 2024

Name_____

Group_____

1. (3 p)

Let $k > 0$ be a fixed parameter. Find the flow $\varphi(t, \eta)$ of the scalar dynamical system $x' = -k(x-17)$. Compute $\lim_{t\to\infty} \varphi(t, \eta)$ for any $\eta \in \mathbb{R}$. Represent the phase portrait.

2. (3 p)

Find the solution of the IVP $x'' + 6x' + 13x = 0$, $x(0) = 0$, $x'(0) = 2$. Represent its graph and describe its long-term behavior.

3. (4 p)

Find the equilibrium points and study their stability using the linearization method of the following nonlinear system $x' = -4y + xy$, $y' = x - y^2$.

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1. (3 p)

Find the solution of the IVP $x' + \frac{2t}{1+t^2}x = -5$, $x(0) = 0$.

2. (3 p)

Let $m \in (0, 4)$ be a fixed parameter. Compute the limit as $t \to -\infty$ of any solution of the differential equation $x'' - 2mx' + 17x = 0$. How many nonconstant periodic solutions has this equation?

3. (4 p)

Find the equilibrium points and study their stability using the linearization method of the following nonlinear system $x' = -y + x^2$, $y' = 27x - 3xy$.