In the following exercises, all coordinates and components are given with respect to an orthonormal frame $\mathcal{K} = (O, \mathcal{B})$. For the 2-dimensional cases $\mathcal{B} = (\mathbf{i}, \mathbf{j})$ and for the 3-dimensional cases $\mathcal{B} = (\mathbf{i}, \mathbf{j}, \mathbf{k})$.

4.1. Let $\mathbf{m}$ and $\mathbf{n}$ be two unit vectors such that $\measuredangle(\mathbf{m}, \mathbf{n}) = 120^\circ$. Determine the angle between the vectors $\mathbf{a} = 2\mathbf{m} + 4\mathbf{n}$ and $\mathbf{b} = \mathbf{m} - \mathbf{n}$.

4.2. Consider the points $A(1,2)$, $B(3,-2)$, $C(5,6)$.

a) Calculate the circumcenter and the orthocenter of the triangle ABC.

b) Determine an equation for the angle bisector $\measuredangle ABC$.

4.3. Determine Cartesian equations for the lines passing through $A(-2,5)$ which intersect the coordinate axes in congruent segments.

4.4. The point $A(2,0)$ is the vertex of an equilateral triangle. The side opposite to A lies on the line $x + y - 1 = 0$. Determine Cartesian equations for the lines containing the other two sides.

4.5. Consider the vector $\mathbf{v}$ which is perpendicular on $\mathbf{a}(4,-2,-3)$ and on $\mathbf{b}(0,1,3)$. If $\mathbf{v}$ describes an acute angle with Ox and $|\mathbf{v}| = 26$ determine the components of $\mathbf{v}$.

4.6. Determine a Cartesian equation of the plane π if $A(1,-1,3)$ is the orthogonal projection of the origin on π.

4.7. Let $A(1,2,-7)$, $B(2,2,-7)$ and $C(3,4,-5)$ be vertices of a triangle. Determine equations for the angle bisector of the angle $\measuredangle CAB$.

4.8. Determine the planes which pass through $P(0,2,0)$ and $Q(-1,0,0)$ and which form an angle of 60° with the z-axis.

4.9 (Law of Cosines)**.** Let ABC be a triangle with $|AB| = c, |BC| = a, |CA| = b$. Show that

$$\langle \overrightarrow{AB}, \overrightarrow{AC} \rangle = \frac{1}{2}(b^2 + c^2 - a^2)$$

and deduce the law of cosines in a triangle.

4.10 (Heron's Formula)**.** Let ABC be a triangle with $|AB| = c, |BC| = a, |CA| = b$ and let p denote the semiperimeter $\frac{1}{2}(a+b+c)$. Show that

$$\langle \overrightarrow{AB}, \overrightarrow{AC} \rangle + bc = 2p(p-a) \quad \text{and} \quad \langle \overrightarrow{AB}, \overrightarrow{AC} \rangle - bc = -2(p-b)(p-c).$$

Deduce that the are of the triangle ABC is given by

$$\text{Area}(ABC) = \sqrt{p(p-a)(p-b)(p-c)}.$$

4.11 (Chord Theorem)**.** Let A, B, A' and B' be points on a circle. Let P be the intersection point of the two chords $[AB]$ and $[A'B']$. Show that

$$|AP| \cdot |BP| = |A'P| \cdot |B'P|.$$