

## Seminar 5

1. We consider the linear planar systems

a)  $\dot{x} = -y, \dot{y} = 5x$ ; b)  $\dot{x} = -x, \dot{y} = 5y$ ;

c)  $\dot{x} = -3x, \dot{y} = -2y$ ; d)  $\dot{x} = x - y, \dot{y} = x + y$ .

(i) Decide the type and stability of the equilibrium point at the origin.

(ii) Decide whether it has a global first integral.

(iii) Find a first integral (global or not). (except for d))

(iv) Represent the phase portrait (using the expression of the first integral).  
(except for d))  $\diamond$

2. We consider the nonlinear planar system

$$\dot{x} = x(1 - x), \dot{y} = y(3 - y).$$

Study the stability of its equilibrium points.  $\diamond$

3. (i) For what values of the real parameter  $a$ , the system  $\dot{x} = ax - 5y, \dot{y} = x - 2y$  has a center? In that cases find the general solution of the system.

(ii) For what values of the real parameter  $a$ , the system from (i) has a line filled with equilibrium points?  $\diamond$