In the following exercises, all coordinates and components are given with respect to a right-oriented orthonormal frame $\mathcal{K}$.

7.1. Consider the vector $\mathbf{v}(2,1,1)$.

a) Give the matrix form for the parallel projection on the plane $\pi : z = 0$ parallel to $\mathbf{v}$.

b) Give the matrix form for the parallel reflection in the plane $\pi : z = 0$ parallel to $\mathbf{v}$.

7.2. Determine the orthogonal projection of the line $\ell : 2x - y - 1 = 0 \cap x + y - z + 1 = 0$ on the plane $\pi : x + 2y - z = 0$. Do this by determining the matrix form of the projection and discuss all other options.

7.3. Give the matrix form for the orthogonal reflections in the planes

$$\pi_1 : 3x - 4z = -1 \quad \text{and} \quad \pi_2 : 10x - 2y + 3z = 4 \quad \text{respectively.}$$

Determine the set of points which are fixed by the composition of these two reflections.

7.4. Let H be a hyperplane and let $\mathbf{v}$ be a vector which is not parallel to H. Use the deduced matrix forms to show that

a) $\mathrm{Pr}_{H,\mathbf{v}} \circ \mathrm{Pr}_{H,\mathbf{v}} = \mathrm{Pr}_{H,\mathbf{v}}$ and

b) $\mathrm{Ref}_{H,\mathbf{v}} \circ \mathrm{Ref}_{H,\mathbf{v}} = \mathrm{Id}$.

7.5. Let $\phi(\mathbf{x}) = A\mathbf{x} + b$ be an affine transformation. Give the homogenous matrix of the inverse transformation ϕ^{-1}.

7.6. Let π be a plane with normal vector $\mathbf{n}$ and equation $\pi : \langle \mathbf{n}, \mathbf{x} \rangle = c$. Show that the composition of the orthogonal reflection in π followed by a translation with a vector $\mathbf{v} || \mathbf{n}$ is a reflection in a plane π' and deduce an equation of π'.

7.7. Show that after a homothety with factor λ the area of any triangle changes by a factor of λ^2 and the volume of any tetrahedron changes by a factor of $|\lambda|^3$.