

10.1. For each of the equations in Table 10.2 , discuss the geometric locus of points satisfying them.

10.2. For each of the following equations write down the associated matrix and bring the equation in isometric canonical form.

a) $-x^2 + xy - y^2 + x = 0,$

b) $6xy + x - y = 0.$

10.3. In each of the following cases, decide the type of the quadratic curve based on the parameter $a \in \mathbb{R}$.

a) $x^2 - 4xy + y^2 = a,$

b) $x^2 + 4xy + y^2 = a.$

10.4. Consider the rotation R_{90° of \mathbb{E}^2 around the origin and the translation $T_{\mathbf{v}}$ of \mathbb{E}^2 by the vector $\mathbf{v}(1, 0)$.

a) Give the matrix form of the isometries R_{90° , $T_{\mathbf{v}}$ and $T_{\mathbf{v}} \circ R_{90^\circ}$.

b) Determine the equations of the hyperbola $\mathcal{H} : \frac{x^2}{4} - \frac{y^2}{9} - 1 = 0$ and the parabola $\mathcal{P} : y^2 - 8x = 0$ after transforming them with R_{90° and with $T_{\mathbf{v}} \circ R_{90^\circ}$ respectively.

10.5. Discuss the type of the curve

$$x^2 + \lambda xy + y^2 - 6x - 16 = 0$$

in terms of $\lambda \in \mathbb{R}$.

10.6. Using the classification of quadrics, decide what surfaces are described by the following equations.

a) $x^2 + 2y^2 + z^2 + xy + yz + zx = 1,$

b) $xy + yz + zx = 1,$

c) $x^2 + xy + yz + zx = 1,$

d) $xy + yz + zx = 0.$