2.1. In each of the following cases, decide if the indicated vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ can be represented with the vertices of a triangle:

a) $\mathbf{u}(7,3)$, $\mathbf{v}(2,8)$, $\mathbf{w}(-5,5)$

b) $\mathbf{u}(1,2,-1)$, $\mathbf{v}(2,-1,0)$, $\mathbf{w}(1,-3,1)$

2.2. In each of the following cases, decide if the given points are collinear:

a) $P(3,-5)$, $Q(-1,2)$, $R(-5,9)$

b) $P(1,0,-1)$, $Q(0,-1,2)$, $R(-1,-2,5)$

2.3. Give the coordinates of the vertices of the parallelepiped whose faces lie in the coordinate planes and in the planes $x = 1$, $y = 3$ and $z = -2$.

2.4. Which of the following sets of vectors form a basis?

a) $\mathbf{v}_1(1,2)$, $\mathbf{v}_2(3,4)$

b) $\mathbf{v}_1(-1,2,1,0)$, $\mathbf{v}_2(2,1,1,0)$, $\mathbf{v}_3(1,0-1,1)$, $\mathbf{v}_4(1,0,0,1)$

2.5. With respect to the basis $\mathcal{B} = (\mathbf{i},\mathbf{j},\mathbf{k})$ consider the vectors $\mathbf{u} = \mathbf{i}+\mathbf{j}$, $\mathbf{v} = \mathbf{j}+\mathbf{k}$ and $\mathbf{w} = \mathbf{i}+\mathbf{k}$. Check that $\mathcal{B}' = (\mathbf{u},\mathbf{v},\mathbf{w})$ is a basis and decide if it is left or right oriented.

2.6. Let $\mathcal{K} = (O,\mathcal{B})$ and $\mathcal{K}' = (O',\mathcal{B}')$ be two frames in $\mathbb{E}^2$, with $\mathcal{B} = (\mathbf{i},\mathbf{j})$ and $\mathcal{B}' = (\mathbf{i}',\mathbf{j}')$. Assume that O', $\mathbf{i}'$ and $\mathbf{j}'$ are known relative to $\mathcal{K}$:

$$[O']_{\mathcal{K}} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}, \quad [\mathbf{i}']_{\mathcal{B}} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}, \quad [\mathbf{j}']_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Determine the base change matrix from $\mathcal{K}$ to $\mathcal{K}'$ and the coordinates with respect to $\mathcal{K}'$ of the points

$$[A]_{\mathcal{K}} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad [B]_{\mathcal{K}} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}, \quad [C]_{\mathcal{K}} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

2.7. Consider the tetrahedron $ABCD$ and the frames

$\mathcal{K}_A = (A,\mathcal{B})$ with $\mathcal{B} = (\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD})$,

$\mathcal{K}_B = (B,\mathcal{B})$ with $\mathcal{B} = (\overrightarrow{BA}, \overrightarrow{BC}, \overrightarrow{BD})$,

$\mathcal{K}'_A = (A,\mathcal{B})$ with $\mathcal{B} = (\overrightarrow{BA}, \overrightarrow{BD}, \overrightarrow{BC})$.

Determine

a) the coordinates of the vertices of the tetrahedron in the three coordinate systems,

b) the base change matrices from $\mathcal{K}'_A$ to $\mathcal{K}_A$ and from $\mathcal{K}_A$ to $\mathcal{K}_B$,

c) the orientations of the three frames.

2.8. Let $\mathcal{K} = (O, \mathcal{B})$ and $\mathcal{K}' = (O', \mathcal{B}')$ be two frames in $\mathbb{E}^3$ with $\mathcal{B} = (\mathbf{i}, \mathbf{j}, \mathbf{k})$ and $\mathcal{B}' = (\mathbf{i}', \mathbf{j}', \mathbf{k}')$. Assume that O', $\mathbf{i}', \mathbf{j}'$ and $\mathbf{k}'$ are known relative to $\mathcal{K}$:

$$[O']_{\mathcal{K}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{i}' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{j}' = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{k}' = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

Determine the coordinates with respect to $\mathcal{K}'$ of the points

$$[A]_{\mathcal{K}} = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}, \quad [B]_{\mathcal{K}} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}, \quad [C]_{\mathcal{K}} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \quad [D]_{\mathcal{K}} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}.$$

2.9. What is the number of Cartesian frames that one can construct with the vertices of a tetrahedron?