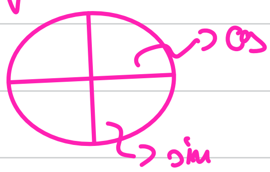
Warm-up - Seminar 4

4.1. Determine a cartesian equation for the line ℓ in the following cases:

a) ℓ contains $A(-2,3)$, $\measuredangle 60^\circ$ w/ OX

A line's direction vector is given by:

$v(\cos\alpha, \sin\alpha)$ $\rightsquigarrow$ 

We have the unit vector,

$u(\cos 60^\circ, \sin 60^\circ) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

in the opposite dir:

$v\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Writing the parametric equations $A(-2,3)$:

$\begin{cases} x = -2 + \frac{1}{2}t \\ y = 3 + \frac{\sqrt{3}}{2}t \end{cases}$ $\Bigg|$ $\begin{cases} x = -2 + \frac{1}{2}t \\ y = 3 - \frac{\sqrt{3}}{2}t \end{cases}$

$\left.\begin{array}{l} t = \dfrac{x+2}{\frac{1}{2}} = 2(x+2) \\ t = \dfrac{y-3}{\frac{\sqrt{3}}{2}} = \dfrac{2(y-3)}{\sqrt{3}} \end{array}\right\} \Rightarrow \ell: \dfrac{x+2}{1} = \dfrac{(y-3)}{\sqrt{3}}$

b) $B(1,7)$ and orthogonal to $n(4,3)$

If a line is perpendicular to a vector $n(A,B) \Rightarrow$

$l: A(x-x_0)+B(y-y_0)=0$

$$\left.\begin{array}{l} B(1,7) \\ n(4,3) \end{array}\right| \Rightarrow l: 4(x-1)+3(y-7)=0$$

$$4x-4+3y-21=0$$

$$l: 4x+3y-25=0$$

4.2. Consider the line l

a) if $v(v_1, v_2)$ is a dir. vector for l, then $n(v_2, -v_1)$ is a <u>normal vector</u>

In dim 2 $\forall$ vector orthogonal to a direction is a normal vector
$\hookrightarrow \perp$

$$\langle v, n\rangle = v_1 v_2 + v_2(-v_1) = 0 \Rightarrow v \perp n \Rightarrow n \text{ - normal}$$

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$J\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -v_2 \\ v_1 \end{bmatrix}$$

! Any vector $\perp$ to a normal vector is a direction

! J-operator is defined as follows:

$\forall\, v \in \mathbb{V}^2 : \exists!\, J(v)$:

a) $J(v) \perp v$

b) $|J(v)| = |v|$

c) $(v, J(v))$ is right-oriented