Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză

Curs: Dynamical Systems

Primăvara 2025

Seminar 6

1. Find the general solution of each of the following equations, looking first for some solutions of the form $x = t^r$, with $r \in \mathbb{R}$.

a) $t^2x'' - 8tx' + 20x = 0$, $t \in (0, \infty)$; b) $t^2x'' - 6x = 0$, $t \in (0, \infty)$;

c) $t^2x'' + tx' + x = 0$, $t \in (0; \infty)$. $\diamond$

2. Find the general solution of the following linear planar system using the reduction method $x' = 2x - 5y$, $y' = x - 2y$. $\diamond$

3. We consider the linear planar system $x' = 2x - 3y$, $y' = x - 2y$.

(i) Find its general solution using the reduction method.

(ii) Specify the type of this system.

(iii) Let A be the matrix of this system. Find the eigenvalues $\lambda_1, \lambda_2 \in \mathbb{R}$ and the corresponding eigenvectors $u_1, u_2 \in \mathbb{R}^2$ (thus $Au_1 = \lambda_1 u_1$ and $Au_2 = \lambda_2 u_2$).

(iv) Justify that $\varphi(t, u_1) = e^{t\lambda_1}u_1$ and $\varphi(t, u_2) = e^{t\lambda_2}u_2$ for all $t \in \mathbb{R}$.

(v) Represent the orbits corresponding to u_1, $-u_1$, u_2 and $-u_2$. $\diamond$

4. Consider the following planar system $x' = -y(x^2 + y^2)$, $y' = x(x^2 + y^2)$.

a) Does this system have other equilibria besides $(0, 0)$? Justify.

b) Verify that $\varphi(t, 1, 0) = (\cos t, \sin t)$, $\varphi(t, 2, 0) = (2\cos 4t; 2\sin 4t)$ for all $t \in \mathbb{R}$. Find $\varphi(t, \eta, 0)$ for each $\eta > 0$. Represent the corresponding orbits.

c) Decide the validity of the statement: "Any solution is periodic." $\diamond$