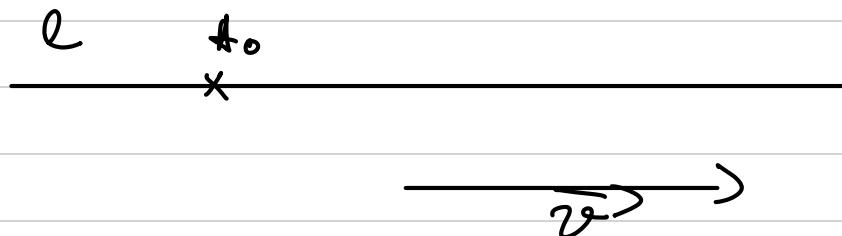


Seminar 3

l - line in \mathbb{E}^2

$A_0 \in l, \vec{v} \parallel l$
 (i.e. $\vec{r} \in D(l)$)
 \hookrightarrow direction subspace



$$\forall m \in l \quad \exists \lambda \in \mathbb{R}; \quad \vec{r}_m = \vec{r}_{A_0} + \lambda \vec{v}$$

\hookrightarrow vector eq. of l

Fix a reference normal:

$$Q: \begin{cases} x = x_{A_0} + \lambda \cdot \vec{v} \\ y = y_{A_0} + \lambda \cdot \vec{v} \end{cases} \quad \text{PARAMETRIC EQ. OF } l$$

\times add as many eq. as dimensions

$$Q_i: \frac{x - x_{A_0}}{\vec{v}} = \frac{y - y_{A_0}}{\vec{v}} \quad \text{SYMMETRIC EQU.}$$

$$Q_i: Ax + By + C = 0 \quad \text{IMPLICIT EQU.}$$

$$l_1: y = mx + b$$

↓ slope

Explicit Eqn

! NO slopes in 3D!

Some side (=) Some sign (a line splits a plane into 2 halves)

3.1. a GR param

$$l_1: 2x + y = 1$$

$$l_2: x + ay = -1$$

Discuss the relative position of the 2 lines in terms of a
Determine a GR: O, P(-2, -2) lie on the same side of l_2 .

$$l_1: y = 1 - 2x \rightarrow m_1 = -2$$

$$l_2: ay = -\frac{1}{a}(1+x) \rightarrow \begin{cases} m_2 = -\frac{1}{a}, a \neq 0 \\ m_2 = -\infty, a = 0 \end{cases}$$

for $a=0$, $l_1 \parallel l_2$

for $a \neq 0$:

$$-\frac{1}{a} = -2 \rightarrow a = \frac{1}{2} \rightarrow l_1 \parallel l_2$$

$$a \neq \frac{1}{2} \rightarrow l_1 \nparallel l_2$$

$$l_2(x, y) = x + ay + 1$$

$$l_2(0, 0) = 1$$

$$l_2(-2, -2) = -2 - 2a + 1$$

O, P are on the one side iff:

$$f_2(-2, -2) > 0 \Leftrightarrow$$

$$\Leftrightarrow -2\alpha - 1 > 0$$

$$\alpha < -\frac{1}{2}$$

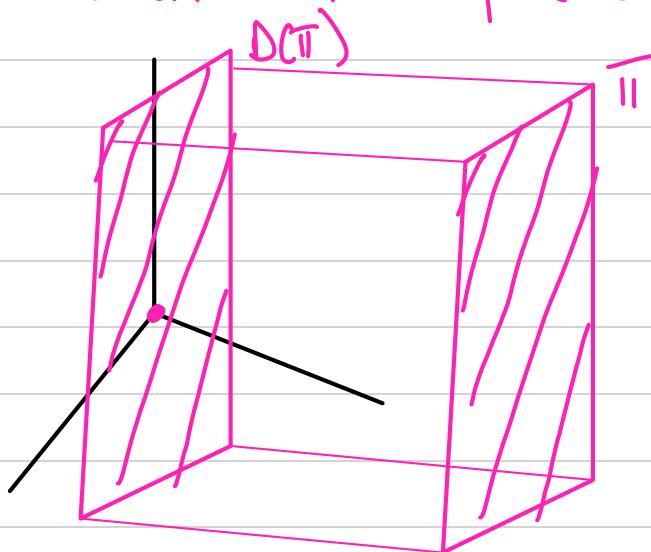
$V - k$ v.d. (ambient space)

A linear variety is a subset $S \subseteq V$ of the form

$$S = a + U = \{a + u \mid u \in U\}$$

where $U \subseteq_k V$
 $a \in V$

The direction space of S is V , denoted $D(S)$



S_1, S_2 linear variety

$S_1 \parallel S_2 \Leftrightarrow D(S_1) \subseteq D(S_2)$

OR

$D(S_2) \subseteq D(S_1)$

Being parallel means the vector is included in the things of the other.

$$3.2. \quad l_1: x - 1 = \frac{y-1}{2} = z - 3$$

$$l_2: \frac{x-3}{3} = \frac{y}{6} = \frac{z-2}{3}$$

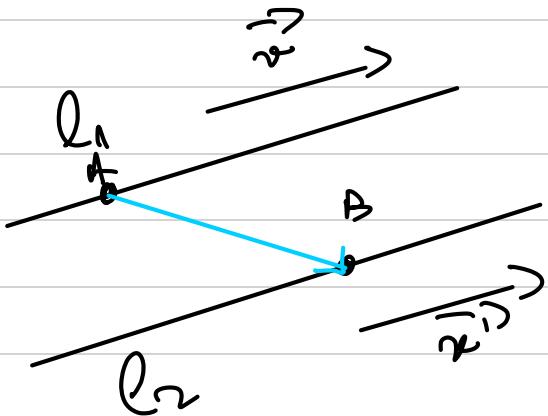
1. Are the lines parallel?

2. If they are, give an eq. for the plane containing them

3. Check if the plane separates the point $P(1,1,1)$ from the origin

The eq. of a plane given by a point A_0 and 2 vectors \vec{v}, \vec{w}

$$\begin{vmatrix} x - x_{A_0} & y - y_{A_0} & z - z_{A_0} \\ \vec{v}_x & \vec{v}_y & \vec{v}_z \\ \vec{w}_x & \vec{w}_y & \vec{w}_z \end{vmatrix} = 0 \quad [...] = Ax + By + Cz + D = 0$$



$$1. \text{ If } A \in \mathbb{Q}, \quad A(1,1,3) \quad \frac{x-1}{1} = \frac{y-1}{2} = \frac{z-3}{3}$$

$$D(l_1) = \langle (1, 2, 1) \rangle$$

$$Q_2: \quad B \in l_2 \Rightarrow B(3, 0, -2)$$

$$D(l_2) = \langle (3, 6, 3) \rangle$$

$$= 3 \times (1, 2, 1) = D(l_1)$$

$$\rightarrow l_1 \parallel l_2$$

$$\overrightarrow{AB}(2, -1, -5)$$

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 2 & -1 & -5 \end{vmatrix} = 0 \Leftrightarrow \{ \dots \}$$

$$1 \rightarrow -5x + 4y - 5z + 14 = 0$$

$$f(x, y, z) = -5x + 4y - 5z + 14$$

$$\left. \begin{array}{l} f(0, 0, 0) = 14 > 0 \\ f(1, 1, 1) = 10 > 0 \end{array} \right\} \Rightarrow \text{no separation}$$

3.3. What is the relative position of the lines?

$$l_1: \begin{cases} x = -3t \\ y = 2+3t \\ z = 1 \end{cases} \quad (\text{LG-R})$$

$$l_2: \begin{cases} x = 1+5s \\ y = 1+13s \\ z = 1+10s \end{cases} \quad (\text{DGR})$$

$$D(l_1) = \langle (-3, 3, 0) \rangle$$

$$D(l_2) = \langle (5, 13, 10) \rangle$$

$(-3, 3, 0)$ and $(5, 13, 10)$ are lin.

indep. $\Rightarrow l_1 \nparallel l_2$

$$l_1 \cap l_2: \begin{cases} x = -3t \\ y = 2+3t \\ z = 1 \\ x = 1+5s \\ y = 1+13s \\ z = 1+10s \end{cases} \quad \leftarrow \begin{cases} -3t = 1+5s \\ 2+3t = 1+13s \\ t = s+10s \end{cases} \Rightarrow \begin{cases} -3t = 1+5s \\ 2+3t = 1+13s \\ t = s+10s \end{cases} \Rightarrow s=0$$

$$\Rightarrow 2+3t = 1 \Rightarrow t = -\frac{1}{3}$$

$$-3t = 1+5s \quad \leftarrow -\frac{1}{3} \cdot -3 = 1$$

$\Rightarrow 1=1$ TRUE

\Rightarrow

$\Rightarrow l_1, l_2$ coplanar \Rightarrow in $A(1,1,1) = l_1 \cap l_2$

3.u. Determine the values a and d for which the line

$$l: \frac{x-2}{3} = \frac{y+1}{2} = \frac{z-3}{-2}$$

is contained in the plane

$$\pi: ax + y - 2z + d = 0$$

$$\cdot D(l) = \langle (3, 2, -2) \rangle$$

$$t = \frac{x-2}{3} = \frac{y+1}{2} = \frac{z-3}{-2}$$

$$\begin{cases} x = 2 + 3t \\ y = -1 + 2t \\ z = 3 - 2t \end{cases}$$

$$a(2 + 3t) + (-1 + 2t) - 2(3 - 2t) + d = 0$$

$$2a + 3at - 1 + 2t - 6 + 4t + d = 0$$

$$(3a + 6)t + (2a - 7 + d) - 0 + t + 1$$

$$\begin{cases} 3a + 6 = 0 \Rightarrow a = -2 \end{cases}$$

$$\begin{cases} 2a - 7 + d = 0 \Rightarrow -4 - 7 + d = 0 \Rightarrow d = 11 \end{cases}$$

3.5. Determine the relative positions of the line

$$l: \frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-1}{-u}$$

and the plane

$$\pi: 2x - y + z - 1 = 0$$

See if the line passes the plane, determine the intersection point

$$t = \frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-1}{-u} \quad M_A(x_1, y_1, z_1) \in l$$

$$\begin{cases} x = t+1 \\ y = -2t+2 \\ z = -ut+1 \end{cases}$$

$$\text{Let } f(x, y, z) = 2x - y + z - 1 \Rightarrow$$

$$\Rightarrow f(A) = 2(t+1) - (-2t+2) + (-ut+1) - 1 =$$

$$= 0 \Rightarrow f(A) = 0, t \notin \mathbb{C} \Rightarrow l \subset \pi$$

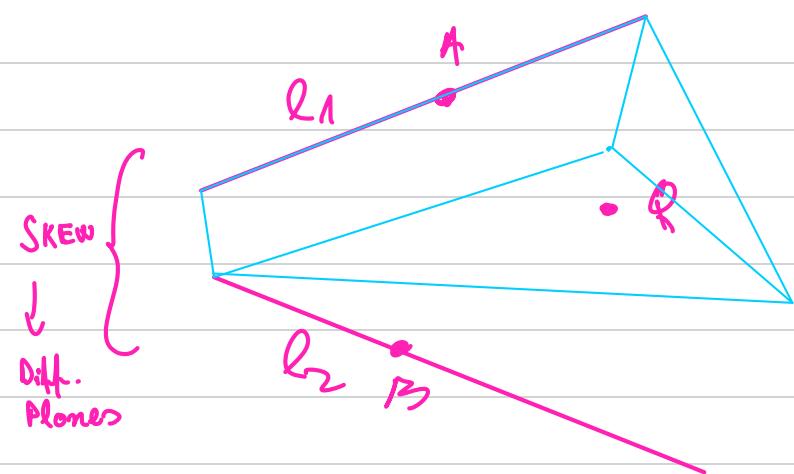
3.6. Determine Cartesian equations for the line ℓ

passing through $Q(1, 1, 2)$ and coplanar to the lines:

$$\ell^1: \begin{cases} 3x - 5y + z = -1 \\ 2x - 3z = -3 \end{cases}$$

$$\ell^2: \begin{cases} x + 5y = 3 \\ 2x + 2y - 2z = -1 \end{cases}$$

ideas:



we need to write

$$\pi_1: Q, \vec{v}_1 \in D(\ell_1), \vec{QR}$$

$$\pi_2: Q, \vec{v}_2 \in D(\ell_2), \vec{QB}$$

$$\ell: \begin{cases} \pi_1: \\ \pi_2: \end{cases}$$