

**10.1.** For each of the equations in Table 10.2 , discuss the geometric locus of points satisfying them.

**10.2.** For each of the following equations write down the associated matrix and bring the equation in isometric canonical form.

a)  $-x^2 + xy - y^2 + x = 0$ ,

b)  $6xy + x - y = 0$ .

**10.3.** In each of the following cases, decide the type of the quadratic curve based on the parameter  $a \in \mathbb{R}$ .

a)  $x^2 - 4xy + y^2 = a$ ,

b)  $x^2 + 4xy + y^2 = a$ .

**10.4.** Consider the rotation  $R_{90^\circ}$  of  $\mathbb{E}^2$  around the origin and the translation  $T_v$  of  $\mathbb{E}^2$  by the vector  $v(1, 0)$ .

a) Give the matrix form of the isometries  $R_{90^\circ}$ ,  $T_v$  and  $T_v \circ R_{90^\circ}$ .

b) Determine the equations of the hyperbola  $\mathcal{H} : \frac{x^2}{4} - \frac{y^2}{9} - 1 = 0$  and the parabola  $\mathcal{P} : y^2 - 8x = 0$  after transforming them with  $R_{90^\circ}$  and with  $T_v \circ R_{90^\circ}$  respectively.

**10.5.** Discuss the type of the curve

$$x^2 + \lambda xy + y^2 - 6x - 16 = 0$$

in terms of  $\lambda \in \mathbb{R}$ .

**10.6.** Using the classification of quadrics, decide what surfaces are described by the following equations.

a)  $x^2 + 2y^2 + z^2 + xy + yz + zx = 1$ ,

b)  $xy + yz + zx = 1$ ,

c)  $x^2 + xy + yz + zx = 1$ ,

d)  $xy + yz + zx = 0$ .