0.1. Let A, B, C and D be points on a circle with center O such that the chords $[AB]$ and $[CD]$ are perpendicular. Let M be the intersection point of the two chords and let N and P be the midpoints of $[AB]$ and $[CD]$ respectively. Show that $MNOP$ is a rectangle.

0.2. Consider a triangle ABC. The triangle whose vertices are the midpoints of the sides is called the *midtriangle* of the triangle ABC. Use this concept to show that the theorems stating that the altitudes, respectively the perpendicular bisectors, of a tringle intersect in a point are equivalent.

0.3. Let ABC be a triangle with midtriangle MNP (see previous exercise for the definition of midtriangle). Show that
$$\mathrm{Area}(ABC) = 4\,\mathrm{Area}(MNP).$$

0.4. Let ABC be a triangle with side lengths a, b and c. Starting from Heron's formula and using the inequality between the geometric and the arithmetic means show that

$$\mathrm{Area}(ABC) \leq \frac{1}{3\sqrt{3}}\left(\frac{a+b+c}{2}\right)^2$$

with equality if and only if the triangle is equilateral.

0.5 (Thales' circle theorem)**.** Prove that if A, B, C are points on a circle, then $[AC]$ is a diameter if and only if $\measuredangle ABC$ is a right angle.

0.6 (Central angle theorem)**.** Let ABC be a triangle and consider the circumcircle with center O. We say that the angle $\varphi = \measuredangle(ACB)$ is subtended by the chord $[AB]$ and that the angle $\psi = \measuredangle AOB$ is the corresponding central angle. Show that $\psi = 2\varphi$.

0.7 (Ptolemy's theorem)**.** A quadrilateral is called *inscribed* if its vertices lie on a circle. Prove that a quadrilateral is inscribed if and only if the sum of the products of the lengths of its two pairs of opposite sides is equal to the product of the lengths of its diagonals.

0.8 (almost ordered Bell numbers)**.** For $n = 1, 2, 3$ what is the number of configurations of not necessarily distinct points on a line. (Answer: $1, 2, 7, 38, \ldots$)