1.1. Let $\lambda = |CA| : |CB|$ be the ratio in which the point $C \in [AB]$ divides the segment $[AB]$. Check that

$$\overrightarrow{OC} = \frac{1}{1+\lambda}\overrightarrow{OA} + \frac{\lambda}{1+\lambda}\overrightarrow{OB}.$$

1.2. Let $ABCD$ be a quadrilateral. Let M, N, P, Q be the midpoints of $[AB]$, $[BC]$, $[CD]$ and $[DA]$ respectively. Show that

$$\overrightarrow{MN} + \overrightarrow{PQ} = 0.$$

Deduce that the midpoints of the sides of an arbitrary quadrilateral form a parallelogram.

1.3. Let $ABCD$ be a quadrilateral. Let E be the midpoint of $[AB]$ and let F be the midpoint of $[CD]$. Show that

$$\overrightarrow{EF} = \frac{1}{2}\left(\overrightarrow{AD} + \overrightarrow{BC}\right).$$

1.4. Let $SABCD$ be a pyramid with apex S and base the parallelogram $ABCD$. Show that

$$\overrightarrow{SA} + \overrightarrow{SB} + \overrightarrow{SC} + \overrightarrow{SD} = 4\overrightarrow{SO}$$

where O is the center of the parallelogram.

1.5. Consider the parallelograms $A_1A_2A_3A_4$ and $B_1B_2B_3B_4$. Show that the midpoints of the segments $[A_1B_1]$, $[A_2B_2]$, $[A_3B_3]$ and $[A_4B_4]$ are the vertices of a parallelogram.

1.6. Starting from Hilbert's Axioms prove that the base angles of an isosceles triangle are congruent.

1.7. Using vectors, show that the medians in a triangle intersect in one point and deduce the ratio in which the common intersection point divides the medians.

1.8 (Euler's line)**.** Prove that the orthocenter H, the centroid G and the circumcenter U of a triangle are collinear. Moreover show that

$$\overrightarrow{HG} = 2\overrightarrow{GU}.$$