

Homework 4
Computer Science
Spring 2017
B351

Steven Myers

April 3, 2017

All the work herein is mine.

Answers

1. (a) i. $\exists y p(y) \vee [\exists y (q(y) \rightarrow (\exists x (p(x) \vee q(x, y, C))))]$
ii. $\exists y p(y) \vee [\exists y (\neg q(y) \vee (\exists x (p(x) \vee q(x, y, C))))] \rightarrow$ replacement
iii. $\exists y p(y) \vee [\exists z (\neg q(z) \vee (\exists x (p(x) \vee q(x, z, C))))]$ standardizing variables
iv. $p(A) \vee \neg q(F(z)) \vee p(G(x)) \vee q(G(x), F(z), C)$ skolemize \exists to constants and functions
- (b) i. $\forall x \forall y \forall z d(x, y) \wedge d(y, z) \rightarrow d(x, z)$
ii. $\forall x \forall y \forall z d(x, y) \wedge \neg d(y, z) \vee d(x, z) \rightarrow$ elimination
iii. $d(x, y) \wedge \neg d(y, z) \vee d(x, z)$ drop universal quantifiers
iv. $(d(x, y) \vee d(x, z)) \wedge (\neg d(y, z) \vee d(x, z))$ distribute \vee over \wedge
- (c) i. $(P \vee Q) \wedge (\neg P \rightarrow (Q \vee R))$
ii. $(P \vee Q) \wedge (P \vee Q \vee R) \rightarrow$ elimination and double negation
2. (a) *True*
(b) *False*
(c) *False.*
(d) *False*
(e) *False*
(f) *True.*
3. (a) *True.* One in three times $p(x)$ evaluates to *false*, and two in three times it is *true*. So, it is exactly twice as often.
(b) *True.* Expands out to $Mym(y, 1) \rightarrow p(1) \wedge Mym(y, 2) \rightarrow p(2) \wedge Mym(y, 3) \rightarrow p(3)$

4. The domain is not specified, so I will assume the domain is the set of dogs Ursala, Kaiser, and Shilah. In plain English, the FOL sentence states that there is some dog, call that dog x , and another dog, call that dog y . x is gray and y is silver, and x loves y . In order to do resolution refutation, we must convert the sentence to CNF or clausal form. Additionally, we'll need to translate our three premises into the formal language. Since we have the three dogs, we'll need an individual constant for each one.

Interpretation (Constants and Domain)

$Domain = \{Ursala, Kaiser, Shilah\}$

$U = Ursala$

$K = Kaiser$

$S = Shilah$

Premises (converted to FOL then to CNF)

- a. $silver(U)$
- b. $gray(S) \wedge loves(S, K)$
- c.
 1. $\neg(gray(K) \leftrightarrow silver(K)) \wedge loves(K, U)$
 2. $(gray(K) \wedge \neg silver(K)) \vee (silver(K) \wedge \neg gray(K)) \wedge loves(K, U)$ \leftrightarrow equivalence rule
 3. $(gray(K) \wedge \neg silver(K) \wedge loves(K, U)) \vee (silver(K) \wedge \neg gray(K) \wedge loves(K, U))$ distribute \vee over \wedge
 4. $\neg[(\neg gray(K) \vee silver(K) \vee \neg loves(K, U)) \wedge (\neg silver(K) \vee gray(K) \vee \neg loves(K, U))]$ DeMorgan's

Conclusion (converted to CNF)

$\exists x \exists y (gray(x) \wedge silver(y) \wedge loves(x, y))$

$\exists y (gray(F(x)) \wedge silver(y) \wedge loves(F(x), y))$ Skolemize or bind $\exists x$ to skolem function $F(x)$

$gray(F(x)) \wedge silver(G(y)) \wedge loves(F(x), G(y))$ Skolemize or bind $\exists y$ to skolem function $G(y)$

Resolution

1. $silver(U)$ Premise
2. $gray(S) \wedge loves(S, K)$ Premise
3. $(gray(K) \vee loves(K, U)) \wedge (\neg silver(K) \vee loves(K, U))$ Premise
4. $\neg(gray(F(x)) \wedge silver(G(y)) \wedge loves(F(x), G(y)))$ Conclusion
5. $\neg gray(F(x)) \vee \neg silver(G(y)) \vee \neg loves(F(x), G(y))$ from 4
6. 7.