Graph Theory
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Undirected and Directed Graphs

Graphs are comprised of vertices and edges. An edge serves as a connection between two vertices. We can also say that if there is an edge between vertices, then there is a pathway to get from one vertex to the next. Some examples of where graphs are applicable are graphs forming relationships between locations. Some vertices under that example could be cities, and their edges would represent the roads or pathways that lead from one city to the next. There are two types of graphs: *undirected* and *directed*. A directed graph is one where edges have a direction. So, an edge from vertex A to vertex B means that there is a connection from A to B, but not from B to A. On the contrary, undirected graphs do not have direction, so an edge between two vertices is bidirectional. If we have an edge between two vertices A and B, then there is a path from A to B, and B to A.

Representing Graphs

We can formally represent graphs as a collection of vertices and edges. A vertex is a singular point or constant, and an edge is tuple of two connected vertices.

$$Vertices = \{V_1, V_2, V_i, \ldots\}$$
$$Edges = \{(V_1, V_2), \ldots\}$$
$$Graph = \{Vertices, Edges\}$$

We say that a graph is *traversable* if every vertex is reachable from any other vertex.

Euler Paths and Circuits

If a graph is traversable, then we know that every vertex in the graph can be visited. If a graph is traversable, and can be traversed by using every edge exactly once, then we say that the graph contains an Euler path. More precisely, an Euler path is a path in which every edge is traversed exactly once. A subset of Euler paths are Euler ciruits, which are Euler paths that begin and end on the same vertex.