CSCI-B 401 Homework #1 Assigned Thursday, August 24th, 2017
Instructor: Wennstrom Due Tuesday, August 29th, 2017

Do problems 0.3 and 0.6 - 0.8 from Sipser (page 26).

1. Prove that for any sets A, B, C, and D, if $A \subseteq B \cap C$, then $A \setminus D \subseteq B \cup D$.

   Use the definitions of $\cap$, $\cup$, $\setminus$, and $\subseteq$ (as given below). This is meant to be an exercise in working with fundamental definitions, not applying theorems, so you are not allowed to use any higher-level facts about unions, intersections, and set differences. (For example, it's true that $X \cap Y \subset X$ and that $X \subset X \cup Y$, but you should not use these theorems in your proof.) Hint: if you haven't introduced any new variables in your proof, then you're on the wrong track.

   Definition. $x \in A \cap B$ means that $x \in A$ and $x \in B$.

   Definition. $x \in A \cup B$ means that $x \in A$ or $x \in B$.

   Definition. $x \in A \setminus B$ means that $x \in A$ and $x \notin B$.

   Definition. $A \subseteq B$ means that if $x \in A$, then $x \in B$.

2. Prove that for all natural numbers $n \geq 7$, $3^n < n!$.

3. Prove that for any positive integer n, $\sum_{i=1}^{n} 2i = n(n+1)$.

4. In your own words, explain why it doesn't make sense talk about "the set of all sets".

5. Let $B = \{s \mid s \text{ is a binary string that starts with } 1\ \}$ (so it contains members like 11011, 101010, and 1, but not 00101, 0, or the empty string). Prove that B is countable.