Homework 4
Computer Science
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B351

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All the work herein is mine.

Answers

1. (a) i. $\exists\ y\ p(y) \lor [\exists\ y\ (q(y) \to (\exists\ x\ (p(x) \lor\ q(x,y,C)))]$
   ii. $\exists\ y\ p(y) \lor [\exists\ y\ (\neg q(y) \lor (\exists\ x\ (p(x) \lor\ q(x,y,C)))]$ $\to$ replacement
   iii. $\exists\ y\ p(y) \lor [\exists\ z\ (\neg q(z) \lor (\exists\ x\ (p(x) \lor\ q(x,z,C)))]$ standardizing variables
   iv. $p(A) \lor \neg q(F(z)) \lor\ p(G(x)) \lor\ q(G(x), F(z), C)$ skolemize $\exists$ to constants and functions

   (b) i. $\forall x \forall y \forall x\ d(x,y) \land d(y,z) \to d(x,z)$
   ii. $\forall x \forall y \forall x\ d(x,y) \land \neg d(y,z) \lor d(x,z)$ $\to$ elimination
   iii. $d(x,y) \land \neg d(y,z) \lor d(x,z)$ drop universal quantifiers
   iv. $(d(x,y) \lor d(x,z)) \land (\neg d(y,z) \lor d(x,z))$ distribute $\lor$ over $\land$

   (c) i. $(P \lor Q) \land (\neg P \to (Q \lor R))$
   ii. $(P \lor Q) \land (P \lor Q \lor R)$ $\to$ elimination and double negation

2. (a) *True*
   (b) *False*
   (c) *False*. In the languages I am aquainted with, x is not a free variable, so this is syntactically incorrect. Moreover, if it were correct syntactically, we would apply the rules by the main connective, so we would have something like $\exists x\ m(1,1) \land \exists x\ m(2,2) \land \exists x\ m(3,3)$ which is also false.
   (d) *False*
   (e) *False*
   (f) *True*. Again, see note for *c*. Here, $\exists x$ is our main connective, so we get $\forall x\ m(1,1) \lor \forall x\ m(2,2) \lor \forall x\ m(3,3)$ and this is true since $m(1,1)$ is true.

3. (a) *True*. One in three times $p(x)$ evaluates to *false*, and two in three times it is *true*. So, it is exactly twice as often.
   (b) *True*. Expands out to $Mym(y,1) \to p(1) \land Mym(y,2) \to p(2) \land Mym(y,3) \to p(3)$

4. The domain is not specified, so I will assume the domain is the set of dogs Ursala, Kaiser, and Shilah. In plain English, the FOL sentence states that there is some dog, call that dog x, and another dog, call that dog y. x is gray and y is silver, and x loves y. In order to do resolution refutation, we must convert the sentence to CNF or clausal form. Additionally, we'll need to translate our three premises into the formal language. Since we have the three dogs, we'll need an individual constant for each one.

Interpretation (Constants and Domain)

$Domain = \{Ursala, Kaiser, Shilah\}$
$U = Ursala$
$K = Kaiser$
$S = Shilah$

Premises (converted to FOL then to CNF)

a. $silver(U)$
b. $gray(S) \wedge loves(S, K)$
c.
   1. $\neg(gray(K) \leftrightarrow silver(K)) \wedge loves(K, U)$
   2. $(gray(K) \wedge \neg silver(K)) \vee (silver(K) \wedge \neg gray(K)) \wedge loves(K, U)$ — $\leftrightarrow$ equivalence rule
   3. $(gray(K) \wedge \neg silver(K) \wedge loves(K, U)) \vee (silver(K) \wedge \neg gray(K) \wedge loves(K, U))$ — distribute $\vee$ over $\wedge$
   4. $(\neg gray(K) \vee silver(K) \vee \neg loves(K, U)) \wedge (\neg silver(K) \vee gray(K) \vee \neg loves(K, U))$ — DeMorgan's

Conclusion (converted to CNF)

$\exists x \exists y (gray(x) \wedge silver(y) \wedge loves(x, y))$	
$\exists y (gray(F(x)) \wedge silver(y) \wedge loves(F(x), y))$	Skolemize or bind $\exists x$ to skolem function $F(x)$
$gray(F(x)) \wedge silver(G(y)) \wedge loves(F(x), G(y))$	Skolemize or bind $\exists y$ to skolem function $G(y)$

Resolution

1.	$silver(U)$	Premise
2.	$gray(S) \wedge loves(S, K)$	Premise
3.	$(gray(K) \vee loves(K, U)) \wedge (\neg silver(K) \vee loves(K, U))$	Premise
4.	$\neg(gray(F(x)) \wedge silver(G(y)) \wedge loves(F(x), G(y)))$	Conclusion
5.	$\neg gray(F(x)) \vee \neg silver(G(y)) \vee \neg loves(F(x), G(y))$	from 4
6.	7.	