Homework 4
Computer Science
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B351

Steven Myers

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All the work herein is mine.

Answers

1. (a) i. $\exists\ y\ p(y) \vee [\exists\ y\ (q(y) \rightarrow (\exists\ x\ (p(x) \vee\ q(x, y, C)))]$
   ii. $\exists\ y\ p(y) \vee [\exists\ y\ (\neg q(y) \vee (\exists\ x\ (p(x) \vee\ q(x, y, C)))]$ $\rightarrow$ replacement
   iii. $\exists\ y\ p(y) \vee [\exists\ z\ (\neg q(z) \vee (\exists\ x\ (p(x) \vee\ q(x, z, C)))]$ standardizing variables
   iv. $p(A) \vee \neg q(F(z)) \vee\ p(G(x)) \vee\ q(G(x), F(z), C)$ skolemize $\exists$ to constants and functions
   v. $[[[p, A], [\neg, q, F(z)], [p, G(x)], [q, G(x), F(z), C]]]$ Pythonic Representation

   (b) i. $\forall x \forall y \forall x\ d(x, y) \wedge d(y, z) \rightarrow d(x, z)$
   ii. $\forall x \forall y \forall x\ d(x, y) \wedge \neg d(y, z) \vee d(x, z)$ $\rightarrow$ elimination
   iii. $d(x, y) \wedge \neg d(y, z) \vee d(x, z)$ drop universal quantifiers
   iv. $(d(x, y) \vee d(x, z)) \wedge (\neg d(y, z) \vee d(x, z))$ distribute $\vee$ over $\wedge$
   v. $[[[d, x, y], [d, x, z]], [[\neg, d, y, z], [d, x, z]]]$ Pythonic Representation

   (c) i. $(P \vee Q) \wedge (\neg P \rightarrow (Q \vee R))$
   ii. $(P \vee Q) \wedge (P \vee Q \vee R)$ $\rightarrow$ elimination and double negation
   iii. $[[[P], [Q]], [[P], [Q], [R]]]$ Pythonic Representation

2. (a) *True.* $m(1, 1) \wedge m(2, 1) \wedge m(3, 2)$ is true, so the universal quantifier is true in all possible values of 'x'.
   (b) *False.* There is no ordered pair in the set of m ordered pairs such that the number 3 is the second element in the ordered pair, thus not all instances of the $\forall x$ are true.
   (c) *False.* In the languages I am aquainted with, x is not a free variable, so this is syntactically incorrect. Moreover, if it were correct syntactically, we would apply the rules by the main connective, so we would have something like $\exists x\ m(1, 1) \wedge \exists x\ m(2, 2) \wedge \exists x\ m(3, 3)$ which is also false.
   (d) *False.* There is no value for x such that it is the first component of every ordered pair in the set of ordered pairs m.
   (e) *False.* There is no value for x such that it is the second component of every ordered pair in the set of ordered pairs m.
   (f) *True.* Again, see note for c. Here, $\exists x$ is our main connective, so we get $\forall x\ m(1, 1) \vee \forall x\ m(2, 2) \vee \forall x\ m(3, 3)$ and this is true since $m(1, 1)$ is true.

3. (a) *True.* One in three times $p(x)$ evaluates to *false*, and two in three times it is *true*. So, it is exactly twice as often.

   (b) *True.* Expands out to $Mym(y,1) \rightarrow p(1) \wedge Mym(y,2) \rightarrow p(2) \wedge Mym(y,3) \rightarrow p(3)$

4. The domain is not specified, so I will assume the domain is the set of dogs Ursala, Kaiser, and Shilah. In plain English, the FOL sentence states that there is some dog, call that dog x, and another dog, call that dog y. x is gray and y is silver, and x loves y. In order to do resolution refutation, we must convert the sentence to CNF or clausal form. Additionally, we'll need to translate our three premises into the formal language. Since we have the three dogs, we'll need an individual constant for each one.

Interpretation (Constants and Domain)

$Domain = \{Ursala, Kaiser, Shilah\}$
$U = Ursala$
$K = Kaiser$
$S = Shilah$

Premises (converted to FOL then to CNF)

a. $silver(U)$
b. $gray(S) \wedge loves(S,K)$
c. 1. $\neg(gray(K) \leftrightarrow silver(K)) \wedge loves(K,U)$
   2. $(gray(K) \wedge \neg silver(K)) \vee (silver(K) \wedge \neg gray(K)) \wedge loves(K,U)$ $\quad\leftrightarrow$ equivalence rule

Conclusion (converted to CNF)

$\exists x \exists y(gray(x) \wedge silver(y) \wedge loves(x,y))$	
$\exists y(gray(F(x)) \wedge silver(y) \wedge loves(F(x),y))$	Skolemize or bind $\exists x$ to skolem function $F(x)$
$gray(F(x)) \wedge silver(G(y)) \wedge loves(F(x),G(y))$	Skolemize or bind $\exists y$ to skolem function $G(y)$

Resolution

1.	$silver(U)$	Premise
2.	$gray(S) \wedge loves(S,K)$	Premise
3.	$(gray(K) \wedge \neg silver(K)) \vee (silver(K) \wedge \neg gray(K)) \wedge loves(K,U)$	Premise
4.	$\neg(gray(F(x)) \wedge silver(G(y)) \wedge loves(F(x),G(y)))$	Conclusion
5.	$\neg gray(F(x)) \vee \neg silver(G(y)) \vee \neg loves(F(x),G(y))$	from 4
6.	$silver(U) \wedge (\neg gray(F(x)) \vee \neg silver(G(y)) \vee \neg loves(F(x),G(y)))$	Addition on lines 1, 5
7.	$silver(U) \wedge (\neg gray(F(x)) \vee \neg silver(U) \vee \neg loves(F(x),U))$	Let $G(y) = U$ and substitute
8.	$(silver(U) \wedge \neg gray(Fx)) \vee (silver(U) \wedge loves(F(x),U) \vee$ $(silver(U) \wedge \neg silver(U))$	Distribute $\wedge$ over $\vee$
9.	$(silver(U) \wedge \neg gray(Fx)) \vee (silver(U) \wedge loves(F(x),U))$	Resolve the last disjunct
10.	...	

5. ***Interpretation (Constants and Domain)***
   $Domain$ = All Objects
   $pushable(x)$ = x is pushable
   $red(x)$ = x is red
   $green(x)$ = x is green
   $C = Object1$ (A cart)
   $O = Object2$ (A pile of ore)

   Translations

   (a) If pushable objects are green, the non-pushable are red.

i.	$\forall x(pushable(x) \rightarrow green(x)) \rightarrow \forall y(\neg pushable(y) \rightarrow red(y))$	FOL
ii.	$\exists x(pushable(x) \wedge \neg green(x)) \vee \forall y(pushable(y) \vee red(y))$	$\rightarrow$ elimination and $\neg\neg$ elimination
iii.	$(pushable(G(x)) \wedge \neg green(G(x)) \vee pushable(y) \vee red(y)$	Drop $\forall y$ and skolemize $\exists x$ with $G(x)$
iiii.	$[[[[pushable, G(x)], [\neg, green, G(x)]], [pushable, y], [red, y]]]$	Pythonic Representation

   (b) All objects are either green or red.

i.	$\forall x(green(x) \vee red(x))$	FOL
ii.	$green(x) \vee red(x)$	CNF
iii.	$[[[green, x], [red, x]]]$	Pythonic Representation

   ****This problem does not state exclusivity of red and green objects***

   (c) Object 1, a cart, is pushable.

i.	$pushable(C)$	FOL and CNF
ii.	$[[pushable, C]]$	Pythonic Representation

   (d) Object 2, a pile of ore, is not pushable.

i.	$\neg pushable(O)$	FOL and CNF
ii.	$[[pushable, O]]$	Pythonic Representation

   Resolution Refutation

i.	$\exists x(red(x))$	Conclusion
ii.	$[[pushable, O], [pushable, C], [[green, x], [red, x]],$ $[[[pushable, G(x)], [\neg, green, G(x)]], [pushable, y], [red, y]]]$	Pythonic Representation