Cylindrical cloaking at oblique incidence with optimized finite multilayer parameters

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We propose multilayer cylindrical invisibility cloaks that are optimized for oblique incidences through a combination of analytic formalism of scattering and genetic optimization. We show that by using only four layers of homogeneous and anisotropic metamaterials without large values of constitutive parameters, the scattering for oblique incidences can be reduced by 2 orders. Although the optimization is done at a single incident angle, the cloak provides reduced scattering over a large range of incident angles.

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Many efforts have been made recently on designing and implementing metamaterial invisibility cloaks using transformation methods [1–9]. A perfect invisibility cloak created from transforming an empty electromagnetic space [1] has been proven theoretically to be perfect in cloaking an arbitrary object [5–7], while the only electromagnetic mechanism so far to detect a perfect cloak within its working band requires extreme conditions [10]. However, the practical development of metamaterial invisibility cloaking is still far from a real application at this stage, because the rigorous requirements of an ideal cloak, such as continuously varying parameters that approach extreme values in some regions, are challenging for practical fabrication. Although a simplified cylindrical cloak that was implemented previously [3] can moderately reduce scattering, it is still inherently visible [11] and works only for normal incidence in 2D geometry. Recently, there have been theoretical suggestions to overcome the drawbacks of continuous and extreme parameters by optimizing the structure of a few layers of homogeneous and anisotropic metamaterials to construct a practical cloak [12,13]. However, similar to previous studies, only normal incidence is considered, and, therefore, the results would only work in purely 2D geometry. The extension of a 2D design to a 3D design will be one step further toward the practically usable realization of invisibility cloaking.

In this Letter, we present an invisibility cloak design with only four layers of metamaterials that can work at a wide range of incident angles. Our work can be treated as the extension and supplement of the design of purely 2D models into more physical and practical 3D models [12,13]. We use genetic optimization to globally search an optimized set of parameters. The optimization of a multilayer cloak is done with an oblique incidence of 30°. Our method can be used as a guide for future implementation of metamaterial invisibility cloaks.

The geometry of the multilayer cylindrical cloak model is shown in Fig. 1, where a cylinder of perfect electric conductor (PEC) with radius R_{M+1} is cloaked by the multilayered cylindrical shell with the innermost radius R_{M+1} and the outermost radius R_1. In between R_1 and R_{M+1} are M layers marked by $m(1 \le m \le M)$ with different outer radii varying from R_1 to R_M. The constitutive parameters (ϵ_ρ, ϵ_ϕ, ϵ_z, μ_ρ, μ_ϕ, and μ_z) of each layer are assumed to be homogeneous. The region of $r > R_1$ is free space. The incident electromagnetic wave is incident on the multilayer cylindrical cloak with an angle of α with respect to the xy plane. Without loss of generality, we consider the vertical polarization as shown in Fig. 1. Therefore, the unit incident wave takes the form of $\bar{E}_i = (-\hat{x}\sin\alpha + \hat{z}\cos\alpha)e^{ikz\sin\alpha + ikx\cos\alpha}$.

For oblique incidence, the difficulty here is that there is no closed-form solution to the wave equation inside each layer. We have provided a method to calculate the scattering from a general cylindrical cloak under oblique incidence in [14] based on the state-variable approach [15]. We apply this method in each layer to calculate each corresponding state propagator matrix, respectively, and their product will be the final state propagator matrix of the entire multilayer structure. If we divide each layer into N sublayers ($N \gg 1$), the state propagator equation will be

$$\bar{V}(R_{M+1}) = \left[\prod_{j=(M-1)N+1}^{MN} (\bar{\bar{I}} + \Delta\rho\bar{\bar{T}}(\rho_j))\right] \cdot \left[\prod_{j=(M-2)N+1}^{(M-1)N} (\bar{\bar{I}} + \Delta\rho\bar{\bar{T}}(\rho_j))\right] \cdots \cdot \left[\prod_{j=1}^{N} (\bar{\bar{I}} + \Delta\rho\bar{\bar{T}}(\rho_j))\right] \cdot \bar{V}(R_1), \qquad (1)$$

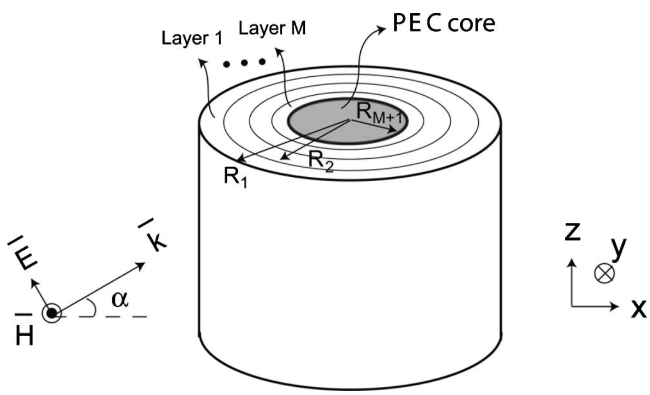

Fig. 1. Configuration of a four-layer cloak when a vertically polarized plane wave is obliquely incident with the incident angle of α.

where the four-dimensional vector $\bar{V}=[E_z \quad E_\phi \quad H_z \quad H_\phi]^T$ is the state vector. After obtaining the state propagator matrix, we can solve the field distribution over the entire space [14]. Now by applying the genetic optimization [12], a multilayer cloak applied at oblique incidence is obtained.

For comparison with the performance of simplified cloaks at oblique incidence, a cylindrical structure with the same size as in [14] is considered, i.e., $R_1 = 1.5\lambda_0 = 2.08R_{M+1}$. We set the number of layers M to be four, where each layer has the same thickness. Another condition is that we force all relative constitutive parameters in each layer to be less than 10. The incident angle of the incident wave is chosen to be $\alpha = 30°$. In the genetic algorithm, the far-field scattering efficiency Q_{sca} [7], or the scattering cross section normalized by the unit geometric cross section of $2R_{M+1}$, is chosen as the function to be minimized.

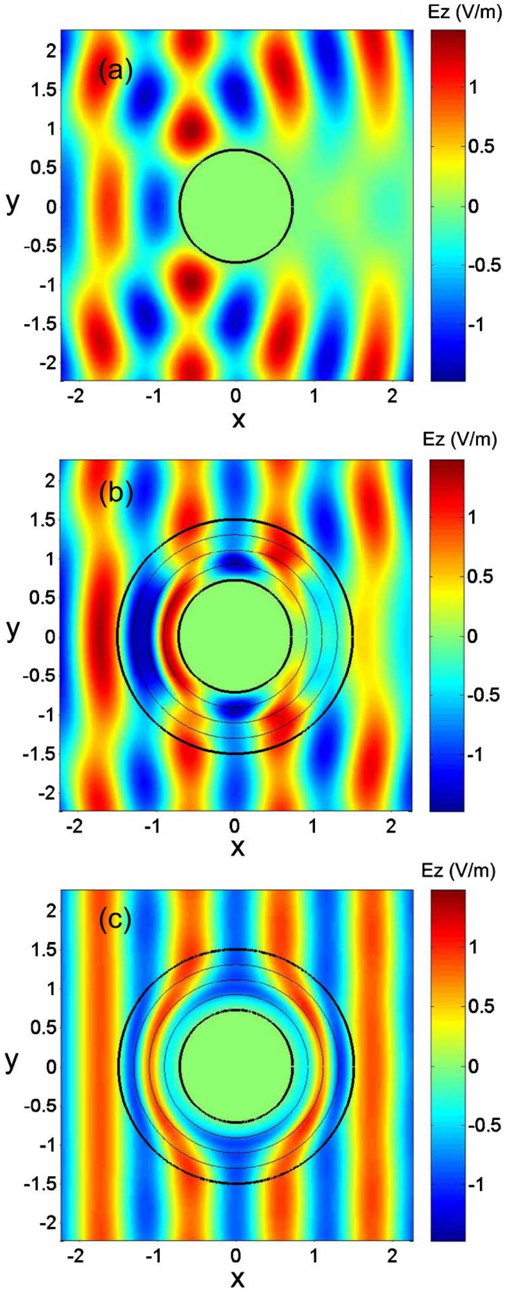

Fig. 2. (Color online) E_z field distribution in xy plane due to scattering from a (a) bare PEC cylinder, (b) four-layer cloak with three parameters (ϵ_z, μ_ρ, and μ_ϕ) optimized, and (c) four-layer cloak with all six parameters optimized, when a vertically polarized plane wave is obliquely incident from left to right with the incident angle of 30°. From (a) to (c), the scattering efficiency Q_{sca} is 1.175, 0.498, and 0.013, respectively.

We first plot the scattering from the bare PEC core (radius: $0.721\lambda_0$) as the reference, where the E_z field in the xy plane is shown in Fig. 2(a). It is shown that the scattering is quite large. The scattering efficiency is 1.175 in this case. Note that although the incident wave is strictly vertically polarized, the reflected wave is generally composed of both vertical and horizontal polarizations, where only the vertical polarization has the E_z component. The next task is to minimize Q_{sca} by optimization.

We have two methods of optimization. One is to control ϵ_z, μ_ρ, and μ_ϕ in each layer (the previous simplified cloak for normal incidence [3] only needs these three parameters), while keeping ϵ_ρ, ϵ_ϕ, and μ_z as a constant of 1. The other method is to control all six constitutive parameters in each layer. The performances of these two optimized four-layer cloaks are shown in Figs. 2(b) and 2(c). Their relative constitutive parameters are summarized in Table 1. It can be seen in Fig. 2(b) that by controlling only three parameters in each layer, the total scattering has been much reduced when compared to the bare PEC core in Fig. 2(a). The scattering efficiency Q_{sca} is reduced from 1.175 to 0.498. After optimizing all six parameters in each layer in the cloak, the scattering efficiency Q_{sca} is 0.013, close to "perfect invisibility," as shown in Fig. 2(c). It is worth mentioning that although the four-layer cloak in Fig. 2(c) is achieved by optimization at the single incident angle of 30°, it is still valid for other incident angles. In Fig. 3, we can see that this cloak has satisfactory performance over a large range of incident angles, even when some loss tangent is included in all constitutive parameters of the cloak.

In principle, we can ascribe this low scattering phenomenon to the destructive interference between the waves from the first reflection on the outermost boundary and the waves from the total transmission from inside the cloak to outside [12]. However, the scenario of oblique incidence is more complicated than the case of normal incidence, because both polarizations need to be optimized at the same time. For normal incidence,

Table 1. Relative Constitutive Parameters for Optimized Four-Layer Cloak by (I) Optimizing Three Parameters in Each Layer and (II) Optimizing Six Parameters in Each Layer

I	Optimizing Three Parameters					
Layer	μ_ρ	μ_ϕ	ϵ_z	ϵ_ρ	ϵ_ϕ	μ_z
1	0.962	1.423	1.154	1	1	1
2	1.128	0.160	0.598	1	1	1
3	0.792	2.754	1.321	1	1	1
4	0.665	1.025	2.625	1	1	1
II	Optimizing Six Parameters					
Layer	μ_ρ	μ_ϕ	ϵ_z	ϵ_ρ	ϵ_ϕ	μ_z
1	0.550	2.320	1.725	0.438	1.781	3.409
2	0.283	3.584	1.657	0.262	3.953	0.037
3	0.224	8.204	0.614	0.189	8.674	1.754
4	0.057	9.994	0.015	1.540	9.069	0.030

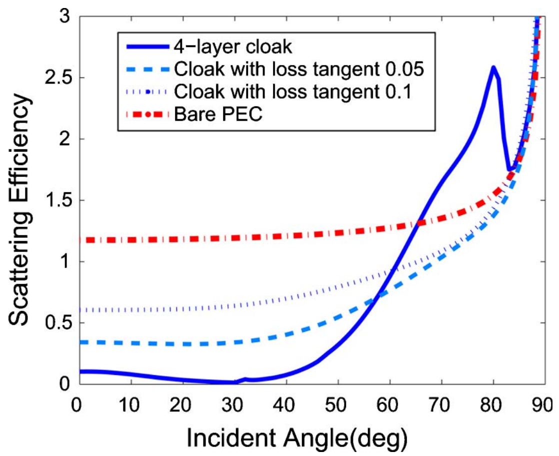

Fig. 3. (Color online) Dependence of scattering efficiency Q_{sca} on the incident angle for the four-layer cloak achieved by optimizing all six parameters in each layer within the cloak. The dotted curves correspond to the scattering efficiencies from the cloak with loss tangents 0.05 and 0.1 and from the bare PEC core without the cloak, respectively.

because vertically polarized waves and horizontally polarized waves are decoupled, reflection and transmission at individual boundaries between adjacent layers will affect one polarization without influence from the other polarization. Therefore, it is relatively easier to achieve destructive interference outside the cloak. When the incident angle is nonzero, vertically polarized and horizontally polarized waves become coupled. Thus there is a need for more degrees of freedom to optimize the structure to achieve destructive interference for both polarizations. This justifies a much better performance of the four-layer cloak with all constitutive parameters optimized in each layer [Fig. 2(c)], as opposed to just three parameters [Fig. 2(b)].

In conclusion, we have proposed a method to achieve multilayered cylindrical cloaks working at oblique incidence by combining the analytic algorithm of 3D scattering and genetic optimization. We show that by using only four layers of homogeneous and anisotropic metamaterials with constitutive parameters of finite values, scattering from the cloak at oblique incidence can be reduced by 2 orders. Although the optimization is performed only at a single incident angle, the performance of the cloak is satisfactory within a large range of incident angles. This work is an extension and supplement of previous 2D models [12,13] in 3D counterparts. We anticipate that the combination of analytic algorithm of 3D scattering and genetic optimization as a powerful tool will be used in more applications in the practical implementation of 3D invisibility cloaks in the future.

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