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## MODELLING AND PID CONTROL DESIGN OF NONLINEAR EDUCATIONAL MODEL BALL & PLATE

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**Abstract:** This paper is focused on modelling and control of nonlinear dynamical system Ball & Plate in language Matlab/Simulink. PID/PSD controller is used in closed loop feedback control structure for the purpose of control. The verification of designed PID control algorithms, the same as nonlinear model of dynamical system, is performed with functional blocks of Simulink environment. This paper includes testing of designed PID control algorithms on real model Ball & Plate using multifunction I/O card MF 614, which communicates with PC by the functions of Real Time Toolbox. Visualization of the simulation results is realized by internet applications, which use Matlab Web Server.

**Keywords:** educational model Ball & Plate, nonlinear system, PID controller, tracking of reference trajectory, simulation

### 1 INTRODUCTION

In this article we concern with modelling of the nonlinear dynamical system Ball & Plate, (an educational model from Humusoft), in language Matlab/Simulink. Next we deal with design of PID/PSD control algorithms in closed loop feedback control structure and its verification on the simulating model of nonlinear dynamical system in Simulink, as well as on the real model, which is situated in Laboratory of Cybernetics at Department of Cybernetics and Artificial Intelligence at FEI TU of Košice.

We targeted to ensure the required position of the ball on the plate, as well as to eliminate disturbance influences at design of PID control algorithms. Another goal was to ensure movement of the ball on the plate according to the reference trajectory. We can note that the educational manual (Educational manual for CE151 Ball & Plate Apparatus by Humusoft 1996 – 2004) doesn't submit this goal as possible as control task.

The paper contains verification of control algorithms on the simulating and the real model and their comparison depending up using different PID controllers, which parameters were designed on the basis of standard type methods of theory of automatic control as method of Naslin, Butterworth, whether Graham-Lathrop.

The simulation results are presented by internet application using technology Matlab Web Server (Jadlovská et al. 2008).

### 2 EDUCATIONAL MODEL BALL & PLATE

The educational model Ball & Plate is an unstable two dimensional system with second-order astaticism. It consists of a plate pivoted at its centre, such that the slope of the plate can be manipulated in two perpendicular directions. A servo system consists a motor controller card and two stepper motors are used for tilting the plate. Intelligent vision system is used for measurement of a ball position (Technical manual for CE151 Ball & Plate Apparatus by Humusoft 1996 – 2004).

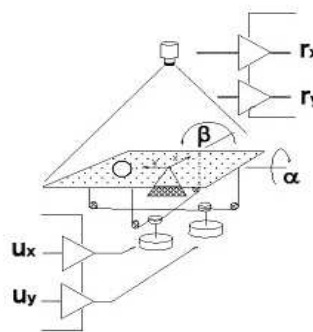


Fig. 1 Educational model Ball & Plate

The model Ball & Plate is dynamical system with four inputs and two outputs (Fig.2). The control inputs of the system Ball & Plate are the voltages of the particular motors  $u_x(t)/u_y(t)$  in the direction of axis  $x/y$ , which are constrained between  $\langle -1, +1 \rangle$  V. They ensure winding of the plate.

The other input to the system (the disturbance variable) is the external force into swing out the plate in the random direction  $z_x(t), z_y(t)$ .

The outputs of the system  $r_x(t), r_y(t)$  are actual positions of the ball on the plate in the direction of an axis  $x/y$  (the controlled variables).

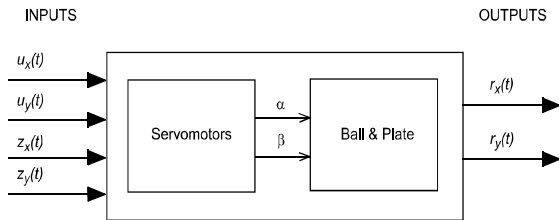


Fig. 2 System Ball & Plate - inputs/outputs

According to (Technical manual for CE151 Ball & Plate Apparatus by Humusoft 1996 – 2004) the both coordinates can be controlled independently as their mutual interactions are negligible. So we acquire two SISO systems of the Ball on the Beam, the first one for an axis  $x$  and the second one for an axis  $y$ .

### 2.1 Mathematical model of the Ball & Beam system

The mathematical model of the Ball and Beam system is described by nonlinear differential equation of second order (Jajčičin 2008):

$$(m + \frac{J}{R^2})\ddot{x}(t) - mg \sin \alpha(t) = 0, \quad (1)$$

where  $m$  - weight of the ball,  $J$  - moment of inertia of the ball,  $R$  - radius of the ball,  $g$  - acceleration of gravity,  $x(t)$  - position of the ball,  $\alpha(t)$  - beam's angle.

The mathematical description of servomotor is according to (Educational manual for CE151 Ball & Plate Apparatus by Humusoft 1996 – 2004) represented by the differential equation:

$$\frac{d\alpha(t)}{dt} = \omega \cdot \text{sign}[u_x(t) - \alpha(t)], \quad (2)$$

$$\text{if } |u_x(t) - \alpha(t)| \geq c,$$

$$\text{in case } \frac{d\alpha(t)}{dt} = 0, \quad \text{if } |u_x(t) - \alpha(t)| < c, \quad (3)$$

where  $\omega$  - an angular velocity of the servomotor,

$c$  - servomotor's dead zone,

$u_x(t)$  - input voltage of the servomotor.

The same as every real system, system Ball & Plate disposes with the certain physical constraints, too. The dimensions of the plate, which ball is moving on determinate the interval of suitable values for the ball's position:

$$x(t) \in \langle -L/2 + R, L/2 + R \rangle, \quad (4)$$

where  $L$  is the length of the plate in one direction.

This fact we have to deliberate with choosing of the desired ball's position or the reference trajectory, which the ball should be followed. Other constraints that are needed comprehend during system modelling are the intervals of the values determining minimal and maximal possible plate's angle or voltage of servomotor.

### 2.2 Analysis of Ball & Plate in Matlab/Simulink

For control algorithms design we created nonlinear dynamical model of the system Ball & Plate using functional blocks of Simulink based on the mathematical description (1), (2) and (3). Servomotors and Ball & Plate blocks are realized as an independent subsystems (Fig. 3). Thus created model enables us to analyze system dynamics with different settings of the model's physical parameters and various initial conditions settings or the disturbances. At the parameters settings of the model based on (Jajčičin 2008), with the initial ball's position in center of the plate, not bending plate and for the choice of the inputs voltages  $u_x = 1V$ ,  $u_y = -0.2V$  without influence of the disturbances we found out that this system is unstable and even relative very quick.

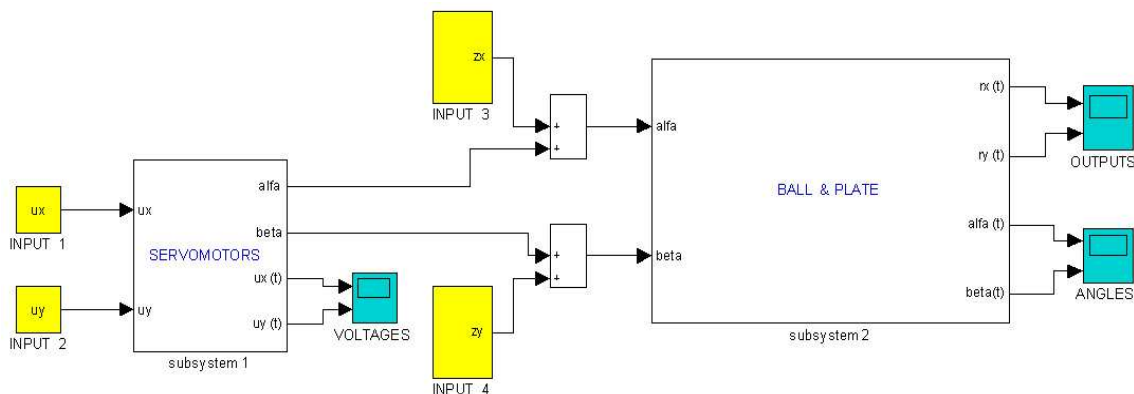


Fig. 3 Scheme of non-linear model of dynamical system Ball & Plate

The analysis results of the dynamical system from simulation in Simulink are on Fig. 4a and Fig. 4b.

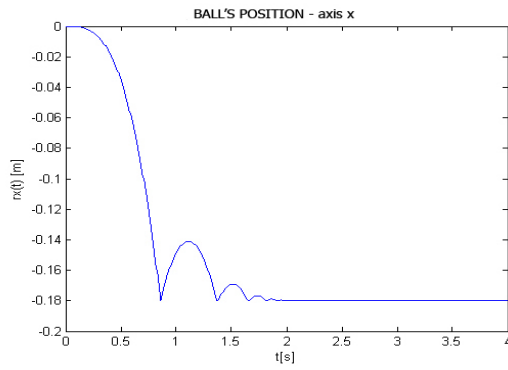


Fig. 4a The position of the ball in direction of x axis

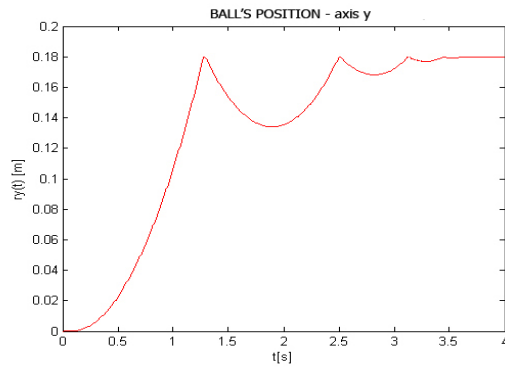


Fig. 4b The position of the ball in direction of y axis

As it is shown on Fig. 4a and Fig. 4b, there is bumping and reflection after the ball bounced to the plate's border. This behaviour is very similar to behaviour of real system, too.

### 3 CONTROL ALGORITHMS

Designed control algorithms are based on the structure of PID/PSD controller in the interactive (5) or component form (6). PID controller is used in the closed loop feedback control architecture (Harsányi et al. 1998):

$$F_r(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s\right), \quad (5)$$

or 
$$F_r(s) = r_0 + \frac{r_{-1}}{s} + r_1 s, \quad (6)$$

where  $K_p$  - proportional gain,  
 $T_i$  - integral time constant,  
 $T_d$  - derivative time constant,  
 $r_0$  - proportional gain ( $r_0 = K_p$ ),  
 $r_{-1}$  - integral gain ( $r_{-1} = K_p/T_i$ ),  
 $r_1$  - derivative gain ( $r_1 = K_p * T_d$ ).

In classical theory of automatic control exists several methods for determining the parameters  $K_p$ ,  $T_i$ ,  $T_d$  or

gains  $r_0$ ,  $r_{-1}$ ,  $r_1$  of the PID controller. Which one of them we use depends on the type of the system transfer function, eventually on the control dynamics requirements, e.g. maximal overshooting, control time, etc. (Bakošová et al. 2003). Because the transfer function of the linear model of Ball & Plate according to (Educational manual for CE151 Ball & Plate Apparatus by Humusoft 1996 – 2004) is

$$F_s(s) = \frac{4,803}{s^2(0.1878s + 1)}, \quad (7)$$

we used standard methods of the characteristic polynomial according to Butterworth, Graham-Lathrop and Naslin's method for PID controller parameters design. Using the other methods (e.g. Ziegler-Nichols method, method of Whiteley) was showed like inadequate, even the simulation results of control were insufficient to reach the required goal of control. The main causality was the double zero root of the characteristic polynomial, hence astatic character of the system.

We designed an algorithm for dynamical analysis of model's behaviour during control process, which is described by block diagram on Fig. 5. We are able to set the parameters, the initial conditions and to choose the operating point of the system. The result of the linearization is the transfer function of the dynamical system, which parameters are computed for chosen operating point. Anyone of before mentioned methods we implemented to Matlab by the *m-functions* (the input parameter is the transfer function of linear system and the outputs parameters are controller coefficients  $r_0$ ,  $r_{-1}$ ,  $r_1$ ). We are able to change the method of PID controller synthesis and so compare the simulation results of control by using the different PID controllers.

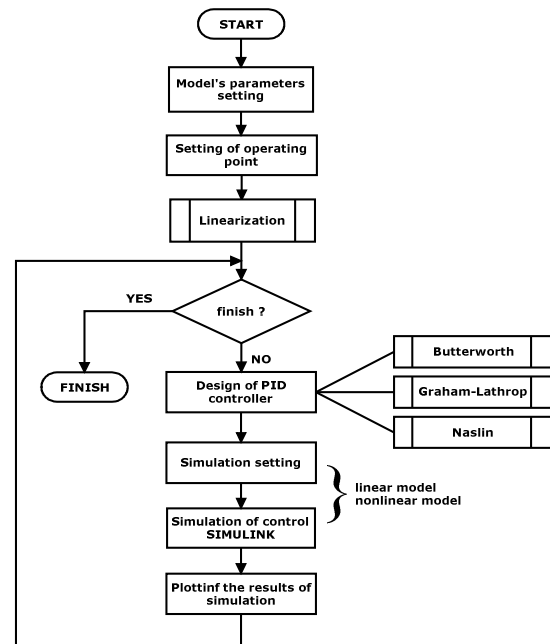


Fig. 5 Algorithm for analysis of dynamics during control process

#### 4 VERIFICATION OF CONTROL ALGORITHMS BY SIMULATION

Verification of the control PID algorithms on the simulation nonlinear model of Ball & Plate was focused to control with the goal to ensure:

- required position of the Ball on the Plate,
- tracking of the reference trajectory of the movement of the ball on the plate,
- eliminate disturbance influence in form of the external force to plate's angle.

Within simulation control algorithms verification is also control of nonlinear dynamical system with discrete PSD controller, which was realized according to (Harsányi et al. 1998) using the velocity algorithm based on trapeziform compensation of integral and replacement the first derivation by the first difference. The parameters of the discrete PSD controller are based on computed parameters of continuous PID controller. The interval of the values for the sampling period  $T_{vz}$  selection we chose on base of an empirical formulas mentioned in the (Švarc 2002).

##### 4.1 Closed loop feedback structure

On Fig. 6 two feedback control loops for the nonlinear system and two PID controllers are shown.

Input parameters of the simulation:

- parameters of the model are introduced in (Educational manual for CE151 Ball & Plate Apparatus by Humusoft 1996 – 2004), (Jajčišin 2008);
- start position of the ball in the coordinate system:  $r_{zacx} = -0.18$ ;  $r_{zacy} = -0.18$ ;
- maximum initial swing out of the plate in both directions:  $\alpha_{poc} = 1$ ;  $\beta_{poc} = 1$ ;
- required position of the ball in the coordinate system XY:  $r_{poz x} = 0$ ;  $r_{poz y} = 0$ ;
- disturbance at the direction of axis  $x$  in time 10s with amplitude 0.03 rad and during 1s (external force into swing out the plate);
- sampling period  $T_{vz}$  for PSD controller: 0.01s.

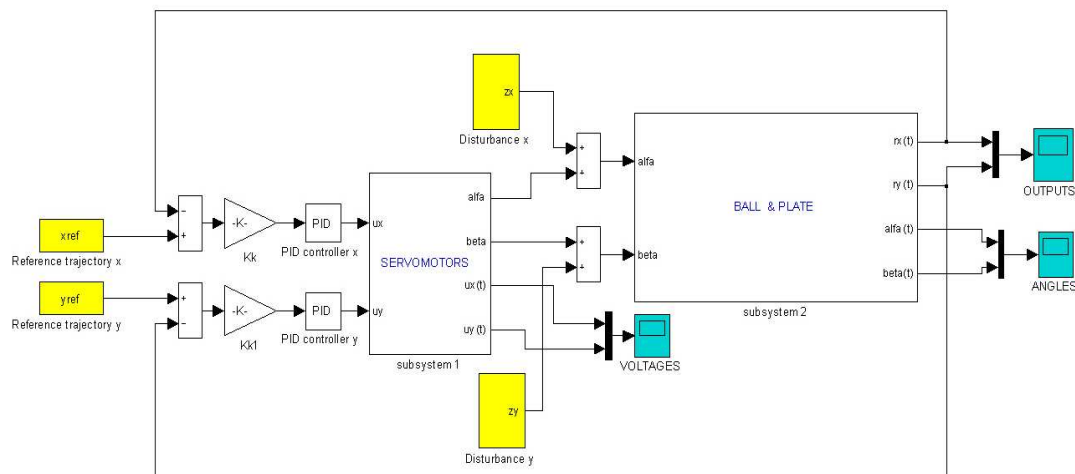


Fig. 6 Control closed loop scheme of the system Ball & Plate in Simulink

##### 4.2 Results of the verification by simulation

The parameters of PID controller, that are results of concrete synthesis method are shown in next tables.

Tab.1 Parameters of PD controller

Method of synthesis	Parameters of PD controller			
	$P$	$D$	$K_p$	$T_d$
Naslin ( $\delta_{\max} = 3\%$ )	0,5587	0,5057	0,5587	0,9051
Graham-Lathrop	1,110	0,781	1,110	0,7036
Butterworth	0,7436	0,5562	0,7436	0,7480

Tab.2 Parameters of PID controller

Method of synthesis	Parameters of PID controller					
	$P$	$I$	$D$	$K_p$	$T_i$	$T_d$
Naslin ( $\delta_{\max} = 3\%$ )	0,56	0,28	0,51	0,56	1,99	0,91
Graham-Lathrop	1,73	1,64	0,86	1,73	1,06	0,5
Butterworth	0,87	0,69	0,56	0,87	1,27	0,64

The Fig. 7 and Fig. 8 show the results of simulation model control using PD controller designed on base of the method per Butterworth. Instead of servomotors' voltages, which represent an actuating input to the model, there is swing out of the plate illustrated on Fig. 8, which better features the model's behavior. On Fig. 9 and Fig. 10 the time responses of the controlled variable using PD and PID controller per of all methods of the synthesis are shown.

If the reference trajectory is equal for axis  $x$  and axis  $y$ , the time responses will be identical for both of axis. So, on Fig. 7 – Fig. 10 there are only time responses for axis  $x$ .

On the time responses of controlled variable we can see approximately 2s time delay. It is caused by the fact, that plate was absolutely swung out at the beginning of control. While servomotors were swinging the plate to other side, the ball has been still in the corner of the plate. When plate's angle reached negative values, the ball started move from the plate's corner.

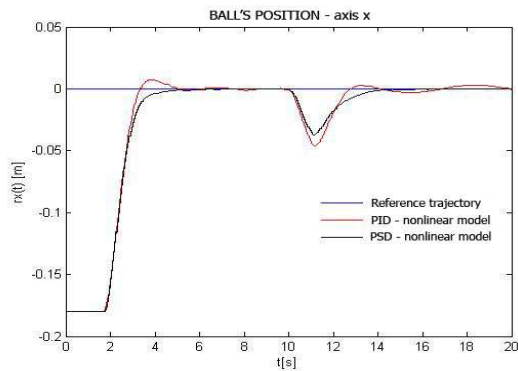


Fig. 7 Position of the ball in direction of axis  $x$  using PD controller – Butterworth

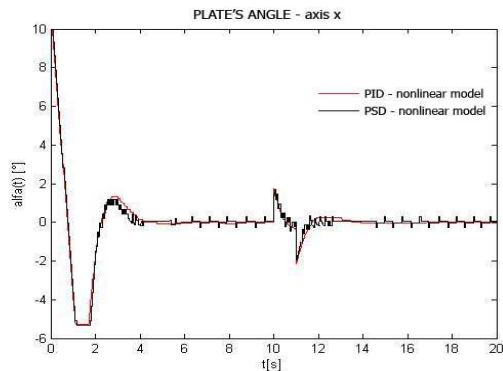


Fig. 8 Plate's angle in direction of axis  $x$  using PD controller – Butterworth

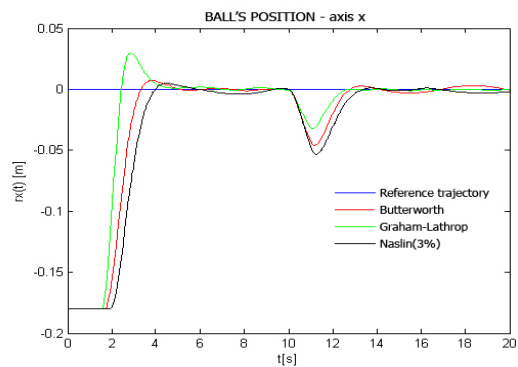


Fig. 9 Position of the ball in direction of axis  $x$  using PD controller – Butterworth, Graham-Lathrop, Naslin

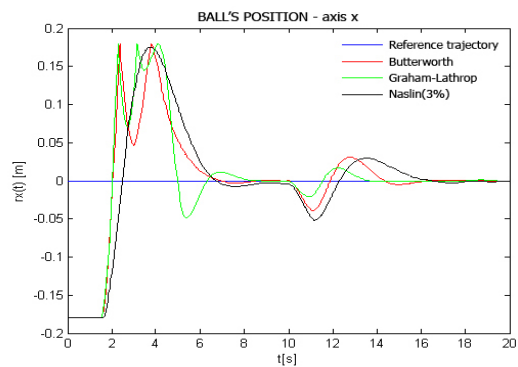


Fig. 10 Position of the ball in direction of axis  $x$  using PD controller – Butterworth, Graham-Lathrop, Naslin

On Fig. 11 – Fig. 13 the simulation results of the control algorithms for tracking of the reference trajectory by the ball on the plate using PD controller based on the standard type method according to Butterworth are shown. The chosen reference trajectory had star shape. Also starting position of the ball is in the corner of the plate ( $r_{zacx} = -0.18$ ;  $r_{zacy} = -0.18$ ) with the plate completely swing ( $\alpha_{poc} = 1$ ;  $\beta_{poc} = 1$ ). Tracking of the reference trajectory we combined from control to the required position to the centre of the plate and from tracking trajectory. In the other case the ball wouldn't be able to observe movement by reference trajectory, as she considers with initial position of the ball in the centre of the plate.

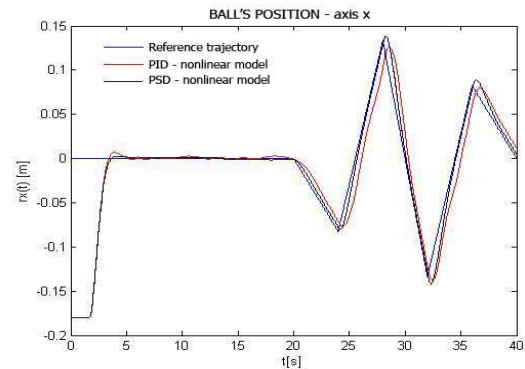


Fig. 11 Position of the ball in direction of axis  $x$  using PD controller – Butterworth

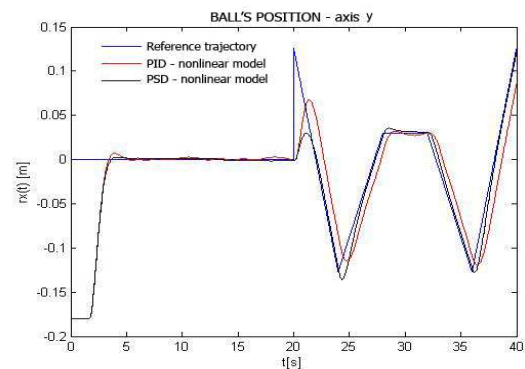


Fig. 12 Position of the ball in direction of axis  $y$  using PD controller – Butterworth

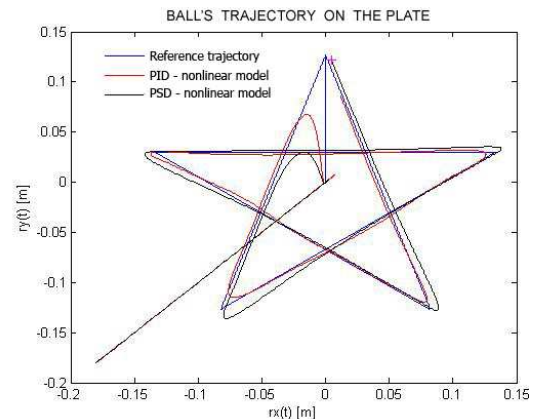


Fig. 13 Position of the ball on the plate during tracking trajectory using PD controller – Butterworth



Because the system has astatism, so it has inclusive integration element, it shows much advantageous usage of PD controller, when doesn't exist big overshooting as it is with PID controller using. Nevertheless PD controller applying for tracking of the reference trajectory results, the ball's motion accounts certain delay towards to the reference trajectory. Otherwise application of PID controller eliminates this deficiency, however an integration element increases overshooting, what results in a fact, that the ball leaves from required trajectory. The deviation of the reference trajectory occurs mainly in the situation, when the reference value suddenly changes the direction. Time responses of control using PID controller are presented in (Jajčičin 2008).

It's a deal using PSD controller that with the miniaturizing of the sampling period results of control approach to results of control with the continuous PID controller. Minimization of the sampling period soft improvements control process.

## 5 VERIFICATION OF CONTROL ALGORITHMS ON REAL MODEL

We used the functional scheme on Fig.14 for verification of designed control algorithms on the real model Ball & Plate, which allows to sense the ball's position through the individual blocks with utilizing of camera, adjustment the signal in required form and determination of ball position on the plate.

Block PID controller generates actuating input depending up variation of the actual and the required ball position. After modification is feeding through block from Real Time Toolbox (Real-Time Toolbox – User's manual by Humusoft 1996 – 2002) *RT Out* to Lab card MF614, and then on real system servo-

motors, too. Block PID controller is actually realization of two independent PSD controllers for an axis  $x$  and an axis  $y$ .

The results of control achieved from the real system using PSD controller with the parameters obtained based on applying the method of standard type according to Butterworth are visible on Fig. 15 - Fig. 19, where they are compared with corresponding results of simulation model control.

On Fig. 15 and Fig. 16 there are visible the time responses of control into the required position of the ball and on Fig. 17 – Fig. 19 are the results of tracking of the reference trajectory with the type of the star.

Because we know that real system adjusts the ball into the plate's corner and completely slopes the plate at start, we proceeded in the same way as in the case of simulation model control.

Because of the limpidity of the pictures, there are trimmed parts of the time responses, which belong to control of the ball's position to the center of the plate.

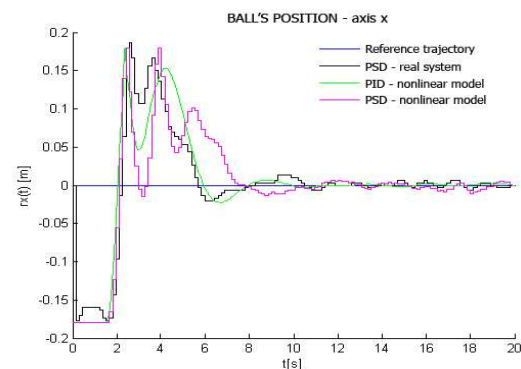


Fig. 15 Position of the ball in direction of axis  $x$  during control of the real system by PID controller

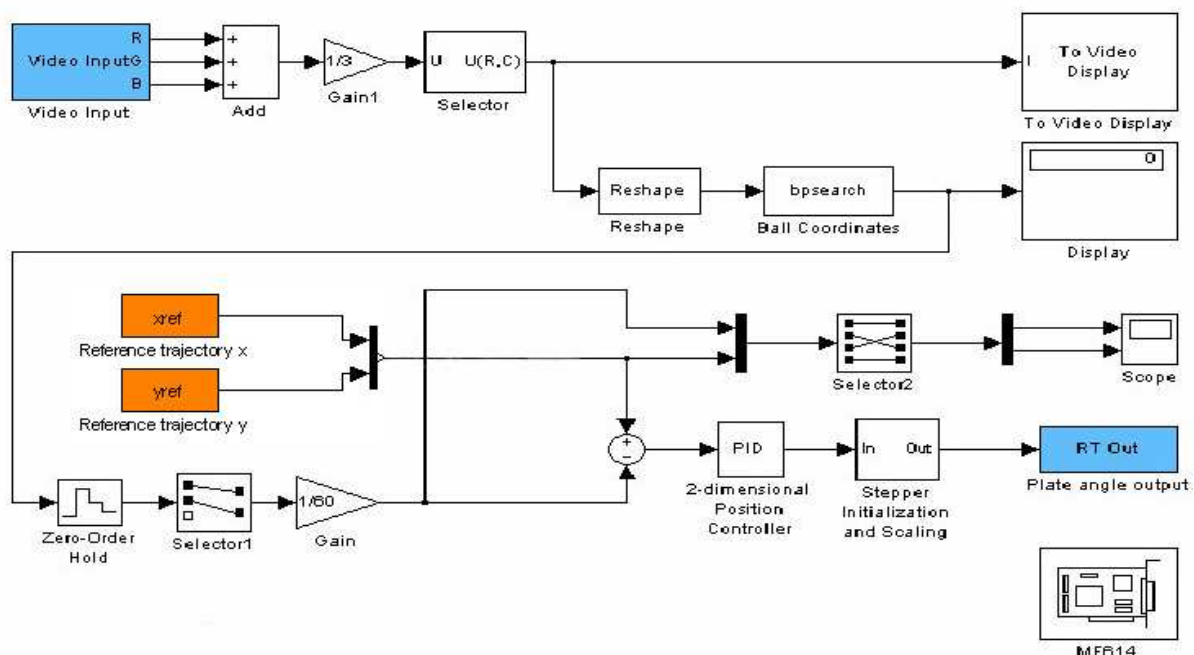


Fig. 14 Scheme for control of the real system Ball & Plate in Simulink

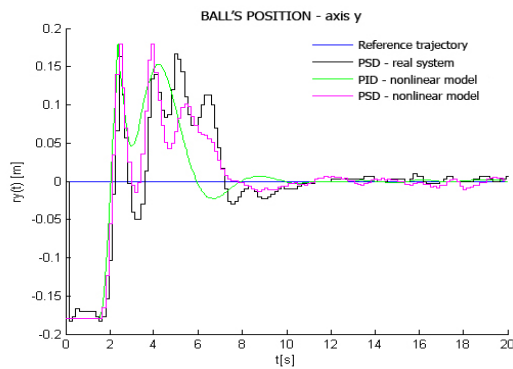


Fig. 16 Position of the ball in direction of axis y during control of the real system by PID controller

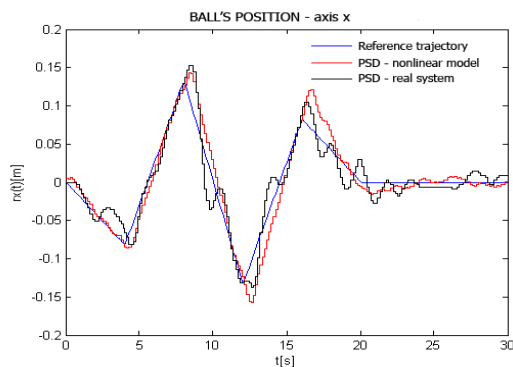


Fig. 17 Position of the ball in direction of axis x during control of the real system by PID controller

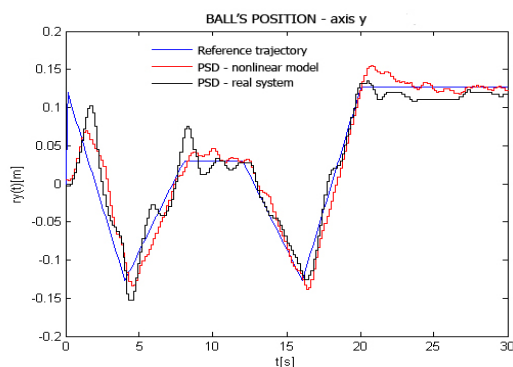


Fig. 18 Position of the ball in direction of axis y during control of the real system by PID controller

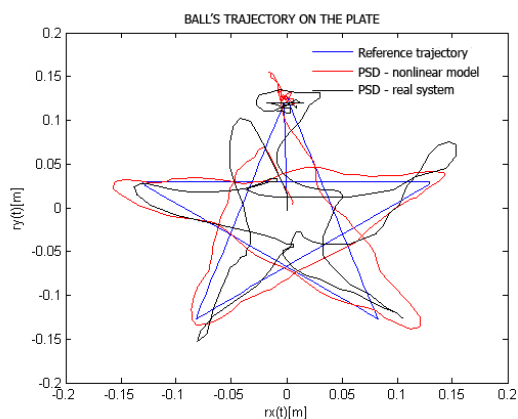


Fig. 19 Position of the ball on the plate during tracking trajectory by real system using PID controller

Because control of the real model is executing by PSD controller sampling period has great impact on the whole system behavior. In our case we used the sampling period  $T_{vz} = 0.15$  seconds, what's the value which is employed in the manual of the model (Educational manual for CE151 Ball & Plate Apparatus by Humusoft 1996 – 2004), too. Additionally the position of the ball is scanning by camera, therefore also illumination of the model or the room has considerable impact on control of system.

Within modelling of nonlinear system and control algorithms design we issued from the supposition of the axis independence as well as from assumption that the ball motion is solely rolling. It can be seen on Fig. 15 and Fig. 16, the ball's position control at the direction of axis x accounts differences against ball's position control at the direction of axis y. It happened during control that the ball reflected away from the plate or from the border of the plate. These facts were why control results of the real model differ from control results of the simulation model, too.

Algorithms of control which the goal have been to follow the reference trajectory was soft affected by the fact, that the first has been needs to realize ball's position control to the centre of the plate. Besides, next factors were earlier mentioned phenomena within the context of the difference of plate's behavior at the direction of the axis x, y during turning and ball's recoiling from plate's surface.

Applying PID/PSD controller by himself accounts the variance from the reference trajectory already by the simulation model. Using application of PSD controller on the real model, ball's motion accounts much difference from the reference trajectory.

## 6 VISUALIZATION OF CONTROL RESULTS BY INTERNET APPLICATION

An internet application, which presents using of designed control algorithms on the nonlinear dynamical systems, was created in the form of HTML pages. The pages contain forms for entering the concrete values of the particular parameters of the dynamical systems. After setting of all necessary values and pressing the button *Simulovat*, all of the attributes are transferred through Matlab Web Server on server into corresponding before programmed functions. These functions execute the particular values and start the equivalent simulations. Particular time responses are saved like images after simulations. The output from the mentioned functions is HTML code, which is generated on base in advance created templates, whereby resorts to concrete modifications and completion of the values according to user settings and the simulation results. HTML code is backward send into user's web browser by server.



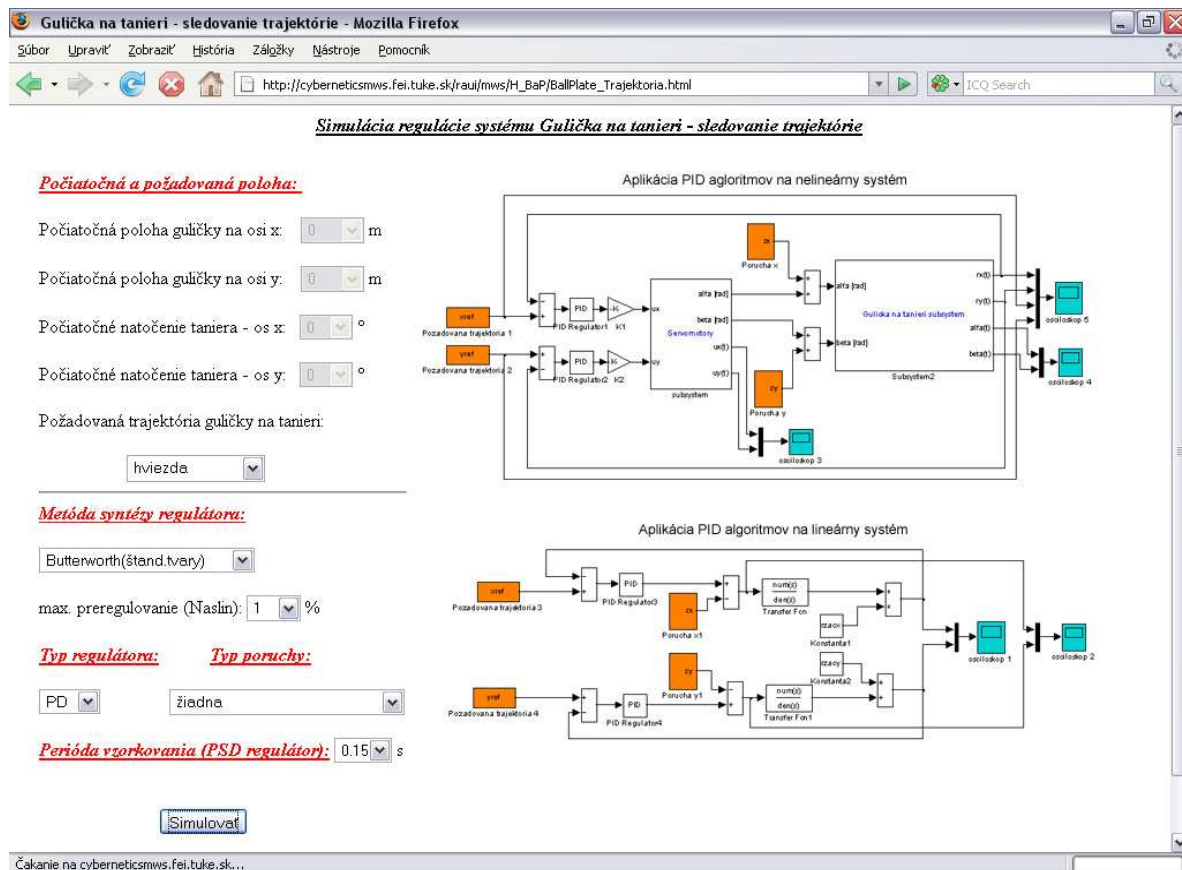


Fig. 20 Tracking of the reference trajectory by Ball & Plate system - internet application

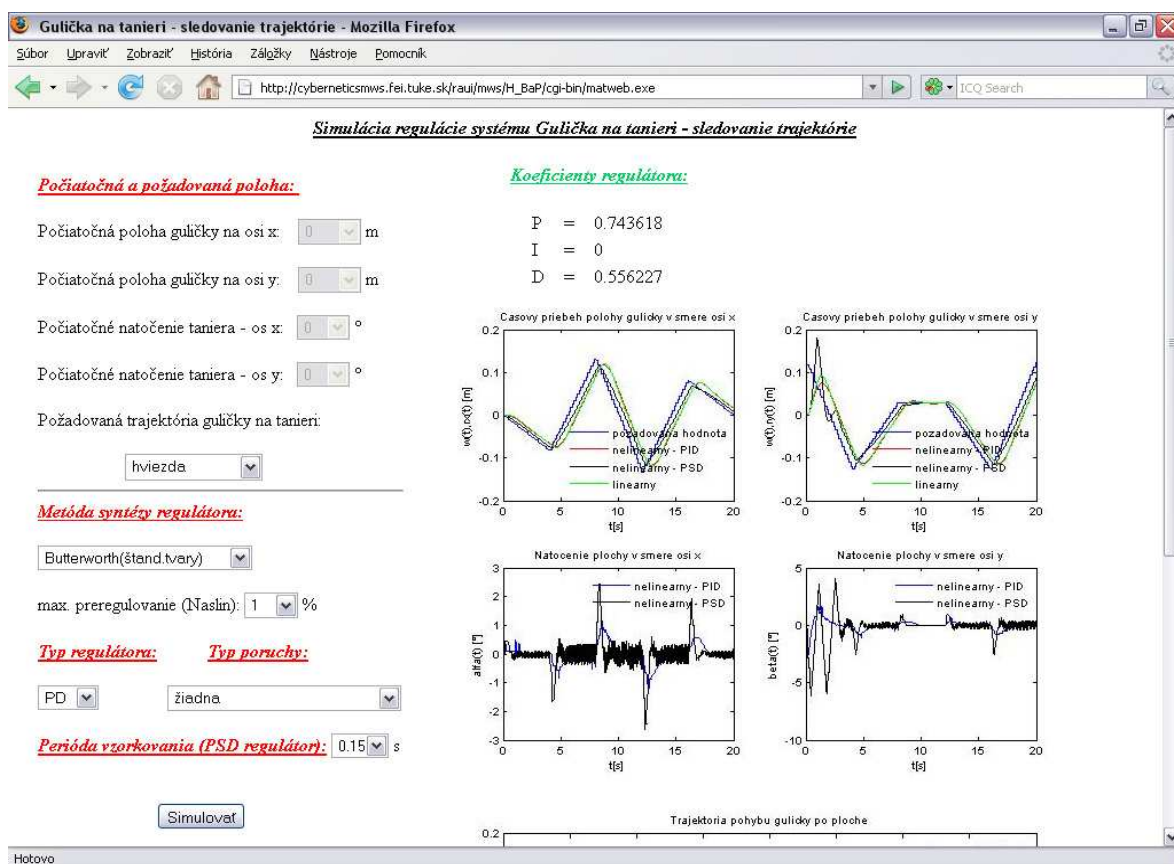


Fig. 21 The page generated on the basis of the simulation results

An internet application is available at the address

[http://cyberneticsmws.feit.tuke.sk/raui/mws/H\\_BaP/](http://cyberneticsmws.feit.tuke.sk/raui/mws/H_BaP/).

Except the system Ball & Plate it also presents function of control algorithms used in simulation control of the hydraulic system.

Internet application serves how teaching tool, reference to it we can find on the page of the subject *Simulation systems* under the name *Analysis and control of simulation models of nonlinear dynamical systems* (*Analýza a riadenie simulačných modelov nelineárnych dynamických systémov*).

(<http://matlab.feit.tuke.sk/ss/aplikacie.aspx>).

The module activity *Reference trajectory tracking of Ball & Plate system* (*Sledovanie referenčnej trajektórie systému Gulička na ploche*) is illustrated on Fig. 20 and Fig. 21. Fig. 20 illustrates the page accessible directly from the main page of an internet application. Scheme built-up in Simulink environment is introduced on it. Fig. 21 draws page or the output of the concrete function, which is being executed at the side of the server and by Matlab Web Server send back to user's browser. Both of these pages enable to change the input attributes of the form and to obtain some results from various simulations (Jadlovská et al. 2008).

## 7 CONCLUSION

This paper presents modelling of the nonlinear dynamical system Ball & Plate in Matlab/Simulink environment with the consequential proposal of control member in the form of PID controller with using *m-functions* and results of control at graph form.

We designed functional schematic for nonlinear model in Simulink. Parameters of model, initial conditions, operating point and PID control algorithms were realized by Matlab. Verification of control algorithms applied to the system Ball & Plate was made according to Fig. 6. Complete logical order of feedback loop control functionality is shown on Fig. 5.

Although applying of PD controller for control of the simulation model showed better result, his usage for control of the real model showed as insufficient. However control algorithms verification on the real model Ball & Plate brought satisfying results in the case of control into required position. At control, which should ensure tracking of the reference trajectory the results are adequate considering at the disposal control type algorithm.

In future control problem solving of the real models of dynamical systems would has been convenient to use modern engineering control methods, e.g. predictive control, whether control using methods of artificial intelligence, for example fuzzy controllers, evolutionary algorithms, neural models, neural controllers (Jadlovská 2003).

## ACKNOWLEDGMENTS

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