Networks & Hierarchies

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Goals for these two lectures:

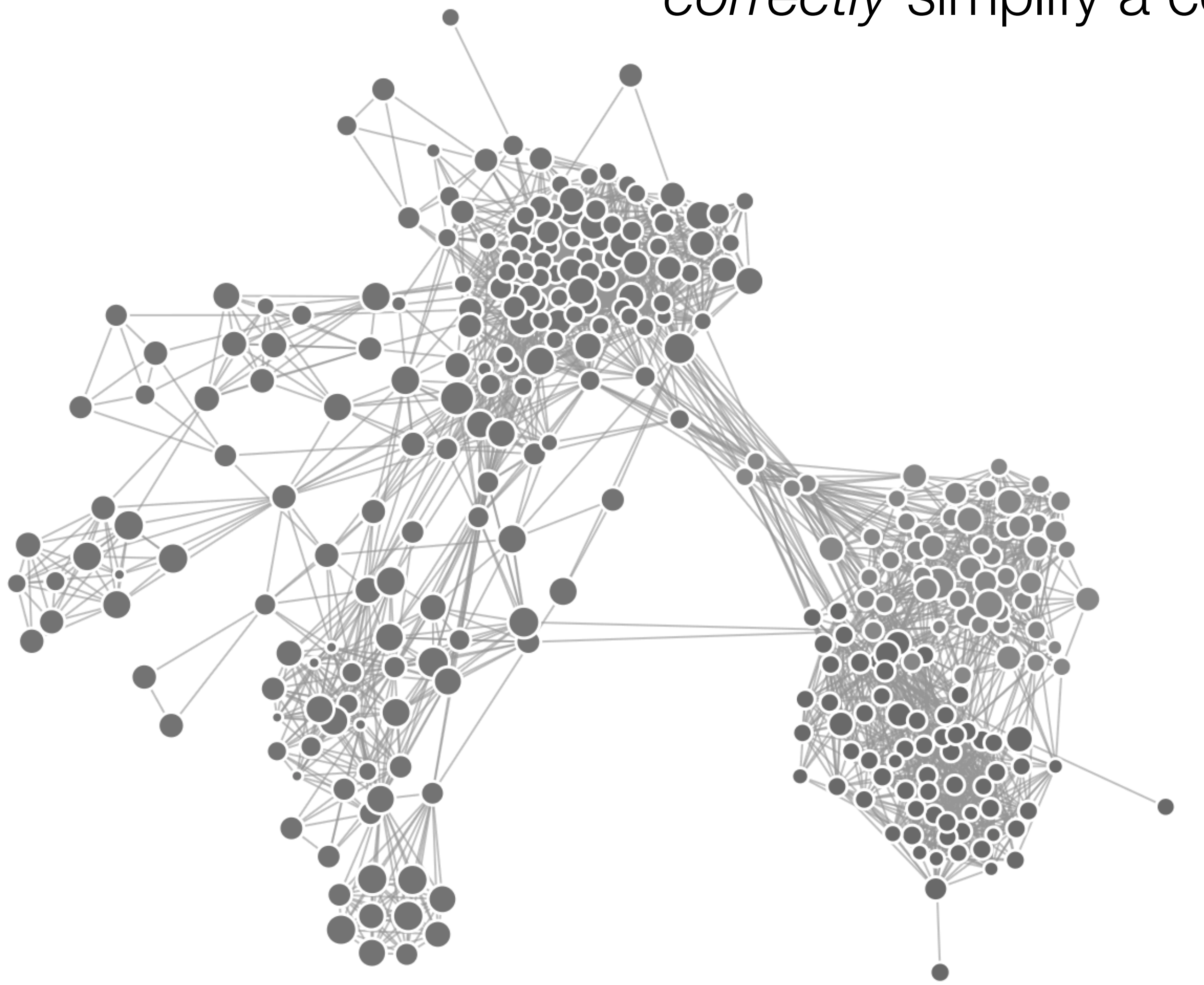
1. **Why** do we look for large-scale structure? 🤔
2. **How** do we find linear hierarchies? 🧐
3. **Where** can we read more details? 📚

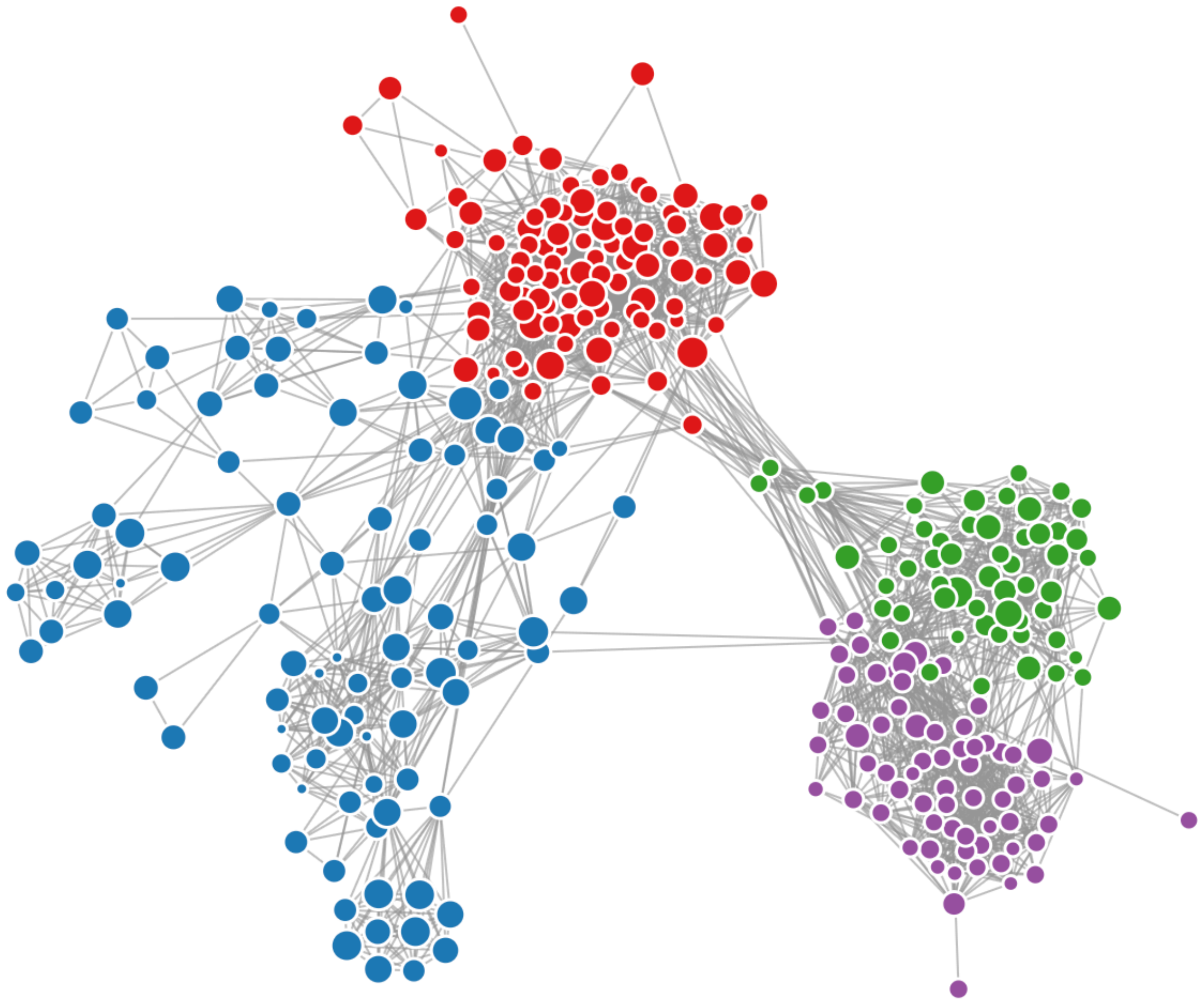
Simplicity is a great virtue but it requires hard work to achieve it and education to appreciate it. And to make matters worse: complexity sells better.
E. W. Dijkstra

We can interpret this in two ways:

The Cynic: Pictures of networks can be *really cool* but our goal is to do good science, not make pretty pictures.

The Scientist: The most beautiful science is when we *correctly* simplify a complex system.





What do we mean by “large-scale structure” ?

Structure is what makes data different from noise.
It’s what makes a network different from a random graph.

Networks are often too large and complex to be adequately summarized by a few scalars, like the number of nodes, the number of edges, or the mean degree.

However, they are also often too large and complex to be analyzed *without* some kind of simplification!

Therefore, understanding what the network *means* requires that we identify key structures.

Searching for large-scale structures in a network reflects a belief that in all the complexity there are patterns that make the network less complicated.

We define these large-scale structures—models, really—to compress complex networks.

Goal: understanding, not a list of parts and dimensions

Finding large-scale structures is the same as anything else:

We want a **simplified model** of something very complicated.

We want to know what the **important pieces** are, and **how they fit together**.

slide more or less stolen from Aaron Clauset

Many uses for models of large-scale structure

Treat the network like a system:

Extrapolation. Make predictions for as-yet unseen nodes (in “space” or time).
Interpolation. Identify missing links.
Generalization. Nodes of this type are like others of the same type.

Treat the network like an artifact:

Mechanisms. How did this network arise? What rules governed its assembly?
Explanations. Coarse-graining or compression.

Treat the network like a means to an end; an intermediate data structure:

Useful division. Need groups so that we can assign treatments in an A/B test.
Simplification. Downstream regression model needs ranks or groups.

intuition: compare this list with the list you would write for regression

Rankings and linear hierarchies

The idea of rankings—pervasive!

Assumptions:

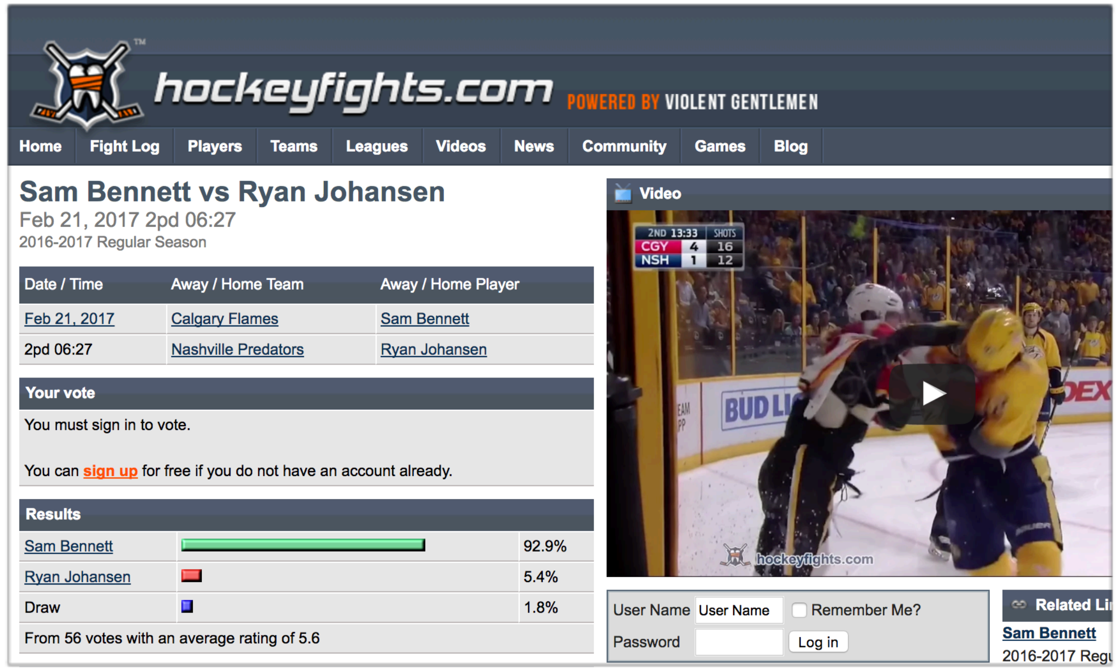
1. Competitors have some intrinsic quality (or vector of qualities).
2. Interactions can (stochastically) reveal differences in qualities.
3. Competitions are pair-wise. (Lee Sedol vs. AlphaGo; Astros vs. Dodgers)

In other words: outcomes are generated by a stochastic process, which is some function of the positions of the competitors.

Systems of dominance

social

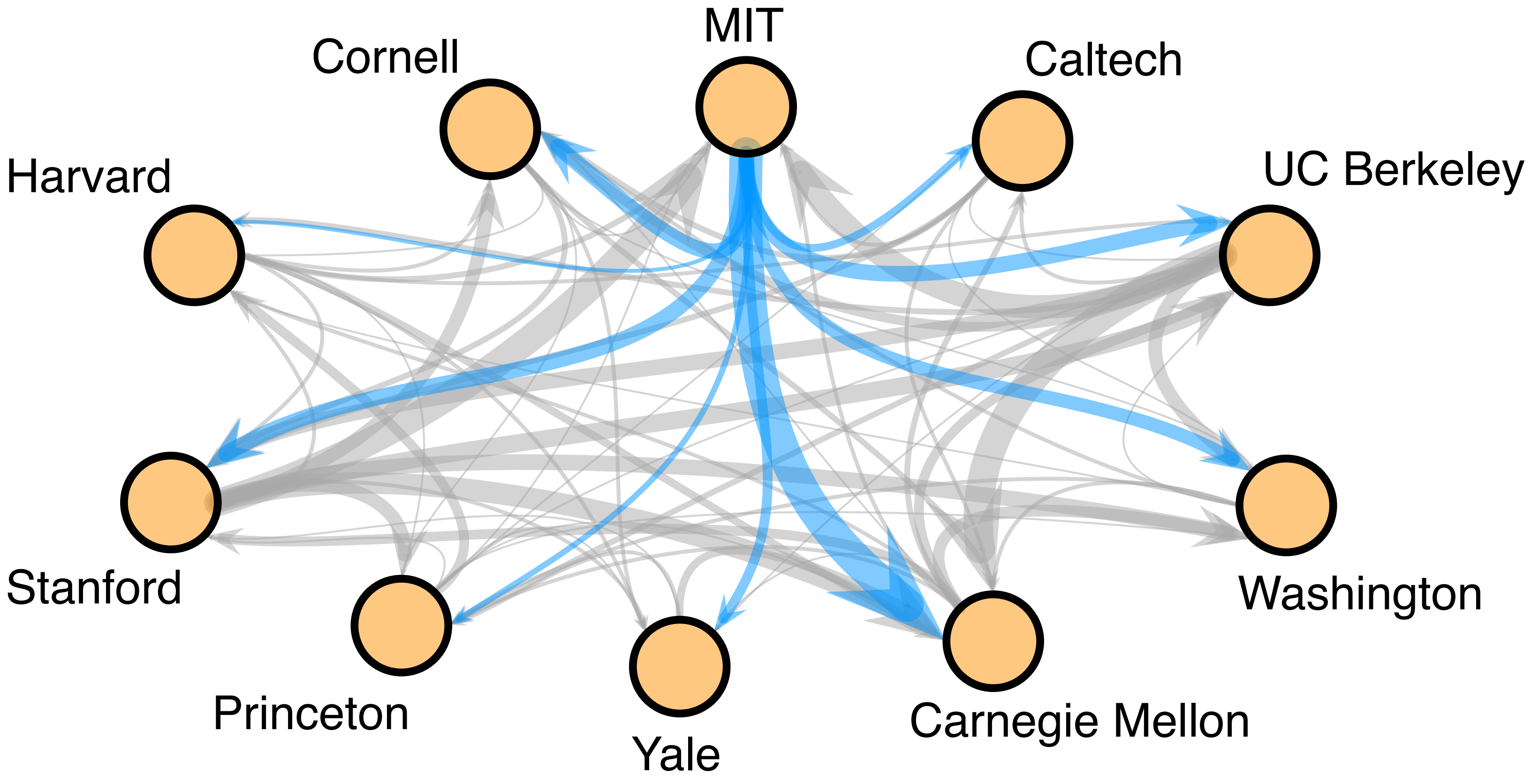
physical

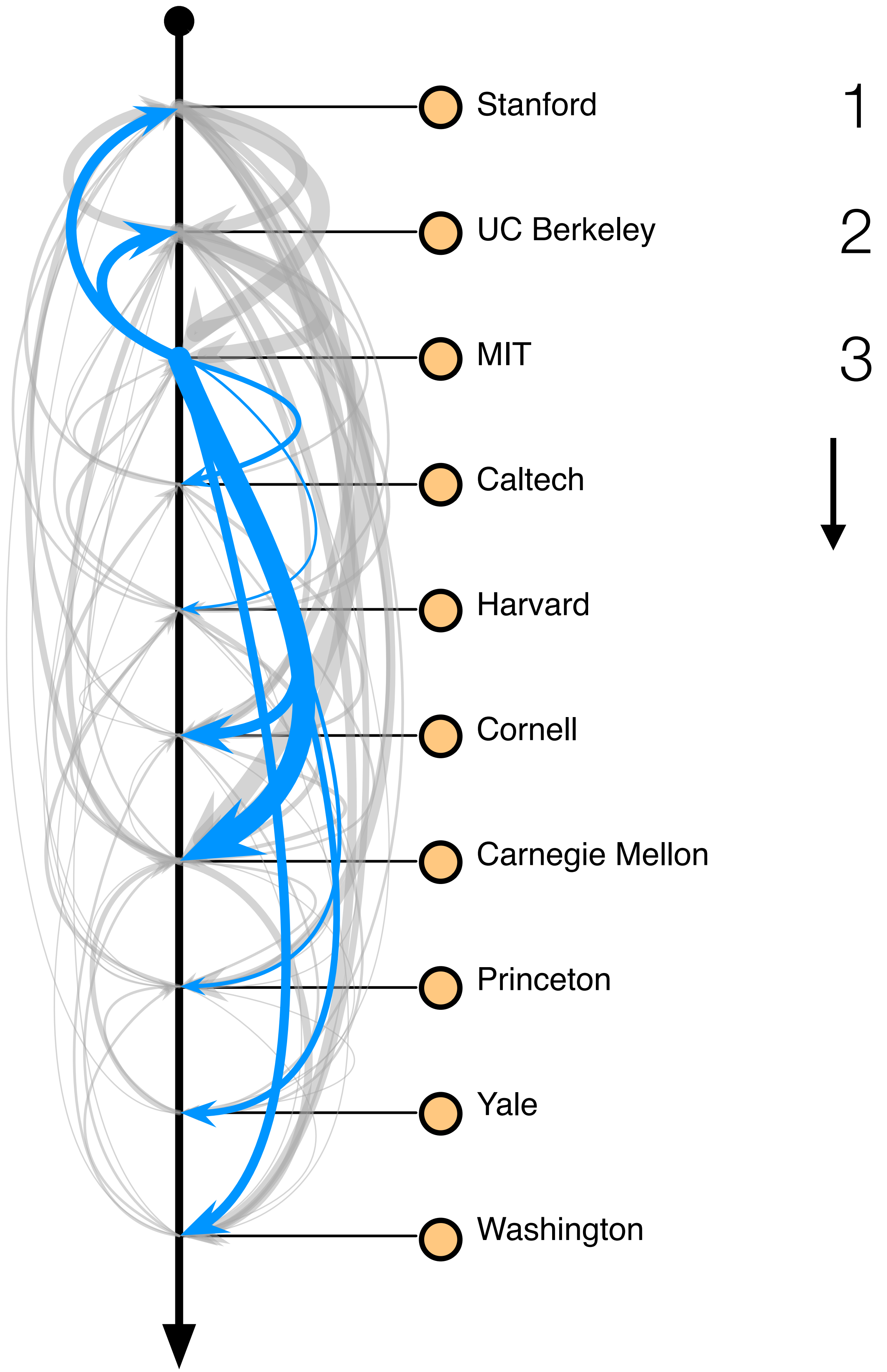


mental

financial

Systems of endorsement



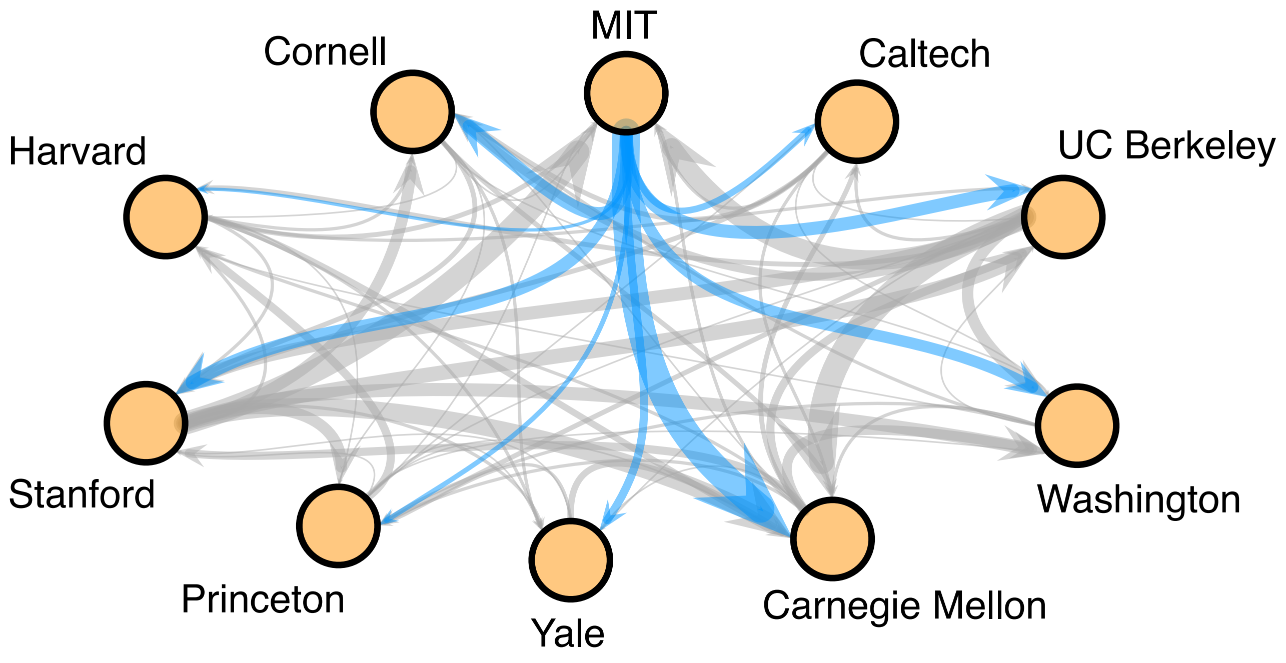


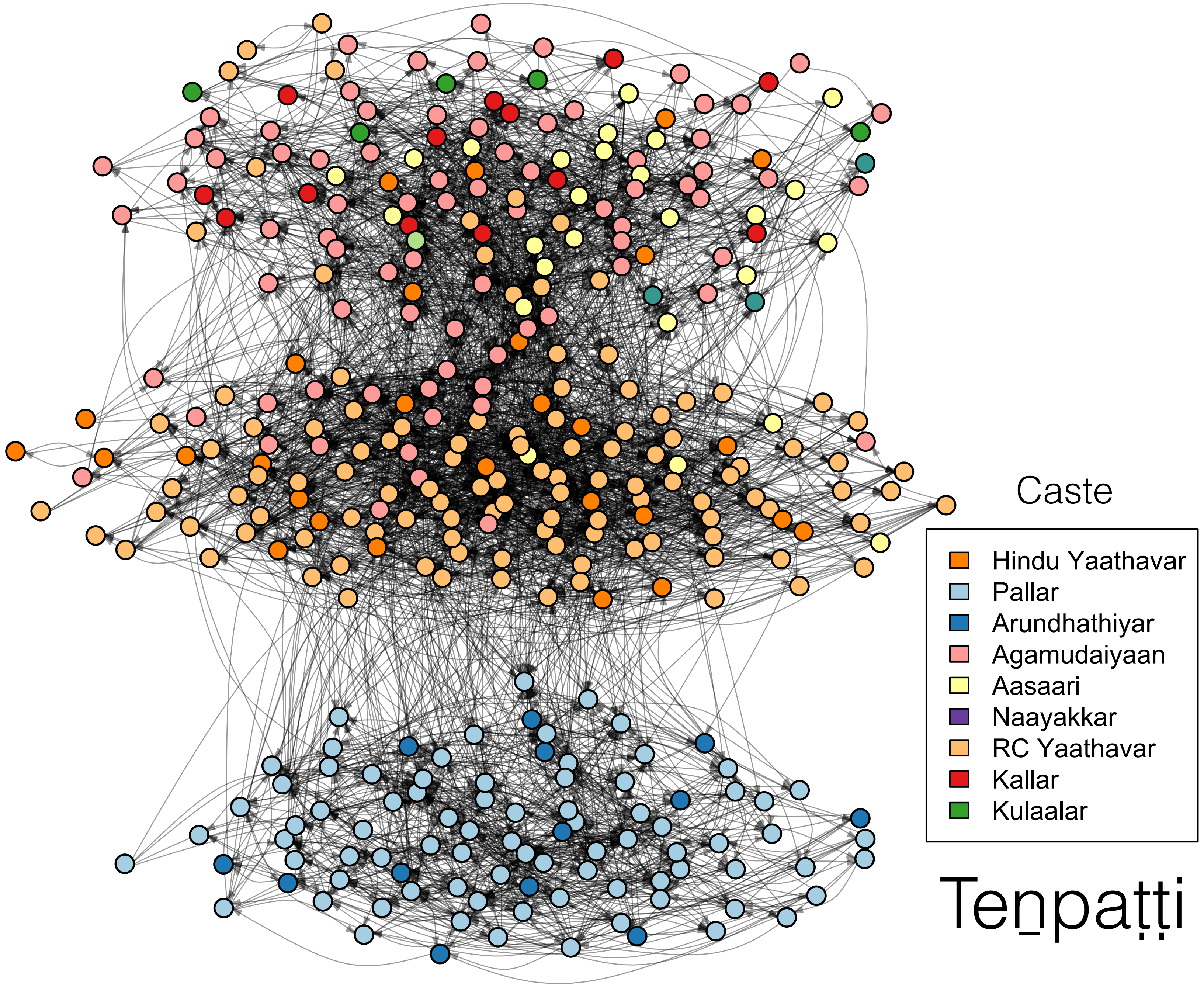
Assumptions:

1. Endorsers have some intrinsic quality.
2. Interactions can reveal differences in qualities.
3. Endorsements are pair-wise.

Clauset, Arbesman, Larremore. *Science Advances* 1, e1400005 (2015).

Systems of endorsement





Latent position can be revealed by **dominance** or **endorsement** interactions.

Figure: Larremore, Hébert-Dufresne, Power. *Draft.* Data: Power. *Nature Human Behaviour* 1, 0057 (2017)

The setup: suppose we have a *directed* network.

Its adjacency matrix is A .

$A_{ij} = A_{i \to j}$ means i beat j or i was endorsed by j

The problem: Rank the nodes.

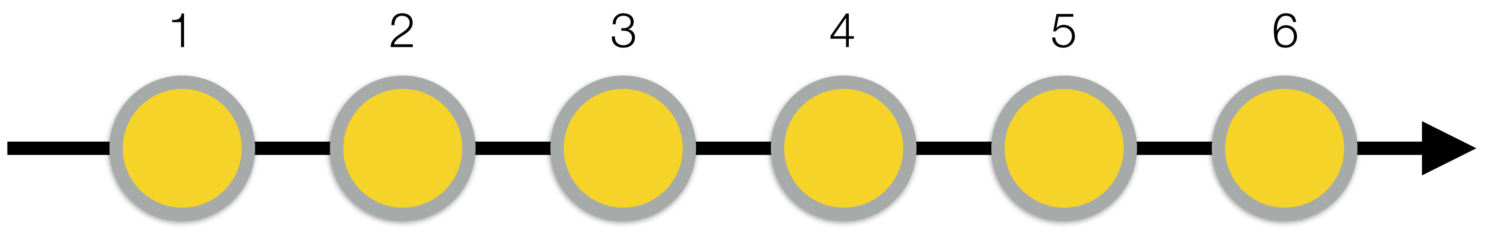
Alternative view: there might be **no network here.** In some cases we're just seeing a network in pairwise comparison data because **networks are a convenient data structure.**

Alternative problem: Which items should be compared next in order to most/best resolve our estimate of the ranks? (sequential tournament design)

Embeddings vs Orderings

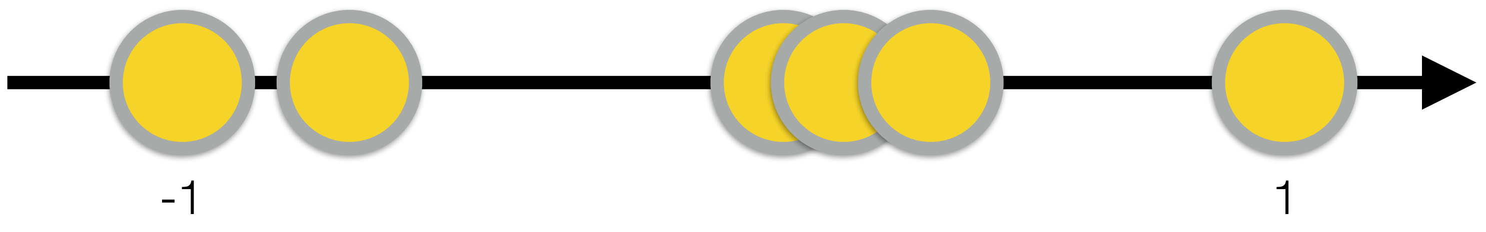
Ordering place the nodes in order:

1, 2, 3, …



Embedding assigns a position to each node:

1, 1.2, 7, 20, 21, 21.2, …



Which one should I use?

> Depends on the use case.

> Is it possible for two nodes to occupy the same rank or position? If so, an embedding is more appropriate. Also better when meaning of 1-rank Δ varies.

> Consider that you can always go from an embedding to an ordering, if you have a rule for breaking ties.

Win-Loss is not satisfactory: schedule matters

Beating the grandmaster counts for more than beating a novice.

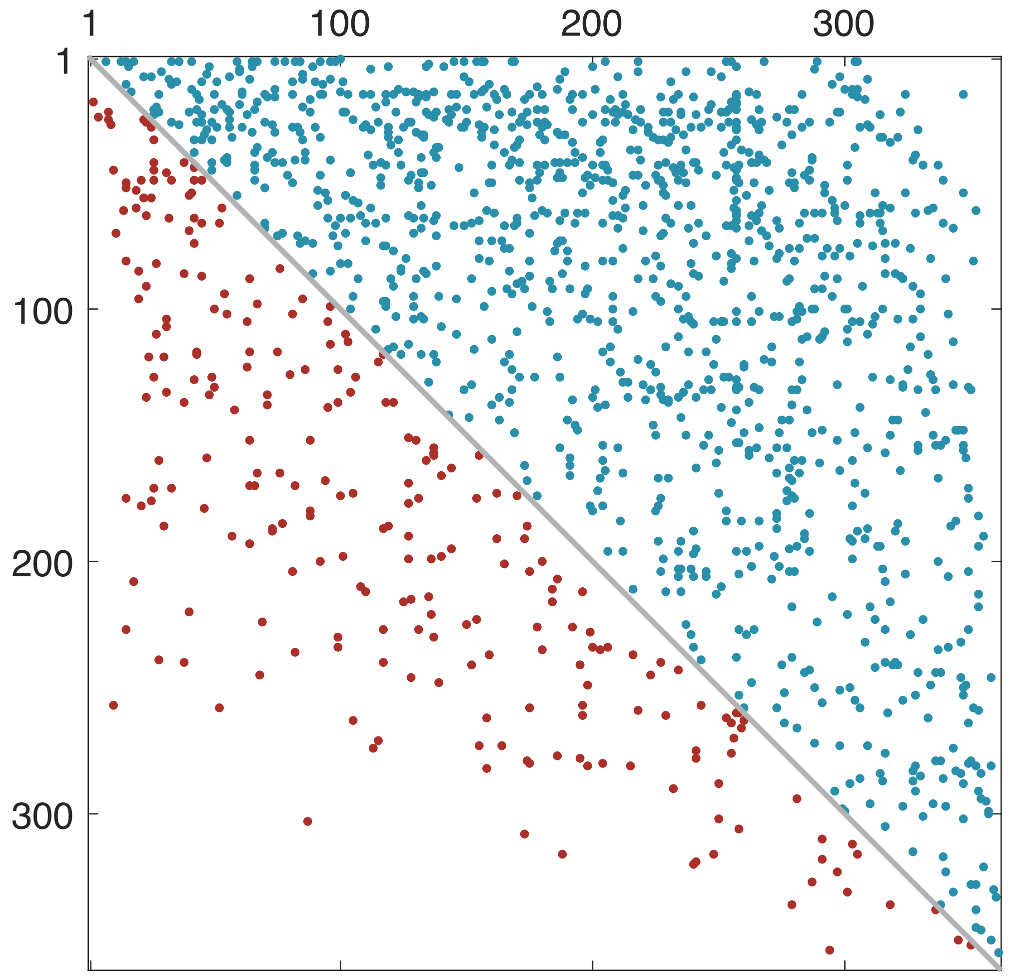
Win and loss tallies don't take this "schedule difficulty" into account. Put differently, win-loss records **leave information on the table.**

One way to make use of this information:

i* beats *j implies $s_i > s_j$

Therefore if we have a whole list of outcomes, we can try to find a total ordering that breaks as few of these implications as possible.

$A_{ij} =$ number of times that *i* beat *j*.



minimum violation ranking: sort *A*.

Win-Loss is not satisfactory: schedule matters

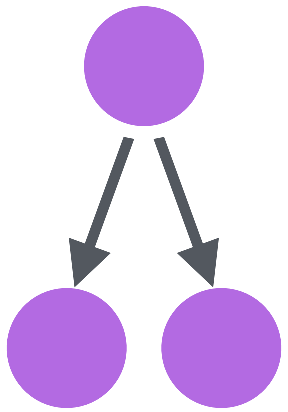
How do we find an ordering that minimizes the number of violations (or upsets) ?

Recipe (MCMC):

1. Order the nodes randomly.
2. Compute the number of violations. In expectation, this should be 50% of edges.
3. Pick two nodes at random and propose to swap their positions.
4. Compute the number of violations in this scenario.
5. If #violations decreases or stays the same, keep the swap. Otherwise, reject.
6. Repeat until….?

Notes:

* The number of violations is non-increasing over time.
* There may be no unique minimum. Consider this scenario:



MVR: non-unique & rough optimization landscape

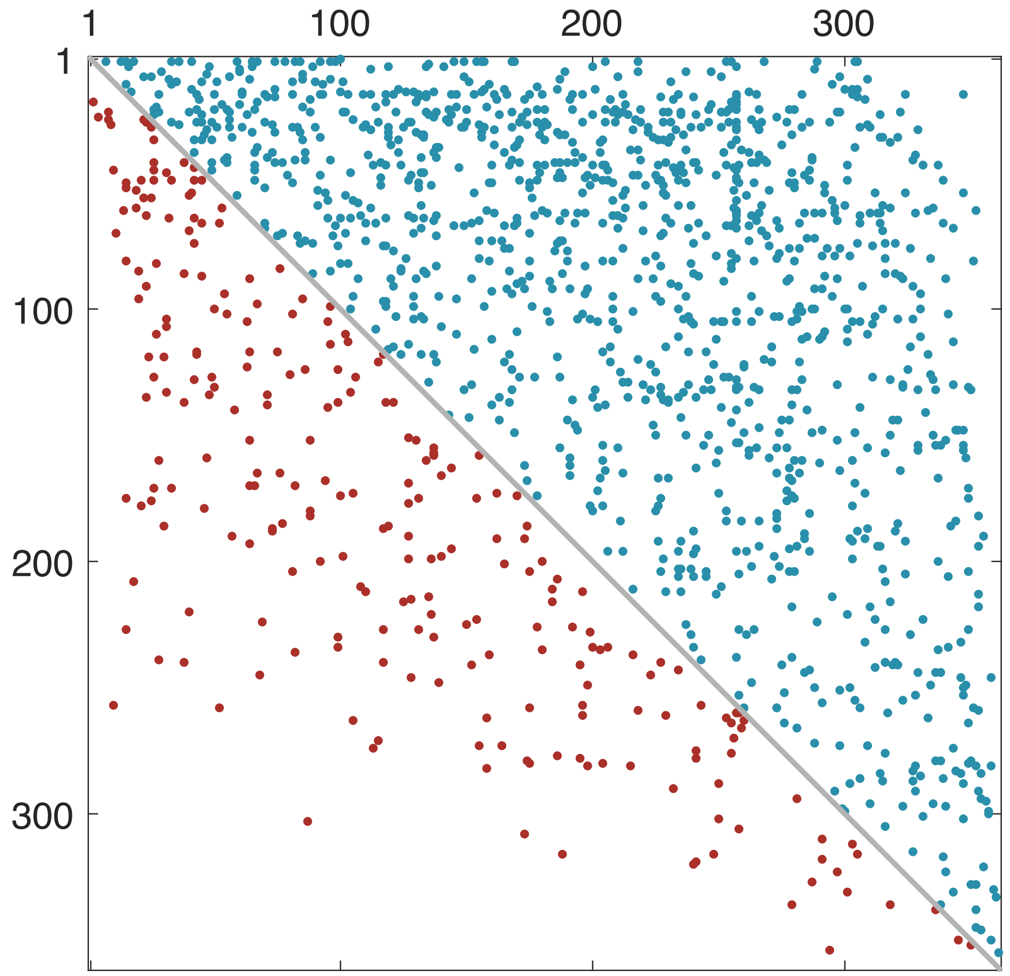
There is no guarantee of a unique minimizing ranking *s.*

Space of ordinal rankings has n! elements—usually use MCMC to search.

Slow.

Ordinal. No ties. No interpretability of rank differences.

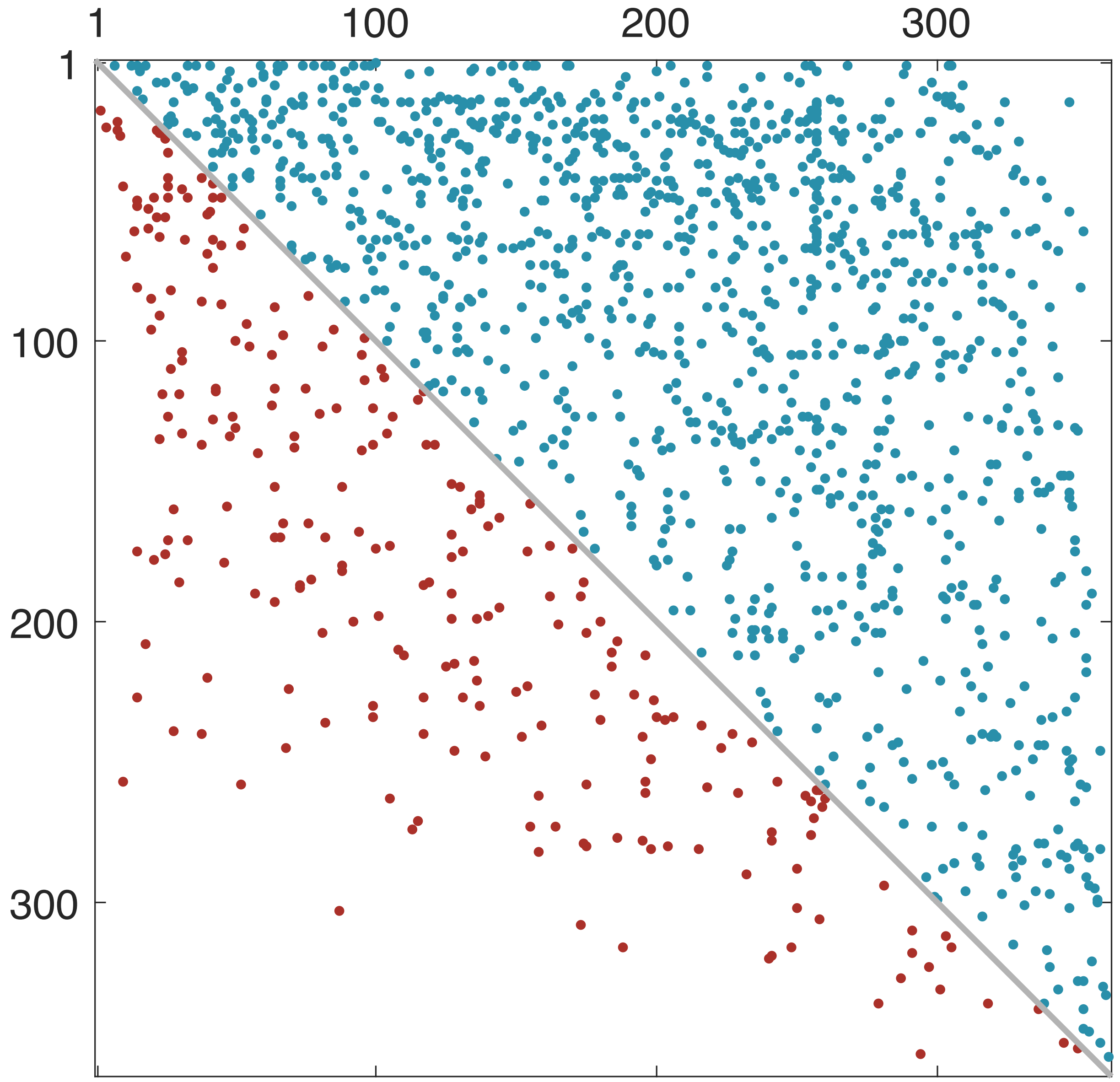
What are other premises on which we can base a ranking model?



minimum violation ranking: sort *A.*

Embeddings and Orderings 1: MVR & Agony

What if you allowed for **ties** and then ran Minimum Violation Ranking (MVR)? What would happen?



MVR: uniform cost (1 per edge).
Agony: generic cost function.
  for example, difference in ranks.

Embeddings and Orderings 1: Discrete choice models

Louis Leon Thurstone and Thelma Thurstone

Embeddings and Orderings 1: Discrete choice models

Instead of rating everything from 1 to 10, try *paired comparisons*.

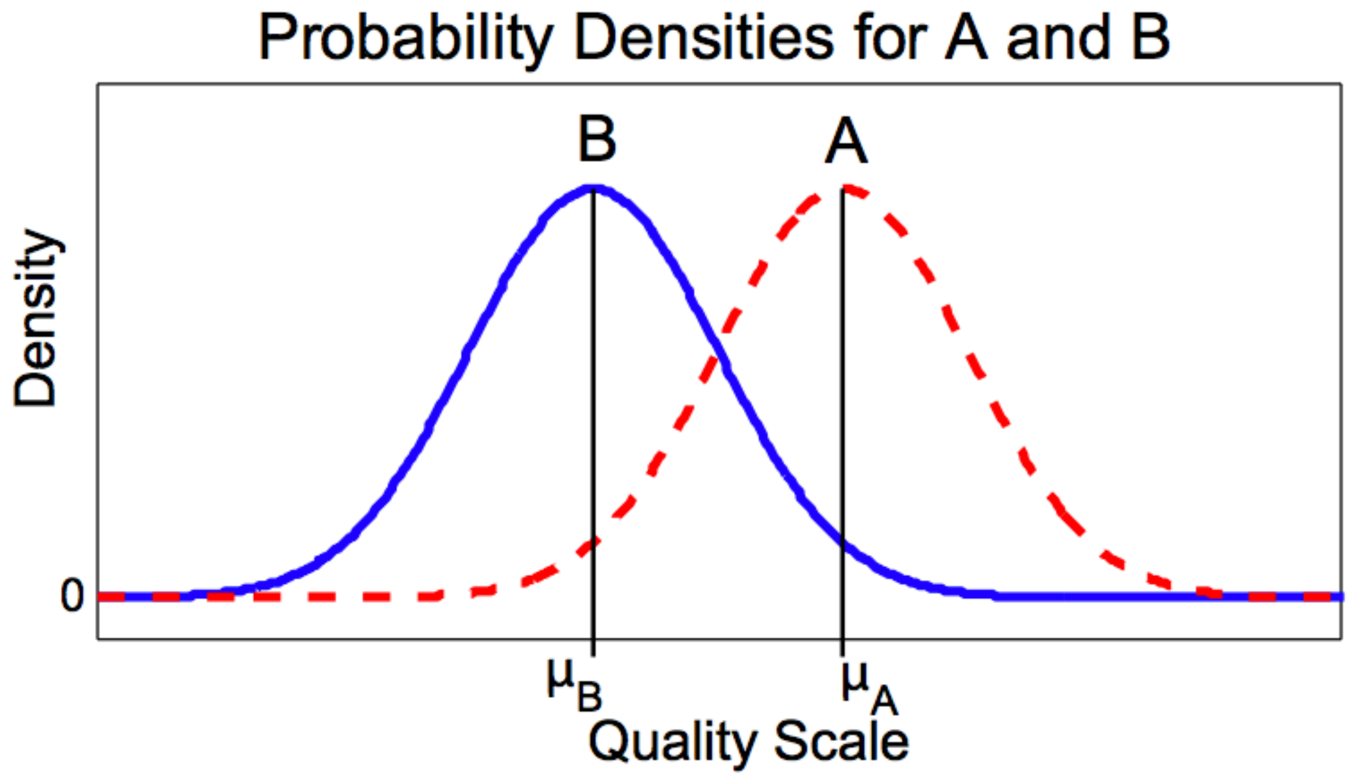
Do you prefer *i* or *j* ?

Why? Consider: My 3 is not your 3. What is 1 and what is 10?

https://xkcd.com/883/

Embeddings and Orderings 1: Discrete choice models

Thurstone: items have quality distributions. When a person judges whether A is better than B they draw from A's distribution and from B's distribution and see which is higher.



Thurstone modeled these as Gaussians.

$$P(A > B) = P(A - B > 0)$$

Difference of Gaussians is Gaussian.

$$\hat{\mu}_{AB} = \Phi^{-1}\left(\frac{C_{A\to B}}{C_{A\to B} + C_{B\to A}}\right)$$

Where $\Phi^{-1}(x)$ is the inverse CDF of standard normal, a.k.a. the *probit*.

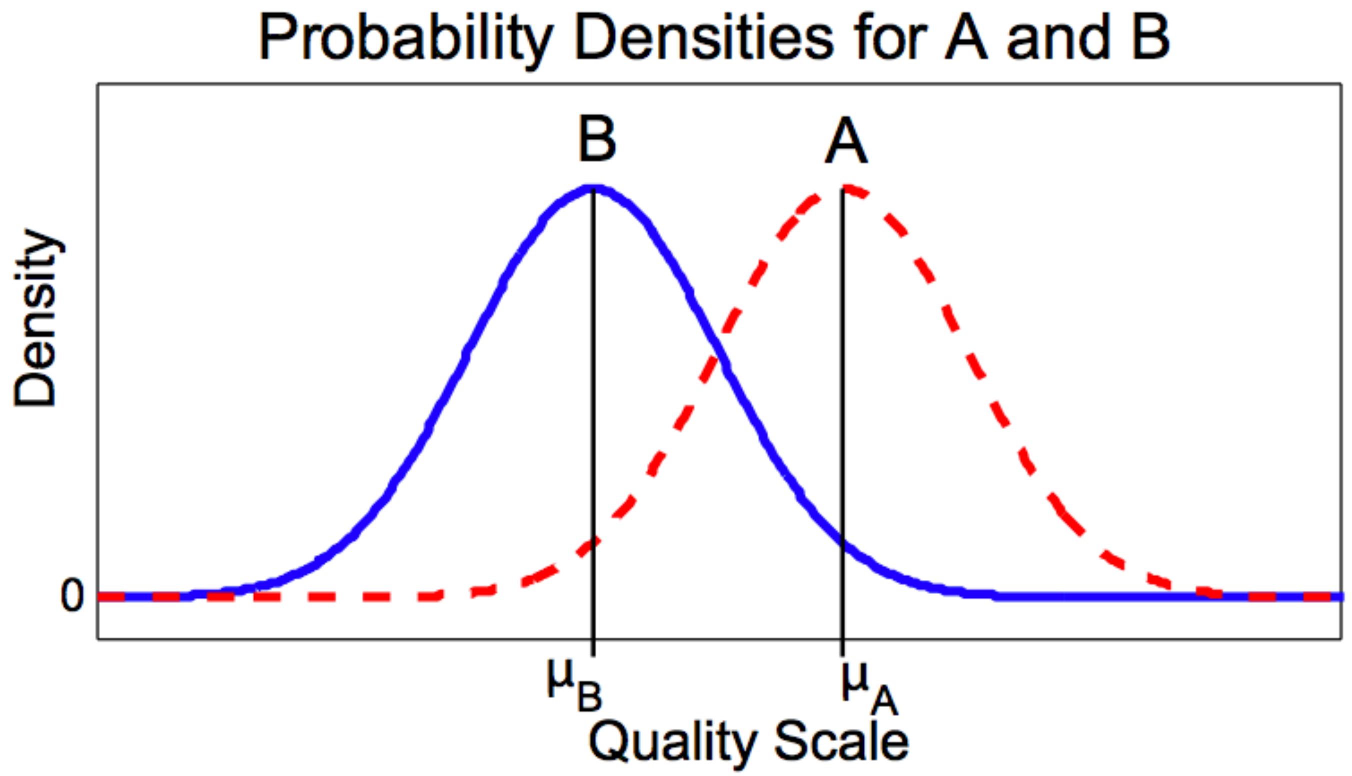
Powerful idea: lots of pairwise comparisons = estimates of all the qualities! An embedding!

Key: pairwise comparisons = directed network. i preferred to j $=$ $i \rightarrow j$

Finding the qualities of items from pairwise comparisons = Finding embedding of nodes.

Embeddings and Orderings 1: Discrete choice models

Bradley-Terry & Luce: items have quality distributions. When a person judges whether A is better than B they draw from A's and from B's distribution and see which is higher.



BTL

$$P(A > B) = \frac{\pi_A}{\pi_A + \pi_B}$$

Or usually:

$$P(A > B) = \frac{e^{\mu_A/s}}{e^{\mu_A/s} + e^{\mu_B/s}} = \frac{1}{1 + e^{-(\mu_A - \mu_B)/s}}$$

Same idea; different distribution. (*logit* instead of *probit*; *Gumbel* instead of *Gaussian*)

Powerful idea: lots of pairwise comparisons = estimates of all the qualities! An embedding!

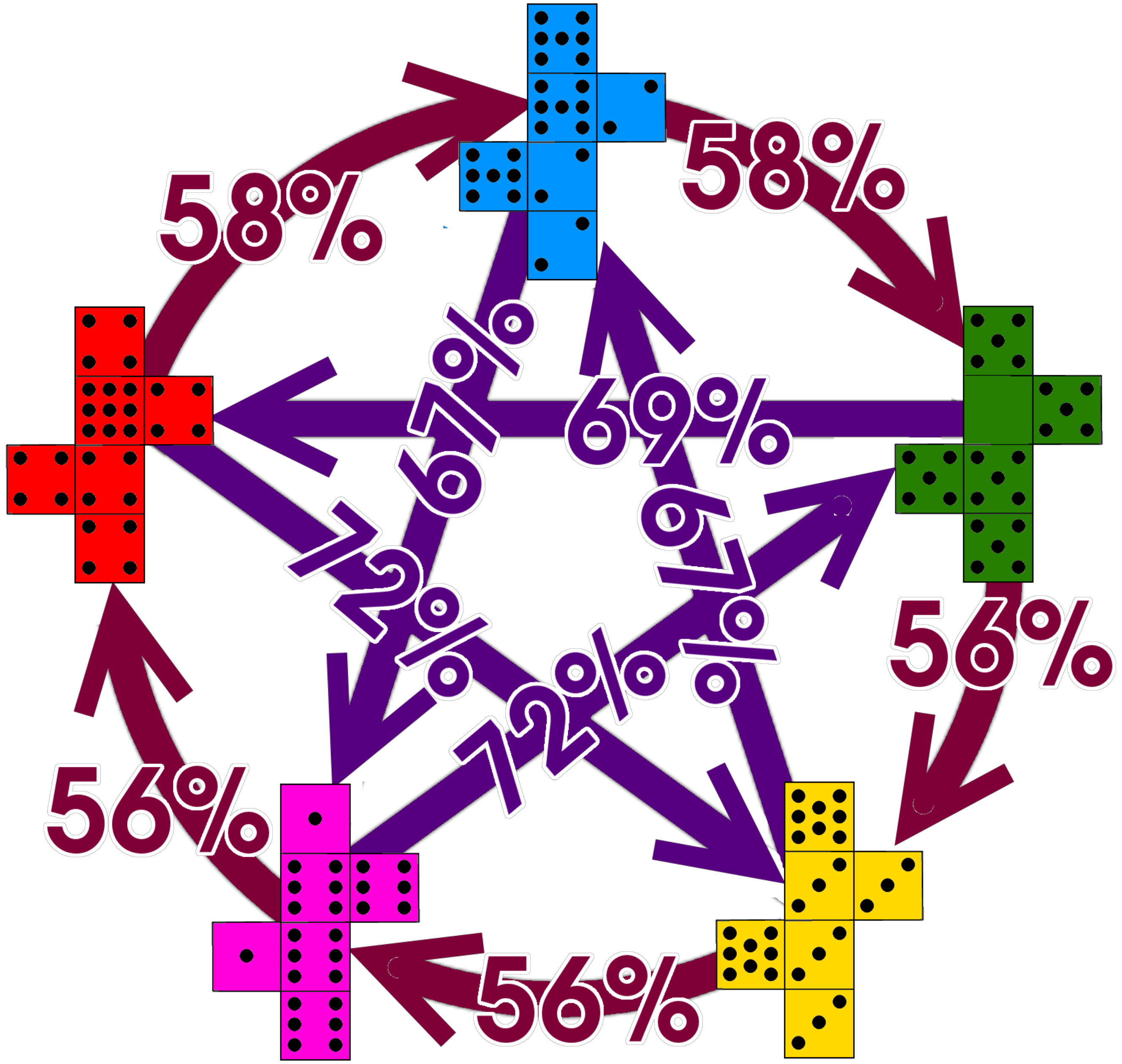
BTL avoids non-transitivities (aka rock-paper-scissors)

Introducing: **non-transitive dice**!

- 3 (or more) dice {A,B,C}
- faces chosen so that they have the property:
  - A>B more than half the time.
  - B>C more than half the time.
  - C>A more than half the time (?!)

https://en.wikipedia.org/wiki/Nontransitive_dice

A great gift for your favorite nerd's desk!
Go to the makerspace and laserbeam your own!



Bradley-Terry-Luce

These methods embed items or players in a 1D space.

- Provably avoids non-transitive properties
- Great when lots of data per interaction.

Pairwise ranking is really nice for ordering large sets of preferences too, and this model specifically models the probability that the preference will be for *i* over *j*.

Iterative algorithms exist. Needs a little regularization so the winningest winners don't fly off to infinity.

$$P(i \to j) = \frac{\gamma_i}{\gamma_i + \gamma_j}$$

Embeddings and Orderings 1: Discrete choice models

Introductory tutorial:

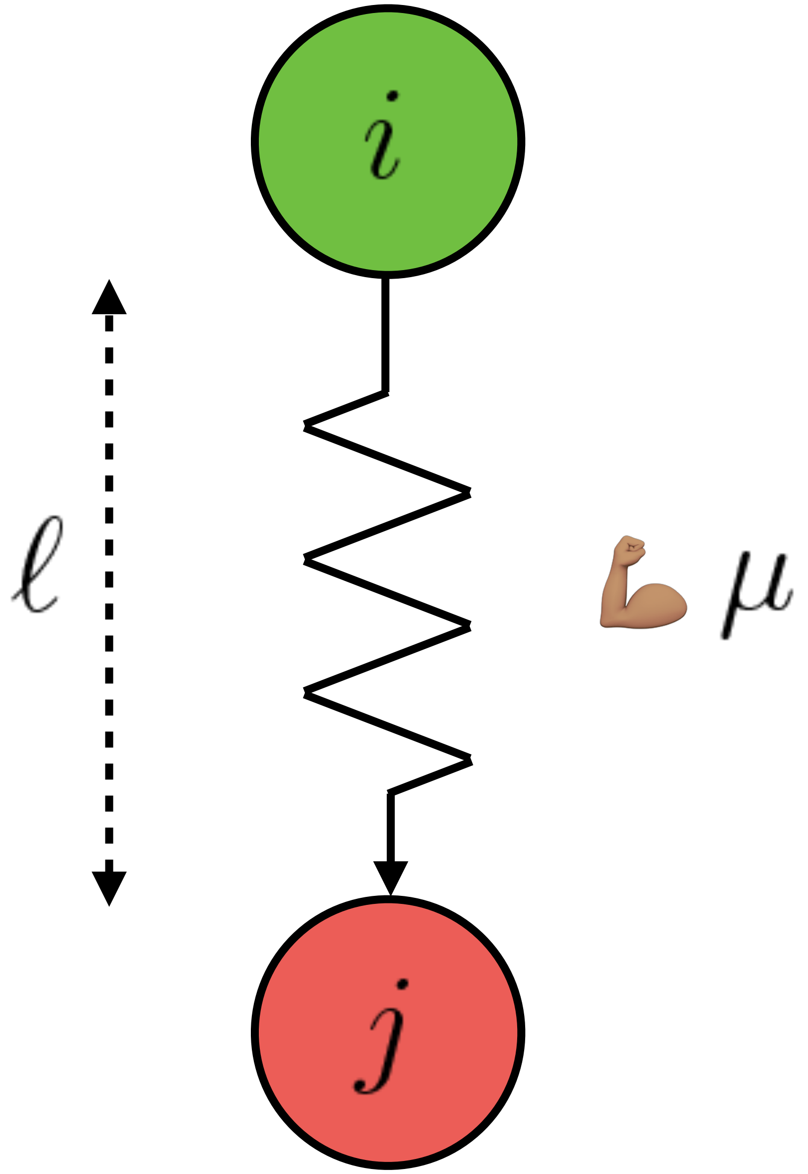
http://mayagupta.org/publications/PairedComparisonTutorialTsukidaGupta.pdf

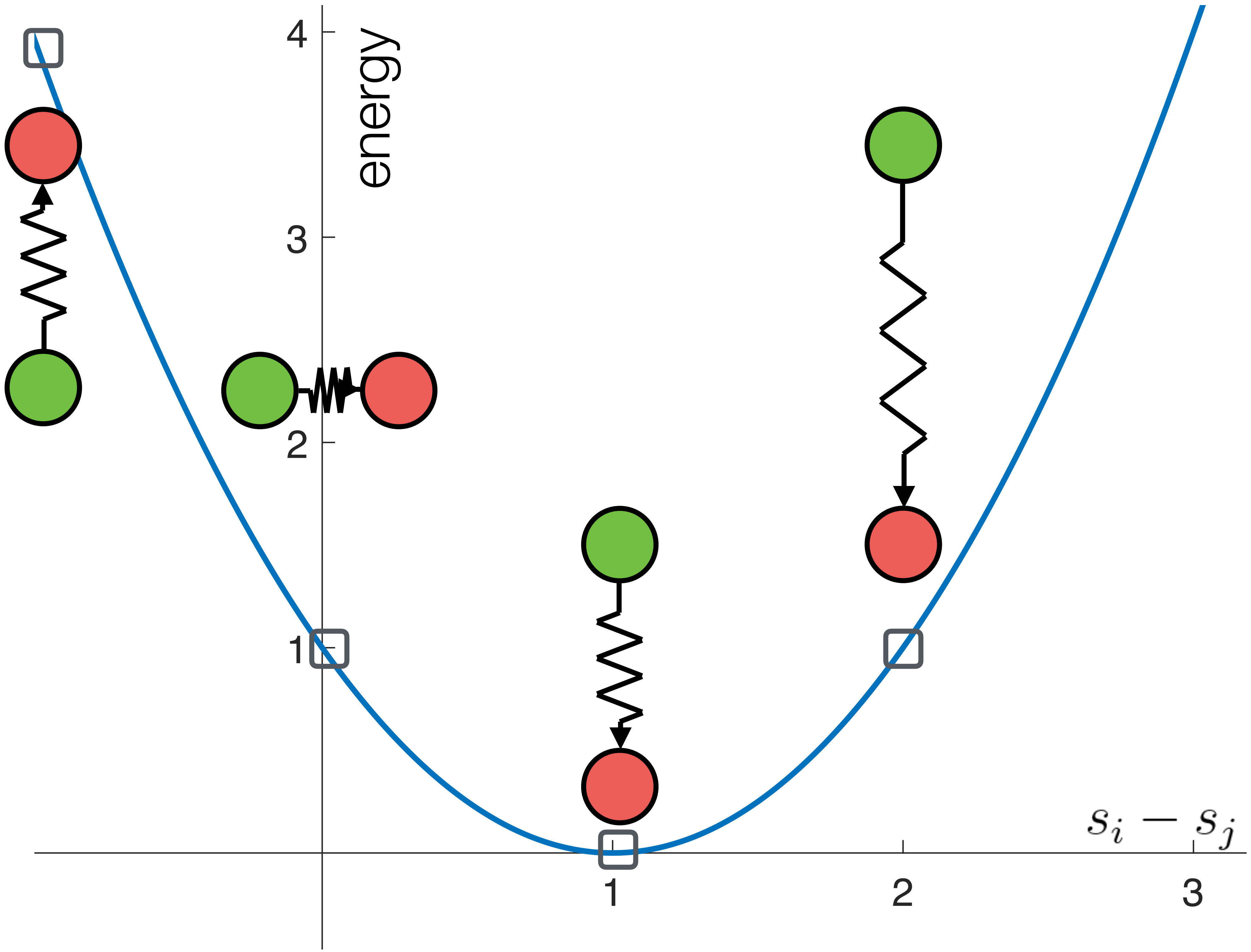
Discrete choice today:

https://web.stanford.edu/~jugander/papers/nips16-pcmc-slides.pdf

Embeddings & Orderings 2: SpringRank

Each directed edge = directed spring





How much energy is this system of springs?

Relax and let the springs decide the ranks

$$H(s) = \frac{1}{2} \sum_{i,j=1}^{N} A_{ij}(s_i - s_j - 1)^2$$

SpringRank Hamiltonian = energy of the system, given the node positions *s.*

Because the springs are linear, the potential is quadratic.

The SR Hamiltonian is *convex* in s.

$$\nabla H(s) = 0$$

The solution is unique…up to an additive constant. (Why?)

Derivatives work out nicely

$$0 = \frac{\partial H}{\partial s_i} = \sum_j A_{ij}(s_i - s_j - 1) - A_{ji}(s_j - s_i - 1)$$

Rewrite as a linear algebra problem.

$$\left[D^{\text{out}} + D^{\text{in}} - \left(A + A^T\right)\right] s^* = \left[D^{\text{out}} - D^{\text{in}}\right] \mathbf{1}$$

We know *a priori* that the matrix on the left is singular: translational invariance of H(s).
[if s is a solution, then $s + k$ is a solution for any constant k; eigenvalue 0, eigenvector **1**]

Notice: the matrix on the left is the *graph Laplacian* of the *undirected* network.

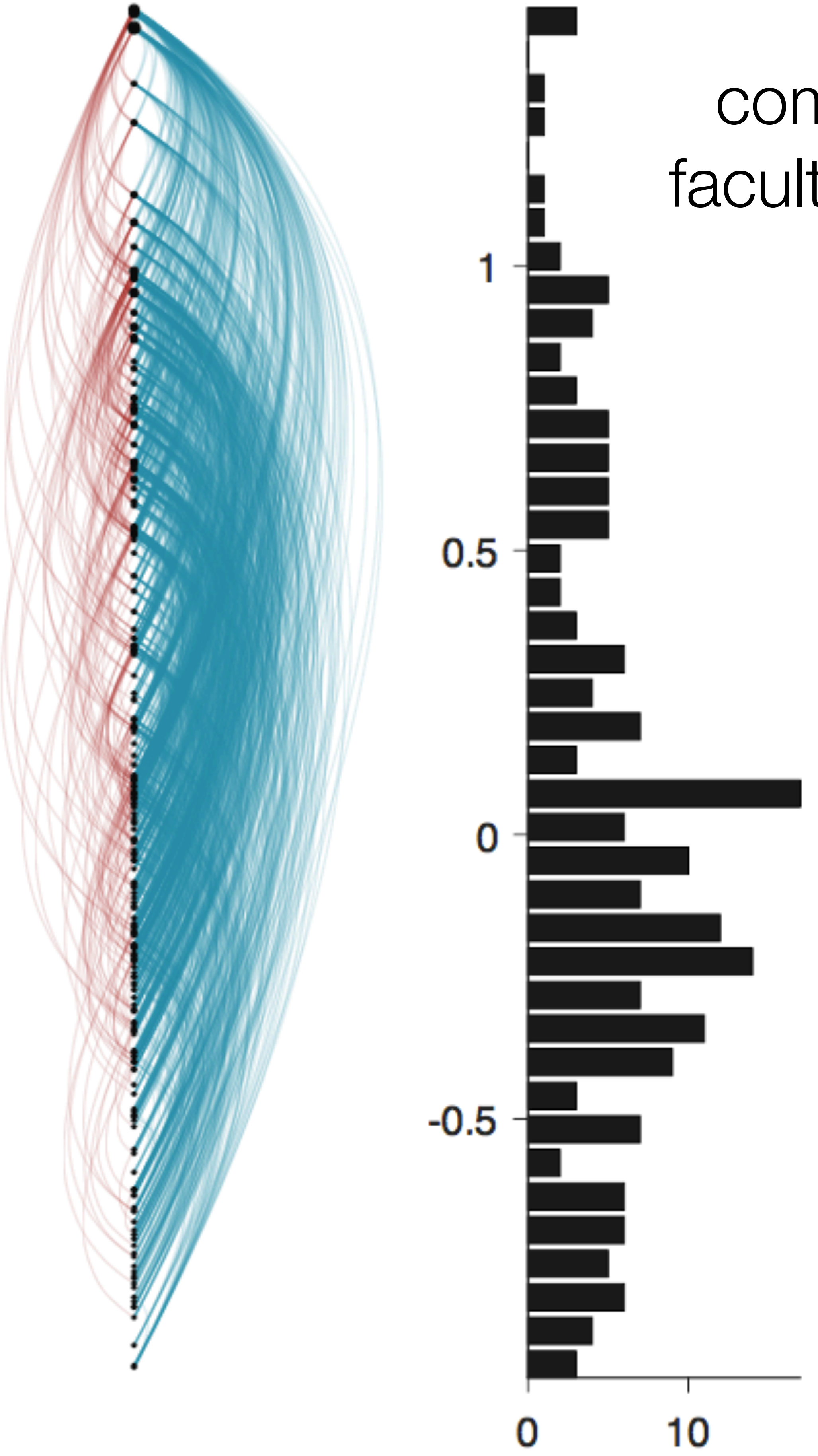
Uniqueness: Set s_1=0, min(s)=0, or mean(s)=0. Or use a pseudoinverse. Or regularize.

It works!

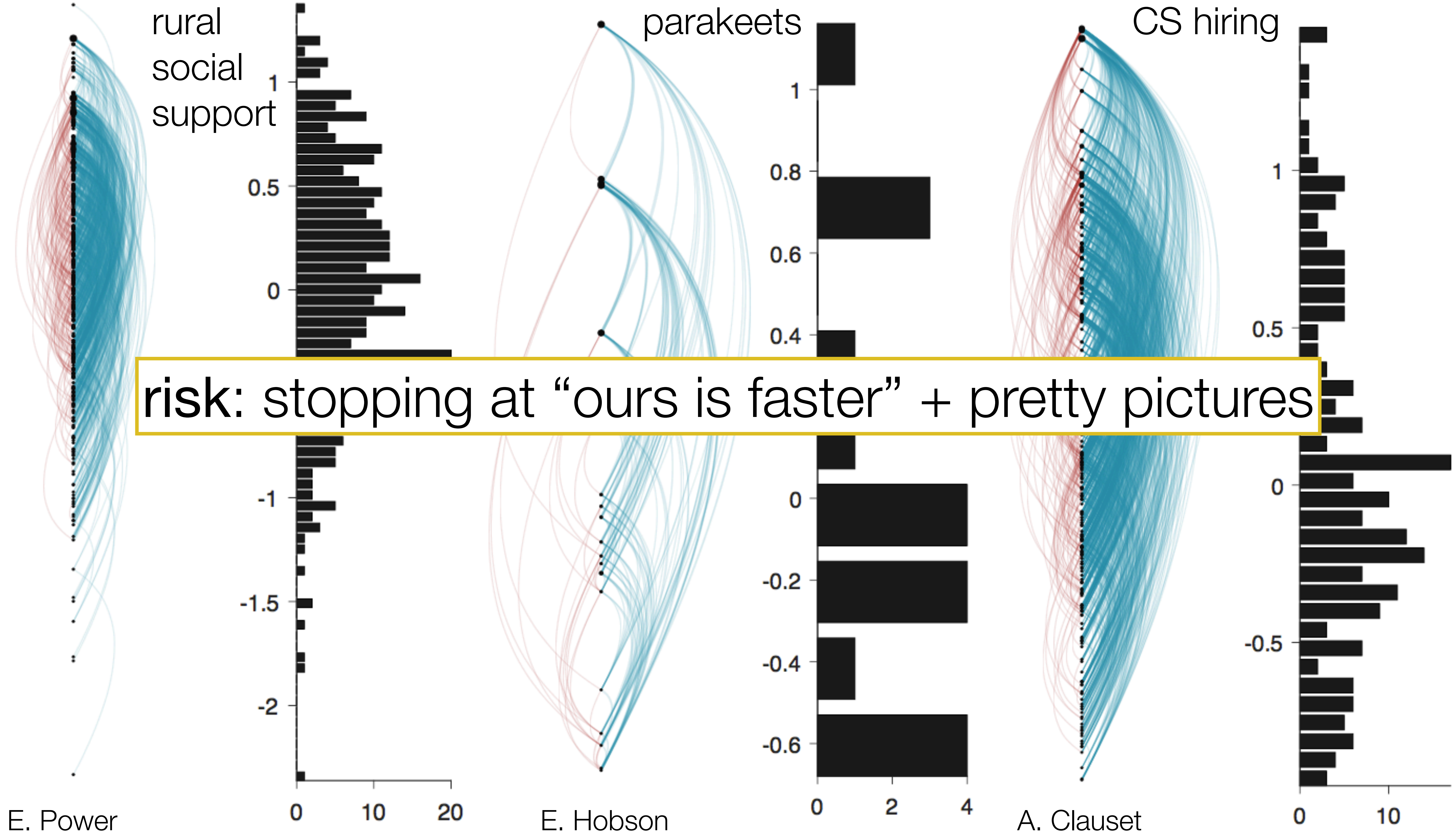
Real networks tend to be sparse…
our linear algebra problem is sparse…
we can use sparse iterative solvers…
millions of edges in **seconds**.

Even better: it's a linear-Laplacian system.
🚀 Near-linear-time (in |edges|) solutions.

Note that node positions can be clumpy,
since this is an *embedding*.



computer science
faculty hiring network



risk: stopping at “ours is faster” + pretty pictures

Cross validation: train on 80%, predict 20%

In a linear hierarchy the key quantity to predict is *edge direction,* given *edge existence*.

If *i* and *j* were to face off, who would win?

I'll give you *undirected(A)*, and you predict *directed(A).*

Setup: learn *s* from 80% of *A*. Then predict edge directions for remaining 20% of *A.*

SpringRank predicts edge direction based on the relative direction probabilities:

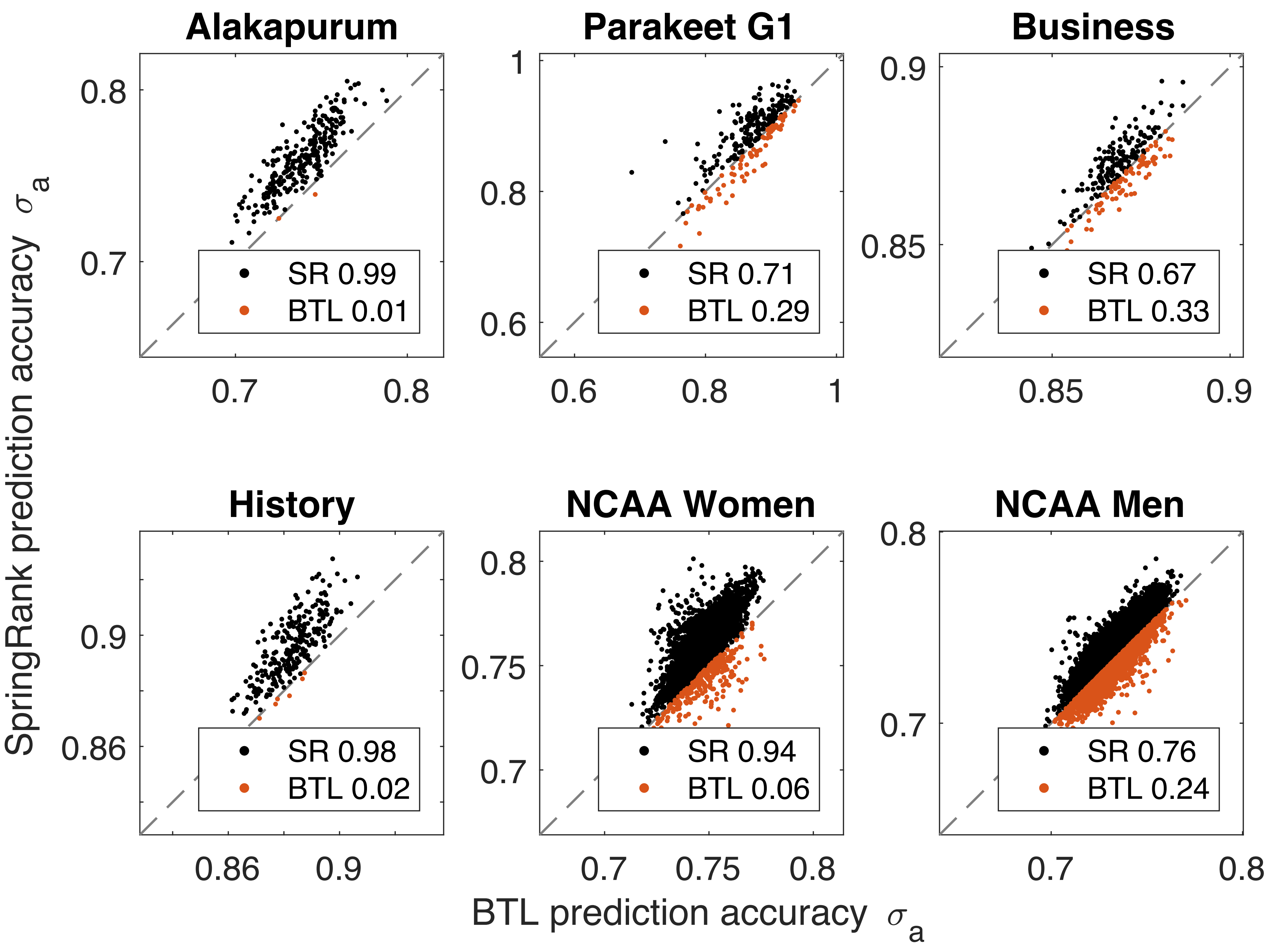
$$P_{ij}(\beta) = \frac{e^{-\beta H_{ij}}}{e^{-\beta H_{ij}} + e^{-\beta H_{ji}}} = \frac{1}{1 + e^{-2\beta(s_i - s_j)}}$$

Cross validation vs BTL: SR makes better predictions

Accuracy:

$$\sigma_a = 1 - \frac{1}{2M} \sum_{i,j} |A_{ij} - (A_{ij} + A_{ji})\, P_{ij}|$$

Goal: maximize the number of correctly predicted edge directions.



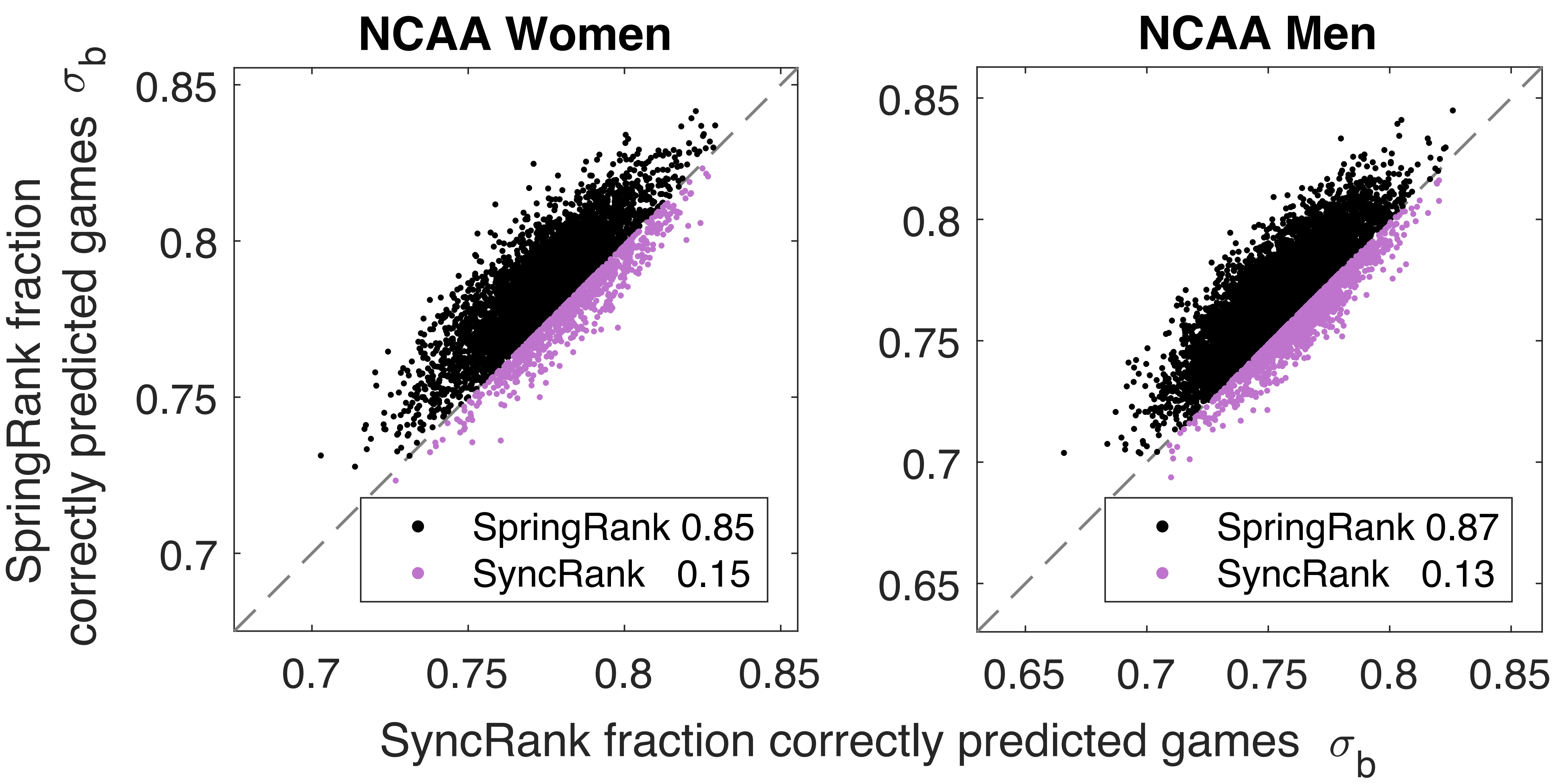
50 independent trials of 5-fold cross validation (250 folds)

Cross validation vs SyncRank: SR makes better predictions

"One-bit" Accuracy:

Higher ranked player always wins.
- No probabilistic prediction.
- Bad for gambling.

Goal: maximize the number of correctly predicted edge directions.



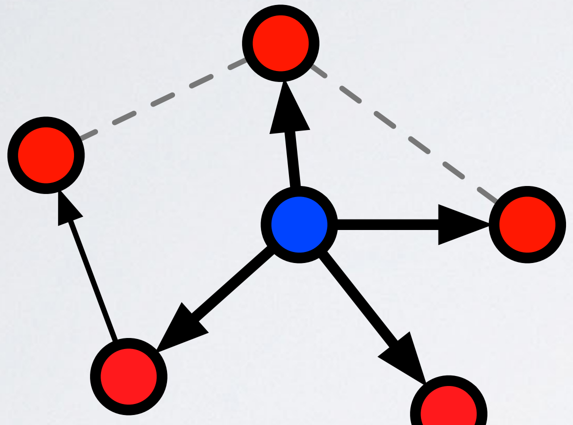
50 independent trials of 5-fold cross validation (250 folds)

Why/when would a model of springs make better predictions than a model of the choices themselves?🧐

Aside on Embeddings and Orderings: Centralities

Clauset lecture 2

describing networks



position = centrality:
structural vs. dynamical importance

geometric	harmonic centrality
	closeness centrality
	betweenness centrality
connectivity	degree centrality
	eigenvector centrality
	PageRank
	Katz centrality
	many many more…

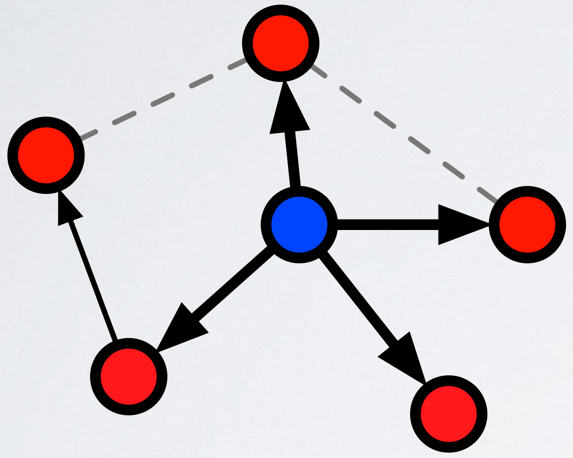
structural importance = cheap estimate of dynamical importance (aka "influence")

Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, *Social Networks* **27**, 55–71 (2005)

Which nodes are important… we've heard this before!

Clauset lecture 2

describing networks



position = centrality:
harmonic, closeness
centrality

importance = being in
"center" of the network

harmonic $$c_i = \frac{1}{n-1}\sum_{j\neq i}\frac{1}{d_{ij}}$$

length of shortest path

distance: $$d_{ij} = \begin{cases} \boxed{\ell_{ij}} & \text{if } j \text{ reachable from } i \\ \infty & \text{otherwise} \end{cases}$$

Boldi & Vigna, arxiv:1308.2140 (2013)
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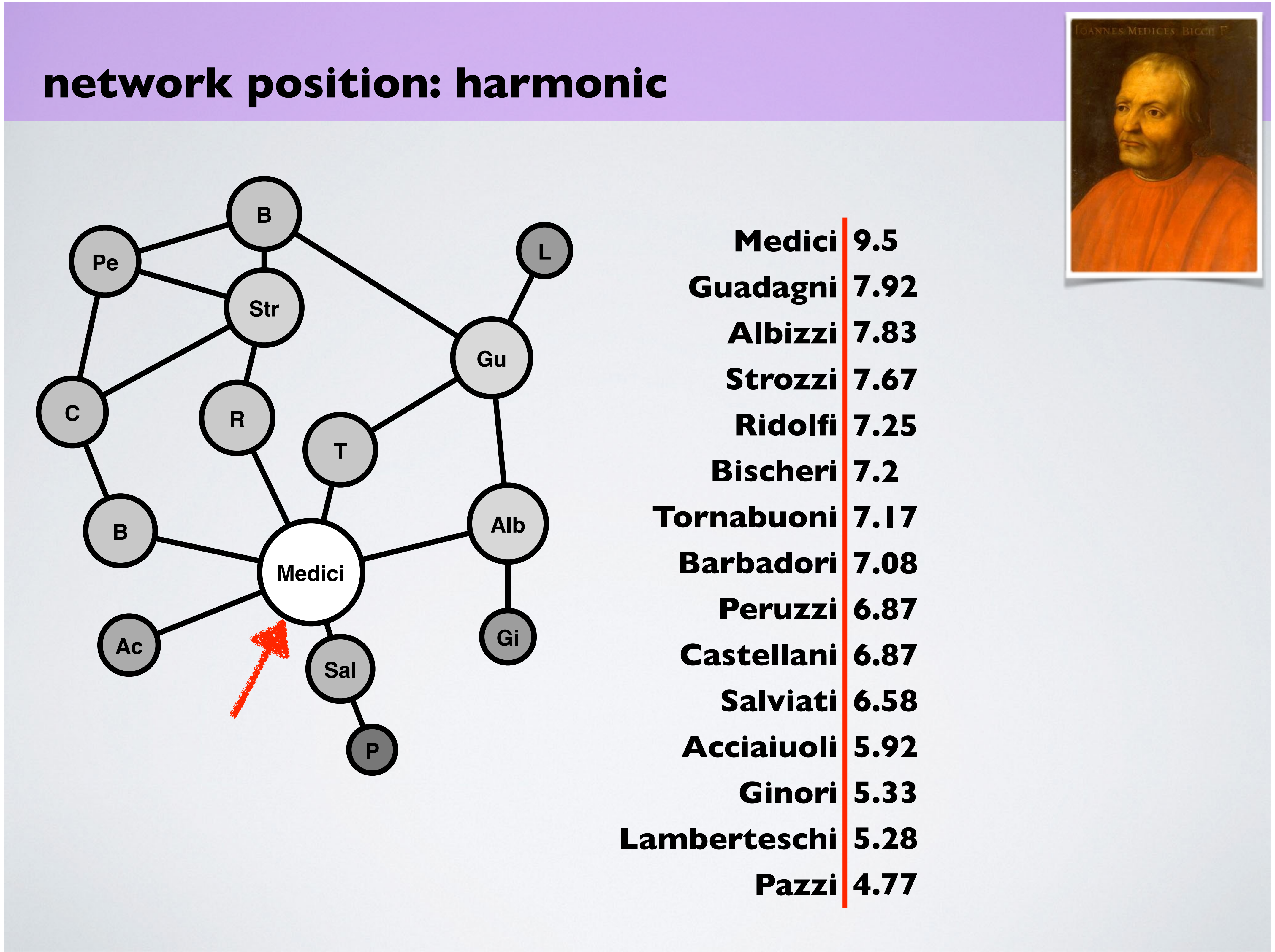
Which nodes are important… we’ve heard this before!

Clauset lecture 2



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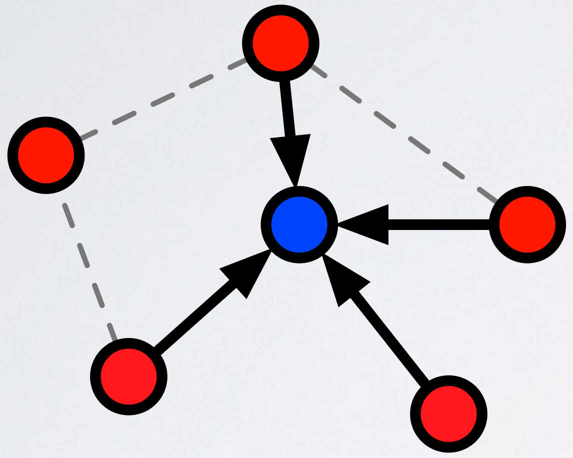
Clauset lecture 2

describing networks



position = centrality:
PageRank, Katz, eigenvector centrality

importance = sum of importances* of nodes that point at you

$$I_i = \sum_{j \to i} \frac{I_j}{k_j}$$

or, the left eigenvector of

$$\mathbf{A}\mathbf{x} = \lambda\,\mathbf{x}$$

Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, *Social Networks* **27**, 55–71 (2005)

*modulo several technical details

Embeddings and Orderings 3: PageRank

PageRank defines scalar rank recursively:

important pages are those that are linked to by important pages.

- Great at finding the top 3 but limited predictions available using the PageRank scores.

The PageRank Citation Ranking: Bringing Order to the Web

January 29, 1998

Abstract

The importance of a Web page is an inherently subjective matter, which depends on the readers interests, knowledge and attitudes. But there is still much that can be said objectively about the relative importance of Web pages. This paper describes PageRank, a method for rating Web pages objectively and mechanically, effectively measuring the human interest and attention devoted to them.

We compare PageRank to an idealized random Web surfer. We show how to efficiently compute PageRank for large numbers of pages. And, we show how to apply PageRank to search and to user navigation.

The Anatomy of a Large-Scale Hypertextual Web Search Engine

Sergey Brin and Lawrence Page

Computer Science Department,
Stanford University, Stanford, CA 94305, USA
sergey@cs.stanford.edu and page@cs.stanford.edu

Abstract

In this paper, we present Google, a prototype of a large-scale search engine which makes heavy use of the structure present in hypertext. Google is designed to crawl and index the Web efficiently and produce much more satisfying search results than existing systems. The prototype with a full

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \frac{1-d}{N} + d \sum_j p_j \pi_{ji}$$

write the equation

$$\mathbf{p} = \left(\frac{1-d}{N} \right) \mathbf{1} + d\pi^T \mathbf{p}$$

matrix-vector form

Alternative: stationary distribution of random walk on the network + weak all-to-all links

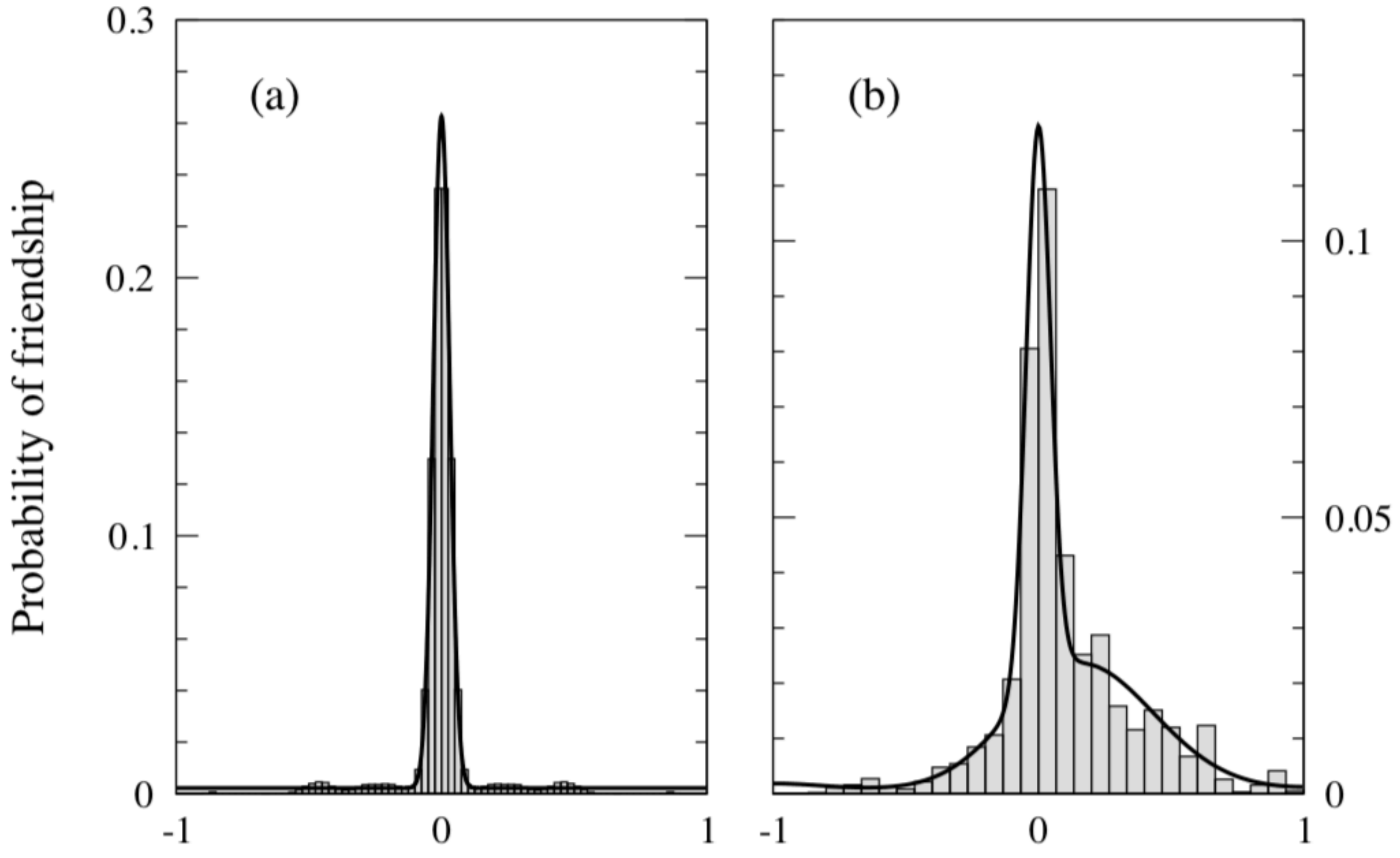
Jeremy Kun: http://www.infinitelooper.com/?v=K3pT0gTaDec&p=n

Embeddings and Orderings 4: Ball & Newman

Generative model:
Generate the patterns that you want to identify.

Create N nodes.
Assign each node an integer rank *r*, from 1 to N.

IRL, not all friendships are reciprocated 😭
So let's generate undirected AND directed edges:



$\alpha(z)$ $\beta(z)$

$$P(i \leftrightarrow j) \;=\; \alpha(r_i - r_j)$$

A gaussian centered at 0

$$P(i \rightarrow j) \;=\; \beta(r_i - r_j)$$

Fourier cosine series, keeping five terms & squaring to enforce nonnegativity, plus an additional Gaussian peak at the origin.

Ball, Newman. Network Science 1, 16-30 (2013)

Embeddings and Orderings 4: Ball & Newman

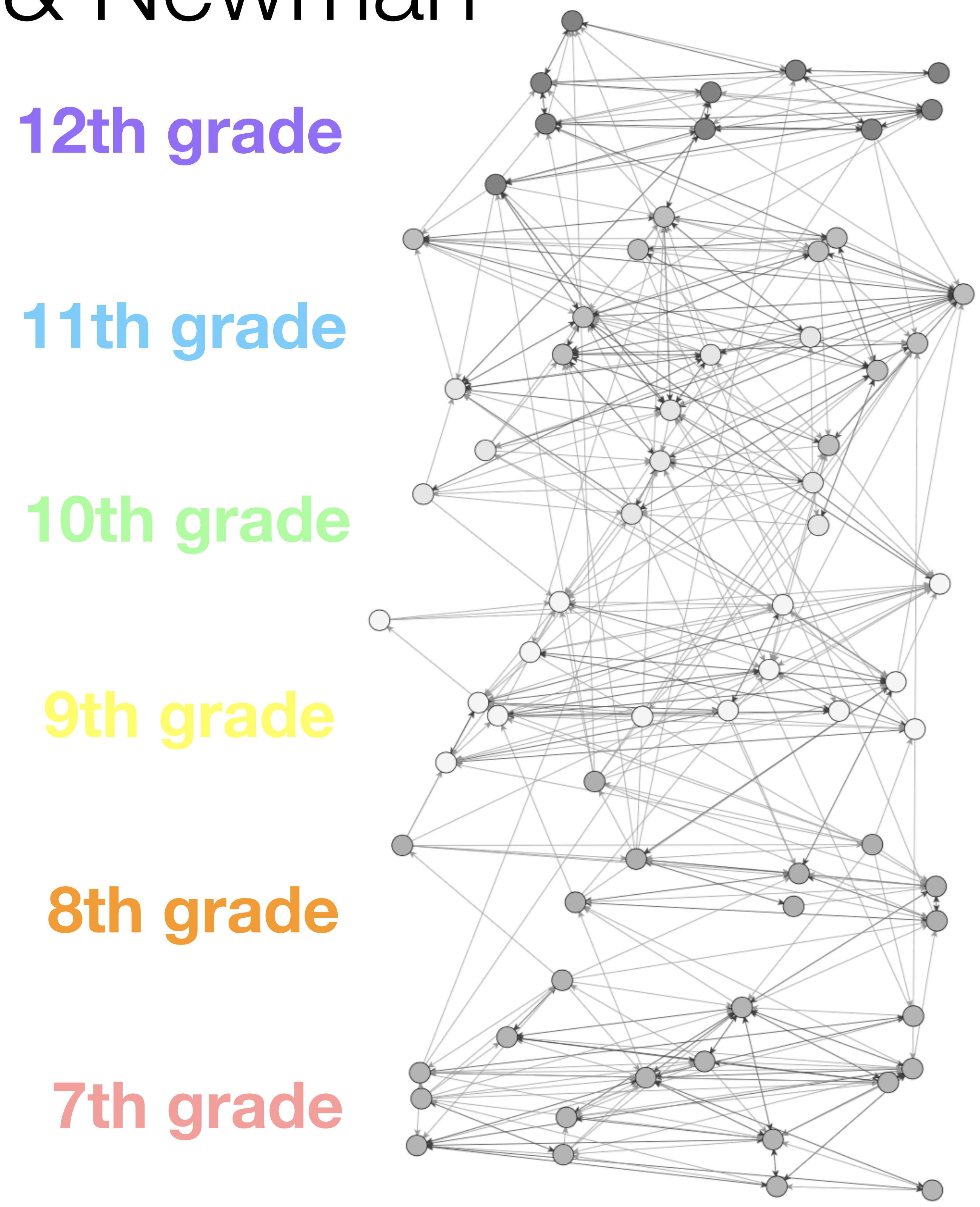
GM + parameters

α

β

ranks

inference ← Data



Inferred parameters of people's attachment preferences & ranks.

- Identified the need to learn from reciprocated friendships.
- Found that in AddHealth data, teens link to others of *nearby* social status.

Ball, Newman. Network Science 1, 16-30 (2013)

Embeddings and Orderings 5: Niche Models

Niche Models embed species in a latent space based on feeding preferences:

most species feed from narrow range in a 1-dim. space (~body size).

- Great for food webs. Inference models v slow for all but small networks.

Want more? Jen Dunne, Cris Moore

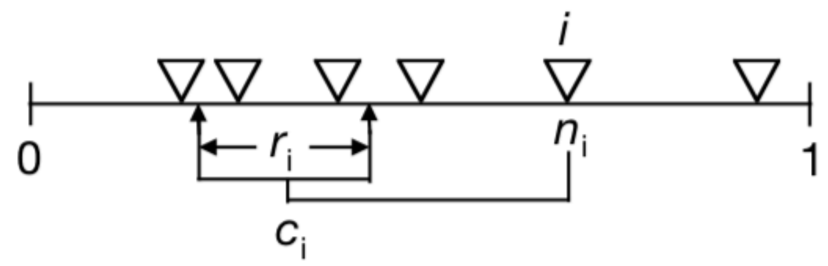


Figure 1 Diagram of the niche model. Each of ***S*** species (for example, ***S*** = 6, each shown as an inverted triangle) is assigned a 'niche value' parameter (n_i) drawn uniformly from the interval [0,1]. Species ***i*** consumes all species falling in a range (r_i) that is placed by uniformly drawing the centre of the range (c_i) from [r_i/2, n_i]. This permits looping and cannibalism by allowing up to half of r_i to include values $\geqslant n_i$. The size of r_i is assigned by using a beta function to randomly draw values from [0,1] whose expected value is 2***C*** and then multiplying that value by n_i [expected $E(n_i) = 0.5$] to obtain the desired ***C***. A beta distribution with $\alpha = 1$ has the form $f(x|1, \beta) = \beta(1-x)^{\beta-1}$, $0 < x < 1$, 0 otherwise, and $E(X) = 1/(1+\beta)$. In this case, $x = 1-(1-y)^{1/\beta}$ is a random variable from the beta distribution if y is a uniform random variable and β is chosen to obtain the desired expected value. We chose this form because of its simplicity and ease of calculation. The fundamental generality of species i is measured by r_i. The number of species falling within r_i measures realized generality. Occasionally, model-generated webs contain completely disconnected species or trophically identical species. Such species are eliminated and replaced until the web is free of such species. The species with the smallest n_i has $r_i = 0$ so that every web has at least one basal species.

Williams & Martinez. Nature 404.6774 (2000).

</methods>

<applications>

Many uses for the same techniques. cf regression

Treat the network like a system:

Extrapolation. Make predictions for as-yet unseen nodes (in “space” or time).
Interpolation. Identify missing links.
Generalization. Nodes of this type are like others of the same type.

Treat the network like an artifact:

Mechanisms. How did this network arise? What rules governed its assembly?
Explanations. Coarse-graining or compression.

Treat the network like a means to an end; an intermediate data structure:

Useful division. Need groups so that we can assign treatments in an A/B test.
Simplification. Downstream regression model needs ranks or groups.

Following

I have a problem with academics using academic data to show something about academics.

9:27 AM - 20 Jun 2018

1 Retweet **4** Likes

1 1 4

Dan Larremore @DanLarremore · Jun 20

Replying to @OverheardAtSFI

Well there goes my research agenda. 😭

6

Structure and inequality in academic hiring

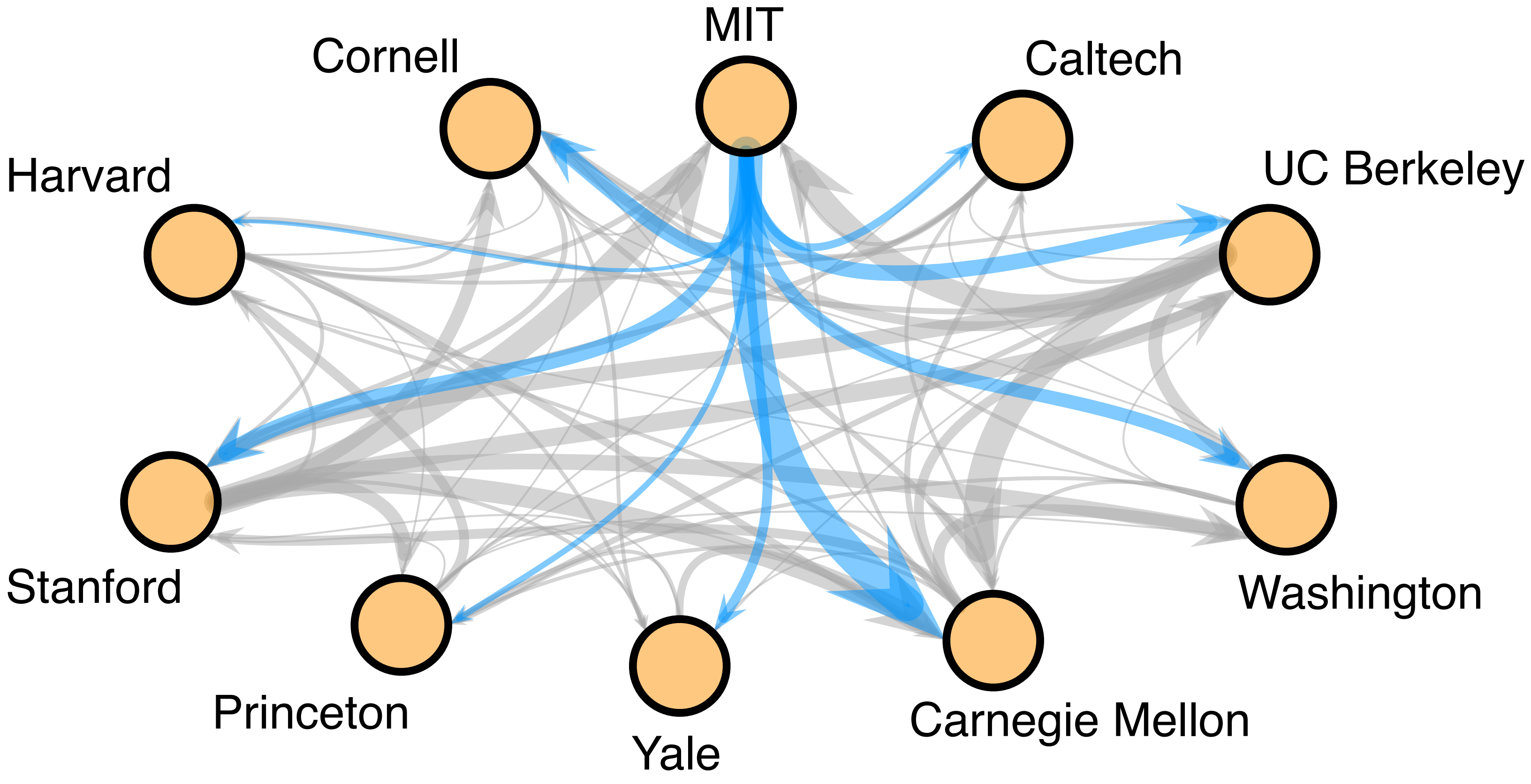
Collect the data (by hand 😭)

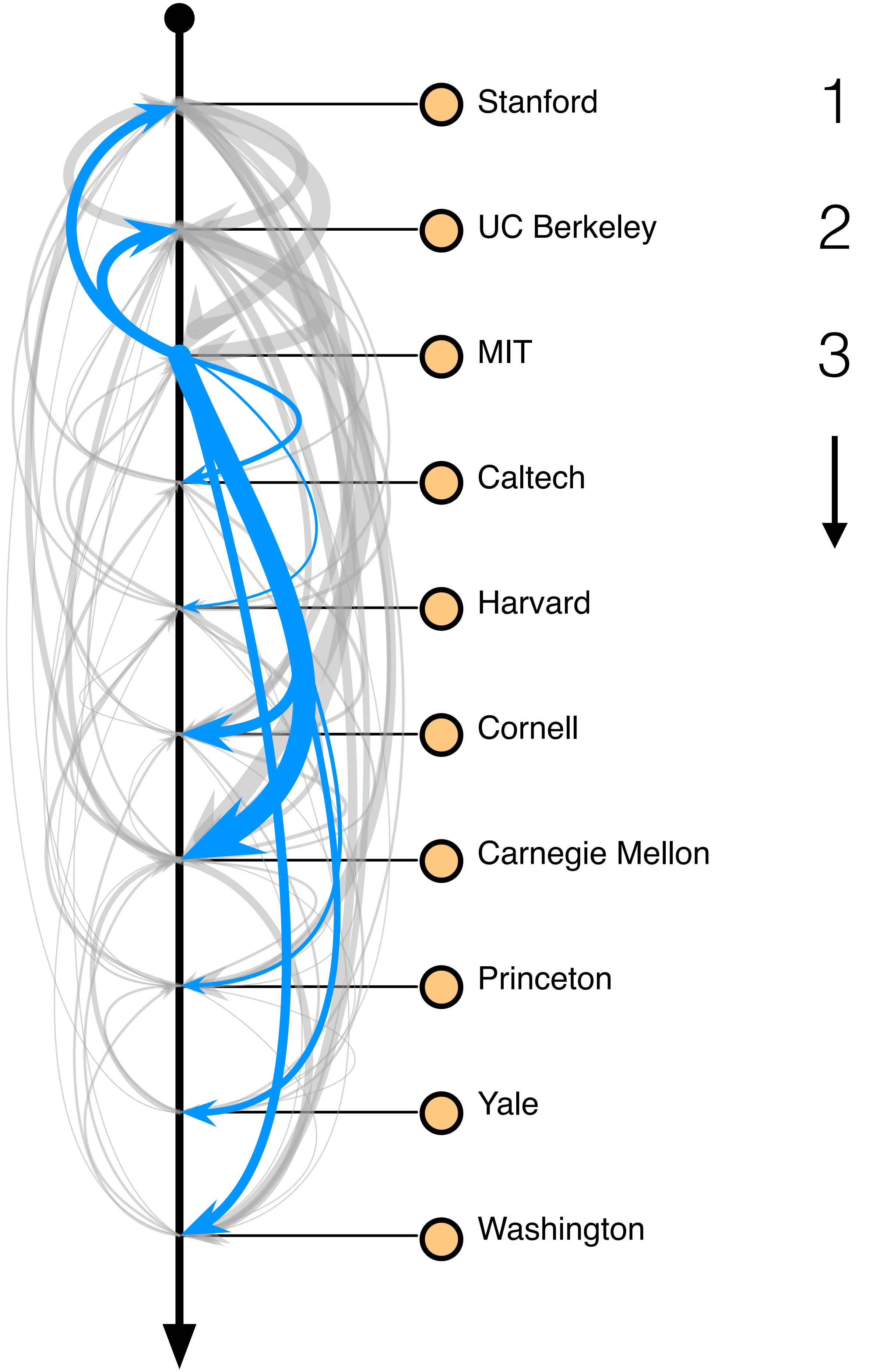
CVs of all US & Canadian tenure-track faculty in CS, Business, History: 2011-2013.

	Computer Science	Business	History
institutions	205	112	144
tenure-track faculty	5032	9336	4556
mean size	25	83	32
female	15%	22%	36%

total: **18,924** CVs

Faculty hiring networks





Premises:

1. Each hiring committee wants to hire **the best**.
2. Entire network reveals **collective preferences**.

Faculty hiring networks

systematic

90% of hiring movement
is “down” the hierarchy

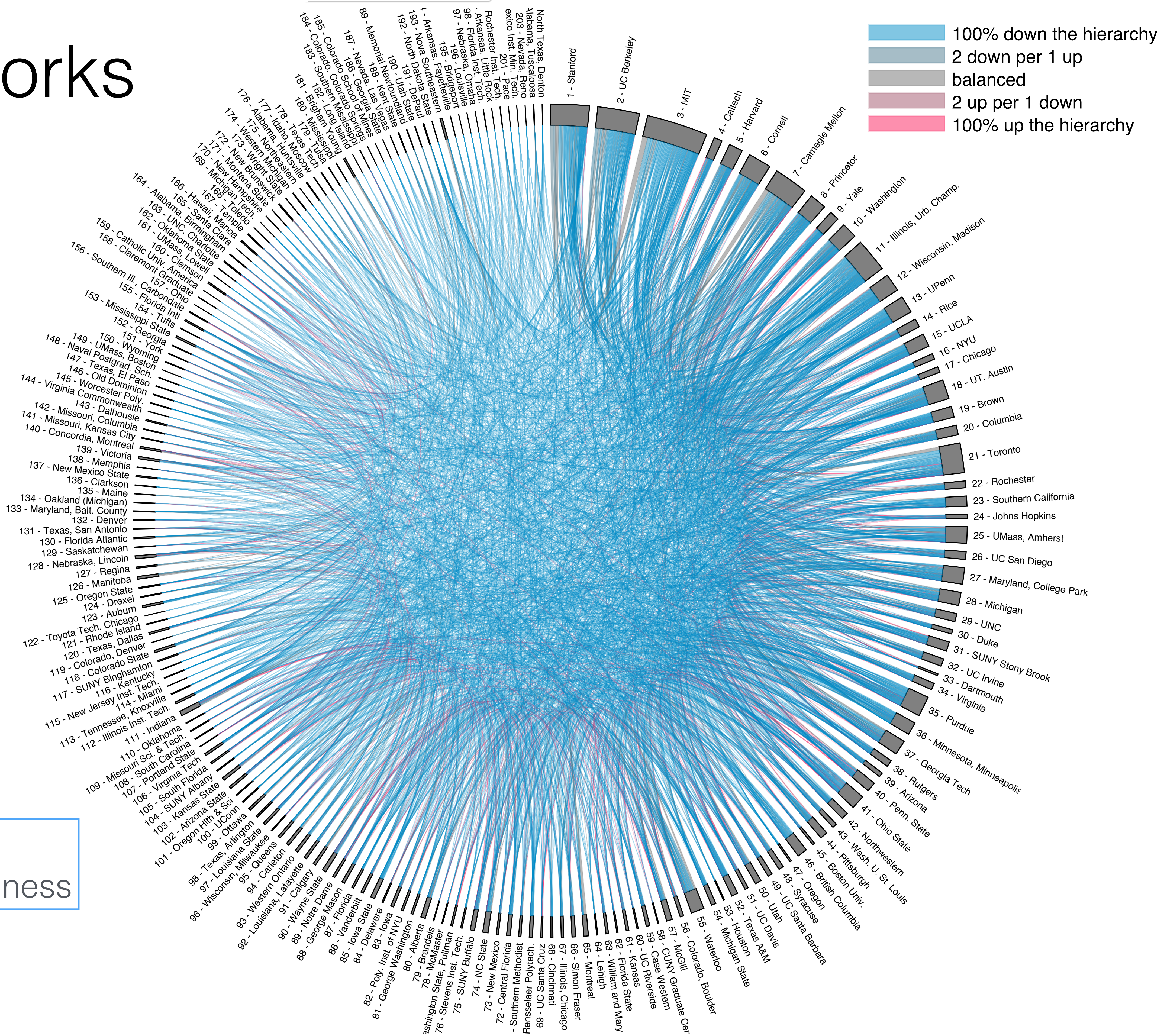
steep

< 7% of faculty have PhD
from lower 75% of universities

biased

median change for women
~3 ranks worse than men

danlarremore.com/**faculty**/
explore 19,000 hires for History, CS, Business

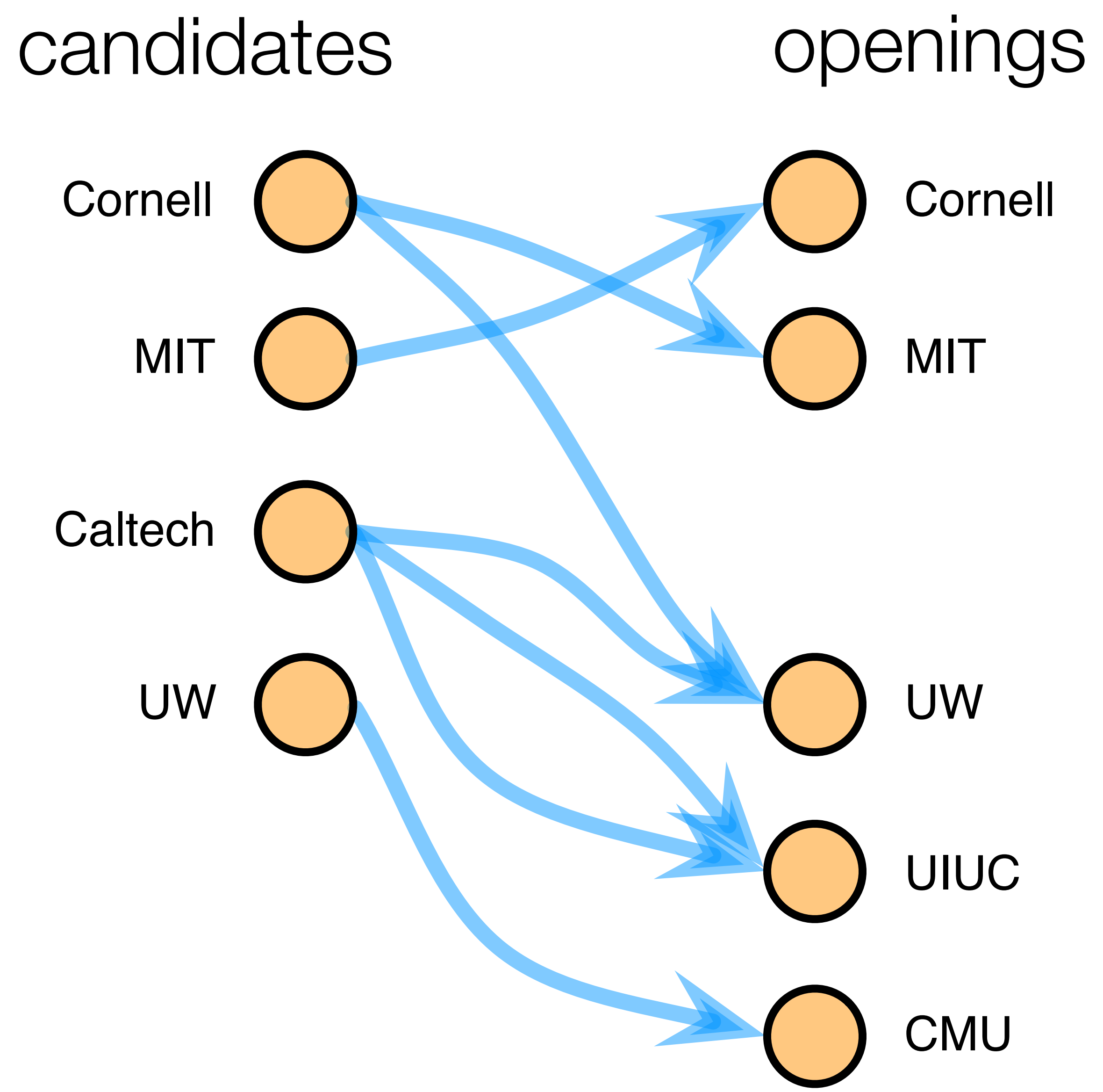


What else explains movement in this labor market?

Generative model:

prestige
productivity
postdoc experience

gender
geography



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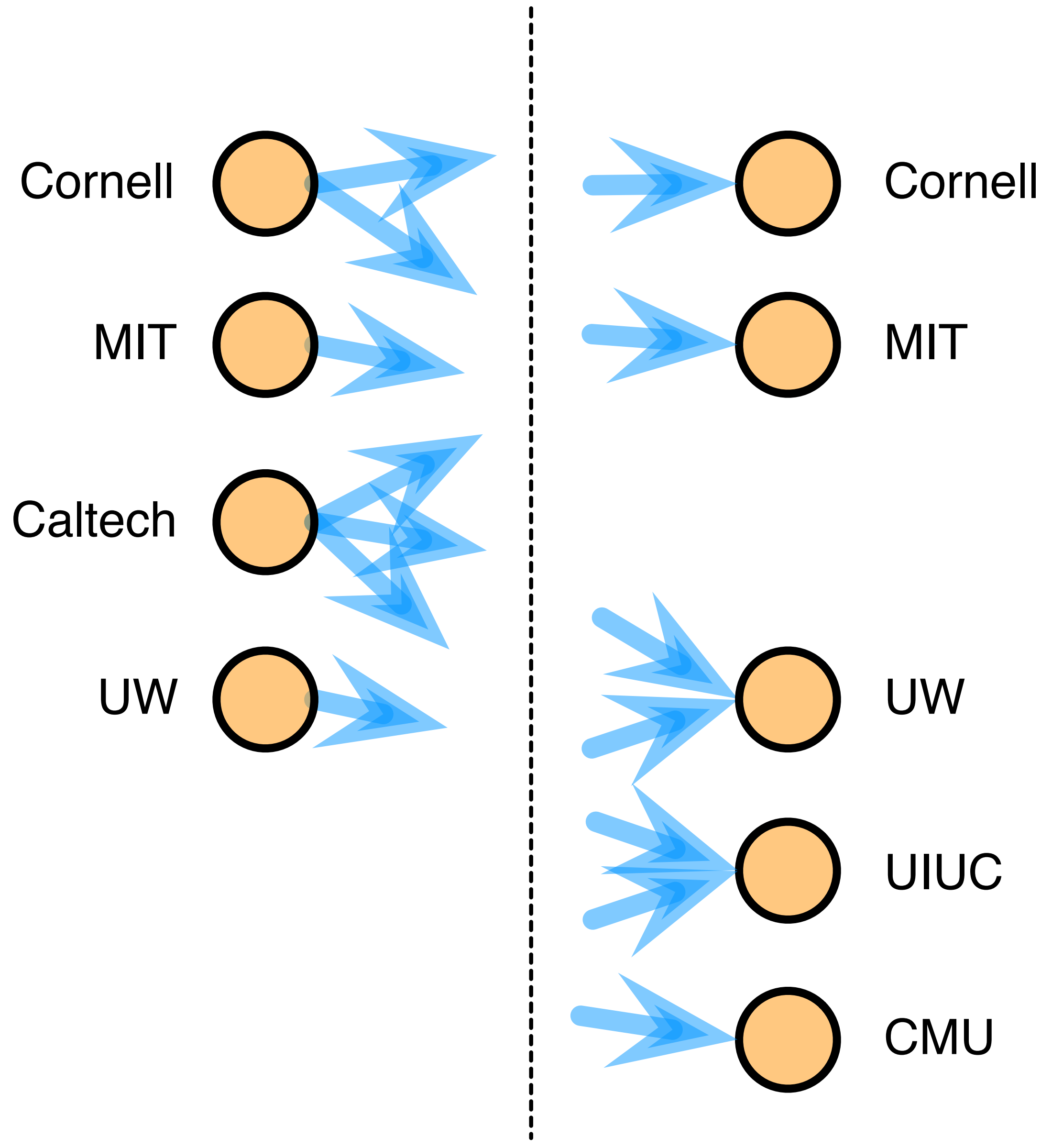


accurately generate the links!

What else explains movement in this labor market?

1. **Prestige difference**: Faculty Job vs PhD
2. **Productivity**
3. **Prestige of Faculty Job**
4. **Postdoc experience** + **geography** (together)
5. **Gender**.

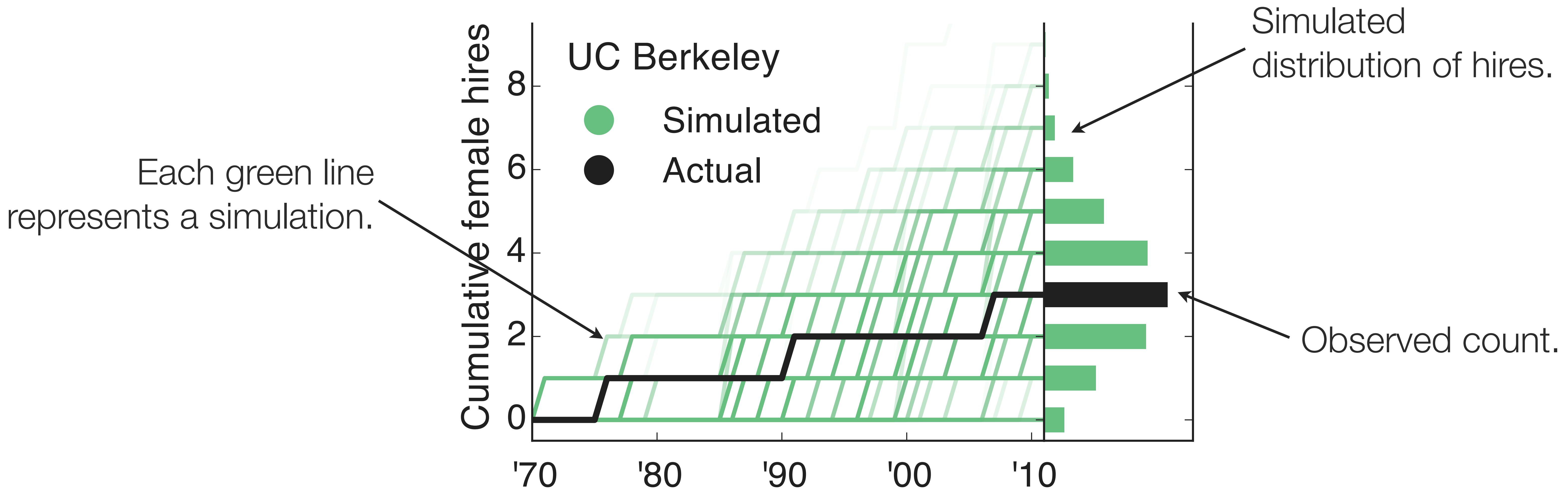
Gender bias is not **uniformly**, systematically affecting all hires. But…

a woman on the job market must have published ~1 additional paper to be placed the same as an equally qualified man.

Institution-level results

Using 40 years of actual hiring data, **simulate** hiring patterns for **each institution.**

Compare actual vs. expected number of female hires.



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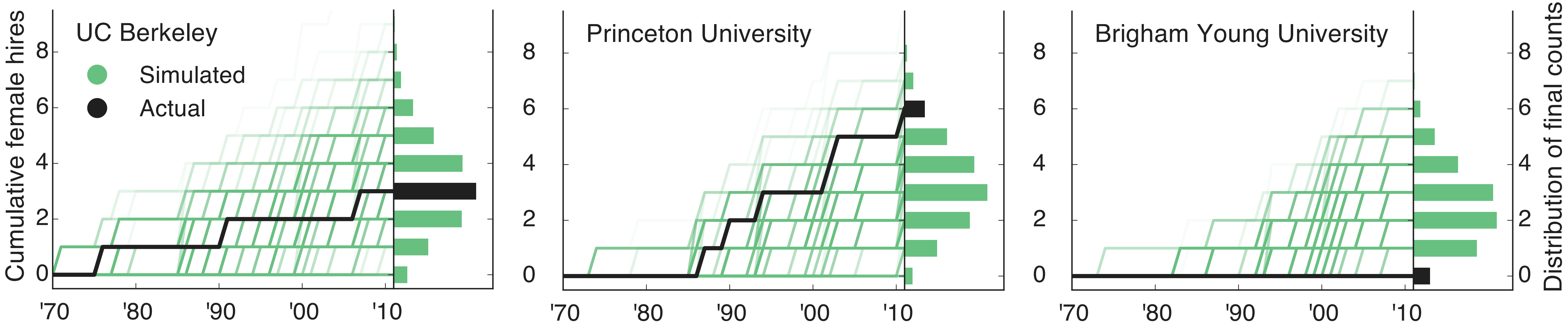
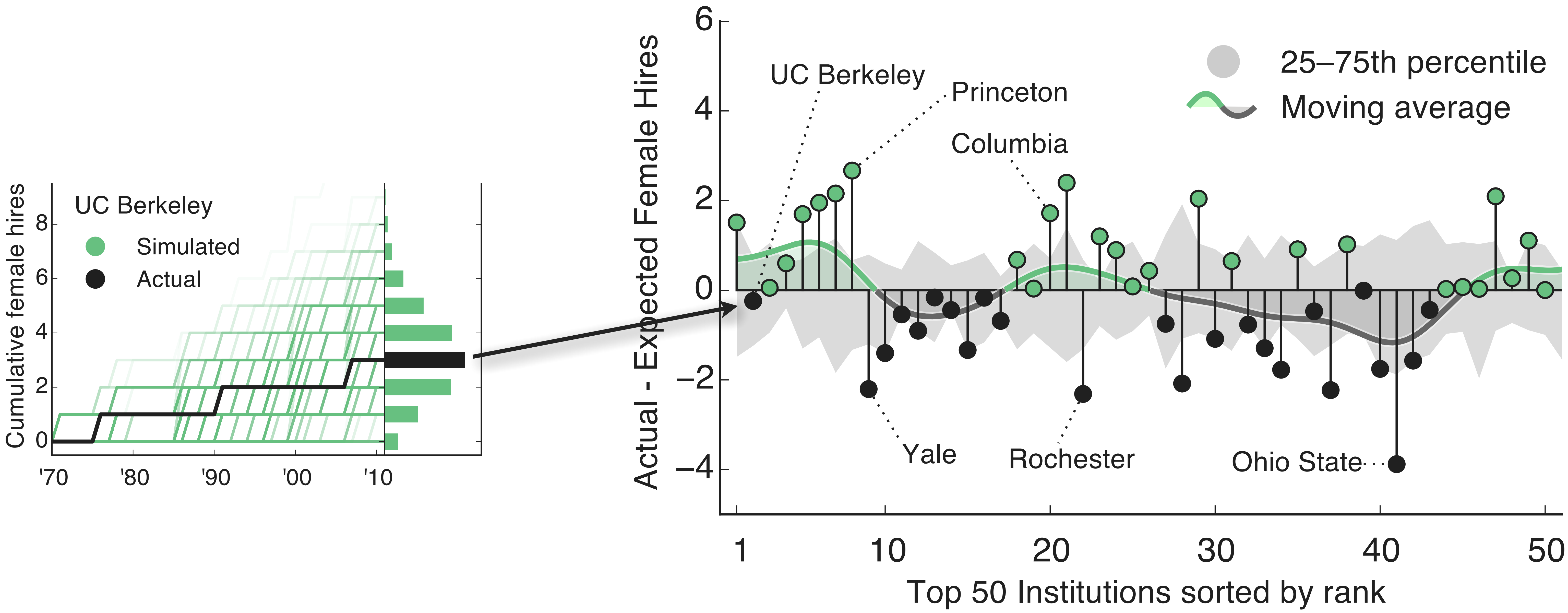
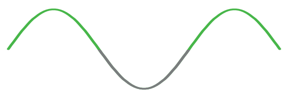


Institution-level results



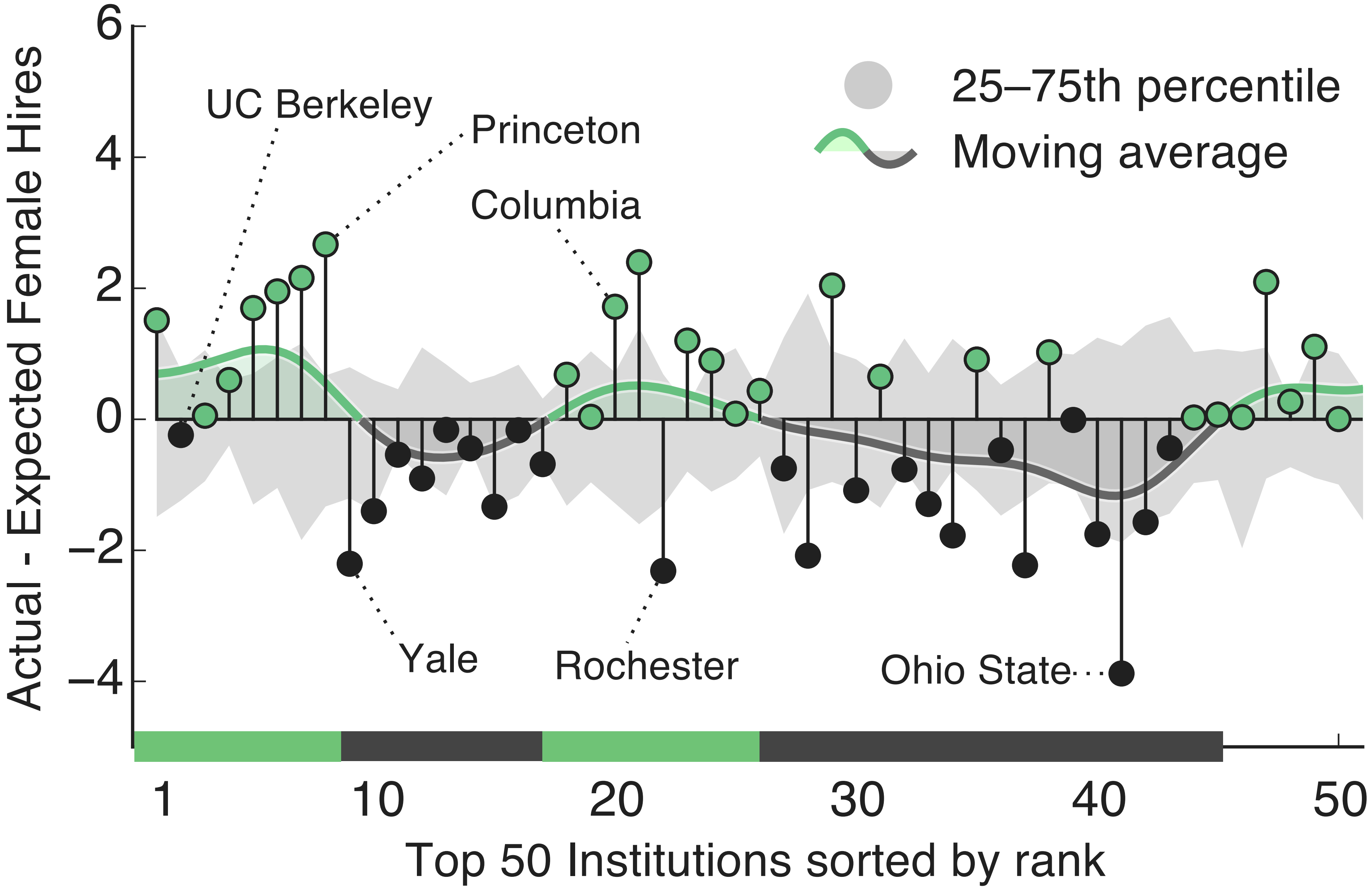
Institution-level results

For the top 50 institutions,
we see an **oscillation**. 

Why?

An interference effect?
Two distinct candidate pools?

Is it real?

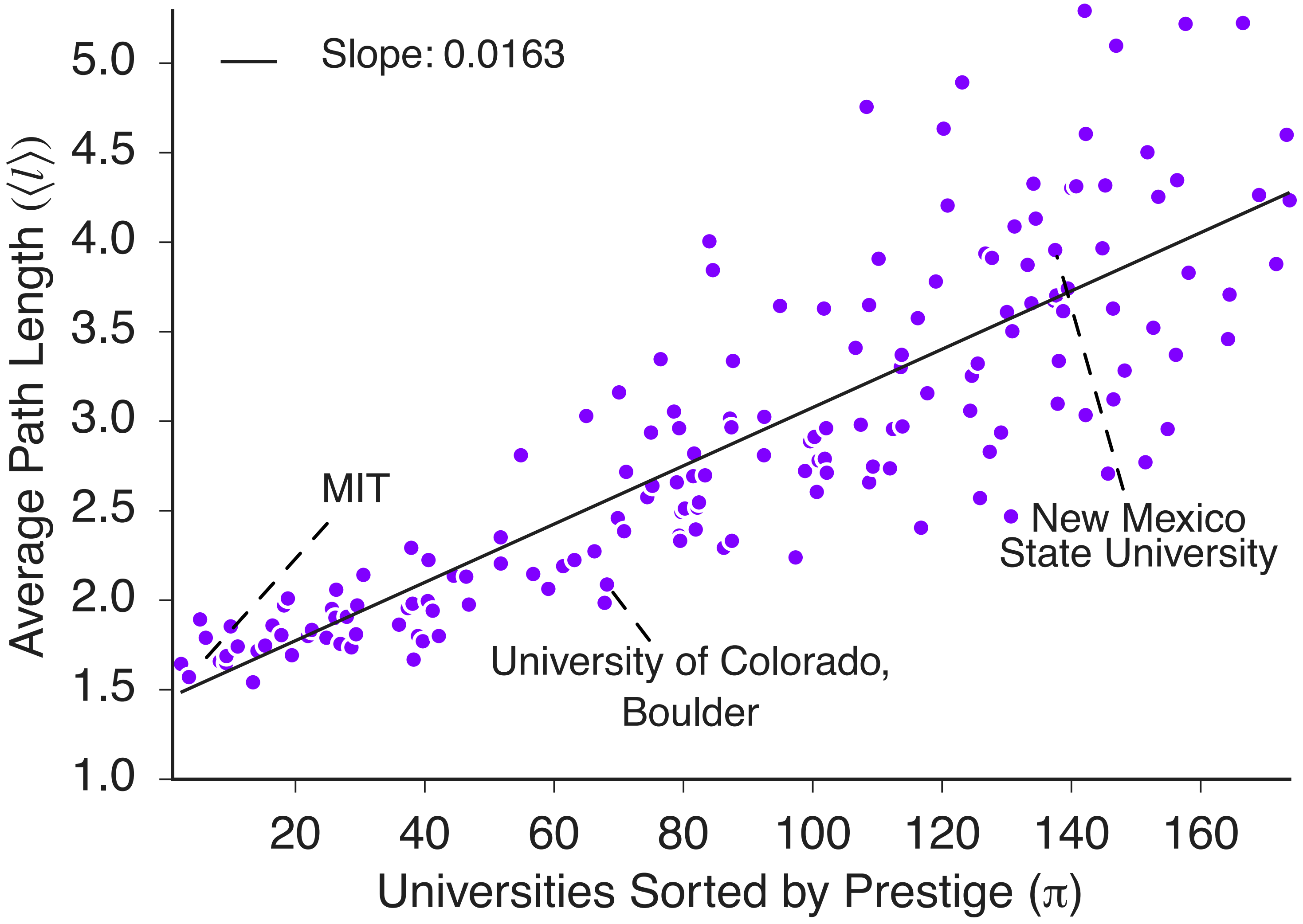


Does the structure of this network affect *ideas?*

Prestigious institutions are **closer** to all other institutions.

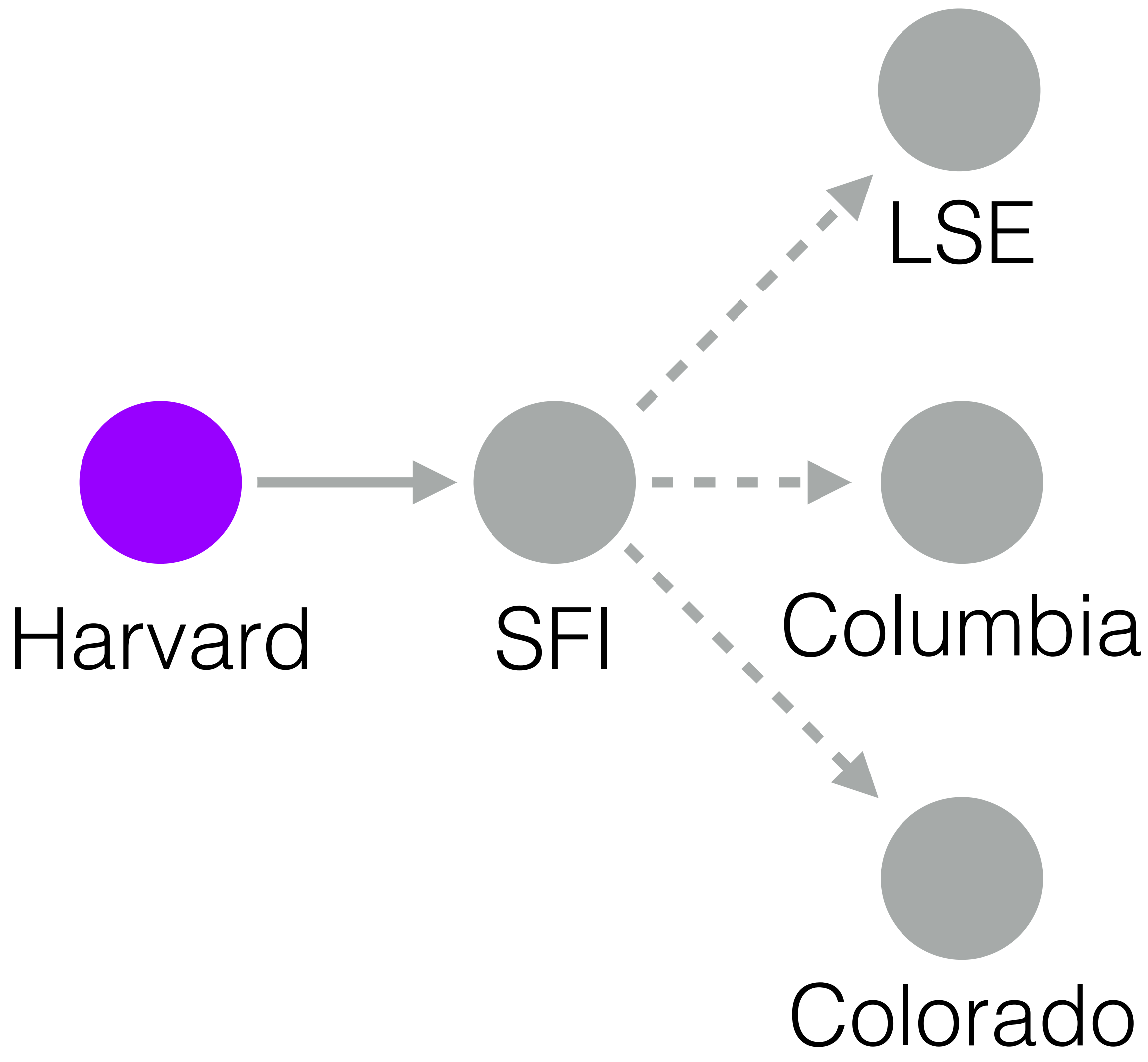
What implications does this have for the **exchange** & **filtration** of ideas?

Does the prestige hierarchy lead to **epistemic inequality**?



Morgan, Economou, Way, Clauset *In preparation* (2018)

New hires as vectors for infectious ideas?



Do new hires *actually* bring ideas with them?
[or would popular topics get there anyway?]

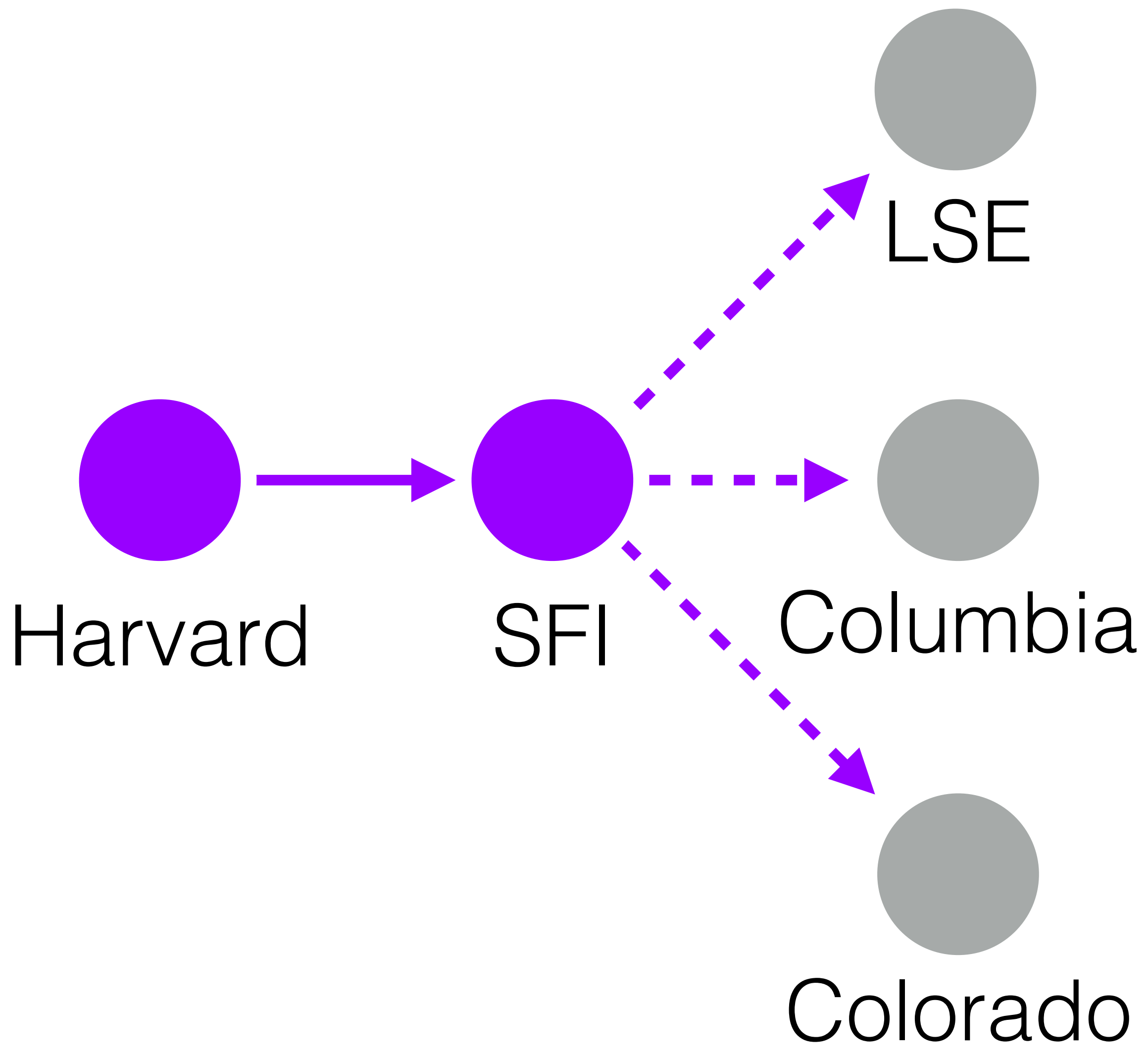
Are some universities better idea exporters?

Epidemic model: treat the idea as an infection, and a new hire as “infectious.”

The probability that a hire transmits the idea to and uninfected university: p [idea quality]

Morgan, Economou, Way, Clauset *In preparation* (2018)

New hires as vectors for infectious ideas?



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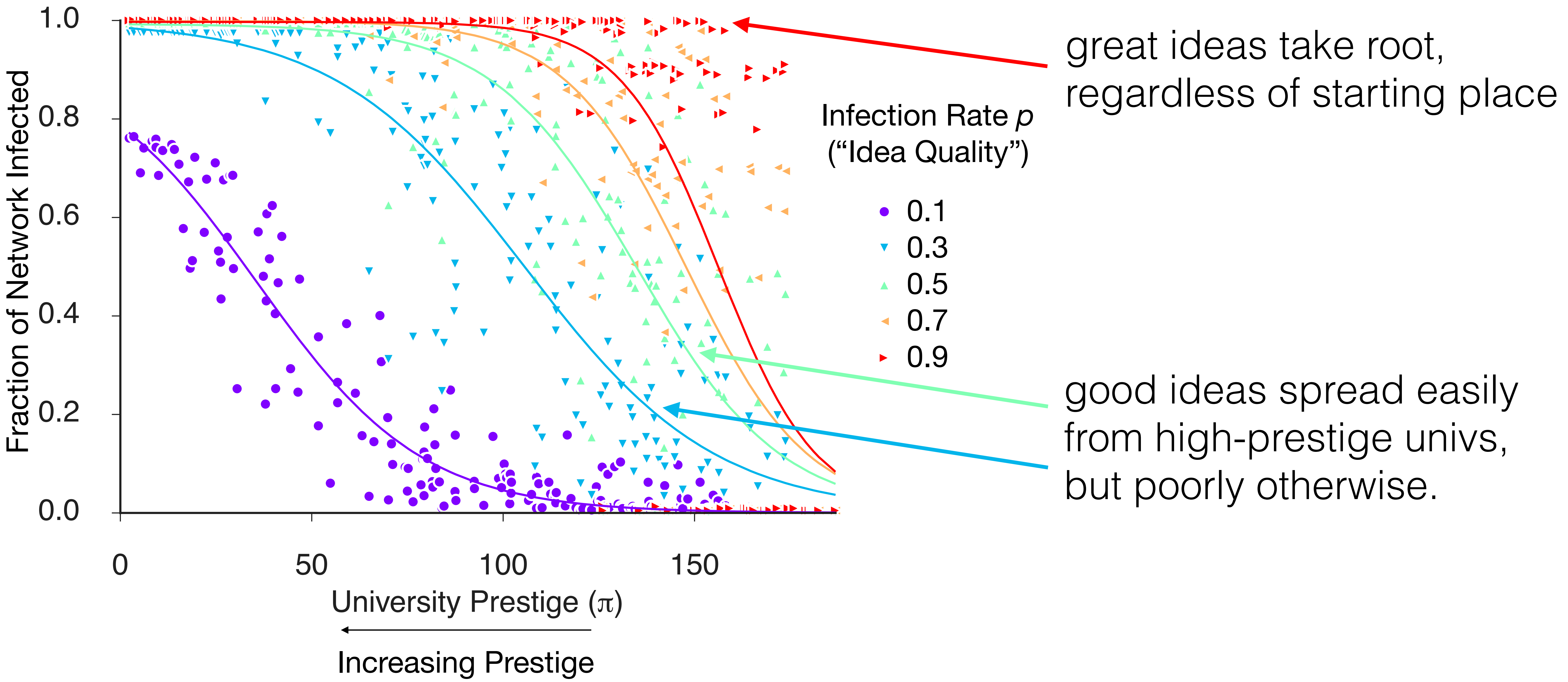
Morgan, Economou, Way, Clauset *In preparation* (2018)

Network position & the spread of ideas



Morgan, Economou, Way, Clauset *In preparation* (2018)

Do *real* ideas spread along hiring links?

Analyzed over 200,000 computer science publications and over 2,500 hires.
Flagged publications on **topic modeling, incremental computing, deep learning**.

Identified faculty who brought [topic] with them when they were hired.
Identified faculty who began working on [topic] only 2+ yrs after being hired.

Compared relative rates of **hiring-link spread** vs **spontaneous spread**. (vs random)

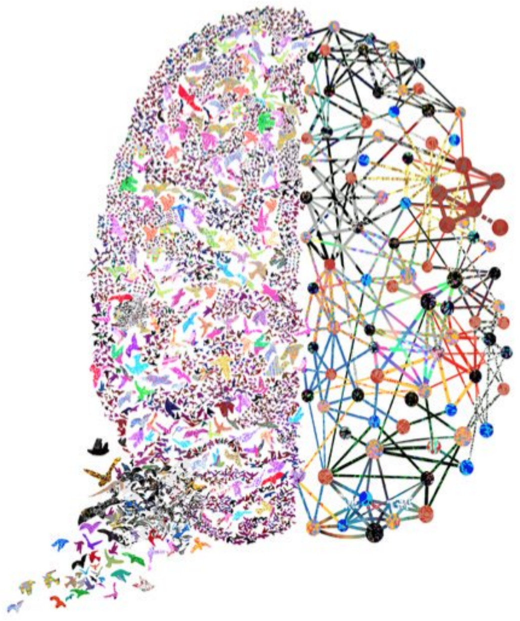
Spread of topic modeling (p=0.01) & incremental computing (p=0.01) significantly tied to infection via hiring. Spread of deep learning (p=0.2) *not* significantly linked to hiring.

Morgan, Economou, Way, Clauset *In preparation* (2018)

Conference on Complex Networks

COMPLENET '18

Hosted by Northeastern University Network Science Institute



BOSTON, MA APRIL 2018

Xindi Wang

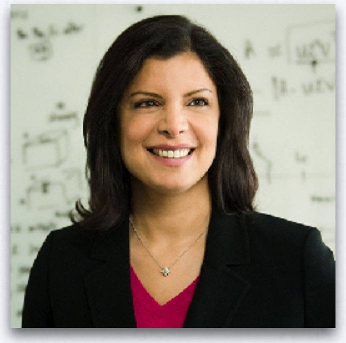
Northeastern University
Network Science Institute

LEARNING TO PLACE OBJECTS: A NETWORK-BASED APPROACH

Xindi Wang

Onur Varol

Tina Eliassi-Rad

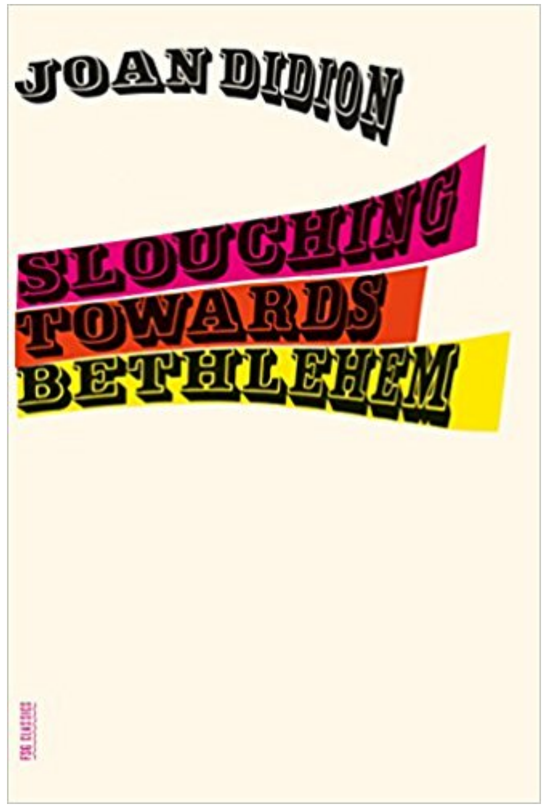


Albert-László Barabási

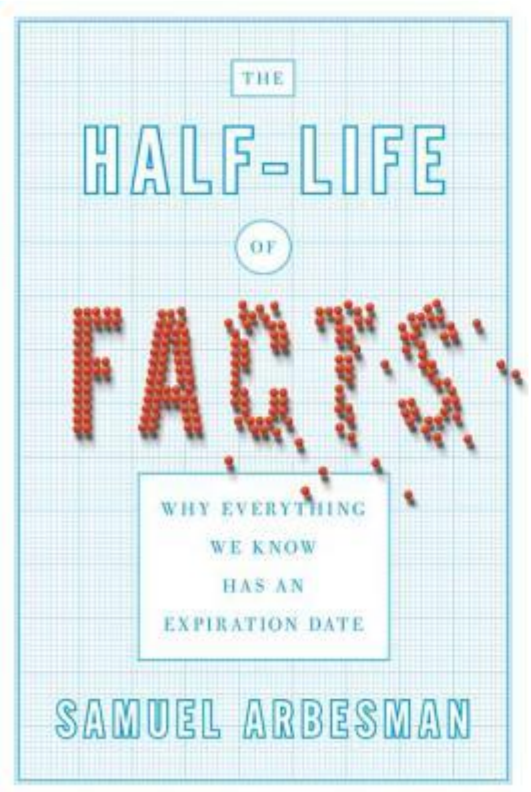
Suppose I give you a book. Predict its sales.

Existing data: books and their sales.

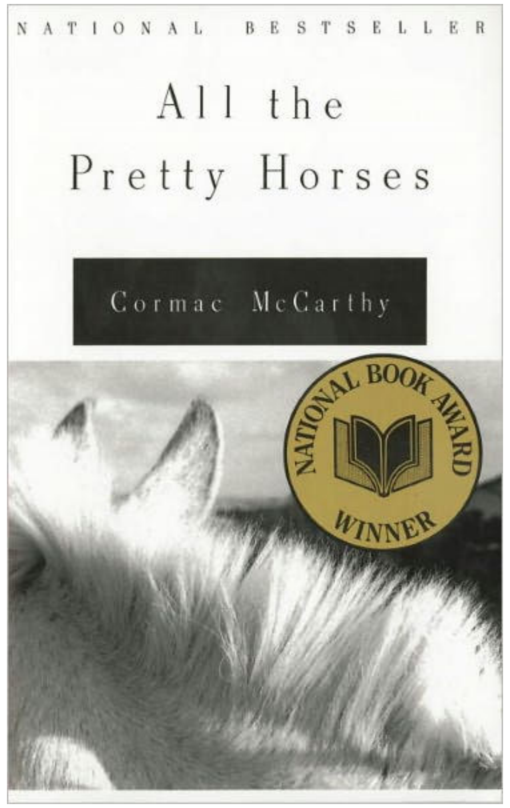
1. turn books into feature vectors.



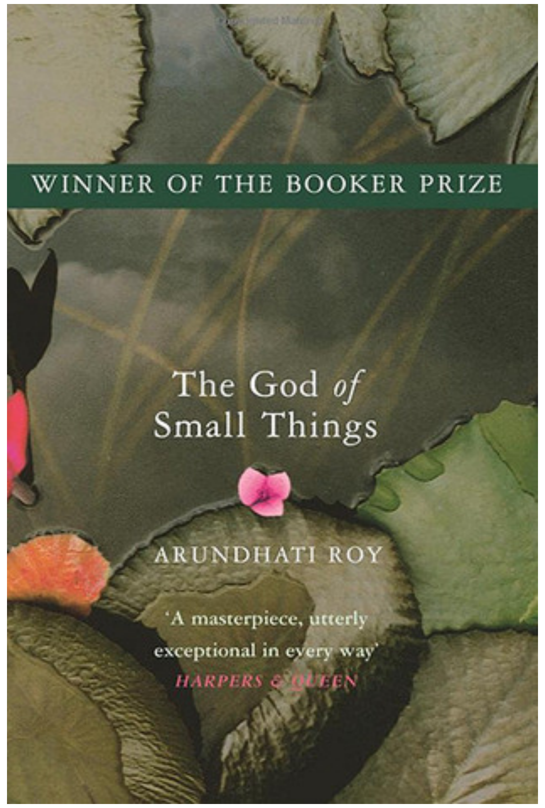
$\vec{x}_1$



$\vec{x}_2$



$\vec{x}_3$



$\vec{x}_4$

2. Train a model:

$$P(\text{book } i > \text{book } j \mid \vec{x}_i, \vec{x}_j, \theta)$$

Suppose I give you a book. Predict its sales.

Existing data: books and their sales.

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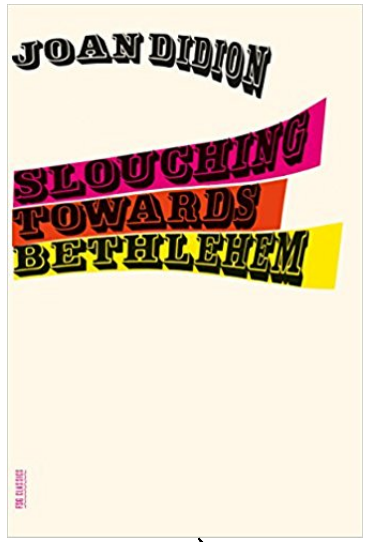
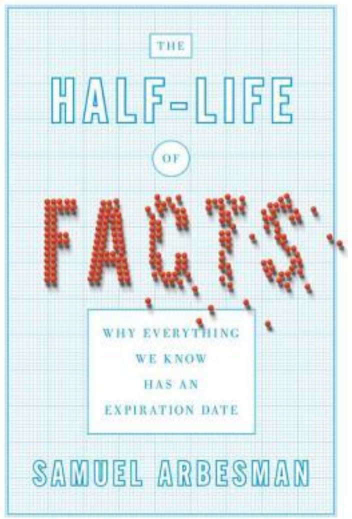
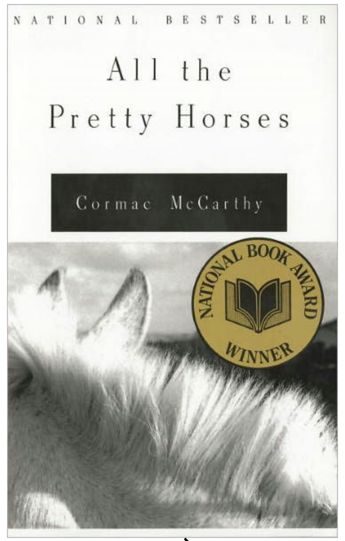


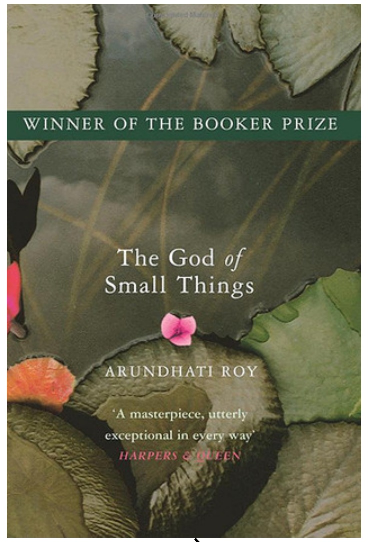
$\vec{x}_1$



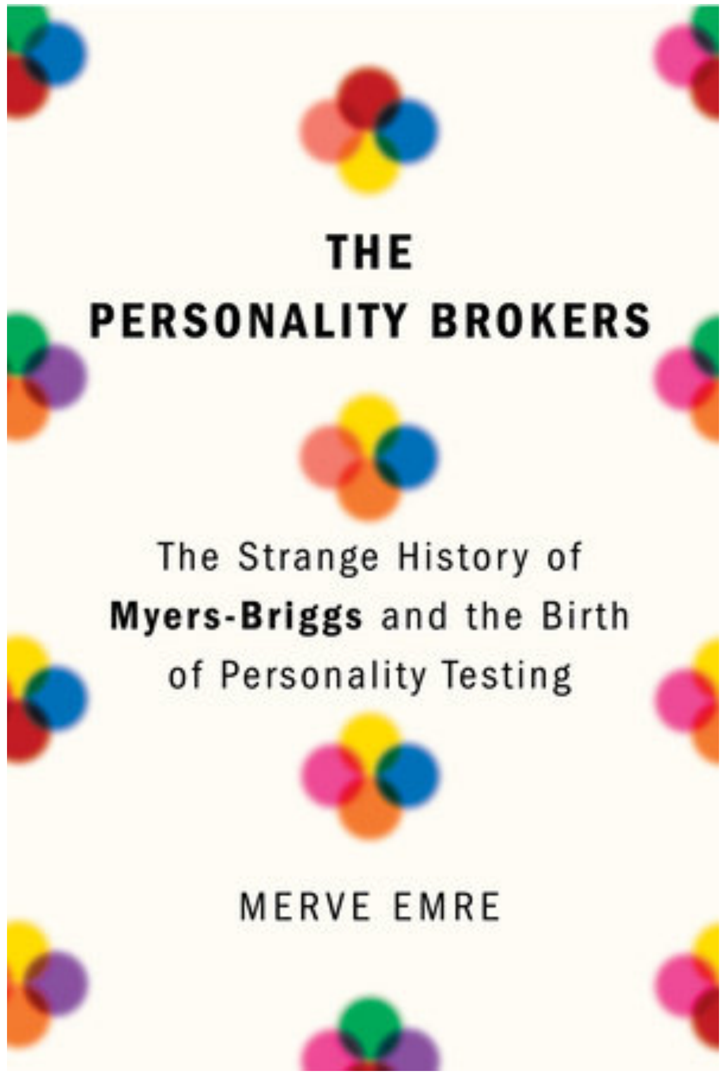
$\vec{x}_2$



$\vec{x}_3$



$\vec{x}_4$



$\vec{x}_5$

2. Train a model.

3. Use the model to simulate pairwise competitions.

$P(\text{book } i > \text{book } 5 \mid \vec{x}_i, \vec{x}_5, \theta)$

4. Use [**your favo(u)rite algorithm**] to infer rank$_5$ from pairwise comparisons.

Rankings rankings

Area under the receiver-operator curve (AUC)

Method		AUC on Fiction	AUC on Biography
	KNN	0.759	0.815
Pairwise +	Cohen et al.	0.892	0.871
	WTG wave	**0.910**	**0.892**
	Voting	**0.915**	**0.891**
	FAS-PIVOT	**0.907**	**0.892**
	SpringRank	**0.908**	**0.893**

Want to learn more? xindi.w1993@gmail.com

But actually… many methods performed well!

Now the question is: why do the top four algorithms perform similarly?

What does that tell us about the **structure of the problem** and
the **structure of the space** over which we are ranking?

Use the consistency of the ranking results across
algorithms to learn about the system itself.

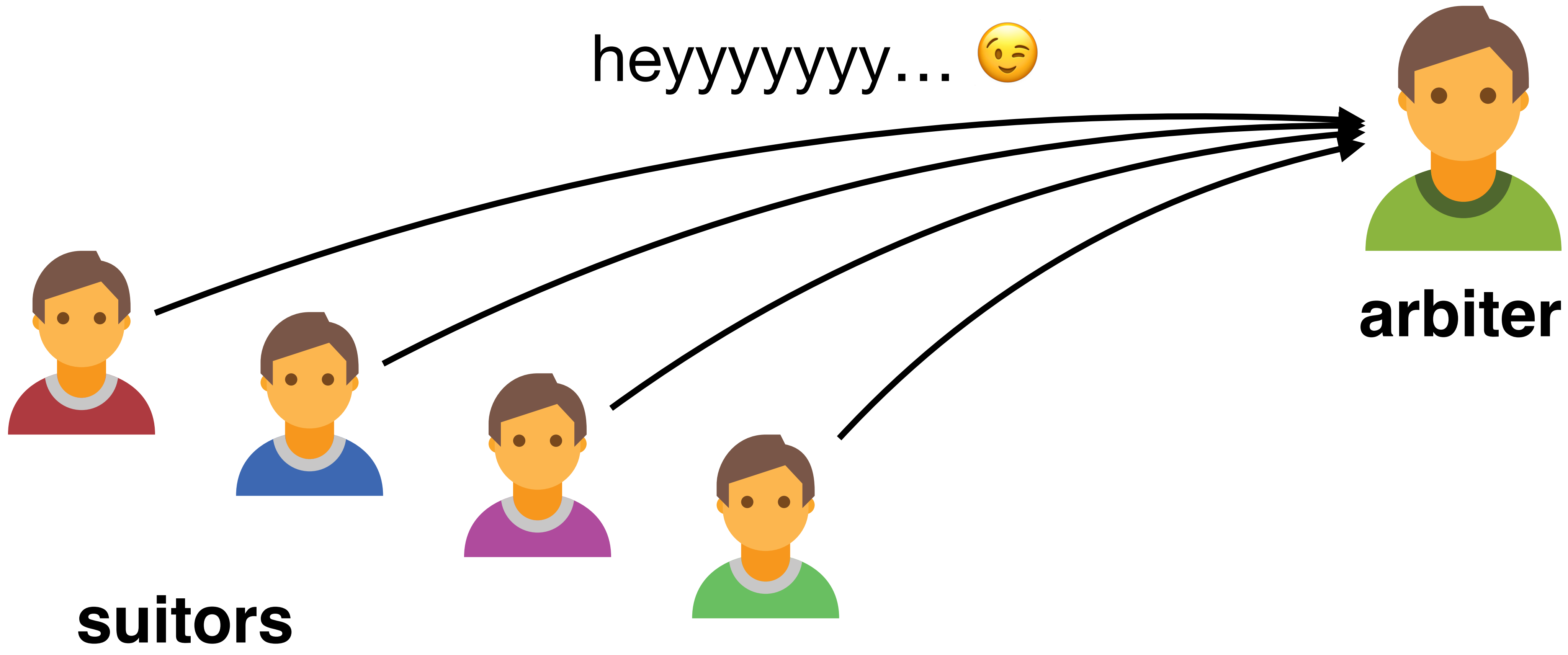
What does it mean for a space or problem or set to be easily ordered or rankable?

Want to learn more? xindi.w1993@gmail.com

Hierarchies of Desirability in Online Dating

Hierarchies of Desirability

Can we learn something about desirability from natural behavior?



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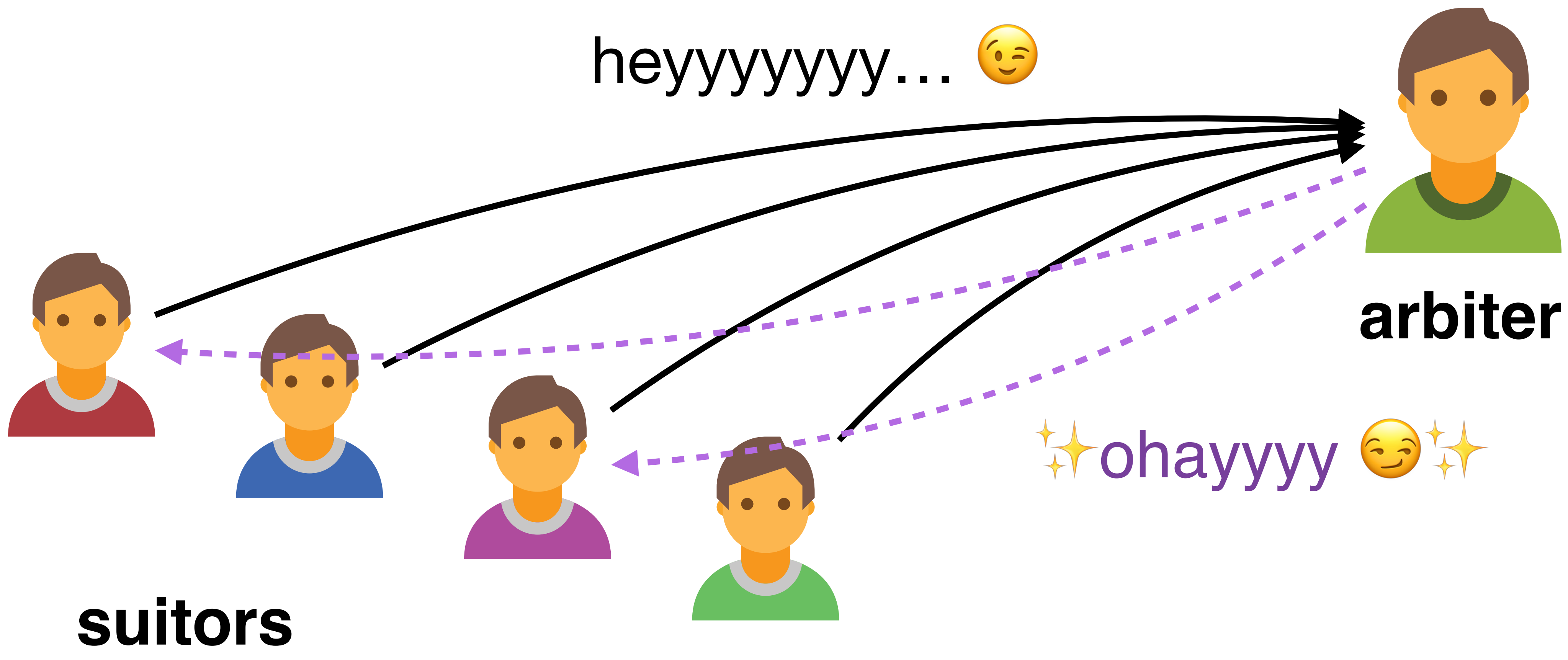

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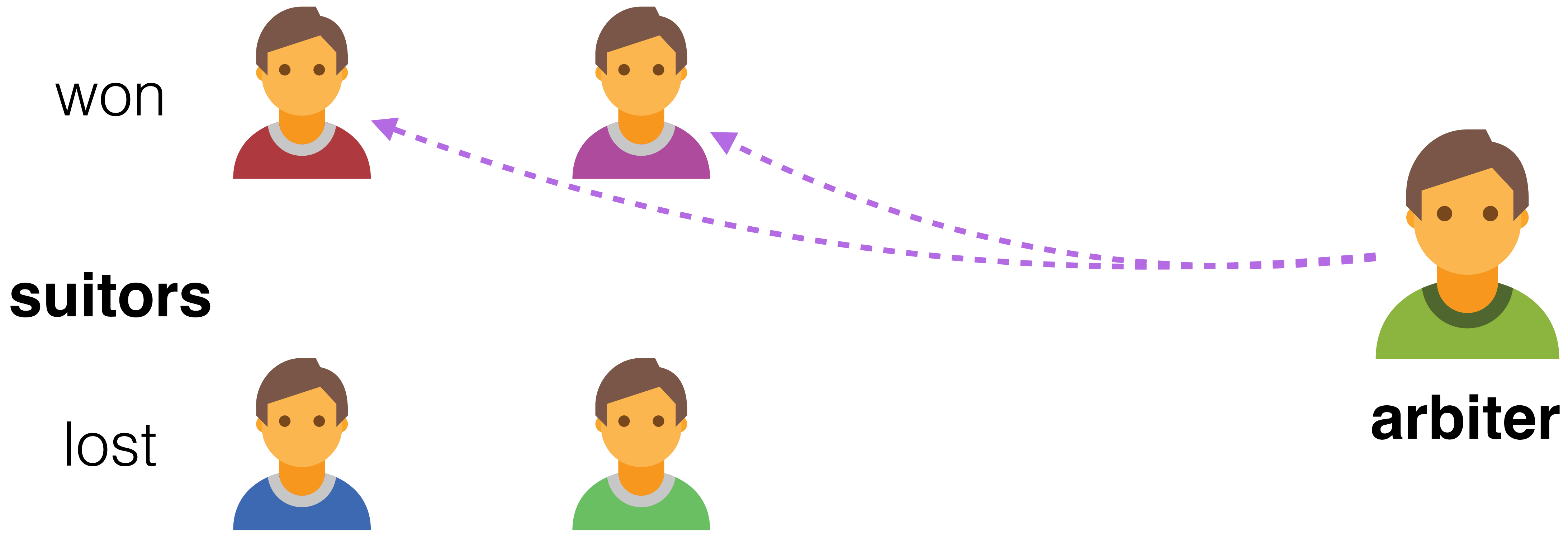
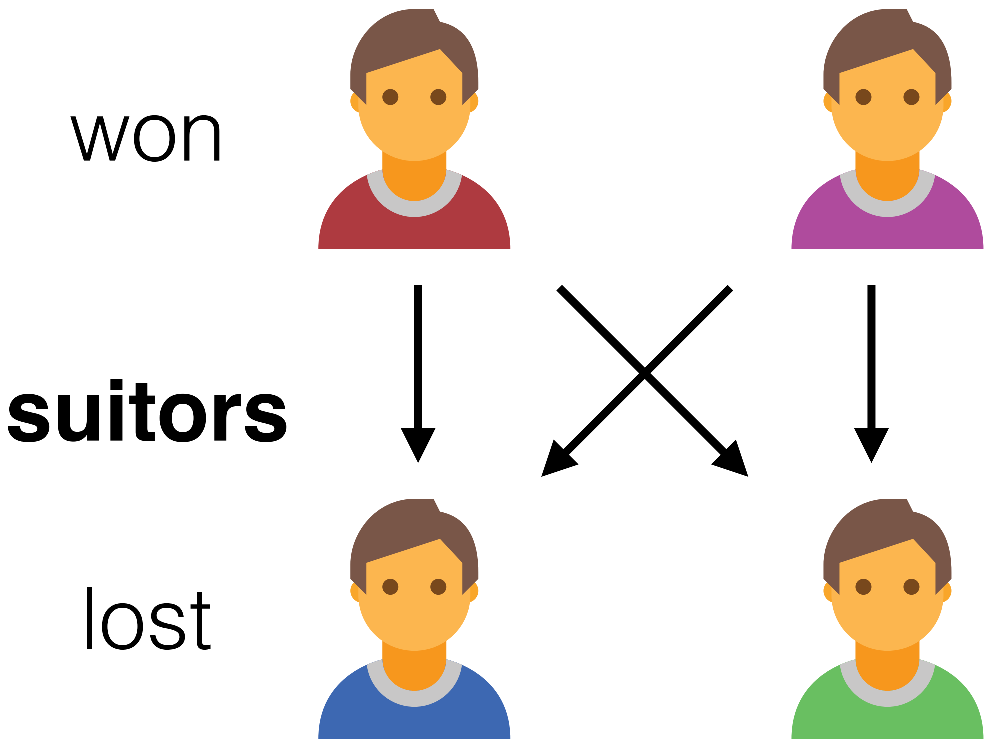


Hierarchies of Desirability

Can we learn something about desirability from natural behavior?



if only we knew how to get a ranking from a directed network…

The Depths of Leagues

Dear Editors,

There are some questions I would like to ask. Firstly, how complex is backgammon compared to other games of skill such as chess or bridge?

Let's start with chess, which has evolved a well-developed rating system over the past 40 years. Chess ratings range from a high of about 2800 to theoretical lows of about 0 (a complete beginner who has just learned the moves). Chess ratings are also designed so that a 200-point rating difference between two players anywhere on the scale means that the higher-rated player has a 70-75% chance of defeating a lower-rated player (discounting draws, which are possible in chess but not in most of the other games we'll consider).

Now consider the following experiment:

(1) Take the best player in the world (in the case of chess, it's Gary Kasparov). Call him player 1.

(2) Find someone that the best player beats 70-75% of the time. Call him player 2.

(3) Call the difference between players 1 and 2 ***one skill differential.***

(4) Find someone that player 2 can beat 70-75% of the time. Call him player 3. The difference between players 2 and 3 is another skill differential.

(5) Continue this process until you have taken the chain down to an absolute beginner.

(6) Count the number of skill differentials involved. This is the ***complexity number*** *of the game.*

In the case of chess, this number is about 14.

Robertie, 1980 *Inside Backgammon*

The Depths of Leagues

COMPLEXITY NUMBERS

Go	*40*
Chess	*14*
Scrabble	*10*
Poker	*10*
Backgammon	*8*
Checkers	*8*
Hearts	*5*
Blackjack	*2*
Craps	*0.001*
Lotteries	*0.0000001*
Roulette	*0*
Online Dating	**???**

Now consider the following experiment:

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In the case of chess, this number is about 14.

Robertie, 1980 *Inside Backgammon*

10,000 Leagues Under the City

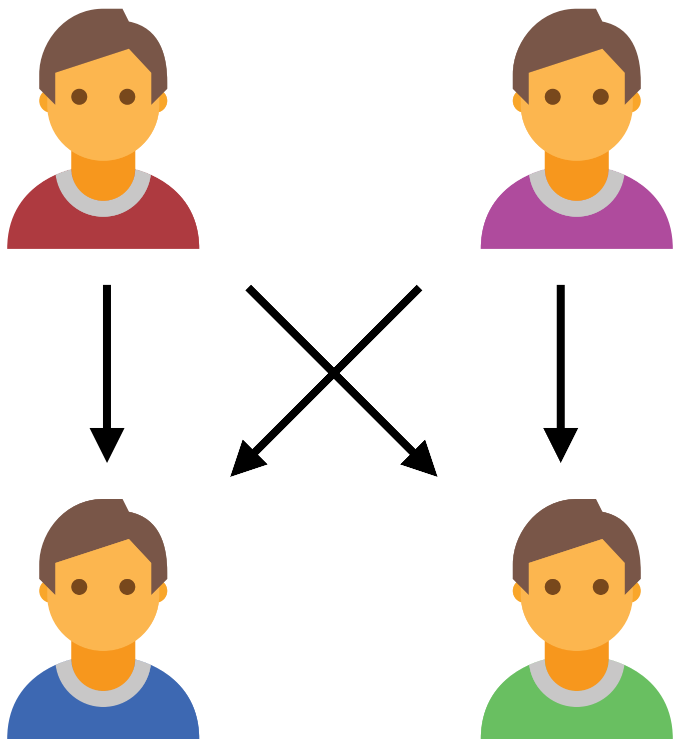
What is the depth of the league in Cityville?

Is the depth in Cityville the same as Townsberg?

What is the desirability **exchange rate** between cities?

What are the correlates of depth?

Do these vary by gender or declared sexual orientation?



Answers forthcoming!

But if you like the questions and topic…

SCIENCE ADVANCES | RESEARCH ARTICLE

SOCIAL SCIENCES

Aspirational pursuit of mates in online dating markets

Elizabeth E. Bruch[1,2]* and M. E. J. Newman[2,3]

Romantic courtship is often described as taking place in a dating market where men and women compete for mates, but the detailed structure and dynamics of dating markets have historically been difficult to quantify for lack of suitable data. In recent years, however, the advent and vigorous growth of the online dating industry has provided a rich new source of information on mate pursuit. We present an empirical analysis of heterosexual dating markets in four large U.S. cities using data from a popular, free online dating service. We show that competition for mates creates a pronounced hierarchy of desirability that correlates strongly with user demographics and is remarkably consistent across cities. We find that both men and women pursue partners who are on average about 25% more desirable than themselves by our measures and that they use different messaging strategies with partners of different desirability. We also find that the probability of receiving a response to an advance drops markedly with increasing difference in desirability between the pursuer and the pursued. Strategic behaviors can improve one's chances of attracting a more desirable mate, although the effects are modest.

Bruch & Newman, *Science Advances.* 2018. http://advances.sciencemag.org/content/4/8/eaap9815/tab-pdf

Hierarchy and cognition

What are the mechanisms that create large-scale patterns from many small interactions?

Confront models with data to reveal cognitively-accessible social mechanisms in parakeets.

Hobson and DeDeo. *PLoS Computational Biology* (2015)

1

2

3

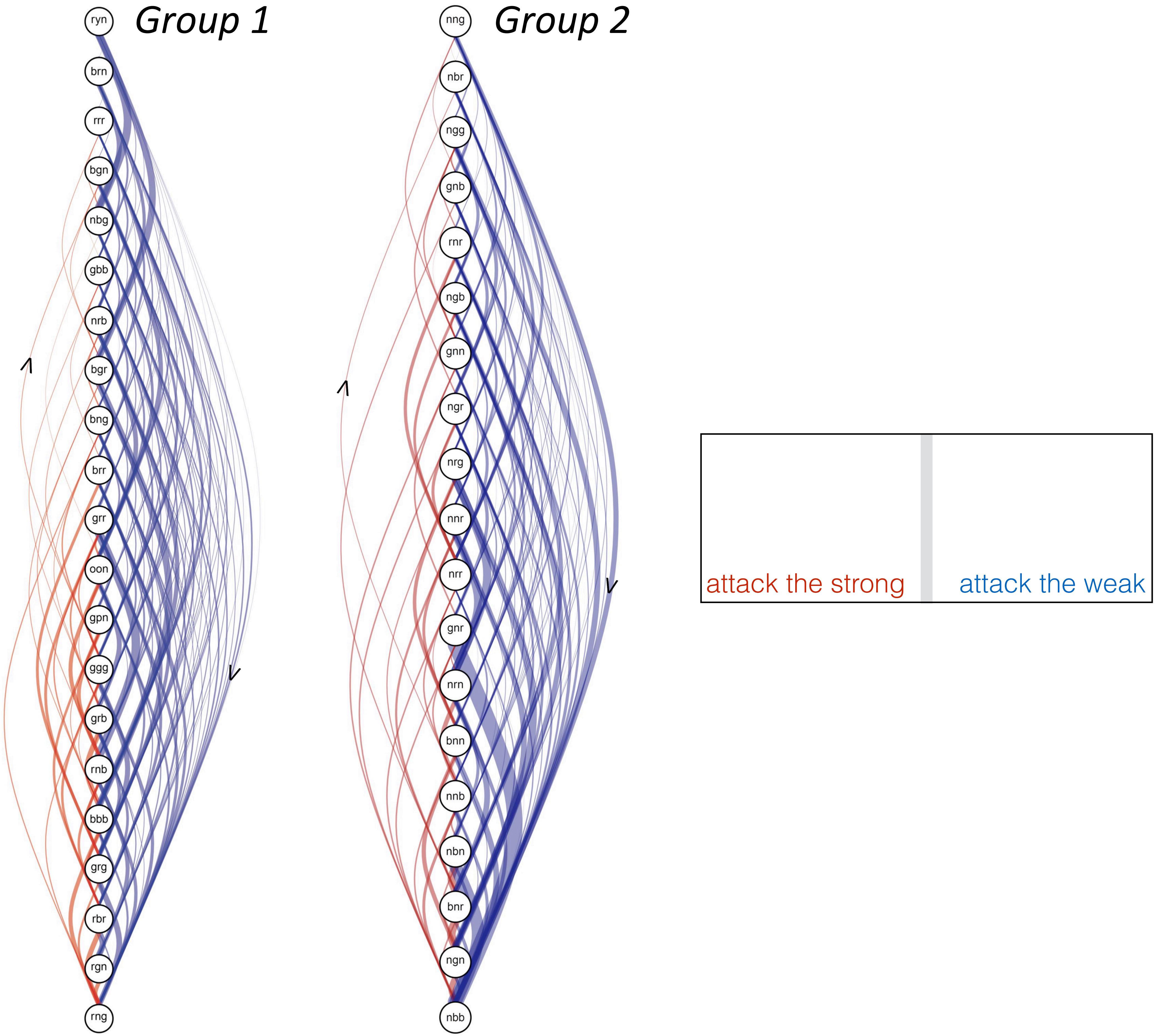
aggressor (winner)

target (loser)

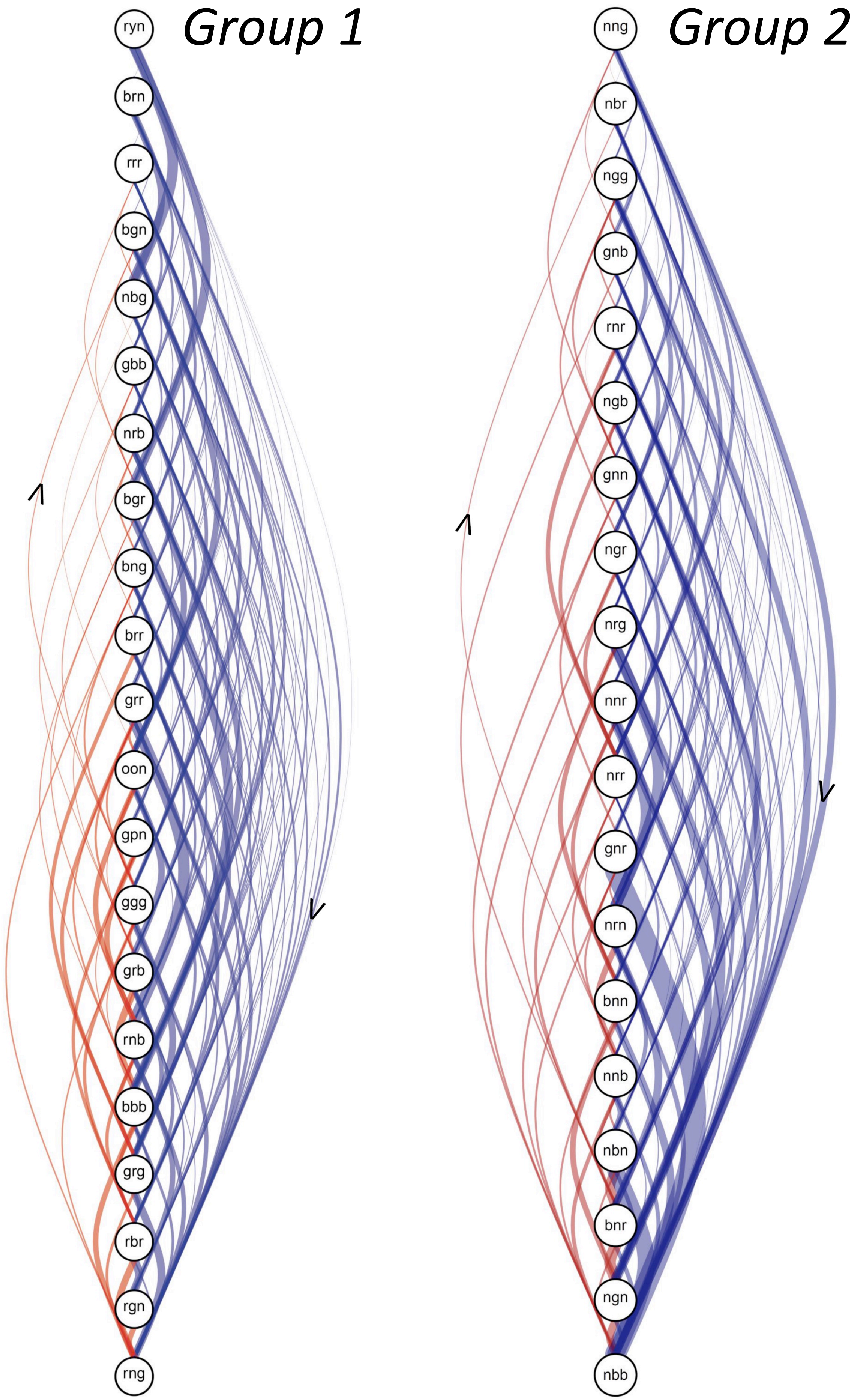
gawking, staring, shameless witnesses!

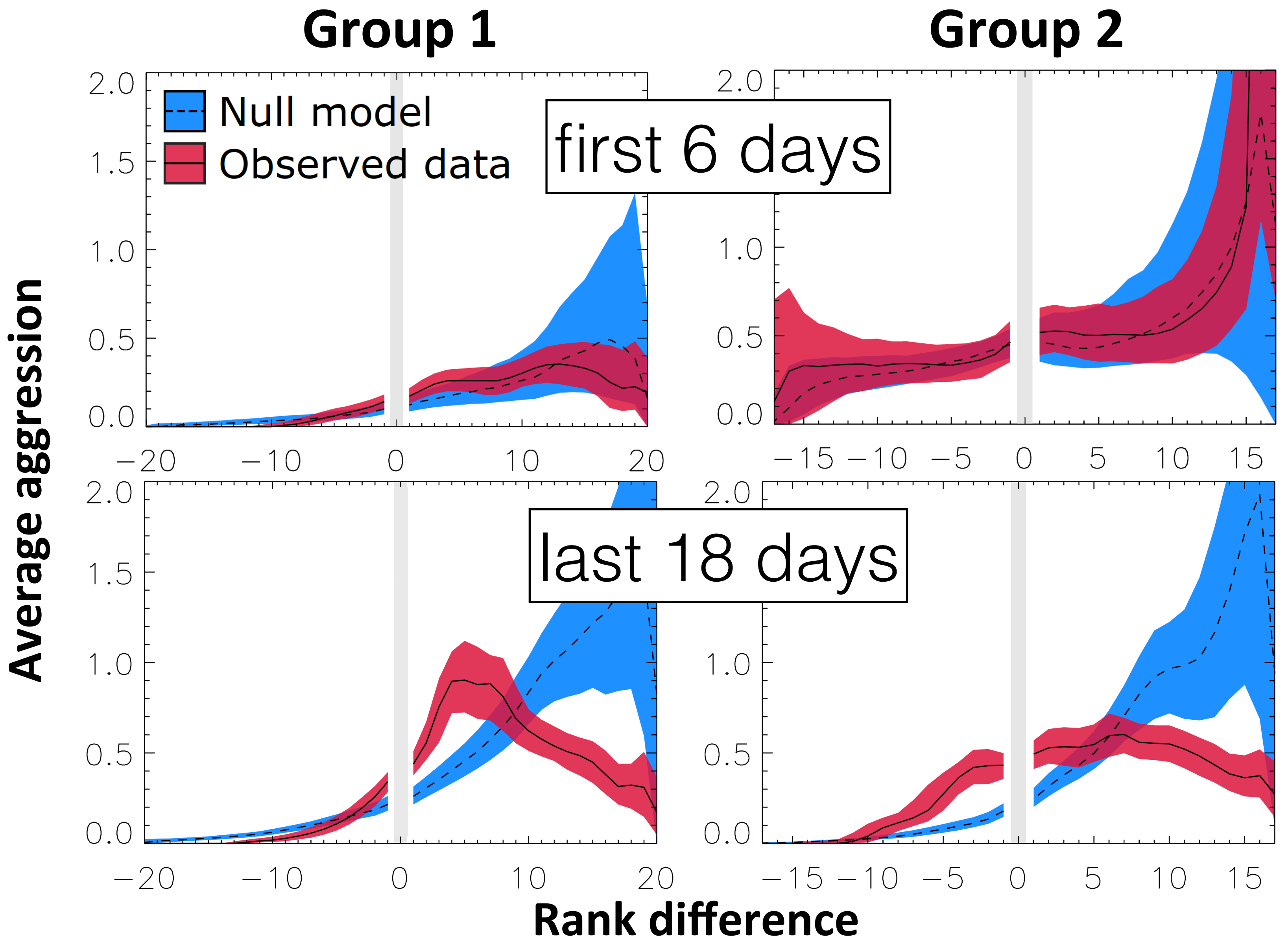
elapsed time < 1 second

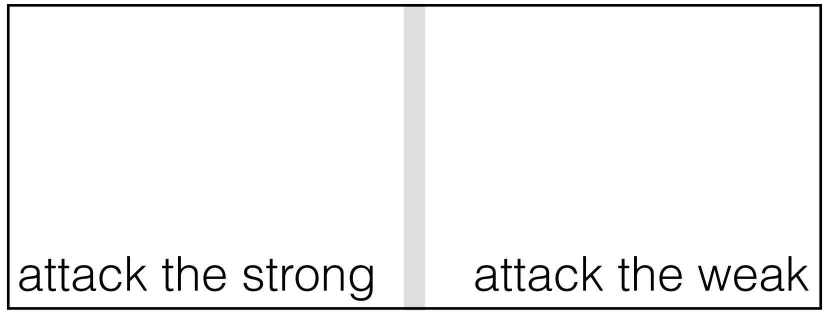
Hobson and DeDeo. *PLoS Computational Biology* (2015)



Hobson and DeDeo. *PLoS Computational Biology* (2015)

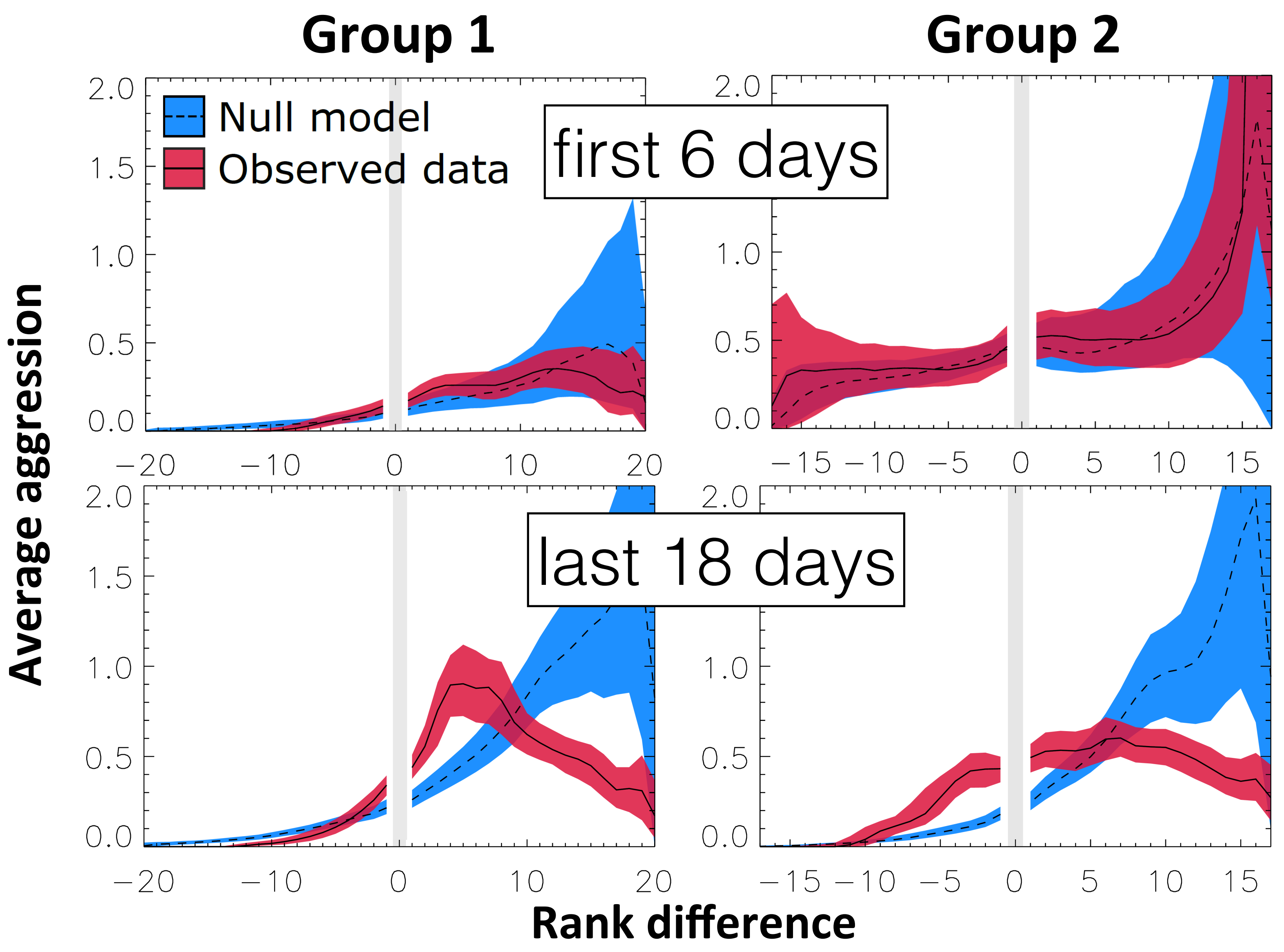

Group 1
ryn
brn
rrr
bgn
nbg
gbb
nrb
bgr
bng
brr
grr
oon
gpn
ggg
grb
rnb
bbb
grg
rbr
rgn
rng
Group 2
nng
nbr
ngg
gnb
rnr
ngb
gnn
ngr
nrg
nnr
nrr
gnr
nrn
bnn
nnb
nbn
bnr
ngn
nbb


Group 1
Group 2
Null model
Observed data
first 6 days
last 18 days
Average aggression
Rank difference
2.0
1.5
1.0
0.5
0.0
−20
−10
0
10
20
−15
−10
−5
5
15


attack the strong
attack the weak

Parakeets know which individuals are ranked above and below themselves.

Parakeets know their own rank and the ranks of others.



Confronting models, which incorporate different complexities of bird-knowledge, with meticulous data, reveals clues about mechanisms of hierarchy formation.

Hobson and DeDeo. *PLoS Computational Biology* (2015)

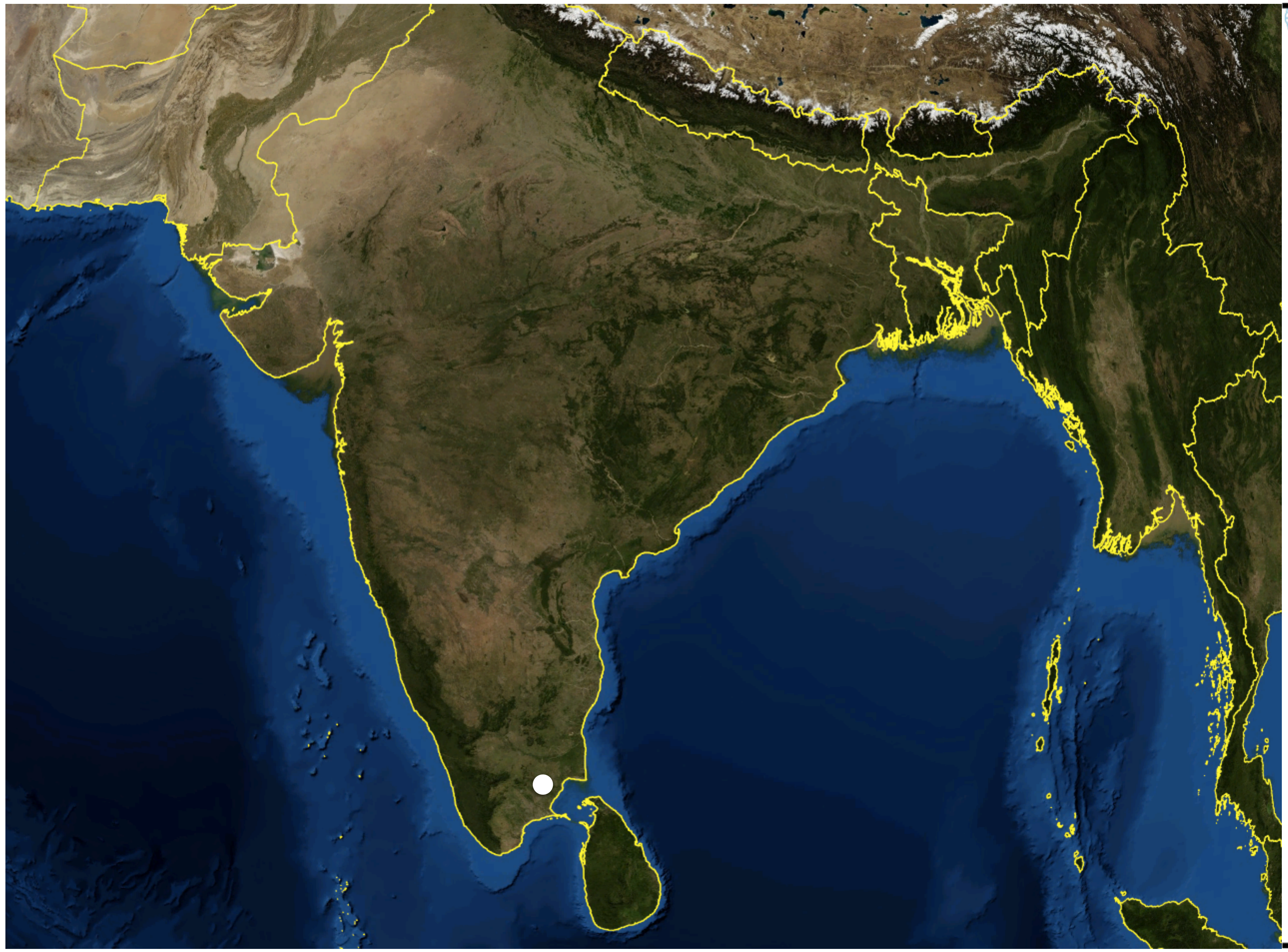
Complex models reveal complex behaviors

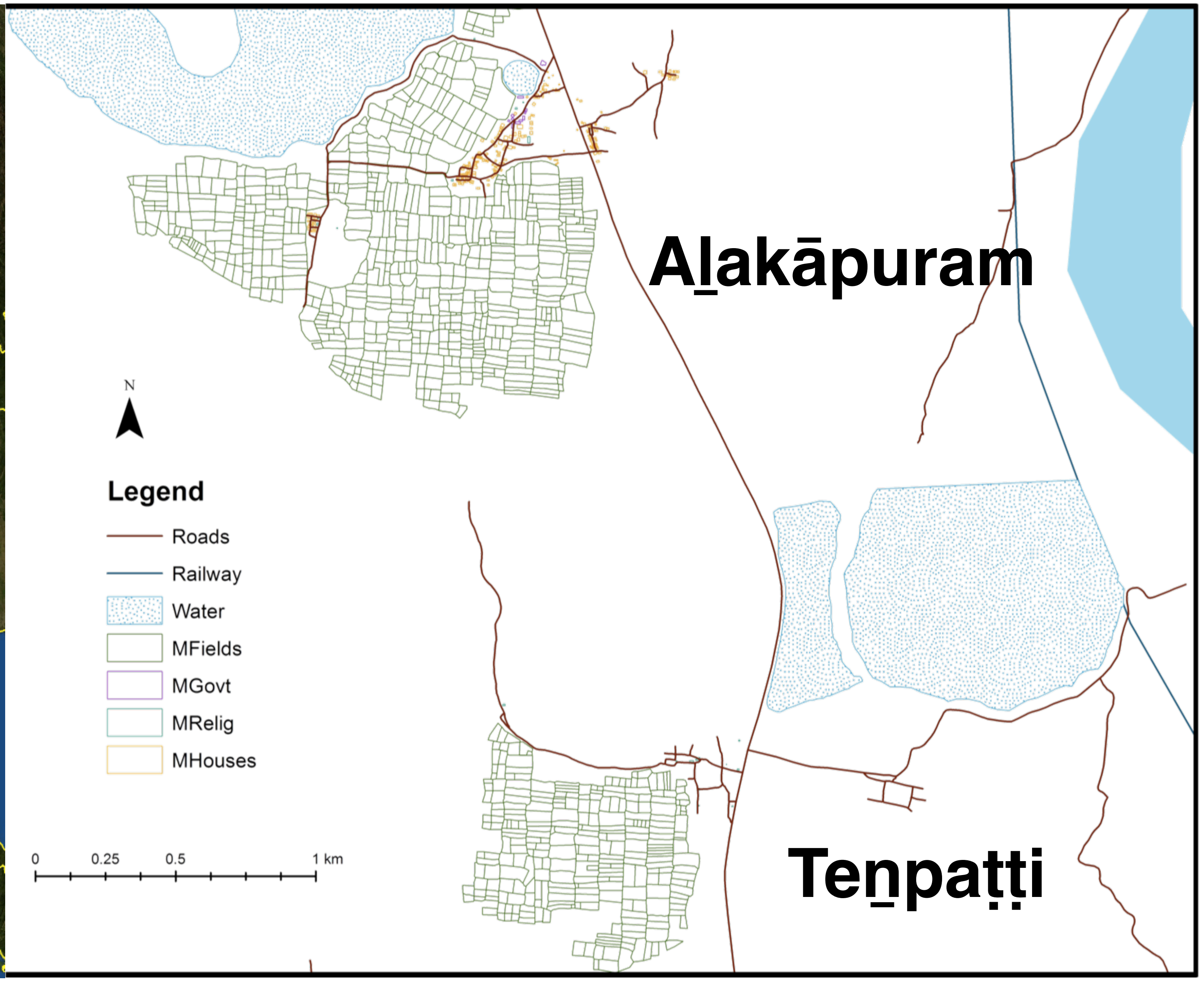
Pile-on	Target the most recent loser.	[kick 'em while they're down]
Pass-along	Target lower-ranked after losing.	[hurt people hurt people]
Opportunism	Target a recent loser of higher rank.	[now's my chance!]

Hobson and DeDeo. *PLoS Computational Biology* (2015)


Groups and ordered structures

South Indian networks: Teṉpaṭṭi and Aḻakāpuram





1964 question of Srinavas and Béteille: beyond ethnographic investigations?

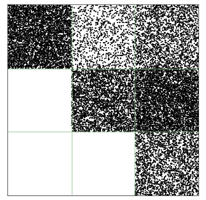
Ranked order quality, R

We propose to measure the **quality** of a ranked ordering by R

$$R = \frac{1}{m} \sum_{ij} (A_{ij} - E_{ij})\, \mathbf{1}_{g_i \leq g_j}$$

\# links

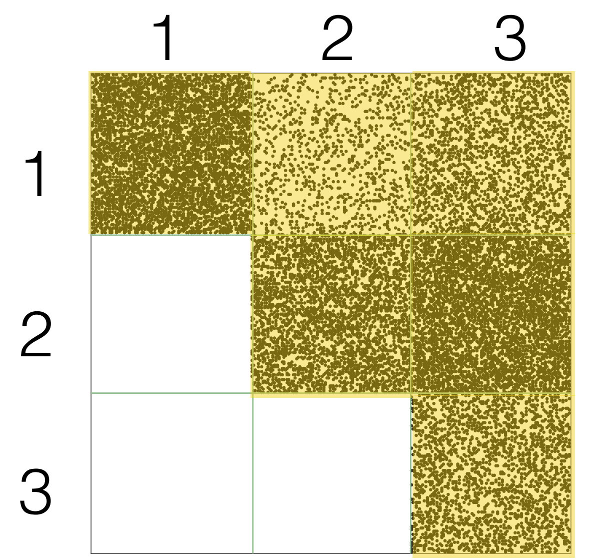
network adjacency matrix



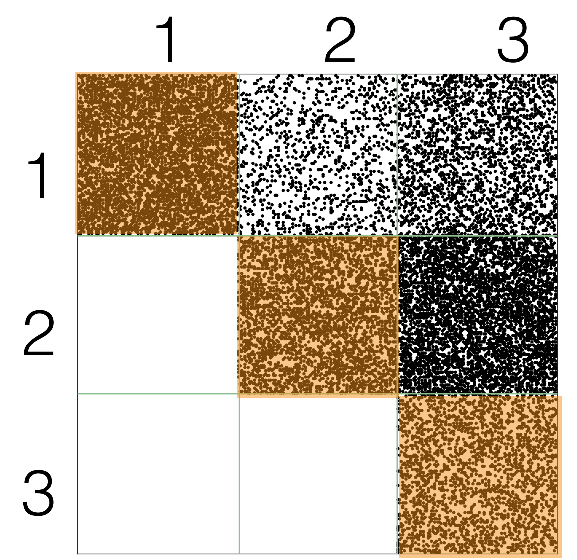
expectation (null model)

indicator

group of i ≤ group of j

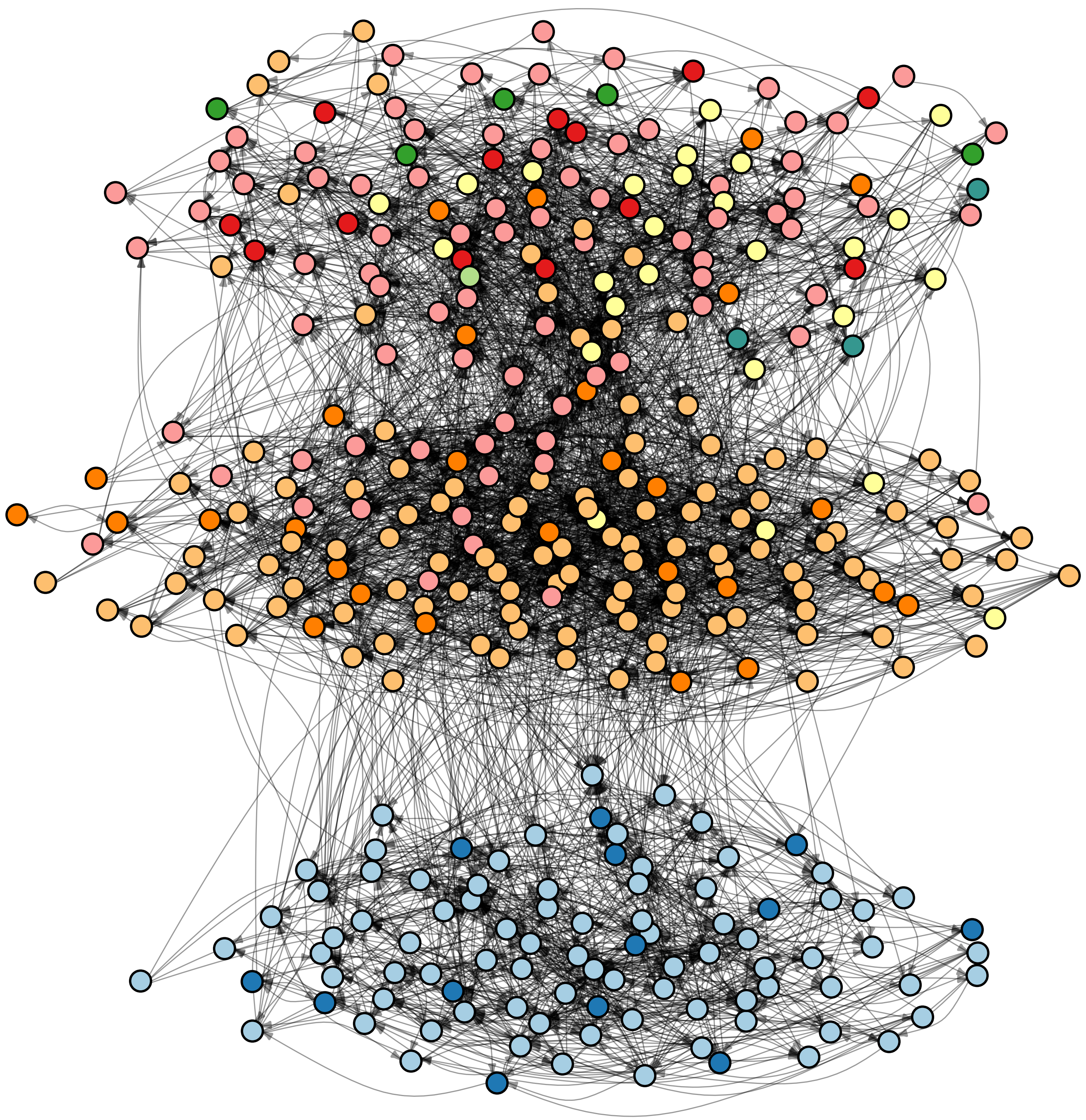


endorsement quality, R



modularity, Q

Teṉpaṭṭi

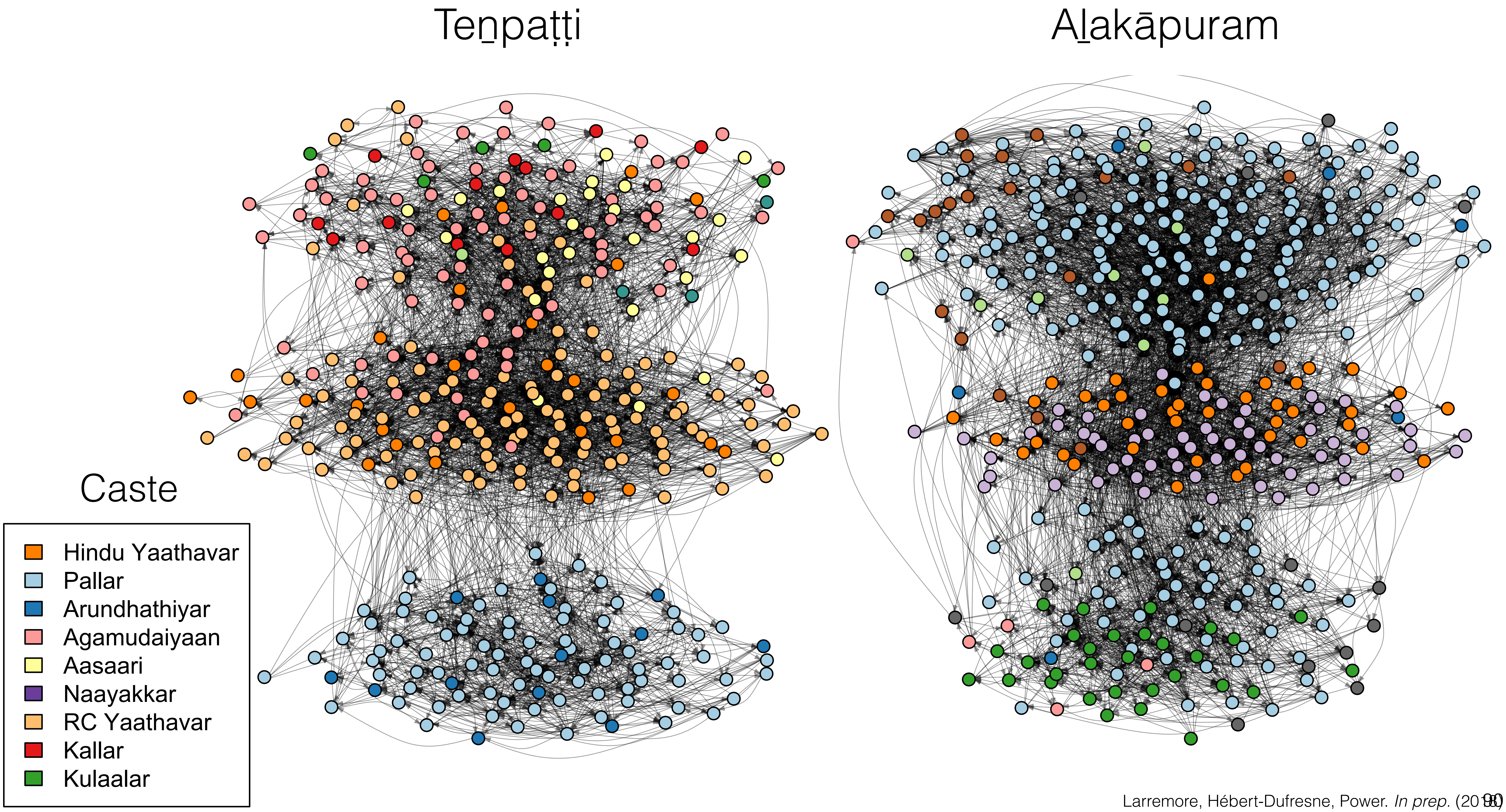


Caste

- Hindu Yaathavar
- Pallar
- Arundhathiyar
- Agamudaiyaan
- Aasaari
- Naayakkar
- RC Yaathavar
- Kallar
- Kulaalar

Scheduled castes—*dalit*
"untouchable"

Larremore, Hébert-Dufresne, Power. *In prep.* (2018)



Larremore, Hébert-Dufresne, Power. *In prep.* (2018)

Many uses for models of large-scale structure

Treat the network like a system:

Extrapolation. Make predictions for as-yet unseen nodes (in “space” or time).
Interpolation. Identify missing links.
Generalization. Nodes of this type are like others of the same type.

Treat the network like an artifact:

Mechanisms. How did this network arise? What rules governed its assembly?
Explanations. Coarse-graining or compression.

Treat the network like a means to an end; an intermediate data structure:

Useful division. Need groups so that we can assign treatments in an A/B test.
Simplification. Downstream regression model needs ranks or groups.

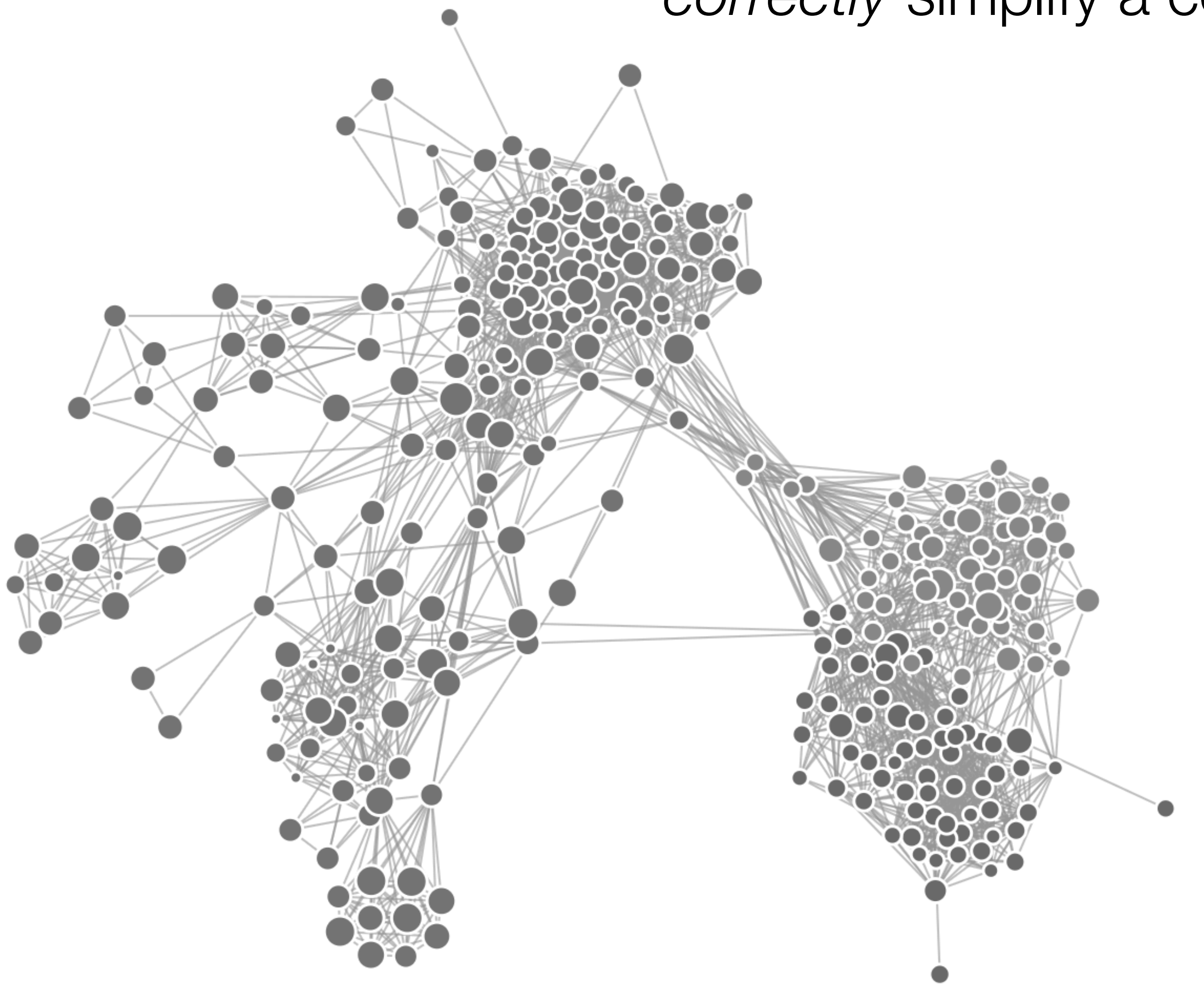
intuition: compare this list with the list you would write for regression

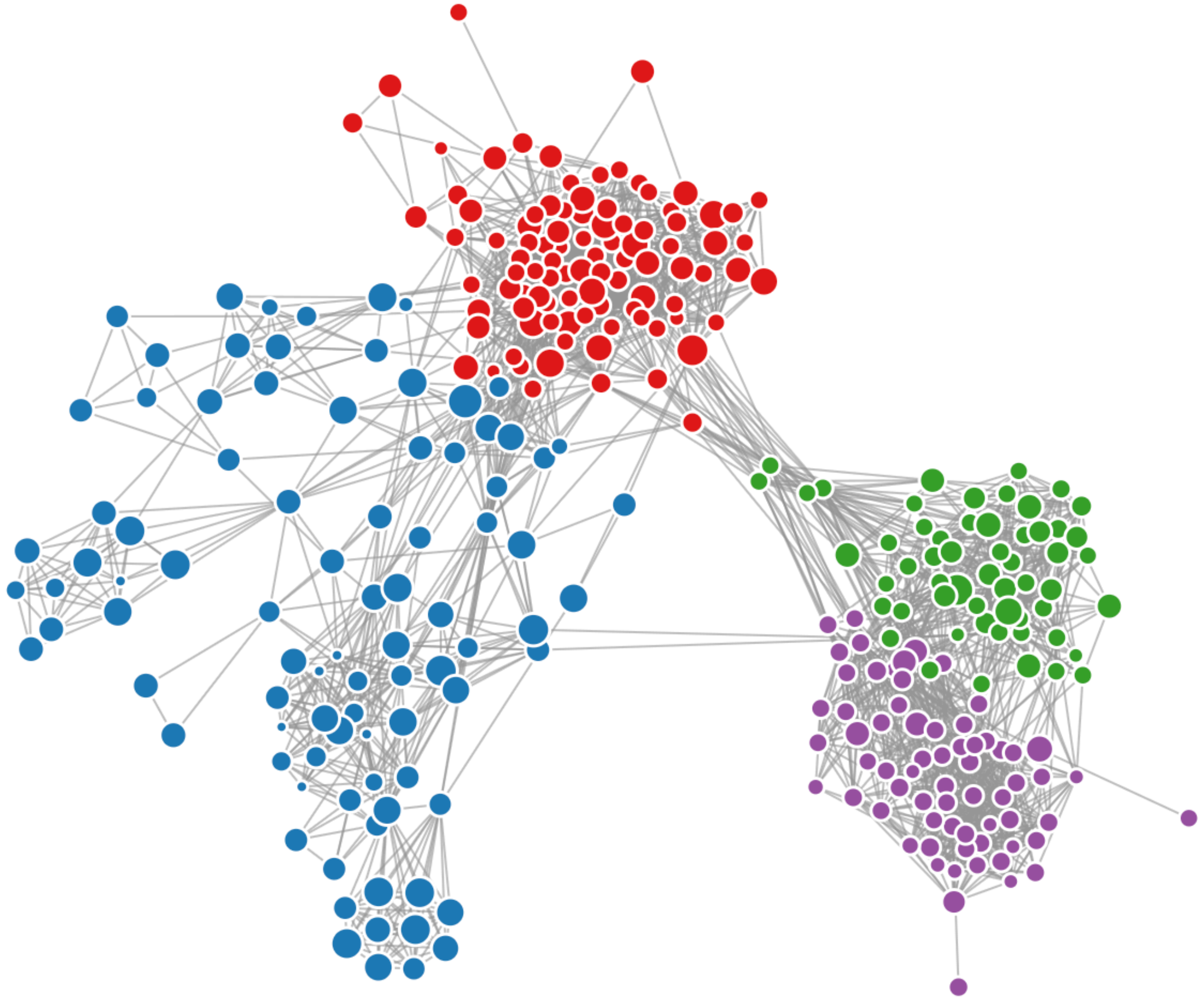
Simplicity is a great virtue but it requires hard work to achieve it and education to appreciate it. And to make matters worse: complexity sells better.
E. W. Dijkstra

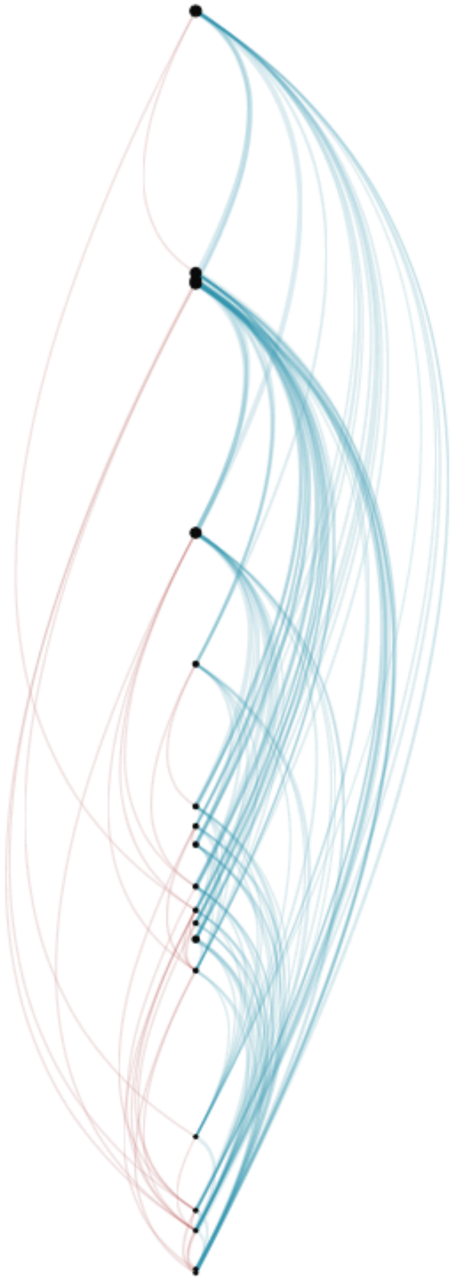
We can interpret this in two ways:

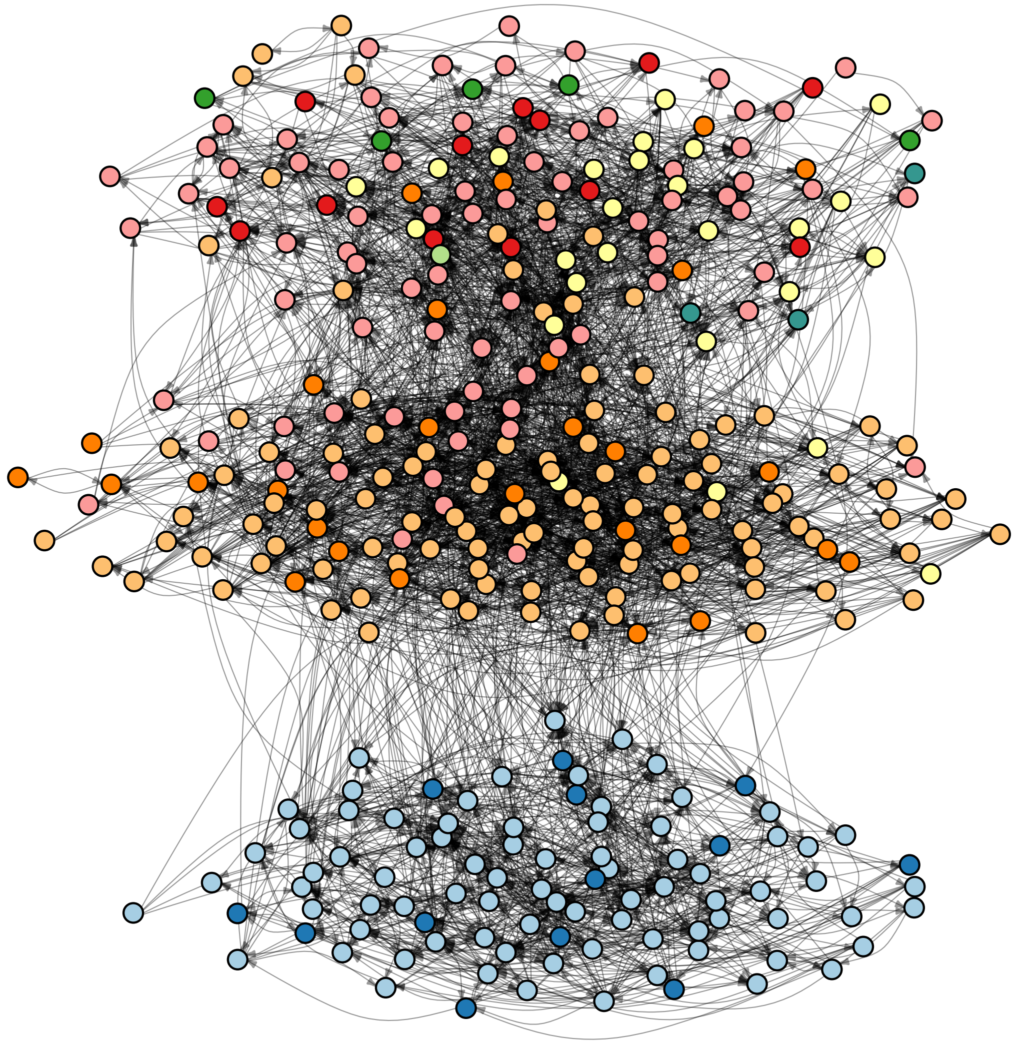
The Cynic: Pictures of networks can be *really cool* but our goal is to do good science, not make pretty pictures.

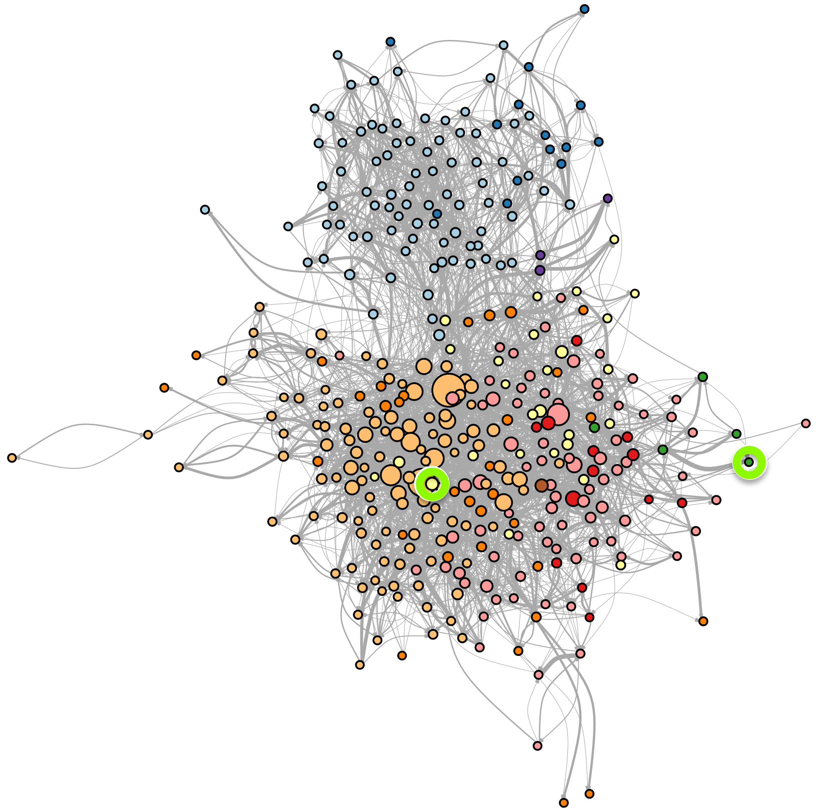
The Scientist: The most beautiful science is when we *correctly* simplify a complex system.

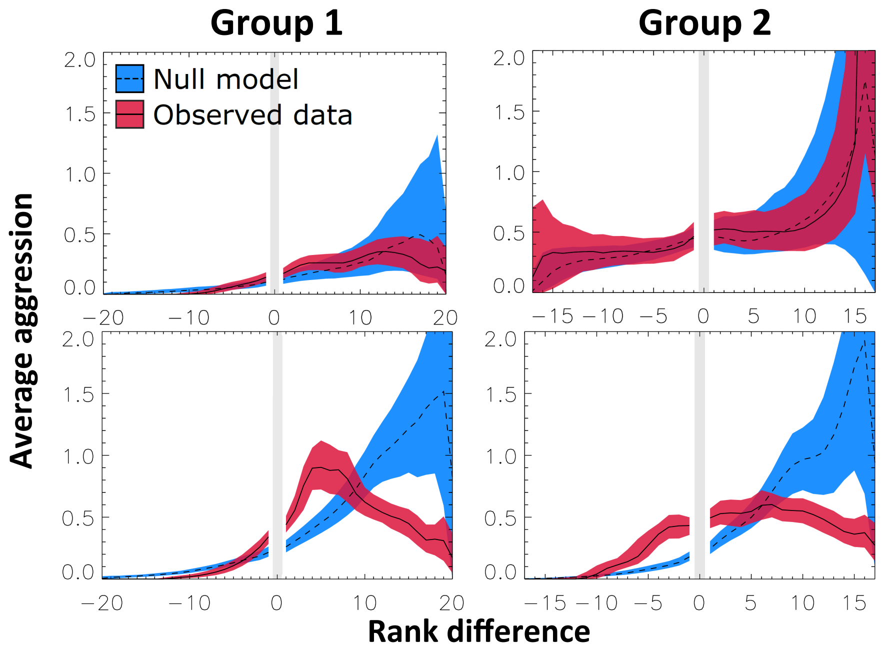


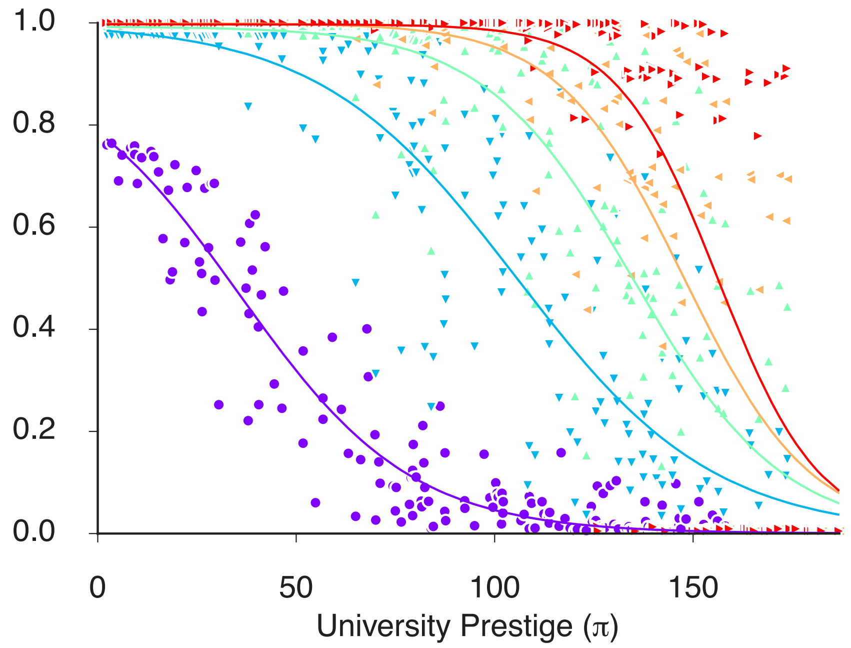


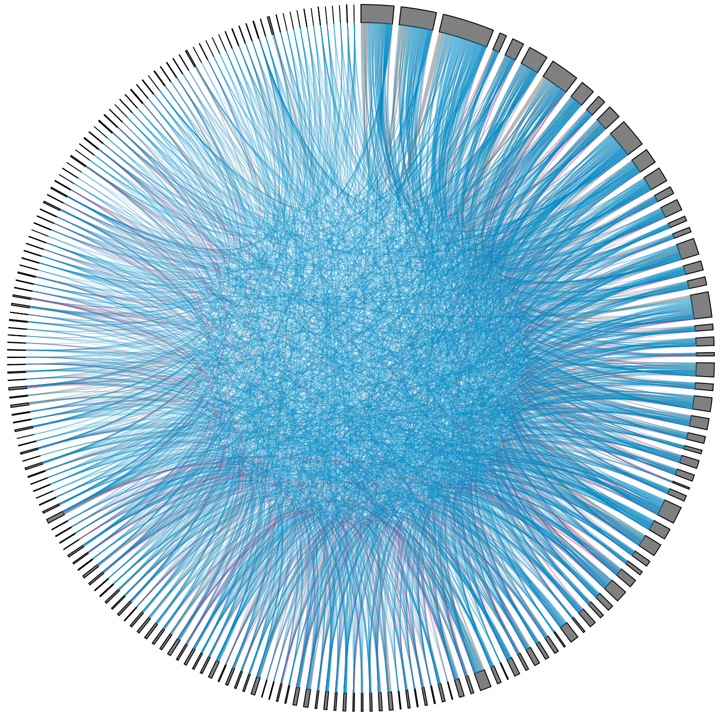












Prestige and status structures emerge in networks & we can identify them.

Beyond pictures: these things matter. traps, formation, ideas, & inequalities.

Thank you

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