Homework 1

Part (a)

void f1(int n) $T(n) = \sum_{i=2}^{n}$

{

int i=2;

while(i<n) { first iteration: $i^2 = 4 = (2)^{2k}$

0=i*i second: $i^4 = 16 = (2)^{2k}$

} third: $i^8 = 256 = (2)^{2k}$

} nth: $i^{2n} = i^{2n}$

$\log_i n = 2n$

iter k: 1, 2, 3, 4, 5 ... kth

δ after iter: 4, 16, 256, 65,536, ... $(2)^{2k}$

2^2, 2^4, 2^8, 2^{16} $k = 2^{2k}$

$(2)^{2^k}$, $(2)^{2^k}$, $(2)^{2^k}$, $(2)^{2^k}$ $\log k = 2^k$

$\log_2 \log_2 k = k$

$\Rightarrow \sum_{k=0}^{\log\log n} (\theta(1))$

$\Rightarrow \theta(\log\log n)$

$(2^2)^2$

$(2^2)^3$

$(4)^3$

Part(b)

```
void f2(int n)
{
    for(int i=1; i<=n; i++){
        if( (i % (int)sqrt(n)) == 0){
            for(int k=0; k<pow(i,3); k++){
                //do something of Θ(1) time.
            }
        }
    }
```

$\Rightarrow \sum_{i=1}^{n} \left(\Theta(1) + O\left(\sum_{k=0}^{i^3-1} \Theta(1) \right) \right)$

$\Rightarrow \Theta(n) + \sum_i \Theta(i^3)$

the if statement triggers 1 times at $n=1$

2 times at $n=4$ (1, 2, 3, 4)

3 times at $n=9$ (1, 2, 3, 4, 5, 6, 7, 8, 9)

$\sqrt{n}$ times at n;

$\Rightarrow \Theta(n) + \sum_{i=1}^{\sqrt{n}} \Theta(i^3)$

$\Rightarrow \Theta(n) + \Theta(n^{3/2})$, arithmetic series.

$\Rightarrow \Theta(n^{3/2})$, final answer.

Part(c)

```
for (int i=1; i<=n; i++) {
    for (int k=1; k<=n; k++) {
        if (A[k] == i) {
            for (int m=1; m<=n; m=m+m) {
                θ(1);
            }
        }
    }
```

$$\Rightarrow \sum_{i=1}^{n} \left(\sum_{k=1}^{n} \left(\theta(1) + \theta\left(\sum_{m} \theta(1) \right) \right) \right)$$

$$\Rightarrow \sum_{i=1}^{n} \left(\theta(n) + \sum_{k} \sum_{m} (\theta(1)) \right); \text{ at}$$

iteration:	1	2	3	...	n	...	log n
	2	4	8	...	2^n	...	n

$$\Rightarrow \theta(n^2) + \sum_{k} \theta(\log n);$$

worst case, the if statement triggers all n times.

$$\Rightarrow \theta(n^2) + \sum_{k=1}^{n} \theta(\log n) = \theta(n \log n)$$

$$\Rightarrow \theta(n^2), \text{ final answer.}$$