Counting:

1. Combinations, so $\frac{7!}{5!(7-5)!} = 21$ for unique subsets.

   Permutations:, so $\frac{7!}{(7-5)!} = 120$ for different songs.

2. 52 cards in deck, 13 sets with 4 cards of same value.

   picking two pairs: $\binom{13}{2}$ sets possible

   two cards per pair: $\binom{4}{2}\binom{4}{2}$

   last card type: $\binom{11}{1}$, with 11 other possible values.

   last card $\binom{4}{1}$, so

   $\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1} = 123,552.$

3. If the couple wants the song,

   15 songs to 6 couples $= \binom{15+7-1}{7-1} = \binom{21}{6} = 84264$

   If not,

   16 songs to 6 couples $= \binom{16+7-1}{7-1} = \binom{22}{6} = 74613 +$

   $\underline{128877}$

   $= \boxed{203490}$

4. Nodes: $0 = 1$

   $1 = 1$

   $2:$ $= 2$

   $3:$ $= 5$

   $4:$ $1 \underset{(3\,combo)}{} + \underset{(2\,combo)}{2} \; {}^4 (2\,combo) \; {}^3_1 + \underset{(3\,combo)}{4} {}^{^{}}_1 = 14$

   $5:$ $(0\,combo \cdot 4\,combo) + (1\,combo \cdot 3\,combo) + \ldots$

   $\ldots (4\,combo \cdot 0\,combo) = (1\cdot4) + (5) + (4) + (5) + (1\cdot4) = 42$

   $\underbrace{\qquad\qquad\qquad}_{2(n-1)}$

   $6: (42) + (14) + (2\cdot5=10) + (10) + (14) + (42) = 132.$

   $(2\,combo)\; \underset{1,2}{}\; \overset{3}{\diagup}\; \underset{(5\,combo)}{\diagdown}\; \overset{9}{\diagdown}\; (3\,combo) = (2\cdot42\cdot5) = \boxed{420}$

   $4,5,6,7,8 \qquad 10,11,12$