COMP 3711: Mathematical Background

Common Log Identities:

$$\begin{aligned}
\log(a \cdot b) &= \log a + \log b \\
\log(a^b) &= b \log a \\
a^{\log_a b} &= b \\
a^{\log_b c} &= c^{\log_b a} \\
\log_a n &= \frac{\log_b n}{\log_b a} = \Theta(\log n) \\
\log(n!) &= \Theta(n \log n) \quad \text{(Stirling's formula)}
\end{aligned}$$

Common Summations: Let $c \neq 1$ be any positive constant and assume $n \geq 0$. The following are the most common summations that arise when analyzing algorithms and data structures.

Name of Series	Formula	Closed-Form Solution	Asymptotic Form
Constant	$\sum_{i=1}^{n} 1$	$= n$	$\Theta(n)$
Arithmetic	$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n$	$= \frac{n(n+1)}{2}$	$\Theta(n^2)$
Polynomial	$\sum_{i=1}^{n} i^c = 1^c + 2^c + \cdots + n^c$	(none for general c)	$\Theta(n^{c+1})$
Geometric	$\sum_{i=0}^{n-1} c^i = 1 + c + c^2 + \cdots + c^{n-1}$	$= \frac{c^n - 1}{c - 1}$	$\Theta(c^n)$ $(c > 1)$ $\Theta(1)$ $(c < 1)$
Harmonic	$\sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$	$= \ln n + O(1)$	$\Theta(\log n)$

Asymptotic Notation

Asymptotic upper bound. Big-Oh: $f(n) = O(g(n))$

There exists constant $c > 0$ and n_0 such that $f(n) \leq c \cdot g(n)$ for $n \geq n_0$.
Equivalent definition: $\lim_{n\to\infty} \frac{f(n)}{g(n)} < \infty$

Asymptotic lower bound. Big-Omega: $f(n) = \Omega(g(n))$

There exists constant $c > 0$ and n_0 such that $f(n) \geq c \cdot g(n)$ for $n \geq n_0$.
Equivalent definition: $\lim_{n\to\infty} \frac{f(n)}{g(n)} > 0$

Asymptotic tight bound. Big-Theta: $f(n) = \Theta(g(n))$

$f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$